

Supporting Online Material

Methods:

Fig S1 shows a schematic of our electronic measurement and cryogenic set-up. We have implemented the *rf* SSET technique[4] where the SSET impedance is monitored by microwave reflectometry. The SSET and microfabricated inductor and capacitor (L_T and C_T) form an RLC tank circuit, where the SSET provides the dissipation; $R = (\partial I_{DS} / \partial V_{DS})^{-1}$ where I_{DS} and V_{DS} are the SSET source drain current and bias voltage. This effective SSET resistance is a complex function of V_{DS} , nearby gate potentials, and the nanomechanical resonator position, which is coupled electrostatically to the SSET by biasing with a large dc voltage, V_{NR} . On resonance at $\omega_0/2\pi = (L_T C_T)^{-1/2}/2\pi = 1.35$ GHz, the tank impedance is given by $Z_T(\omega = \omega_0) = L_T / (RC_T)$. This impedance network terminates an ultra-low dissipation, $Z_0 = 50 \Omega$, Nb-Nb coax and controls the reflection coefficient: $\Gamma = (Z_T - Z_0) / (Z_T + Z_0)$. The bandwidth of this resonant network determines the readout bandwidth. By biasing the SSET far above the Coulomb blockade potential, we directly detect the resonant circuit using the shot noise of the SSET to excite the tank. We find *rf* SSET bandwidth of approximately 70 MHz, which is close to the bandwidth expected from design.

Using a directional-coupler placed at 1K, 1.35 GHz microwaves are launched at the device, and the reflected wave is detected and amplified by an ultra-low temperature cryogenic preamplifier located in the bath at 4.2 K. A mechanical displacement produces a change in the charge at the SSET, which produces a change in the SSET impedance, which changes the tank impedance Z_T , which changes the reflected microwave signal. It is this microwave signal that is our measured quantity.

In order to compare signals measured with different device conditions, we apply a 1 MHz modulation voltage to the gate for system calibration. This signal induces a charge at the SSET corresponding to $\sim 10 me_{\text{RMS}}$, where e is the electron charge. This charge signal modulates the reflected microwave signal, forming sidebands 1 MHz about the 1.35 GHz carrier. The amplitude of these sidebands is used to measure the charge responsivity. This calibration is essential for comparing data measured under different conditions (temperature, bias point, ect.) or for making long term averages of the noise.

Due to low frequency charge drifts at the device, active control of the system responsivity is essential to both make the SSET a stable, usable amplifier at high frequencies and to achieve the lowest noise performance. To sense the change in charge responsivity for a small change in bias point, an additional 50 kHz modulation signal of amplitude $\sim 10 me_{\text{RMS}}$ is applied to the resonator gate. The response of the 1 MHz side band to this modulation is analyzed and a feedback signal is applied to the resonator gate to hold the SSET at maximum charge responsivity. Finally, it should be mentioned that the SSET is biased near the Double Josephson Quasiparticle Resonance (DJQP). It is at this point that the system responsivity is found to be maximum. It is also the bias point where the SSET is expected to act as an ideal amplifier[12].

Great care is taken to isolate the SSET from electromagnetic noise. The samples are mounted in a light-tight package on a dilution refrigerator which has been isolated within an electro-magnetically shielded enclosure ($>100\text{db}$ of attenuation at 1GHz). Only battery powered analog electronics are located within the enclosure, all other electronics for stimulation and analysis of the device are located outside the enclosure. All low frequency signals ($<50\text{kHz}$) are feed-through the shielded room using optical isolators.

High frequency signals are feed-through the shielded enclosure using high-pass filters and dc blocks on the center and outer conductors. A Nb-Nb coax is used for connecting the device located at millikelvin temperatures, to the microwave amplifier in the liquid He bath at 4.2 K. Due to the superconductivity, this coax provides both an ultra-low loss transmission line and very low thermal conductivity between the amplifier and the sample. The Nb-Nb coax is thermalized on a bias-tee at millikelvin, and thermalized at 1.7 K by inserting a 2 cm long, 50Ω characteristic impedance, 200 nm thick Au microstrip running over a $350 \mu\text{m}$ thick quartz substrate.

To minimize the effects of seismic vibrations which can add microphonic noise to our dc bias leads, the dilution refrigerator is fixed to a 1000 Kg steel table, which is supported by 4, 1000 Kg legs which are filled with a mixture of lead bricks and sand. These massive legs rest upon a stack of alternating rubber pads and aluminum plates which attenuate acoustic vibrations emanating from the floor. The interior of the shielded enclosure is lined with acoustically adsorbing material.

SET Displacement Detection Calibration:

The two central results of this paper are:

1. The noise temperature of the nanomechanical oscillator reaches $T_N^{NR} = 55\text{mK}$.
2. We achieve displacement detection a factor of 4.3 from the quantum limit, or equivalently, the SSET is a near-ideal linear amplifier, with noise temperature of $T_N^{SSET} = 16\text{mK} = 18T_{QL}$ at 20 MHz.

Both assertions rely on the proper calibration of the noise temperature of the mechanical system and the noise temperature of our displacement detection scheme. A very important point is that this calibration rests upon the behavior of the measured noise versus temperature, which allows for a completely geometric method to calibrate the noise temperature of both the mechanics and the position detection scheme. This geometric scheme, described below, can be performed to a high accuracy, much higher than one could achieve by using estimates of the device parameters (gaps, capacitances, charge sensitivity, ect..)

Below, we first discuss the expected noise signals and dependence of signal size on coupling voltage. We then discuss the calibration of the noise temperature of the mechanical resonator, which ultimately establishes point #1. Next, we describe how the mechanical noise temperature is used to calibrate the SSET position detector noise temperature, establishing point #2.

The expected noise signals: The charge induced at the SSET island from the voltage biased nanomechanical resonator is given by $Q_{SSET}=C_G V_{NR}$, where C_G is the resonator-SSET capacitance. Motion of the resonator, δx , will modulate the capacitance, C_G , which

will change the SSET charge by $\delta Q_{SSET} = \frac{\partial C_G}{\partial x} V_{NR} \delta x$. Thus mechanical noise will

produce charge noise, $S_Q^{NR}(\omega) = \left(\frac{\partial C_G}{\partial x} V_{NR} \right)^2 S_X(\omega)$, where $S_Q^{NR}(\omega)$ and $S_X(\omega)$ are the

charge and position noise power spectral densities. Thermal motion of the resonator is

expected to have a spectral density given by: $S_X(\omega) = \frac{4k_B T \omega_0}{Q m_{eff}} \cdot \frac{1}{(\omega^2 - \omega_0^2)^2 + \left(\frac{\omega \omega_0}{Q} \right)^2}$

where $\int_0^{+\infty} S_x(\omega) \frac{d\omega}{2\pi} = \langle x^2 \rangle = \frac{k_B T}{m_{\text{eff}} \omega_0^2}$ obeying the equipartition of energy, where k_B is the

Boltzmann constant. The expected total charge noise spectrum is

$$S_Q(\omega) = S_Q^{NR}(\omega) + S_{SSET} = \left(\frac{\partial C_{NR}}{\partial x} V_{NR} \right)^2 S_x(\omega) + S_{SSET},$$

where S_{SSET} is the white, SSET charge noise originating from the cryogenic preamplifier in our setup.

Nanomechanical noise temperature calibration: First, we measure the charge noise of the *rf* SSET detector around the mechanical resonance for temperatures from 30 mK to 550 mK. We find a noise peak at the expected mechanical resonance frequency (identified earlier by driving the nanomechanical resonator), sitting upon a white background, S_{SSET} . The charge noise power data accurately fits the expected harmonic oscillator response function. We extract both the background noise power, S_{SSET} and the integral of the resonator noise power which is a measure of the resonator position

$$\text{variance: } P_{NR} = \int S_Q^{NR}(\omega) \frac{d\omega}{2\pi} \propto \langle x^2 \rangle.$$

Regardless of coupling voltage and for temperatures above 100mK, the measured peak areas, P_{NR} , scale linearly with temperature with an intercept through the origin, as seen in Fig 4. This is the expected behavior from the equipartition of energy. Using estimates of the device parameters, our measured signals, $S_Q^{NR}(\omega)$ and P_{NR} , are found to be within 33% of the expected signal size. In addition, the plots of P_{NR} vs T can be collapsed to a single curve by dividing by P_{NR} by V_{NR}^2 . This collapse of the data is shown in Fig. S2, and confirms the expected dependence of device sensitivity on coupling voltage, V_{NR} .

From this behavior we conclude that for temperatures above 100mK the integrated peak noise power, P_{NR} , is proportional to the thermodynamic temperature of the fundamental in-plane mode of the oscillator, and can be used as a noise thermometer, T_N^{NR} .

For temperatures below 100mK, we find that P_{NR} deviates from the linear temperature dependence. Since this deviation is independent of coupling voltage, we conclude that it is not a result of back-action and is more likely a result of the loss of thermal contact to the refrigerator bath. Although in this temperature range P_{NR} no longer corresponds to the refrigerator temperature, we can use the higher temperature data to scale P_{NR} and calculate a noise temperature of the resonator, T_N^{NR} . Our conclusion that the mechanical noise temperature reached a low of $T_N^{NR}=55\text{mK}$ is based upon this noise thermometry.

SSET noise temperature calibration: Given that P_{NR} is a measure of the mechanical noise temperature, the noise temperature of the *rf* SSET amplifier scheme, T_N^{SSET} , can be determined easily. The noise temperature of the amplifier is defined to be the temperature at which one would have to cool the load such that the thermal noise of the load equals the amplitude of the amplifier noise. In our case with a resonator as the load, the SSET noise temperature at the mechanical resonance is defined by scaling the peak in the mechanical charge noise, $S_Q^{NR}(\omega_0, T_N^{NR})$, to the white SSET charge noise:

$$T_N^{SSET} = \left[\frac{S_{SSET}}{S_Q^{NR}(\omega_0, T_N^{NR})} \right] \times T_N^{NR}. \text{ In this way, } T_N^{SSET} \text{ is geometrically determined from the}$$

measured power spectrum to a high accuracy.

The lowest SSET noise temperature is expected to be found for the coupling voltage approaching the optimal value, expected to be ~ 13 V for our device. We made measurements with the refrigerator at 35 mK and at 100 mK, with $V_{NR}=15$ V. From Fig. 4 it is clear that the 100 mK point ($V_{NR}=15$ V) plots on top of the other data taken at lower coupling voltages, confirming that the mechanical noise temperature is indeed 100mK. Using the scaling of the peak height to noise floor at 100mK, the noise temperature of the SSET under these conditions is $T_N^{SSET}=20$ mK. By cooling the refrigerator to 35 mK, we find that T_N^{NR} falls to 73 mK giving $T_N^{SSET}=16$ mK= $18 T_{QL}$, where the improvement in T_N^{SSET} compared to 100 mK is due to the increase in mechanical quality factor.

Finally, our displacement sensitivity, Δx , defined as the position standard deviation due to the SSET white charge noise, S_{SSET} , is computed by taking the position noise power of $S_x=3.8$ fm/Hz^{1/2} and integrating against a normalized simple harmonic oscillator response function, $R(\omega)$: $R(\omega) = \frac{\omega_0^4}{Q^2} \frac{1}{(\omega^2 - \omega_0^2)^2 + \left(\frac{\omega\omega_0}{Q}\right)^2}$, note $R(\omega_0)=1$.

Using this response function, the standard deviation in position from a white noise is

given by: $\langle \Delta x^2 \rangle = \int_0^\infty S_x R(\omega) \frac{d\omega}{2\pi} = \frac{\omega_0}{4Q} S_x$, which defines the effective noise bandwidth of

$$\Delta f = \frac{\omega_0}{4Q}.$$

Figure S1

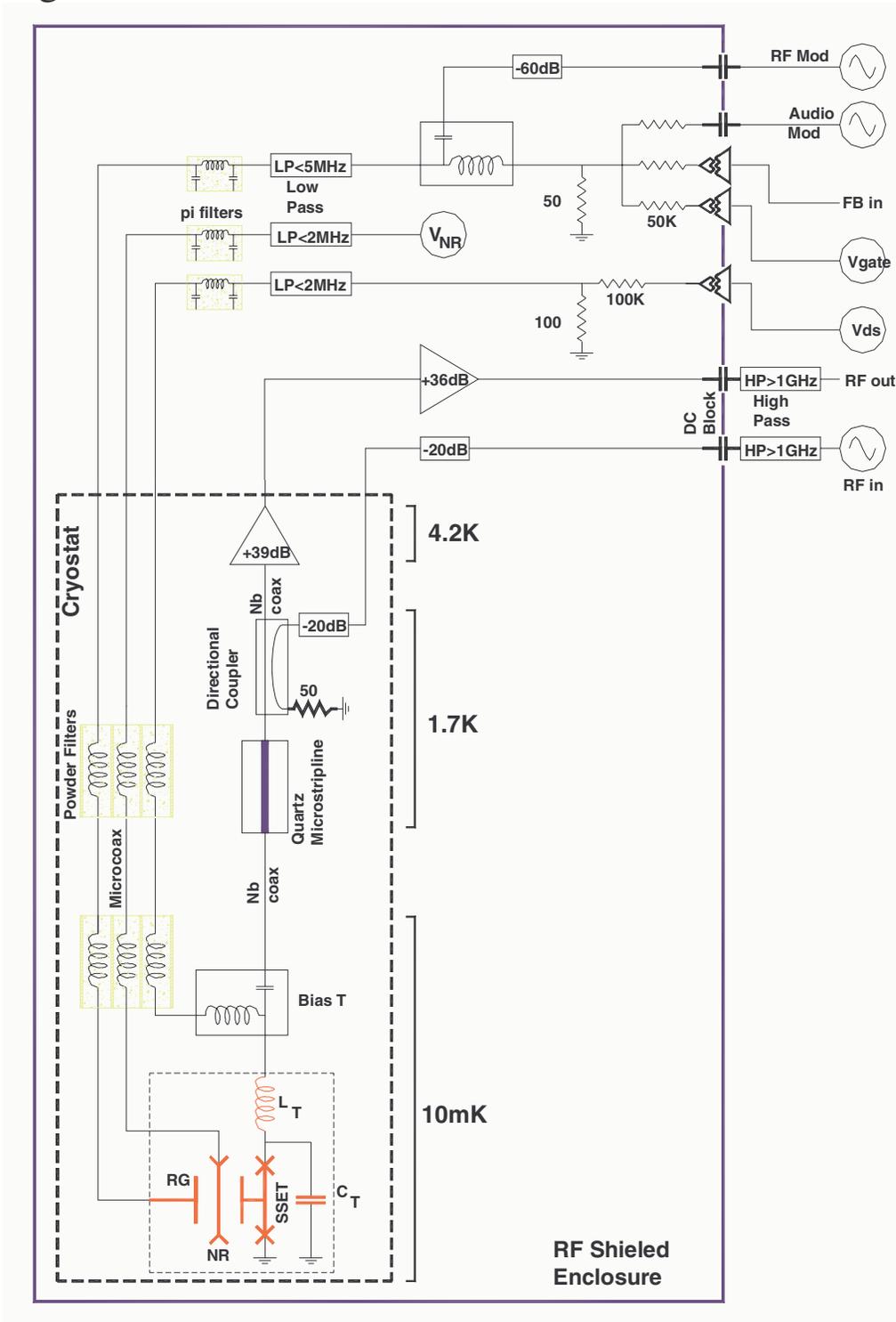


Figure S1: Measurement setup enclosed in an rf-shielded enclosure (blue box). The sample (dotted-line box) consists of the SSET, nanomechanical resonator (NR), resonator gate (RG), and an on-chip tank circuit (L_T, C_T). Low frequency lines (V_{DS} , V_{NR} , V_{RG}) run through low-pass filters (LP) and pi-filters (1MHz cut-off) at room temperature, and microwave adsorbing powder filters at 1.7K and at milli-kelvin, which are connected by lossy microcoax. Low frequency leads are fed-through the shielded enclosure using optical isolators. High frequency leads are fed-through using high pass filters and dc blocks (on the outer and inner conductor) to avoid ground loops. Microwave reflectometry of the SSET is accomplished by a cryogenic preamp located in the He bath at 4.2K, a directional coupler at 1.7K, and a bias-tee at the sample. This microwave circuit uses Nb-Nb coax, for ultra-low loss and thermal isolation between the amplifier and sample. The center of this coax is thermalized to 1.7K with a quartz microstripline.