Proc. Natl. Acad. Sci. USA
Vol. 90, pp. 5901–5903, July 1993
Physics

Analysis of the ground-state band and a closely related intercalated band of $^{235}_{92}U_{143}$ by the two-revolving-cluster model with consideration of symmetric and antisymmetric resonance of the two dissimilar clusters

(unquantized angular-momentum vector/vector model/Poinsot ellipsoid/parity/radius of revolution)

LINUS PAULING* AND BARCLAY KAMB†

*Linus Pauling Institute of Science and Medicine, 440 Page Mill Road, Palo Alto, CA 94306-2025; and †Division of Geological and Planetary Sciences, California Institute of Technology, 1201 East California Street, Pasadena, CA 91125

Contributed by Linus Pauling, March 5, 1993

ABSTRACT **The reported ground-state band of $^{235}_{92}U_{143}$ extending from J = 7/2⁻ to 51/2⁻ is found to be two intercalated bands, one beginning with 7/2⁻ and the other with 9/2⁻, each with $\Delta J = 2$. Analysis by the two-resonating-cluster model leads to 3875 Da·fm² for the moment of inertia for the first few levels, then increasing by centrifugal stretching. This value is interpreted by the structure $p^{70}n^{112}$ for the central sphere with clusters $p^{11}n^{16}$ and $p^{11}n^{15}$ with radius of revolution R = 8.55 fm. The major principal axis of the Poinsot ellipsoid is taken to be determined by an unquantized number K, with vector intermediate in orientation between L and J. The values of K are found by empirical analysis to equal L + 0.28, with the theory of rotation of the ellipsoid giving L + 0.28 for 7/2⁻, slowly decreasing to L + 0.251 for 51/2⁻. The 7/2⁻ band is based on odd values of L (negative parity) and the 9/2⁻ band on even values (positive parity). Negative parity for both bands is achieved by symmetric resonance of the two dissimilar clusters in the 7/2⁻ band and antisymmetric resonance in the 9/2⁻ band.**

The polyspheron model of nuclear structure (1) is based upon the idea (1, 2) that the shell-model orbitals can be hybridized into a set of localized 1*s* orbitals, each of which can be occupied by no more than two protons and two neutrons to form spherons. The principal spherons are the helion, p^2n^2, and the triton, pn^2, with the deuteron, pn, and others sometimes playing a part. The model has had considerable success in providing simple interpretations of many properties of nuclei. Levels of rotational bands have been analyzed by assuming that they correspond to revolution of a cluster about a central sphere (3–5). It has been reported recently by one of us (6) that 35 ground-state bands of lanthanon nuclei show a transition with increasing value of the angular momentum quantum number J from revolution of a single cluster to revolution of two clusters, giving segments of superdeformed bands, and also (7) that 56 reported excited bands of lanthanon and Hg–Tl–Pb nuclei could be interpreted on the basis of the two-revolving-cluster model.

The small energy differences of the levels of the reported ground-state band of $^{235}_{92}U_{143}$, extending from J = 7/2⁻ to 51/2⁻, with $\Delta J = 1$ (8), led us to conclude that this band (which we found to be two intercalated bands with $\Delta J = 2$) is based on a superdeformed two-revolving-cluster model. The methods of analysis, both empirical and theoretical (vector-model theory, theory of rotational motion of a torus), are presented in the following paragraphs.

The publication costs of this article were defrayed in part by page charge payment. This article must therefore be hereby marked "*advertisement*" in accordance with 18 U.S.C. §1734 solely to indicate this fact.

Analysis of the Apparent Ground-State Band: Its Resolution into Two Bands

There are a reported 23 negative-parity energy levels from the ground state J = 7/2⁻ to J = 51/2⁻, with $\Delta J = 1$ (8). The first five values of the second difference in energy, $\Delta^2E(\Delta J = 1)$, are nearly equal; they are 10.62, 10.85, 10.75, 10.96, and 10.40 keV, as expected for a single band with constant moment of inertia. However, it is to be seen that lower and higher values alternate, and this alternation becomes more pronounced with increasing J, reaching 45.7 and −40.2 for the largest values of J. Accordingly, the apparent band with $\Delta J = 1$ is in fact two closely related bands, one (with lower energy values) beginning with 7/2⁻ and the other with 9/2⁻, each with $\Delta J = 2$.

Values of the Moment of Inertia. Values of the energy, $E(J)$, for each of the two bands are given in Table 1. For the 7/2⁻ band the first three values of the moment of inertia calculated by the rigid-rotator equation from the second difference in the energy $\Delta^2E(\Delta J = 2)$ are 3884, 3862, and 3886, mean 3877 Da·fm², and for the 9/2⁻ band they are 3851, 3803, and 3923, mean 3859 Da·fm². The near equality of these values, mean 3868 Da·fm², indicates that the two bands correspond to closely similar structures.

The Nature of the Structures. The revolving-cluster model permits the intrinsic structure of the nucleus to be either a central sphere with one revolving cluster or a central sphere with two revolving clusters. The ground-state bands of even–even lanthanon nuclei show a transition from one revolving cluster to two revolving clusters (6), and the lanthanons and Hg–Tl–Pb nuclei have excited superdeformed bands with two revolving clusters (7). In a following section we present the arguments to support the conclusion that the 7/2⁻ band and the 9/2⁻ band have the two-revolving-cluster structure, probably with the doubly-semimagic structure $p^{70}n^{112}$ for the sphere and with clusters $p^{11}n^{16}$ and $p^{11}n^{15}$.

Explanation of the Negative Parity. The model that we assign for these bands involves three angular-momentum quantum numbers: L, the orbital angular-momentum quantum number of the two clusters (equal to 3⁻, 5⁻, 7⁻, . . . for the 7/2⁻ band and to 4⁺, 6⁺, 8⁺, . . . for the 9/2⁻ band), the spin angular-momentum quantum number S (with value ½ for the odd neutron), and the total angular-momentum quantum number J (equal to L + ½ for all levels). In addition, there is resonance between the two dissimilar clusters, $p^{11}n^{16}$ and $p^{11}n^{15}$, which is either S (symmetric, positive parity) or A (antisymmetric, negative parity). Thus both the 7/2⁻ band, with L^-S^+, and the 9/2⁻ band, with L^+S^-, have negative parity.

Table 1. Comparison of energy values of the levels of the $7/2^-$ and $9/2^-$ band of $^{235}_{92}U_{143}$ calculated here with those of ref. 8

J*	$E(J)$†	$I(E)$‡	$I(\Delta E)$§	J*	$E(J)$†	$I(E)$‡	$I(\Delta E)$§
$7/2^-$							
			3878	$9/2^-$	46.204		
$11/2^-$	103.032	3878					3881
			3880	$13/2^-$	170.705	3881	
$15/2^-$	249.130	3879					3873
			3876	$17/2^-$	338.52	3878	
$19/2^-$	438.5	3878					3856
			3878	$21/2^-$	550.4	3870	
$23/2^-$	670.9	3878					3870
			3901	$25/2^-$	804.9	3870	
$27/2^-$	944.8	3884					3899
			3948	$29/2^-$	1100.4	3878	
$31/2^-$	1257.8	3900					3956
			4031	$33/2^-$	1433.9	3897	
$35/2^-$	1605.8	3929					4037
			4123	$37/2^-$	1802.1	3926	
$39/2^-$	1986.6	3966					4143
			4235	$41/2^-$	2201.2	3966	
$43/2^-$	2395.8	4014					4273
			4390	$45/2^-$	2527.3	4017	
$47/2^-$	2829.6	4071					4415
			4543	$49/2^-$	3077.6	4076	
$51/2^-$	3285.6	4137					

Values of the energy of the levels of the $7/2^-$ band and the $9/2^-$ band of $^{235}_{92}U_{143}$ from ref. 8 are given in columns 2 and 6, and values of the moment of inertia in Da·fm^2, calculated by the rigid-rotation equation, are in columns 3, 4, 7, and 8.

*Values of L are equal to $J - 1/2$.

†Units are keV.

‡$I(E)$, units Da·fm^2, calculated as $K(K + 1)\hbar^2/2[E(J) - E(7/2^-)]$ or $K(K + 1)\hbar^2/2[E(J) - E(9/2^-)]$, with $K = L + 0.28$ (see text).

§$I(\Delta E)$, with $\Delta E = E(J + 1) - E(J - 1)$.

It is expected that S^+ resonance is stabilizing and A^- resonance is destabilizing. This model accordingly explains the fact that the $9/2^-$ band levels lie higher than the $7/2^-$ levels.

Evaluation of the Moment of Inertia from $\Delta E(\Delta J = 2)$, the First Differences in the Energy Levels

With the two-revolving-cluster model there are several energy quantities that contribute to the total energy $E(J)$ of the Jth state. The largest contribution is made by the revolution of the two clusters around their angular-momentum vector L. Values of this energy term can be calculated for the first few levels by the rigid-rotator equation with use of the $I(\Delta^2E)$ value for the moment of inertia, 3868 Da·fm^2. The next largest contribution is that of the precession of the structure about the vector J. We have analyzed the values of ΔE to obtain an empirical treatment of the sum of these energy terms.

Let K be an angular-momentum quantum number such that the energy difference $E(K + 1) - E(K - 1)$ is given by the rigid-rotator expression $(2K + 1)\hbar^2/I$, with I assumed to have the Δ^2E value 3868 Da·fm^2. The first eight corresponding values of K (four for each of the two bands) are found to be equal to $L + 0.28$, with mean deviation 0.01. We have evaluated $I(\Delta E)$ with the assumption that $K = L + 0.28$, with the results given in Table 1. The first four values for each band are nearly equal; the values then begin to increase because of centrifugal stretching. The means of the first four values are 3878 for the $7/2^-$ band and 3859 for the $9/2^-$ band.

Evaluation of the Moment of Inertia from the Energy Levels

Experience has shown that application of the rigid-rotator equation to the difference in energy of the Jth level and the lowest level of the band leads to nearly constant values of I for the first few levels, with centrifugal stretching then being revealed by the later steady increase in these values. In calculating the values of $I(E)$ given in Table 1, the rigid-rotor equation was used with the factor $K(K + 1)$, the values of K being taken as $L + 0.28$. For each of the two bands the first five values are nearly the same, mean 3879 for the $7/2^-$ band and 3875 for the $9/2^-$ band.

The weighted mean of all of the quoted values of I is 3875 Da·fm^2. This value is used in the following discussion.

The Structure of the Clusters and the Central Sphere

The fact that the centrifugal stretching is small, as expected for small quadrupolarizability (7), shows that there are two revolving clusters. For 24 excited superdeformed bands of Hg, Tl, and Pb with central sphere $p^{56}n^{82}$ and total nucleon number 50 to 56 for the two clusters, the values of the radius of revolution lie between 8.71 and 8.93 fm (7). The assignment of the doubly semimagic composition $p^{70}n^{112}$ to the central sphere of the two ^{235}U bands, total nucleon number of the clusters 53, leads to $R = 8.55$ fm. This value and the somewhat larger values for the Hg–Tl–Pb bands show that in this region R decreases with increase in A and N, as is found for some other regions.

The two clusters may be taken to be $p^{11}n^{16}$ and $p^{11}n^{15}$, which may be described as α^3t^5 and α^3t^4d. With this description, the S^+ and A^- character of the two bands results from the resonance of one neutron between the two clusters; that is, from pd–dp resonance.

It is expected that S^+ resonance between the two clusters would lower the energy levels and A^- would raise them. The fact that the levels of the $9/2^-$ band lie higher than the values interpolated from those of the $7/2^-$ band thus provides some support for the proposed interpretation of the negative parity of all of the levels.

The Nature of the Ground State and the Relation of the Revolving-Cluster Model to the Shell Model

The foregoing interpretation of the energy levels, with use of the effective orbital angular-momentum quantum number $K = L + 0.28$, indicates that the value of L for the ground state is 3^- and that the orbital angular momentum corresponds to the revolution of all 53 nucleons outside of the central sphere. This interpretation differs from that of the shell model, which would assign the level $J = 7/2^-$ to a single neutron in the state $^2f7/2^-$ as the most stable level not occupied by pairs of neutrons. The relation between the shell model and the polyspheron model is such that the same argument about the quantum numbers of the normal state can be applied also to the revolving-cluster model.

Further Discussion of the Theory

Application of the vector model provides some further insight. In this model the vectors **L**, **S**, and **J** have magnitudes $[L(L + 1)]^{1/2}$, $[S(S + 1)]^{1/2}$, and $[J(J + 1)]^{1/2}$. These vectors represent angular momenta. S can contribute either positively or negatively to L to give J. In our model there are two clusters revolving about a central sphere. The orientation of the plane of revolution determines the direction of the angular-momentum vector corresponding to the major principal axis of the Poinsot ellipsoid, which for our model has major axis A and two minor axes $B = C = 1/2A$. We have designated the major axis as K. It is not quantized, but L and J are quantized, as well as S. We assume that S does not contribute significantly to the energy, and we see no reason to distinguish between L and J in their relation to K. Accordingly the vector **K** can be taken to be directed toward the midpoint of **S**, which leads to $K = L + 0.25$ and $L - 0.25$.

A small correction for the polhode and herpolhode (precession) needs to be made. The energy of the rotating nucleus needs to be corrected because the moment of inertia about an axis at angle θ with the major principal axis is different from A by the divisor $1 + \sin^2 \theta$. This correction is made to K rather than to L and J because L and J are quantized. With values of θ given by the vector model, $K = L + 0.25$ becomes $L + 0.28$ for $J = 7/2^-$, decreasing slowly to $L + 0.27$ for $15/2^-$ and on to $L + 0.251$ for $51/2^-$.

The agreement of this theory with the results of our empirical analysis is striking. We obtained $K = L + 0.28$ for small values of L, without noticing the decrease, which is masked by our neglect of centrifugal stretching. This agreement provides strong support for the two-revolving-cluster model.

Discussion

Success in analyzing 35 ground-state bands and 56 excited bands with the model in which two clusters revolve about a central sphere (6, 7) suggested to us that the model be applied to the reported ground-state band of $^{235}_{92}U_{143}$, with levels $J = 7/2^-, 9/2^-, \ldots, 51/2^-$. The first result of our analysis was that this sequence consists of two intercalated bands, each with $\Delta J = 2$, one beginning with $7/2^-$ and the other with $9/2^-$. The analysis gave the values 3879 and 3875 Da·fm^2 for the first few levels of the two bands, which we interpret as showing that the composition of the two clusters is $p^{11}n^{16}$ and $p^{11}n^{15}$, the sphere $p^{70}n^{112}$ being doubly semimagic. The radius of revolution, 8.55 fm, is compatible with values for other nuclei. To account for the parity of the levels, we invoked the concept that there is symmetric resonance (positive parity) of the two clusters, which are not identical, for the $7/2^-$ band (which involves odd values of the orbital quantum number L) and antisymmetric resonance (negative parity) for the $9/2^-$ band (with even values of L). In the course of the empirical analysis it was found necessary to introduce an auxiliary orbital angular-momentum quantum number K, which is not quantized but was found to be equal to $L + 0.28$. We considered this empirical analysis, which led us to introduce some interesting ideas, to be reasonable, but we then decided to attempt to develop a theory of these bands on the basis of the vector model, incorporating the quantized vectors **L**, **S**, and **J**, and of the properties of a rotating rigid body with two principal axes of the Poinsot ellipsoid different from the third. This effort was successful. It required the introduction of another quantum number K, with calculated values changing slowly from $L + 0.28$ for the $7/2^-$ level to $L + 0.251$ for the $51/2^-$ level, in excellent substantiation of the results of our empirical analysis.

We conclude that this study shows that the polyspheron model in its special application, the two-revolving-cluster model, has value in the interpretation of nuclear data. Similar results might well be obtained by application of the shell model, the liquid-drop model, or other models, but probably only with much greater effort.

1. Pauling, L. (1965) *Science* **150**, 297–305.
2. Nesbet, R. K. (1955) *Phys. Rev.* **100**, 228–234.
3. Pauling, L. (1969) *Proc. Natl. Acad. Sci. USA* **61**, 807–809.
4. Pauling, L. (1981) *Proc. Natl. Acad. Sci. USA* **78**, 5296–5298.
5. Pauling, L. (1991) *Proc. Natl. Acad. Sci. USA* **88**, 4401–4403.
6. Pauling, L. (1991) *Proc. Natl. Acad. Sci. USA* **88**, 820–823.
7. Pauling, L. (1992) *Proc. Natl. Acad. Sci. USA* **89**, 7277–7281.
8. Schmorak, M. R. (1983) *Nuclear Data Sheets* **40**, 1–85.