³He Spin-Dependent Cross Sections and Sum Rules

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We present a measurement of the spin-dependent cross sections for the ${}^{3}\vec{He}(\vec{e}, e')X$ reaction in the quasielastic and resonance regions at a four-momentum transfer $0.1 \le Q^2 \le 0.9$ GeV². The spinstructure functions have been extracted and used to evaluate the nuclear Burkhardt-Cottingham and extended Gerasimov-Drell-Hearn sum rules for the first time. The data are also compared to an impulse approximation calculation and an exact three-body Faddeev calculation in the quasielastic region.

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In recent years, a large amount of high-quality spindependent data has become available from a new generation of inclusive electron scattering experiments [1]. These data enable a deeper understanding of the theory of strong interactions, quantum chromodynamics (QCD), via tests of fundamental sum rule predictions. These predictions are typically derived from an effective theory or perturbative expansion of QCD, with the choice of appropriate implementation depending on the four-momentum transfer Q^2 of the interaction. At low Q^2 , an effective approach known as chiral perturbation theory [2] (γ PT) has been tested by several recent spin-dependent measurements in the simplest systems [3-8], and larger Q^2 data provide strict tests of future lattice QCD calculations. It is crucially important to evaluate these predictions over a wide range of Q^2 to determine their limitations and range of applicability.

Many famous sum rules have been tested with nucleon data, but the assumptions made in deriving these relations often apply regardless of whether the target is a nucleon or a nucleus. For example, the Gerasimov-Drell-Hearn (GDH) sum rule [9] for a target of spin S, mass M, and anomalous magnetic moment κ reads

$$\int_{\nu_{\rm th}}^{\infty} \frac{\sigma_{\rm A}(\nu) - \sigma_{\rm P}(\nu)}{\nu} d\nu = -4\pi^2 S \alpha \left(\frac{\kappa}{M}\right)^2.$$
(1)

Here $\sigma_A(\sigma_P)$ represents the cross section for absorption of a real photon ($Q^2 = 0$) of energy ν which is polarized antiparallel (parallel) to the target spin, and α is the finestructure constant. The inelastic threshold is signified by $v_{\rm th}$, which is pion production (photodisintegration) for a nucleonic (nuclear) target. Because of the $1/\nu$ weighting, states with lower invariant mass provide the most significant contribution to the sum rule. The GDH predictions for the neutron and ³He are -234 and -496μ b, respectively. To gauge the relative strength of the nuclear contribution to Eq. (1), we divide the ³He integral into two excitation energy regions. Region I extends from two-body breakup to the pion production threshold, and region II extends from threshold to ∞ . Polarized ³He at first order appears as a free polarized neutron due to the spin pairing of the two protons, so the contribution from region II should be approximately $-234 \mu b$. Therefore, the contribution from disintegration, which is the only reaction available to real photons in region I, is necessarily quite large in order to satisfy the sum rule prediction for ³He. Similarly, in the case of virtual-photon scattering, Ref. [10] indicates the growing importance of threshold disintegration at low Q^2 for the lightest nuclear systems.

Ji and Osborne [11] suggest a generalization of the GDH sum rule based on the relationship between the forward virtual Compton amplitudes S_1 and S_2 and the spindependent structure functions g_1 and g_2 . Since Eq. (1) is derived from the dispersion relation for S_1 at the real photon point, a generalized sum rule can also be constructed from the same relation at nonzero Q^2 . This leads to a set of Q^2 -dependent dispersion relations [12] for the spin-structure functions. In particular, the dispersion relation for S_1 leads to the following extension of the GDH sum rule to virtual-photon scattering:

$$\bar{\Gamma}_1(Q^2) \equiv \int_0^{1-\epsilon} g_1(x, Q^2) dx = \frac{Q^2}{8} \bar{S}_1(0, Q^2).$$
 (2)

The infinitesimal ϵ ensures that only inelastic contributions are included, which is indicated by the overbar, and $x = Q^2/2M\nu$ is the Bjorken scaling variable. However, an alternate extension is often presented [13]:

$$I(Q^{2}) = \int_{\nu_{th}}^{\infty} \frac{\sigma_{A}(\nu, Q^{2}) - \sigma_{P}(\nu, Q^{2})}{\nu} d\nu$$

= $2 \int_{\nu_{th}}^{\infty} \frac{K}{\nu} \frac{\sigma_{TT'}(\nu, Q^{2})}{\nu} d\nu,$ (3)

where K is the virtual-photon flux factor. The spindependent contributions to the inclusive cross section of a spin-1/2 system are contained in g_1 and g_2 , or equivalently the cross sections σ'_{TT} and σ'_{LT} , which are the transverse-transverse and longitudinal-transverse, respectively, cross sections relevant to scattering with the target spin aligned with, or perpendicular to, the direction of the momentum transfer \vec{q} .

The dispersion relation for S_2 leads [12] to the following superconvergence relation:

$$\Gamma_2(Q^2) \equiv \int_0^1 g_2(x, Q^2) dx = 0,$$
 (4)

which is the Burkhardt-Cottingham (BC) sum rule [14]. The derivation of the BC sum rule depends on the convergence of the integral and assumes that g_2 is a well-behaved function [15] as $x \rightarrow 0$.

This Letter details a test of the sum rules described above via an inclusive cross-section measurement in the quasielastic (QE) and resonance regions. The experiment was performed in Hall A [16] of the Thomas Jefferson National Accelerator Facility (JLab). Longitudinally polarized electrons at six incident energies (0.9, 1.7, 2.6, 3.4, 4.3, and 5.1 GeV) were scattered from a high-density polarized ³He target. Longitudinal and transverse target polarizations were maintained, allowing a precision determination of both $g_1^{^{3}\text{He}}(x, Q^2)$ and $g_2^{^{3}\text{He}}(x, Q^2)$ or, alternatively, $\sigma_{TT'}(\nu, Q^2)$ and $\sigma_{LT'}(\nu, Q^2)$. Full experimental details can be found in Refs. [3,7,8].

The measured spin-structure functions were interpolated (or extrapolated for a few data points at large ν) to constant Q^2 [7] from 0.1 to 0.9 GeV². Figure 1 displays the first moments of $g_1^{^{3}\text{He}}$ and $g_2^{^{3}\text{He}}$, along with the extended GDH sum $I(Q^2)$. In all panels, the circles represent the ³He data integrated to W = 2 GeV. The invariant mass W is defined here in terms of the proton mass: $W^2 = M_p^2 - Q^2 + 2M_p\nu$. Squares include an estimate (discussed below) of any unmeasured contributions. Statistical uncertainties are shown on the data points, while the systematic uncertainty



FIG. 1 (color online). ³He spin-structure moments. Top: $\overline{\Gamma}_1(Q^2)$ compared to the PWIA model described in text (dotted-dashed line) and the GDH sum rule slope (solid line). Middle: $\Gamma_2(Q^2)$ along with the elastic contribution [17] (solid line) to the moment. Bottom: $I(Q^2)$ with $K = \nu$, compared to the PWIA model.

of the measured (total) integral is represented by the light (dark) band. The absolute cross sections contribute 5% uncertainty, while the beam and target polarization each contribute 4%. The radiative corrections are assigned 20%uncertainty to reflect the variation seen from choosing different initial models for our unfolding procedure. This uncertainty is doubled for the 0.9 GeV incident energy spectra to reflect the lack of lower energy data. A separate contribution to the radiative corrections uncertainty arises from the subtraction of the ³He elastic radiative tail, which is significant only for the lowest incident energy. The light band represents the quadratic sum of the above errors including a contribution from interpolation. The full systematic band includes an estimate of the uncertainty of the unmeasured contribution to the integrals. The Γ_2 full systematic error includes a 5% uncertainty from the elastic contribution (solid black curve) which was evaluated using previously measured form factors [17].

The total integral of $\overline{\Gamma}_1$ includes an estimate [18] of the unmeasured region above W = 2 GeV, and the uncertainty arising from this is reflected in the total error band. Reference [18] was shown to be consistent with existing deep inelastic scattering data [19] in our previous publication [8]. The data show some hint of a turnover at low Q^2 , where we have also plotted the slope predicted by Eq. (1)for ³He. To obtain the dotted-dashed curve, we have summed the MAID model [20] proton and neutron predictions using an effective polarization procedure [21]. To this we add an estimate of the contribution below the pion threshold using the plane-wave impulse approximation (PWIA) model [22,23]. This model contains contributions for $W \leq 2$ GeV, so it should be compared directly with the open symbols. At large momentum transfer, $\Gamma_1(Q^2)$ appears to be nearly independent of Q^2 , which would seem to indicate the diminishing importance of higher twist effects, consistent with other recent findings (e.g., [24,25]) in this kinematic range.

Experimental measurements of g_2 are scarce, and only recently has the BC sum rule been evaluated for the first time. The SLAC E155 Collaboration [26] measured Γ_2 at $Q^2 = 5$ GeV². They found the BC sum rule to be satisfied within a large uncertainty for the deuteron, while a violation of almost 3σ was found for the more precise proton measurement. In Fig. 1 (middle), we plot Γ_2 . The unmeasured contribution was estimated using the method described in Ref. [26], which assumes the validity of the Wandzura-Wilczek relation [27]. All six data points are consistent with the Burkhardt-Cottingham prediction. Results from this same experiment have been used to test the BC sum rule for the neutron [8], using only data for which W > 1.073 GeV, and with nuclear corrections applied. It was found that the neutron BC sum rule is satisfied primarily due to the cancellation of the resonance and nucleon elastic contributions. It is interesting to find that for ³He a balance is struck between the positive inelastic contribution above the pion threshold and the negative contribution from the elastic and quasielastic regions, with the elastic becoming important below $Q^2 \approx 0.2 \text{ GeV}^2$.

Figure 1 (bottom) displays the extended GDH sum as defined in Eq. (3). We follow the convention $K = \nu$ for the virtual-photon flux. Accounting for the unmeasured contribution [18] has only a minor effect due to the $1/\nu$ weighting of the integrand. The phenomenological model (dotted-dashed curve) tracks the data well, but the negative sum rule prediction at $Q^2 = 0$ (black star) stands in contrast to the large positive value of our lowest point. The ³He GDH integral is dominated by a positive QE contribution which largely outweighs the negative contribution of the resonances. Assuming the continuity of the integrand as $Q^2 \rightarrow 0$, as in the nucleonic case [28], our results indicate the necessity of a dramatic turnover in $I(Q^2)$ at very low Q^2 . The only possible reaction channel available to accommodate such a turnover is electrodisintegration at threshold. Indeed, our σ'_{TT} data [29] show an indication of a growing negative contribution to the sum in the threshold region as Q^2 approaches zero. A recently completed experiment [30] may shed light on this behavior.

We focus now on the quasielastic region, where ³He can be treated with exact nonrelativistic Faddeev calculations. This approach describes existing data [31-33] well at low Q^2 . At larger Q^2 , modern applications [22,23] of the PWIA have had good success reproducing data. The measured quasielastic differential cross section $d\sigma/d\Omega dE'$ is displayed in Fig. 2 as a function of W. In addition, Fig. 3 displays the ³He polarized cross sections $\sigma_{TT'}$ and $\sigma_{LT'}$. Radiative corrections have been applied to the data as discussed in Refs. [7,29]. The data are compared to a PWIA calculation [22,23] and an exact nonrelativistic Faddeev calculation [32-34]. The latter includes both final state interactions (FSI) and meson exchange currents (MEC). Both groups utilize the Höhler [35] parametrization for the single nucleon current and the AV18 [36] nucleon-nucleon potential. We also display the PWIA curves that result when the Reid soft-core (RSC) [22,23] potential is used instead of AV18 or if the Galster [37] form factor parametrization is used instead of Höhler. In the Faddeev calculation, the three-nucleon current operator consists of the single nucleon current and the π - and ρ -like meson exchange contributions consistent with AV18.

The Faddeev calculation does not address relativistic effects and as such was performed only for the lowest Q^2 data. The agreement with data is, in general, quite good, but we find a small discrepancy from the data on the high energy side of the QE peak at $\langle Q^2 \rangle = 0.2 \text{ GeV}^2$. This may indicate the increasing importance of relativistic effects, along with the growth in relative strength of the Δ resonance tail in the QE region as Q^2 increases. We note that $\sigma_{LT'}$, which is not sensitive to the Δ resonance, generally



FIG. 2 (color online). ³He unpolarized cross sections in the QE region compared to PWIA [22,23] with AV18 (dashed line) or RSC (dotted and dotted-dashed lines) potential and to the Faddeev calculation [34] (solid line). The error bars (bands) represent statistical (systematic) uncertainties. $\langle Q^2 \rangle$ is in GeV².

shows better agreement with the Faddeev calculation on the high energy side of the quasielastic peak.

At very low Q^2 , the PWIA calculation fails but improves as expected with increasing momentum transfer, in part because it takes the relativistic kinematics into account. The fact that the Faddeev and PWIA calculations differ less as Q^2 increases seems to indicate that FSI and MEC (neglected in the PWIA) become less important for these observables as Q^2 increases. It also appears that the PWIA calculation is more sensitive to the choice of the form factor parametrization than to the nucleon-nucleon potential utilized.

The Faddeev calculation reproduces the polarized data well at the lowest Q^2 , and the PWIA does well at the highest, but there remains an intermediate zone where both approaches are unsatisfactory. References [32,33] previously reported that this same PWIA calculation reproduced well the measured ³He quasielastic asymmetry A'_T in this kinematic region. As such, we compared this calculation directly to the transverse asymmetry A'_T data from our experiment and found good agreement, consistent with the previous results but only in a narrow window centered on the QE peak.





FIG. 3 (color). ³He polarized cross sections in the QE region. Curves and notations are the same as in Fig. 2.

In summary, we find the Burkhardt-Cottingham sum rule to hold for ³He. The GDH integral and Γ_1 display intriguing behavior at low Q^2 and will provide valuable constraints on future χ PT and lattice QCD calculations. We have measured the first precision polarized cross sections in the quasielastic and resonance regions of ³He. A full three-body Faddeev calculation agrees well with the data but starts to exhibit discrepancies as the energy increases, possibly due to growing relativistic effects. As the momentum transfer increases, the PWIA approach reproduces the data well, but there exists an intermediate range where neither calculation succeeds.

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