Achievable Throughput in Two-Scale Wireless Networks

Radhika Gowaikar and Babak Hassibi

***Abstract*—We propose a new model of wireless networks which we refer to as "two-scale networks." At a local scale, characterised by nodes being within a distance r, channel strengths are drawn independently and identically from a distance-independent distribution. At a global scale, characterised by nodes being further apart from each other than a distance r, channel connections are governed by a Rayleigh distribution, with the power satisfying a distance-based decay law. Thus, at a local scale, channel strengths are determined primarily by random effects such as obstacles and scatterers whereas at the global scale channel strengths depend on distance.**

For such networks, we propose a hybrid communications scheme, combining elements of distance-dependent networks and random networks. For particular classes of two-scale networks with N nodes, we show that an aggregate throughput that is slightly sublinear in N, for instance, of the form $N/\log^4 N$ is achievable. This offers a significant improvement over a throughput scaling behaviour of $O(\sqrt{N})$ that is obtained in other work.

***Index Terms*—Wireless networks, ad hoc networks, i.i.d. connections, decay law, throughput.**

I. Introduction

SENSOR and ad hoc networks have seen much research activity in recent years. Throughput, delay, routing protocols, scalability, resource allocation, efficiency, connectivity and so on are some of the aspects that have been the focus of investigation. The first major result of the field was by Kumar and Gupta [18] in which the throughput of a network of n nodes was studied. Strengths of the connections between two nodes were determined entirely by the distance between them and followed a deterministic power scaling law. With this model, for networks called 'Random Networks,' it was shown that a total throughput that scaled like $\sqrt{n}/\log n$ was the best possible. This implied that the throughput per user fell like $\frac{1}{\sqrt{n \log n}}$ which was quite discouraging. Similar scaling laws were shown to hold in other settings as well [15], [4], [9], [11], [14], [19], [21], [22], [16], [17]. The recent result of Özgür, Leveque and Tse improves this significantly and achieves linearly scaling throughput in certain cases, using a hierarchical communication scheme. The other cases in which scaling laws that are better than $O(\sqrt{n})$ are obtained are where nodes are allowed to approach each other [12], or when the attenuation is very low [22].

In all of the above results, the network connections are governed by a distance-based decay law. A different network model is proposed in [23], [2]. In it, the channel strengths are independent of distance and geometry and are instead drawn identically and independently (i.i.d.) from a probability distribution function (pdf). This model is suitable for networks over a small area, where multipath and physical obstructions dominate and the decay laws associated with far-field effects do not kick in.

Though the throughput that is possible with this model depends very strongly on the distribution that the channel strengths are drawn from, several distributions, including the Bernoulli and some heavy-tailed distributions lead to throughputs that are almost linear in n. Thus the introduction of randomness changes the behaviour of the system significantly.

In practice, we expect neither the deterministic model of [18] nor the random model of [2] to hold. Work in the area of link-level modeling and network modeling tells us that a combination of distance-dependent connections and random connections makes for a more realistic model. The next section introduces existing network models and puts into context the two-scale model that is proposed and analysed in this work.

A. Network Characterisation

The analysis of ad hoc networks and the results obtained are strongly dependent on the network model that is under consideration. Several experimental and analytical results regarding network models can be found in the literature [6], [7]. The pathloss phenomenon is the underlying aspect of many of these. This phenomenon dictates that the signal power decays according to a power law. This means that over a distance d, it decays by a factor proportional to d^{-m} where m is a a constant that depends on the environment. Typically, m is expected to vary between 2 and 6 as the environment varies from free space to urban areas characterised by tall buildings and obstructions. The pathloss phenomenon is taken into account in models such as the Okumura Model, the Hata Model and its COST-231 extension and the Walfisch and Bertoni model which have been widely adopted in industry and by standards bodies [8].

An important point to note is that these models have been developed under the cellular communications concept which assumes that communication occurs between a base station that is at a much greater height than the users. This is not a valid assumption for most ad hoc networks and therefore these models are of limited use for us. Furthermore, the pathloss

Manuscript received 22 August 2008; revised 31 January 2009. This work is supported in part by the National Science Foundation under grant nos. CCR-0133818 and CCR-0326554, by the David and Lucille Packard Foundation, and by Caltech's Lee Center for Advanced Networking.

Radhika Gowaikar is with Qualcomm Incorporated (e-mail: gowaikar@systems.caltech.edu).

Babak Hassibi is with the California Institute of Technology (e-mail: hassibi@caltech.edu).

Digital Object Identifier 10.1109/JSAC.2009.090913.

model is a far-field effect and the models mentioned above are known to hold only over distances between 1 km and 20 km and in certain ranges of frequency. Since the nodes in an ad hoc network are likely to be within 1 km of each other, these models often do not hold in this setting. Models for shorter ranges are still in development, but the general observation is that they are very strongly dependent on the terrain. For example, if the landscape consists of buildings, the propagation model is dependent on even the specific building materials [7]. This makes it very difficult to come up with short range models that hold with any amount of generality.

Another feature, apart from pathloss, that is typically incorporated in network models is that of shadowing. Under the pathloss model, all receivers at a distance d from the transmitter are expected to receive the same average power. However, because of the occurrence of obstacles such as buildings, trees and other properties of the environment this is not what is observed. At every point the mean received power is a random variable that follows a log-normal distribution with a mean equal to the received power dictated by the pathloss model and a variance that can depend on the terrain. As explained in [13], this produces an interesting connectivity phenomenon where nodes that are further apart are sometimes more likely to be connected than nodes that are closer. This result also implies that channel strengths between the transmitter and two receivers that are close to each other are uncorrelated. Such models, or the connectivity thereof, is captured by geometric random graphs, in which the connections are drawn independently but not identically, and the parameters governing the connections are chosen to reflect an underlying physical phenomenon, for example, the distance between a pair of nodes. Thus we see that a combination of distance-dependent behaviour and random variations makes for a more complete model. In summary, over short distances, where the pathloss model does not apply or the pathloss given by it is small compared to the variance of the shadow fading, it is the shadow fading that determines the channel strength and connectivity. Over longer distances, as the pathloss becomes large, the shadow fading plays a smaller role and the distance-decay dominates.

Thus, for our purposes of analysis, a suitable model is one that incorporates the far field effects at a global level through the decay law, but also recognizes that channel strengths look much more random at a local level. In this work, we propose and analyze such a model, which we call the two-scale model. We discuss a communication strategy for this model and derive the throughput that is achieved under it. In Section VI we describe some other models that also incorporate propagation effects over the short range and long range.

The rest of the paper is organised as follows. A precise description of the model and the problem statement is in Section II. Sections III and IV study the scheduling and error-free communication properties of this model and the main result is stated in Section V. A simplified throughput expression is presented in Section V-A, examples are presented in V-B and conclusions and directions for future work are presented in Section VI.

II. Network Model

In the two-scale model, we assume that nodes that are within a distance r of each other are connected by channels that are distance-independent. These channel strengths are assumed to be drawn i.i.d. from a specified distribution. For nodes that are further apart than r, the channel connections obey a Rayleigh distribution with a mean power that depends on the distance between the nodes and follows a distance-decay law.

More specifically, consider a network with N nodes that are uniformly and randomly distributed on the surface of a sphere of radius R. We use a sphere rather than a planar disk to separate edge effects and to have symmetry between all nodes. We follow the standard convention of measuring distances along great circles.

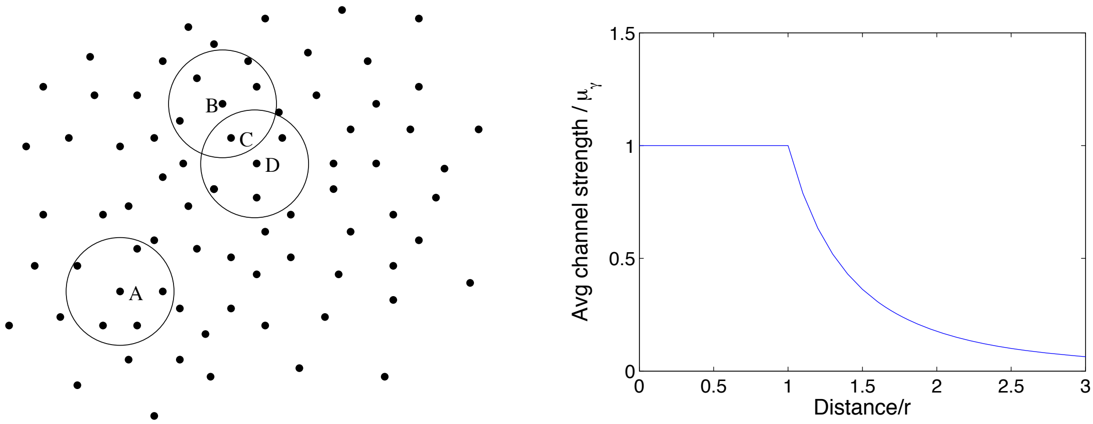
The channel between nodes i and j is denoted by $h_{i,j} = h_{j,i}$. Define the channel strength to be $\gamma_{i,j} = |h_{i,j}|^2$. For nodes that are within a distance r, the channel strengths, or $\gamma_{i,j}$, are drawn i.i.d., according to a pdf, say $f(\gamma)$. Let the expected value corresponding to this distribution be denoted by μ_γ. If nodes i and j are at a distance of $l(i,j) > r$ from each other, we model $h_{i,j}$ to be a Rayleigh distributed random variable with its power (or second moment), $E|h_{i,j}|^2$, given by $cg(l(i,j))$ where $g(x)$ is used to model the distance-dependence and c is a constant. This gives us that the corresponding $\gamma_{i,j}$ is drawn from an exponential distribution with $cg(x)$ as its mean. Thus the distribution is given by $cg(x)\exp(-\gamma/cg(x))$. Typically, $g(x)$ is a decreasing function such as $\frac{1}{x^m}$ or $\frac{e^{-\delta x}}{x^m}$ with $m > 2$, $\delta > 0$ and c is chosen such that $cg(r)$ equals μ_γ. This is done to ensure that the expected value of $\gamma_{i,j}$ does not change abruptly as the distance between i and j changes from being less than r to being greater than r. Therefore, $c = \frac{\mu_\gamma}{g(r)}$.

Thus the formal definition of the channel strengths is as follows. Denote by $p_x(\gamma)$ the distribution from which the channel strength between two nodes with distance x between them is drawn. Then we have

$$p_x(\gamma) = \begin{cases} f(\gamma) & \text{if } x \le r \\ \frac{\mu_\gamma g(x)}{g(r)} \exp(-\gamma \frac{g(r)}{\mu_\gamma g(x)}) & \text{if } x > r \end{cases}. \quad (1)$$

Figure 1(a) shows a sample network and the channel strengths are as explained in the caption. Figure 1(b) plots the mean channel strength as a function of the distance between two nodes. The model of equation (1) ensures that the mean channel strength is a continuous function at distance r.

We allow r and R to be functions of the number of nodes N. This makes the model versatile and it can be used to subsume existing models. For example, appropriate choices of r and R can help model a full range of networks, from the purely geometric ones of [18], to the purely random ones of [2]. The former are obtained when r is small enough to ensure that at most a finite number of nodes (preferably no more than one) lie inside any circle of radius r and the latter are obtained when $r = R$. The dependence of R on N can give networks of different densities. In addition, it is also possible to choose $f(\gamma)$ to be a function of N.



(a) Example of a network (b) Average channel strength as a function of distance

Fig. 1. (a) Node A had i.i.d. channel strengths to the nodes inside the circle surrounding it and distance-dependent channel strengths otherwise. The channel strength between nodes B and D is distance-dependent. Node C has i.i.d. channel strengths to both nodes D and B. (b) Average channel strength as a function of distance is constant and equal to μ_γ up to a distance r and follows a decay law beyond that.

A. Successful Communication

Next, we define the notion of successful communication between two nodes. Assume that node i wishes to transmit signal x_i. We assume that x_i is a complex Gaussian random process with zero mean and unit variance. Each node is permitted a maximum power of P watts.

We incorporate interference and additive noise in our model as follows. Assume that l nodes $i_1, i_2, \ldots, i_l$ are simultaneously transmitting signals $x_{i_1}, x_{i_2}, \ldots, x_{i_l}$ respectively. Suppose that node j is the intended receiver of the signal x_{i_1}. Then, the signal received by node $j (\neq i_1, \ldots, i_l)$ is given by

$$y_j = \sum_{t=1}^{l} \sqrt{P} h_{i_t, j} x_{i_t} + w_j \qquad (2)$$

where w_j represents additive noise for node j. The additive noise variables $w_1, \ldots, w_N$ are i.i.d., drawn from a complex Gaussian distribution of zero mean and variance σ^2. That is, $w_i \sim \mathcal{CN}(0, \sigma^2)$. The noise is statistically independent of x_i.

In equation (2), assume that only node i_1 wishes to communicate with node j and the signals $x_{i_2}, \ldots, x_{i_l}$ are interference. Then the signal-to-interference-plus-noise ratio (SINR) for node j is given by

$$\rho_j = \frac{P \gamma_{i_1, j}}{\sigma^2 + P \sum_{t=2}^{l} \gamma_{i_t, j}}$$

Note that some of the interference terms will come from the exponential distribution and the others will be drawn from $f(\gamma)$, depending upon the distance of the interferer from j. We assume that transmission is successful when the SINR exceeds some ρ_0. If the SINR is less than ρ_0, we say that an error has been made.

B. Network Operation and Throughput

We suppose that K nodes $s_1, \ldots, s_K$ are randomly chosen as sources. For every s_i, a destination node, say d_i, is chosen at random, thus making K source-destination pairs. We assume that these $2K$ nodes are all distinct and therefore $K \leq N/2$. Source s_i wishes to transmit message W_i to destination d_i and has encoded it as signal x_i.

Communications are assumed to occur using a series of hops. Every source-destination pair (s_i, d_i) uses a sequence of relay nodes to transmit message x_i. Each relay node is expected to decode the message x_i and retransmit it in a future time slot, using power P. We expect several messages to be making hops simultaneously and therefore the relay nodes have to decode in the presence of interference. With this in mind, we impose the constraint that no relay node be asked to decode two messages simultaneously. We also assume that no relay node can receive and transmit in the same time slot. These properties will define a *non-colliding* schedule of relaying.

Assume that all K messages reach the intended destinations in (at most) H time slots. Assume that a fraction ϵ of messages fail to reach the intended destination due to decoding or scheduling errors. Each message contains at least $\log(1+\rho_0)$ bits of information since ρ_0 is the SINR threshold. Therefore, we define the throughput as

$$T = (1-\epsilon) \frac{K}{H} \log(1+\rho_0) \qquad (3)$$

Note that all the quantities above may depend on N. Typically, we force ϵ to go to zero. In the rest of this paper, we present a scheme of scheduling and communicating and analyze the throughput as well as performance of this scheme. Our concern will primarily be with arbitrarily large values of N. Thus, we will obtain an asymptotic achievability result for the throughput T. We state our main result next.

C. Main Result

We state the main result assuming a decay law of $g(x) = \frac{1}{x^m}$ with $m > 2$.

Theorem 1: For the network described above, a throughput of

$$T = (1-\epsilon) \cdot K \cdot \frac{\log\left(1 + \frac{P\beta}{\sigma^2 + \eta P K \mu_\gamma \left(\sin^2 \frac{r}{2R} + \frac{r^2}{2R^2} \frac{1}{m-2}\right)}\right)}{\frac{\log n}{\alpha \log np} \cdot c_2 \frac{R}{r} \cdot \log N}$$

is achievable. Here, n is bounded by $c_5 \cdot \frac{1}{2} N \sin^2 \frac{r}{24R} \leq n \leq c_5 \cdot 2N \sin^2 \frac{r}{12R}$, α, c_2 and c_5 are known constants and β, K and $\eta \geq 1$ are chosen such that the following conditions are satisfied:

1) $K \leq \frac{N}{32 \cos^2 \frac{r}{24R}}$.
2) $p = \mathrm{P}(\gamma \geq \beta) = \frac{\log n + \omega_n}{n}$, where $\omega_n \to \infty$.
3) $K \leq \alpha N \cdot \frac{\log(c_5 \cdot \frac{1}{2} N \sin^2 \frac{r}{24R} p)}{\log(c_5 \cdot \frac{1}{2} N \sin^2 \frac{r}{24R})} \frac{1}{16 \cos^2 \frac{r}{24R}}$.
4) $\epsilon \leq \frac{\log n}{\alpha \log np} \cdot \frac{R}{r} \cdot \frac{1}{\eta} \to 0$.
5) $\frac{r}{R}\sqrt{N} \to \infty$.

Note that the theorem is general and can be applied to any pdf $f(\gamma)$. For different choices of r and $f(\gamma)$ the theorem gives different achievable throughputs. In Section V-B we see that these can range from throughputs that are rapidly going to zero to throughputs that increase almost as fast as N. The result is derived in Section V and is based on results from Sections III and IV.

III. Relaying Scheme

In this section we determine the scheduling of the relay nodes for the multihop protocol. We do this through various constructions, including Voronoi tessellations, a superschedule and many subschedules. We will borrow techniques from [18] and [2] and put them together in a suitable manner to perform scheduling for the proposed hybrid, or two-scale, model. The general approach is as follows. We first divide the network into cells and determine a sequence of adjacent cells that a particular message will have to pass through. This is called a superschedule. Next, we find a subschedule that determines which precise nodes will perform the task of carrying a message from one cell to the next. During the subscheduling, cells of the network form aggregates and communication only takes place among nodes within an aggregate. Details of this relaying scheme are presented in the following subsections.

A. Tessellations and cell-aggregates

Recall the concept of a Voronoi tessellation, used extensively in [18]. Lemma 4.1 of [18] establishes the existence of a Voronoi tessellation of the surface of the unit sphere where each Voronoi cell contains a disk of radius δ_1 and is contained in a disk of radius $2\delta_1$ for any $\delta_1 > 0$. We will use this result for the surface of the sphere of radius R. (This can be done by using the original result for δ_1/R rather than δ_1 and then scaling the obtained tessellation by a factor of R.) Denote by $\mathcal{T}(x)$ a tessellation of the surface of the sphere of radius R where each Voronoi cell contains a disk of radius x and is contained in a disc of radius $2x$. In particular, consider a tessellation $\mathcal{T}(r/12)$ where r is the radius within which channel strengths are distance-independent and are drawn i.i.d. from $f(\gamma)$. Cells of this tessellation are labelled S_i. If two cells share an edge or a vertex they are called neighbors. It is easy to show that in such a tessellation, for any cell, S_i, it and all its neighboring cells are contained in a disk of diameter r. (A similar but slightly different result is shown in Lemma 4.2 of [18].) This means that all the nodes within this group of cells are within a distance r of each other. Therefore, all the connection strengths within this set of cells are independent of distance and are drawn i.i.d. from the distribution $f(\gamma)$. Recall that the area of a circle of radius x on the surface of a sphere of radius R is given by $A(x) = 4\pi R^2 \sin^2 \frac{x}{2R}$. Using this fact, it is possible to show that the number of cells that are neighbors to a given cell is bounded by a constant, say c_1. This is similar to Lemma 4.3 of [18]. We will use this fact in what follows.

B. Determining a Non-colliding Superschedule

Assume that a tessellation such as one mentioned in the previous subsection is done and kept fixed. We refer to this as $\mathcal{T}_0(r/12)$. If the cells of this tessellation are labelled S_j, every node belongs to some S_j. (Nodes lying on cell boundaries can be assigned arbitrarily.) Consider the source-destination pair (s_i, d_i). Consider the great circle containing s_i and d_i. Denote by L_i the segment of this circle that connects s_i and d_i. (We will take the shorter of the two segments that form the great circle.) This segment passes through a sequence of cells as it traverses from s_i to d_i. Since the radius of the sphere is R and each cell is big enough to contain a disc of radius $r/12$, the maximum number of cells that the segment has to pass through is $M = c_2 \frac{R}{r}$ where c_2 is a constant. Denote these cells, in sequence, by $s_i \in S_{i,0}, S_{i,1}, S_{i,2}, \ldots, S_{i,M} \ni d_i$. We can obtain such a sequence of cells for each of the K source-destination pairs. Some sequences may, in actuality, be shorter than M. We refer to the set of cells $S_{1,t}, S_{2,t}, \ldots, S_{K,t}$ as the t-th *layer* of cells. We aim to design a schedule in which the messages from the sources have to progressively pass through at most M layers in order to reach the intended destinations.

The aforementioned scheme only tells us the cells that a message has to pass through in a certain layer. We now decide which node in a particular cell is responsible for a certain message in a given layer of transmission. We refer to this schedule of nodes as the *superschedule*.

In particular, we seek a *non-colliding superschedule*. The non-colliding condition requires that the K nodes that act as relay nodes in one layer be distinct from each other as well as distinct from the K nodes that occur in the previous layer. Clearly, this condition can be imposed at the level of each cell: we require the relay nodes in each cell of the t-th layer to be distinct from each other as well as distinct from the relay nodes in the same cell that occur in the $(t-1)$-th layer. (Note that in the zeroth layer of transmission, this condition is trivially met since the K source nodes are assumed to be distinct and there is no previous layer.) We wish to have such distinct nodes for the i-th layer assuming that such nodes for each layer up to the $(i-1)$-th have already been determined. Let us determine the conditions under which this is possible.

We first estimate the number of sources and the number of nodes we expect to find in an arbitrary cell. There are at least $\frac{4\pi R^2}{A(r/6)} = 1/\sin^2 \frac{r}{12R}$ cells in $\mathcal{T}_0(r/12)$. The K sources are assumed to be uniformly distributed on the surface of the sphere. Therefore we expect each cell to contain around $K \sin^2 \frac{r}{12R}$ sources. This is made more rigourous in the following lemma.

Lemma 1: With probability going to 1, each cell contains at most $k_1 = 2K \sin^2 \frac{r}{12R}$ and at least $k_2 = \frac{1}{2} K \sin^2 \frac{r}{24R}$ sources.

Proof: The area of each cell is at most $A(r/6)$ and at least $A(r/12)$. The K source-destination pairs are uniformly distributed over the surface of the sphere. For a particular cell C with area $A(r/12) \leq x_C \leq A(r/6)$, let y be the random variable representing the actual number of sources in cell C. Let $\mu_y = K\frac{x_C}{4\pi R^2}$ represent the average value of y. Now $K\frac{A(r/12)}{4\pi R^2} \leq \mu_y \leq K\frac{A(r/6)}{4\pi R^2}$. Now consider the following sequence of inequalities for some $\delta_2 > 0$ (see equation top of following page).

The second inequality comes from a Chernoff bound and the other two follow from the above discussion. Therefore, for $\delta_2 = 1$, we have Eq. 4. Conversely, consider the following sequence of inequalities for some $0 < \delta_3 < 1$ (see equation below Eq. 4 on the following page).

The second inequality comes from a Chernoff bound and the other two follow from the above discussion. Therefore, for $\delta_3 = 1/2$, we have Eq. 5. Since $\frac{e}{4}$ and $\frac{2}{e}$ are both less than 1, for large values of K, the probabilities in (4) and (5) go to zero, giving us the lemma. ■

Thus, every cell occurs in the zeroth layer no more than k_1 times. By symmetry, a cell occurs in the t-th layer no more than k_1 times. A similar argument can be made for the minimum number of nodes that are contained in a cell to give us the following lemma.

Lemma 2: With probability going to 1, each cell contains at least $n_1 = \frac{1}{2}N\sin^2\frac{r}{24R}$ and at most $n_2 = 2N\sin^2\frac{r}{12R}$ nodes.

To go back to the problem of finding distinct relays, consider a specific cell in $\mathcal{T}_0(r/12)$. This is expected to have no more than k_1 distinct nodes that are the chosen relays in the $(i-1)$-th layer. This cell also occurs k_1 times in the i-th layer and we wish to assign a further k_1 distinct relay nodes for each occurrence. The total number of nodes in this cell is at least n_1. Therefore the condition of distinct relays can be met if $2k_1 \leq n_1$. Substituting for k_1 and n_1 and simplifying, we have the following lemma:

Lemma 3: It is possible to obtain a non-colliding superschedule of nodes provided the following condition is met:

$$K \leq N/(32\cos^2\frac{r}{24R}).$$

Once this condition is satisfied, we can assign a distinct relay node for each of the K messages in each layer. This can be done in an arbitrary manner. The relay node in layer t that is responsible for message i will be called $s_{i,t}$. The K sequences $s_i = s_{i,0}, s_{i,1}, \ldots, s_{i,M} = d_i$ for $i = 1, \ldots, K$ give us the non-colliding superschedule. It now remains to decide how to route the message i from its relay node in layer t, namely $s_{i,t}$ to its relay node in layer $(t+1)$, namely, $s_{i,t+1}$. We refer to this as subscheduling and address it next.

C. Non-colliding Subschedules

We consider time slots in blocks of size h, where h denotes the (maximum) number of hops required for a message to be transmitted from $s_{i,j}$ to $s_{i,j+1}$. The value of h is quantified later. In a specific block of time slots, say from $vh+1$ to $(v+1)h$, some constant fraction c_3 of all cells will be chosen at random and called active cells. Denote the set of chosen cells by T_v. Consider the cells that are not in T_v. Let j be such a cell. If one of the neighbors of j is in T_v, assign j to it. If more than one of the neighbors of j are in T_v, this assignment can be done randomly. Thus, for each of the $|T_v|$ originally chosen cells, we now have $|T_v|$ *cell-aggregates* that are active. (Some of these may consist of just one cell, namely, the originally chosen cell.) Figure 2 demonstrates this. In the v-th block of time slots, communications will occur only within the T_v cell aggregates and not between nodes in different aggregates. Since any cell and its neighbors can together be put inside a circle of diameter r (see Section III-A), connections within an aggregate are drawn i.i.d. from $f(\gamma)$. We will make use of this fact in determining h and a non-colliding subschedule in Lemma 4.

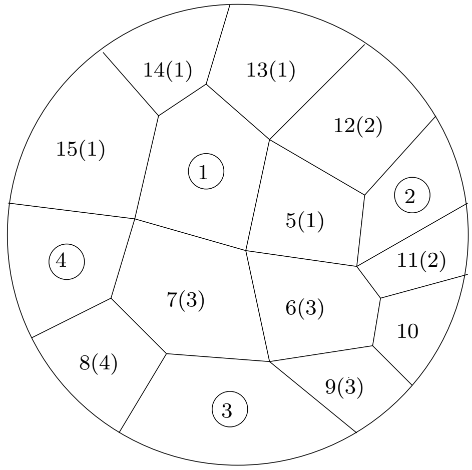


Fig. 2. Cells 1, 2, 3, 4 (circled) are originally chosen to be in T_v. The remaining cells are then assigned as indicated in parentheses. For example, 13 gets assigned to 1 and 6 to 3. Cell 10 remains unassigned. The aggregate corresponding to cell 3 consists of cells 3, 6, 7 and 9.

A particular choice of T_v leads to some pairs of adjacent cells not being in the same cell-aggregate. For a pair that gets split into two cell-aggregates, the relays in one cell that have the next relay in the other cell are unable to communicate with each other in the v-th block of time slots. However, there is a probability that in another set, say T_w, this pair does not get split up. Let B be the number of sets we have to choose in order for every pair of adjacent cells to have been chosen in the same aggregate at least once.

Let i and j be adjacent cells. They can be in the same cell-aggregate in a randomly obtained T_v if ($i \in T_v$, $j \notin T_v$ and j gets assigned to i) or vice versa. By symmetry, both cases are equally likely. Therefore,

$$\begin{aligned} &\mathrm{P}(i, j \text{ are in the same cell-aggregate}) \\ &= 2\ \mathrm{P}(i \in T_v, j \notin T_v, j \text{ gets assigned to } i) \\ &= 2\ \mathrm{P}(i \in T_v) \cdot \\ &\quad \mathrm{P}(j \notin T_v | i \in T_v) \cdot \mathrm{P}(j \text{ is assigned to } i | i \in T_v, j \notin T_v) \\ &\geq 2\ c_3 \cdot (1 - c_3) \cdot \frac{1}{c_1} \end{aligned}$$

The last expression comes from the fact that a fraction c_3 of cells are chosen at random to be in T_v. Therefore i is in T_v with probability c_3 and j is not in T_v with probability $(1-c_3)$ independently of i. Finally, j has at most c_1 neighbors, including i (see Section III-A). If some w of them are chosen

$$P\left(y > (1+\delta_2)K\frac{A(r/6)}{4\pi R^2}\right) \le P\left(y > (1+\delta_2)\mu_y\right) \le \left(\frac{e^{\delta_2}}{(1+\delta_2)^{(1+\delta_2)}}\right)^{\mu_y} \le \left(\frac{e^{\delta_2}}{(1+\delta_2)^{(1+\delta_2)}}\right)^{K\frac{A(r/12)}{4\pi R^2}}$$

$$P\left(y > 2K\sin^2\frac{r}{12R}\right) = P\left(y > 2K\frac{A(r/6)}{4\pi R^2}\right) \le \left(\frac{e}{4}\right)^{K\frac{A(r/12)}{4\pi R^2}} \quad (4)$$

$$P\left(y < (1-\delta_3)K\frac{A(r/12)}{4\pi R^2}\right) \le P\left(y < (1-\delta_3)\mu_y\right) \le \left(\frac{e^{-\delta_3}}{(1-\delta_3)^{(1-\delta_3)}}\right)^{\mu_y} \le \left(\frac{e^{-\delta_3}}{(1-\delta_3)^{(1-\delta_3)}}\right)^{K\frac{A(r/12)}{4\pi R^2}}$$

$$P\left(y < \frac{1}{2}K\sin^2\frac{r}{24R}\right) = P\left(y < \frac{1}{2}K\frac{A(r/12)}{4\pi R^2}\right) \le \left(\frac{2}{e}\right)^{K\frac{A(r/12)}{4\pi R^2}} \quad (5)$$

in T_v (and i is one of them), the probability of j being assigned to i is $1/w \ge 1/c_1$.

Let $c_4 = 2c_3(1-c_3)\frac{1}{c_1}$. Any choice of $c_3 < c_1/2$ ensures that $c_4 < 1$. Therefore, the probability that i and j are not in the same cell-aggregate in B choices for sets of cell-aggregates is bounded above by $(1-c_4)^B = e^{B\log(1-c_4)}$. If we choose B to be $\log N$, this behaves as $N^{\log(1-c_4)}$ which goes to zero as N goes to infinity. (It is enough to choose B to be a function that goes to infinity for large N.) Therefore, with probability going to 1, each pair of adjacent cells will be chosen to be in the same cell aggregate at least once in B sets of h blocks each.

Consider a block of h time slots in which a particular cell-aggregate is active. Assume that it consists of $c_5 \le 1 + c_1$ cells. Each cell has around k_1 relays that wish to transmit and k_1 relays that wish to receive in a particular layer. Thus, we expect there to be no more than c_5k_1 transmissions that need to take place while that cell-aggregate is active. We denote the actual number of transmissions by $k \le c_5k_1$. In addition, the cell-aggregate lies entirely in a circle of diameter r, therefore all the connection strengths within it are drawn i.i.d. from the distribution $f(\gamma)$. Let $n \ge c_5n_1$ be the total number of nodes in the aggregate.

In this subnetwork of n nodes with i.i.d. connections we seek a schedule of k non-colliding paths from the set of transmitting relays to the set of receiving relays. But this is exactly the problem that is addressed in [2].

D. Good edges and vertex-disjoint paths

We reproduce the solution presented there. The channels that are stronger than a chosen parameter β are called *good.* All communications take place over good channels. Since channels are drawn i.i.d. from $f(\gamma)$, for every channel, there is a probability $p = P(\gamma \ge \beta)$ of its being good. We now construct a graph on n vertices where each vertex represents a node of the network. An edge is drawn between two vertices if the channel between the corresponding nodes is good. Thus, we obtain a graph on n vertices where edges are drawn i.i.d. from a Bernoulli distribution of parameter p.

Such a graph fits a standard random graph model called $\mathcal{G}(n,p)$ [5]. This model is well-studied and we appeal to an existing result in the literature to help us with our scheduling. We seek k non-colliding paths that go from the set of t-th layer relay nodes to the respective $(t+1)$-th layer relay nodes. In [1], an identical problem is studied, but the condition on the paths is stricter still – no two paths can share a vertex. In other words, the paths must be vertex-disjoint. We state here the result of [1] as it applies to our problem.

Lemma 4: Suppose that $G = G(n,p)$ and $p \ge \frac{\log n + \omega_n}{n}$, where $\omega_n \to \infty$. Then there exists a constant $\alpha > 0$ such that, with probability approaching 1, there are vertex-disjoint paths connecting x_i to y_i for any set of disjoint, randomly chosen node pairs

$$F = \{(x_i, y_i) | x_i, y_i \in \{1,\ldots,n\}, i = 1,\ldots,k\}$$

provided $k = |F|$ is not greater than $\alpha n\frac{\log np}{\log n}$.
The x_is of the result above are the transmitting relays (from the t-th layer) and the y_is are the corresponding receiving relays (from the $(t+1)$-layer). From Section III-B, we know that these are all distinct nodes. We have $k \le c_5k_1 \le c_5 \cdot 2K\sin^2\frac{r}{12R}$ and $n \ge c_5n_1 \ge c_5 \cdot \frac{1}{2}N\sin^2\frac{r}{24R}$ and need to impose the condition $k \le \alpha n\frac{\log np}{\log n}$. Under the condition on p mentioned in the statement of the lemma, $\alpha n\frac{\log np}{\log n}$ is an increasing function for sufficiently large n. Therefore it is sufficient to impose the equation on the top of the following page, which simplifies to

$$K \le \alpha N \cdot \frac{\log(c_5 \cdot \frac{1}{2}N\sin^2\frac{r}{24R}p)}{\log(c_5 \cdot \frac{1}{2}N\sin^2\frac{r}{24R})}\frac{1}{16\cos^2\frac{r}{24R}}.$$

Finally, the theorem above is an asymptotic result and we need $n \to \infty$ before we can apply it. It is enough to impose the condition $n_1 = \frac{1}{2}N\sin^2\frac{r}{24R} \to \infty$. For r and R being functions of N, this means that $\frac{r}{R}\sqrt{N} \to \infty$ or $\frac{r}{R}$ cannot decrease faster than $1/\sqrt{N}$.

Recall that for every block of h time slots, we have certain active cell-aggregates. Each time a cell-aggregate is active, we can appeal to the above theorem to get a satisfactory subschedule. Additionally, following the treatment in [2], it is possible to show that the lengths of the vertex-disjoint paths grow no

$$c_5 \cdot 2K \sin^2 \frac{r}{12R} \leq \alpha \left(c_5 \cdot \frac{1}{2} N \sin^2 \frac{r}{24R} \right) \frac{\log(c_5 \cdot \frac{1}{2} N \sin^2 \frac{r}{24R} p)}{\log c_5 \cdot \frac{1}{2} N \sin^2 \frac{r}{24R})}$$

faster than $\frac{\log n}{\alpha \log np}$. Except for the case when p is a constant, this is an increasing function of n. When p is a constant, this is a decreasing function of n. Therefore, the time slots required, h, can be bounded above by $h \leq \frac{\log n}{\alpha \log np} \leq \frac{\log c_5 n_i}{\alpha \log c_5 n_i p}$ where n_i can be n_1 or n_2 from Lemma 2.

Putting the results of this section together, we have the following result.

Theorem 2: All K communications can be scheduled in $H = hMB \leq \frac{\log n}{\alpha \log np} \cdot c_2 \frac{R}{r} \cdot \log N$ time slots using non-colliding paths of length $hM \leq \frac{\log n}{\alpha \log np} \cdot c_2 \frac{R}{r}$ provided the following conditions hold.

1) $K \leq \frac{N}{32 \cos^2 \frac{r}{24R}}$.
2) $K \leq \alpha N \frac{\log(c_5 \cdot \frac{1}{2} N \sin^2 \frac{r}{24R} p)}{\log(c_5 \cdot \frac{1}{2} N \sin^2 \frac{r}{24R})} \frac{1}{16 \cos^2 \frac{r}{24R}}$
3) $\frac{r}{R}\sqrt{N} \to \infty$

Here, $\frac{\log n + \omega_n}{n} \leq p \leq 1$ is a probability, n is bounded as $c_5 \cdot \frac{1}{2} N \sin^2 \frac{r}{24R} \leq n \leq c_5 \cdot 2N \sin^2 \frac{r}{12R}$, α and c_2 are constants, and ω_n can be any function that goes to infinity.

Proof: The first condition is as obtained in Lemma 3. Recall that a message has to traverse M cells or layers as indicated by the superschedule. In order to go from one cell to the next, those two cells need to be part of the same cell-aggregate and it may take as many as B choices of cell aggregates before this happens. Once the two cells are part of the same cell-aggregate, it takes at most h hops for the message to go from the designated relay in one cell to that in the next cell. Thus the message makes up to hM hops in up to hMB time-slots. The conditions in the theorem are as derived in the earlier sections. ■

Thus, the hybrid model allows us to schedule non-colliding paths using a combination of ideas from the deterministic model of [18] and the random model of [2]. At the same time, the last condition above prevents us from making r so small that it contains at most one or a constant number of nodes. Because of this, the above result and hence the main result are not applicable to the deterministic, purely distance-dependent model of [18]. Note that the network model itself permits the purely geometric models, but our method of obtaining a non-colliding schedule does not apply for those models. The next question to investigate is that of an appropriate SINR threshold, ρ_0, that determines the rate of the transmissions.

IV. Rate of Transmissions and Error Probability

All transmissions take place in the presence of noise and interference. Successful transmissions require a threshold of ρ_0 and occur at a rate of $\log(1 + \rho_0)$ nats per channel use. The SINR threshold ρ_0 has to be carefully set so that it is not too low, but is low enough to ensure that most communications are successful. Let us investigate the SINR at any particular hop. Let us assume that node a is transmitting to node b. The power of the transmission is P. All communications take place on channels that are good, that is, where $\gamma \geq \beta$. Therefore, the signal power is at least $P\beta$. The additive noise power is σ^2. There is interference from all other transmissions that occur in the same time slot. Some of these transmitting nodes lie within a distance r of the receiving node b and others lie further. We now calculate the expected interference.

Consider a node d that is causing interference to b. This location of this interferer follows a uniform distribution on the surface of the sphere. Let the distance between b and d be denoted by the random variable X. We determine the pdf of X through its cumulative density function (cdf) as follows.

$$\begin{aligned} p_X(x) &= \frac{d}{dx} F_X(x) = \frac{d}{dx} P(X < x) \\ &= \frac{d}{dx} \frac{A(x)}{4\pi R^2} = \frac{1}{2R} \sin \frac{x}{R}. \end{aligned}$$

Recall from (1) that when the distance x, drawn according to $p_X(x)$, satisfies $x \leq r$, the channel strength between d and b is drawn from $f(\gamma)$ and has mean μ_γ and when $x > r$, the channel strength is an exponential random variable with mean $\mu_\gamma \frac{g(x)}{g(r)}$. Let us calculate the expected value of the interference, assuming that d transmits with power P. Denote the interference by I in Eq. 6 (top of next page).

In the last two lines we have used the specific distance decay law of $g(x) = \frac{1}{x^m}$. This makes the remaining paper easier to read, but similar results can be obtained with more general laws as well. Note that there are at most K interferers, since there are only K messages. So the expected value of the total interference is at most $\mathrm{E}I_{\text{total}} = K \cdot \mathrm{E}I$. (In fact, it is possible to show that only about $\frac{K}{B} = \frac{K}{\log N}$ interferers are active at any time. Therefore the interference above and the throughput results later are actually pessimistic by a factor of $\log N$. However, for simplicity, we consider that there are at most K interferers at any given time.)

Since the strength of the received signal is at least $P\beta$, we have this bound on the SINR for node b.

$$\rho_b \geq \frac{P\beta}{\sigma^2 + I_{\text{total}}}.$$

The probability that the SINR falls below some threshold ρ_0 is bounded as follows.

$$\begin{aligned} \mathrm{P}(\rho_b \leq \rho_0) &\leq \mathrm{P}\left(\frac{P\beta}{\sigma^2 + I_{\text{total}}} \leq \rho_0 \right) \\ &= \mathrm{P}\left(I_{\text{total}} \geq \frac{P\beta}{\rho_0} - \sigma^2 \right) \\ &\leq \frac{E(I_{\text{total}})}{\frac{P\beta}{\rho_0} - \sigma^2} \leq \frac{PK\mu_\gamma(\sin^2 \frac{r}{2R} + \frac{r^2}{2R^2}\frac{1}{m-2})}{\frac{P\beta}{\rho_0} - \sigma^2} \end{aligned} \quad (7)$$

where the Markov inequality and the expected value of the interference have been used in the last line.

We will set the SINR threshold to

$$\rho_0 = \frac{P\beta}{\sigma^2 + \eta PK\mu_\gamma(\sin^2 \frac{r}{2R} + \frac{r^2}{2R^2}\frac{1}{m-2})} \quad (8)$$

where $\eta \geq 1$ can be suitably chosen to make transmissions error free. This value of ρ_0 is chosen keeping in mind that the

$$\begin{aligned}
\mathrm{E}I &= \mathrm{E}(I|X \le r)\cdot P(X \le r) + \mathrm{E}(I, X > r) \\
&= P\mu_\gamma \cdot \frac{A(r)}{4\pi R^2} + P\int_r^{\pi R} \mathrm{E}(I, X = x)P(X = x)dx \\
&= P\mu_\gamma \cdot \sin^2\frac{r}{2R} + P\int_r^{\pi R} \mu_\gamma \frac{g(x)}{g(r)}\cdot\frac{1}{2R}\sin\frac{x}{R}dx \\
&\le P\mu_\gamma \cdot \sin^2\frac{r}{2R} + P\int_r^{\infty} \mu_\gamma \frac{g(x)}{g(r)}\cdot\frac{1}{2R}\frac{x}{R}dx \quad \text{since } \sin x \le x \text{ for } x \ge 0 \\
&= P\mu_\gamma \cdot \sin^2\frac{r}{2R} + P\int_r^{\infty} \mu_\gamma \frac{r^m}{x^m}\cdot\frac{1}{2R}\frac{x}{R}dx \\
&= P\mu_\gamma \cdot \sin^2\frac{r}{2R} + P\mu_\gamma\cdot\frac{r^2}{2R^2}\frac{1}{m-2}
\end{aligned} \tag{6}$$

interference terms are expected to behave like their expected values for large networks. We use η to keep the threshold conservative. Substituting for ρ_0 in (7), we have for the probability of error, $\mathrm{P}(\rho_b \le \rho_0) \le \frac{1}{\eta}$.

Finally, we know from Theorem 2 in Section III that every message makes $hM = \frac{\log n}{\alpha \log np}\cdot c_2\frac{R}{r}$ hops. At each hop, the probability that the SINR falls below the threshold ρ_0 is as calculated above. With a simple union bound, we find the probability that a message fails to reach its destination. Denote this by ϵ. The destination fails to receive the intended message if the SINR falls below ρ_0 at any of the hM hops. Denote by E_t the event that the t-th hop does not have an SINR greater than ρ_0. Note that the events $E_1, \ldots, E_{hM}$ are identical. Therefore we have Eq. 9 (next page), where the inequality comes from the union bound. Note that the value of ρ_0 as given in (8) has been used. Typically, we will force ϵ to go to zero by making the upperbound obtained above go to zero.

V. Derivation of the Main Result

We now have all the pieces we need to obtain the final result. Section III tells us the conditions for the existence of a non-colliding schedule, obtained by means of superschedules and subschedules and Section IV tells us the conditions for successful communication under this schedule. We thus have the following result.

Theorem 3: Consider a network of N nodes that are uniformly and randomly distributed over the surface of a sphere of radius R. For two nodes that are within a distance r of each other, the channel strength between them is drawn i.i.d. from a pdf $f(\gamma)$ with mean μ_γ. For two nodes that are a distance $x > r$ apart, it is drawn from an exponential distribution with a mean of $\mu_\gamma \frac{r^m}{x^m}$, where $x > r$ is the distance between them. Each node transmits with power P. For this network, a throughput of

$$T = (1-\epsilon)\cdot K \cdot \frac{\log\left(1 + \frac{P\beta}{\sigma^2 + \eta P K \mu_\gamma\left(\sin^2\frac{r}{2R} + \frac{r^2}{2R^2}\frac{1}{m-2}\right)}\right)}{\frac{\log n}{\alpha\log np}\cdot c_2\frac{R}{r}\cdot\log N} \tag{10}$$

is achievable. Here, n is bounded by $c_5 \cdot \frac{1}{2}N\sin^2\frac{r}{24R} \le n \le c_5 \cdot 2N\sin^2\frac{r}{12R}$, α, c_2 and c_5 are known constants and β, K and $\eta \ge 1$ are chosen such that the following conditions are satisfied:

1) $K \le \frac{N}{32\cos^2\frac{r}{24R}}$.
2) $p = \mathrm{P}(\gamma \ge \beta) = \frac{\log n + \omega_n}{n}$, where $\omega_n \to \infty$.
3) $K \le \alpha N \cdot \frac{\log(c_5\cdot\frac{1}{2}N\sin^2\frac{r}{24R}p)}{\log(c_5\cdot\frac{1}{2}N\sin^2\frac{r}{24R})}\frac{1}{16\cos^2\frac{r}{24R}}$.
4) $\epsilon \le \frac{\log n}{\alpha\log np}\cdot\frac{R}{r}\cdot\frac{1}{\eta} \to 0$.
5) $\frac{r}{R}\sqrt{N} \to \infty$.

Proof: The throughput expression is obtained from (3) by substituting for H and ρ_0 from Theorem 2 and (8) respectively. The conditions of Theorem 2 are reproduced here as conditions 1, 3 and 5. The condition on p also comes from Theorem 2 and condition 4 comes from forcing the upperbound given in (9) to go to zero. Thus, all the conditions that are necessary for scheduling and for ensuring that the probability of error of the communications goes to zero are summarised above. This gives us the achievable throughput mentioned in the theorem. ■

Note that the theorem allows r and R to be functions of N so long as condition 5 is satisfied. Another observation regarding r and R is that they always occur as a ratio, $\frac{r}{R}$, and never independently.

A. Simplifying the expression for the Throughput

Note that Theorem 3 is general and can be applied to a network with arbitrary $f(\gamma)$. Even the distance decay law that applies for distances greater than r can be arbitrary, but we have used the specific case of the $\frac{1}{x^m}$ law in our derivation of the expected value of the interference. While determining the exact achievable throughput requires a precise evaluation of the expression in (10), for the purposes of scaling law behaviour, it is often enough to consider a simpler expression. This is obtained next through a series of simplifying assumptions that often hold.

Consider the throughput term as given by (10) which arises from $T = (1-\epsilon)\frac{K\log(1+\rho_0)}{H}$. In most examples, the term $(1-\epsilon)$ can be ignored since we force $\epsilon \to 0$. Also, in the ρ_0 expression given in (8), the interference term, as given by $\eta P K \mu_\gamma\left(\sin^2\frac{r}{2R} + \frac{r^2}{2R^2}\frac{1}{m-2}\right)$, typically goes to infinity, therefore the noise term, as given by σ^2, can be ignored and ρ_0 itself goes to zero. (Note that in order to ignore the noise term,

$$\epsilon = \mathrm{P}(\bigcup_{t=1}^{hM} E_t) \leq \sum_{t=1}^{hM} \mathrm{P}(E_t) = hM \cdot \mathrm{P}(E_1) \leq hM\frac{1}{\eta} \leq \frac{\log n}{\alpha \log np} c_2 \frac{R}{r}\frac{1}{\eta} \tag{9}$$

it is enough for the interference term to asymptotically be nonzero.) Therefore, the $\log(1+\rho_0)$ term in the throughput can be approximated by ρ_0. Furthermore, we can use $\sin^2 \frac{r}{2R} \leq \frac{r^2}{4R^2}$ in the denominator of ρ_0 (8).

Then, to enforce condition 4 of the theorem, assume that $\frac{\log n}{\alpha \log np} \cdot \frac{R}{r} \cdot \frac{1}{\eta} = \frac{1}{\theta_N}$ where θ_N is some function of N that goes to infinity. This gives an expression for η. Using it and the above simplifications for $\log(1+\rho_0)$, and ignoring some constants, we get the following expression for the throughput:

$$T = \frac{\beta(m-2)\log^2 np}{\log^2 n \cdot \log N \cdot \mu_\gamma \cdot \theta_N}.$$

With the condition on p, specified in condition 2 of Theorem 3, and with an appropriate choice of θ_N, the scaling law for T can be further simplified to

$$T = \frac{\beta(m-2)}{\log^3 N \cdot \mu_\gamma}.$$

Since we have assumed a simple decay law of the form $\frac{1}{x^m}$, it is not surprising that m is the only parameter relating to it that appears in the throughput expression. As for the dependence of the throughput on $f(\gamma)$, we note that β and μ_γ are the main parameters that affect throughput. Thus the throughput is strongly dependent on the tail of the distribution as indicated by β. Note that this in turn is related to the notion of good channels over which all the hops were scheduled. Now μ_γ may be a function of n or N since $f(\gamma)$ may depend on N or n and β typically depends on n through condition 2 on p. Since n asymptotically behaves like $N\frac{r^2}{R^2}$, the throughput depends on r, R and N even though these do not appear in the throughput expression directly. Also, the conditions of Theorem 3 still need to hold and these involve r, R, K and N. Furthermore, in order for the simplifying assumptions made above to be valid, additional conditions, such as those required to have $\rho_0 \to 0$, will need to be met. These have to be looked at on a case by case basis. We formalise the simplified throughput expression in the following corollary.

Corollary 1: The asymptotic achievable throughput of the network given in Theorem 3 can be simplified to

$$T = \frac{\beta(m-2)}{\log^3 N \cdot \mu_\gamma}.$$

provided the following conditions are satisfied:

1) Conditions 1, 2, 3, 4 and 5 of Theorem 3.
2) $\rho_0 = \frac{P\beta}{\sigma^2 + \eta PK\mu_\gamma(\sin^2 \frac{r}{2R} + \frac{r^2}{2R^2}\frac{1}{m-2})} \to 0.$
3) $\eta PK\mu_\gamma(\sin^2 \frac{r}{2R} + \frac{r^2}{2R^2}\frac{1}{m-2}) \nrightarrow 0.$

B. Examples

In this section, we briefly state some examples of networks and the throughput obtained for them.

1) Consider $f(\gamma) = \frac{t-1}{(1+\gamma)^t}$ with $t > 2$ as the distribution from which the channel strengths are drawn i.i.d. for nodes within a distance r from each other. We need $t > 2$ for μ_γ to be finite. We will assume that the other connections are exponential with the mean following a distance decay law of $g(x) = 1/x^m$ for $m > 2$. Choosing $p = \frac{2\log n}{n}$, we get a β that behaves like $(n/2\log n)^{\frac{1}{t-1}} - 1$. It is possible to ensure that the simplifying assumptions of Section V-A hold and a throughput that scales as $\frac{\left(N\frac{r^2}{R^2}\right)^{\frac{1}{t-1}}}{\log^{3+\frac{1}{t-1}} N}$ is achievable. For $\frac{r}{R}$ being constant, the throughput is almost linear in N for t just greater than 2, but for $t > 3$, it falls below $\sqrt{N}$. If $\frac{r}{R}$ varies as $N^{-\nu}$ (we need $\nu < \frac{1}{2}$ to satisfy condition 5 of Theorem 3), the throughput behaves as $\frac{N^{\frac{1-2\nu}{t-1}}}{\log^{3+\frac{1}{t-1}} N}$ which scales better than $\sqrt{N}$ when $t + 4\nu < 3$.
2) Another interesting distribution is the shadow fading distribution. In this the connections within distance r satisfy the Bernoulli distribution $f(\gamma) = (1-p)\cdot\Delta(\gamma) + p\cdot\Delta(\gamma-1)$ where $\Delta(\cdot)$ is the Dirac delta-function. The natural choice for β is 1. Depending on the value of p, the throughput can vary widely. Note that we need p to be at least $\frac{\log n + \omega_n}{n}$ in order for our results to hold. Also, $\mu_\gamma = p$ and therefore the interference term of ρ_0 does not necessarily go to infinity and we need be careful about applying the simplifications of Section V-A. It is possible to show that a throughput of $\frac{n}{\log^4 N} = \frac{N\frac{r^2}{R^2}}{\log^4 N}$ is achievable. Thus the throughput scaling is completely dependent on how n scales. It scales as $\frac{w_N^2}{\log^4 N}$ if $\frac{r}{R}$ decays as $\frac{w_N}{\sqrt{N}}$ for any $w_N \to \infty$. This means that the throughput can go to zero (for example, if $w_N = \log N$) or can be just sublinear in N (for example, if $w_N = \sqrt{N}$) depending on the behaviour of $\frac{r}{R}$.
3) If we apply the Theorem to a network where the i.i.d. connections are drawn from an exponential distribution, $f(\gamma) = e^{-\gamma}$, we see that the transmissions get heavily dominated by interference. We obtain a very low throughput that scales as $\frac{\log n}{\log^3 N}$ and goes to zero.

We see that the throughput obtained in the examples is often directly dependent on n. Therefore, as $\frac{r}{R}$ decreases, the throughput takes a hit. To understand this intuitively, consider that the scheduling is over the good or strong short range i.i.d. links (defined as being stronger than β). As $\frac{r}{R}$ decreases, the overall number of short range links decreases and, therefore, we are forced to lower β to maintain connectivity and obtain a schedule. This decreases the SINR threshold, ρ_0, and the throughput.

For the case when the distance decay law is given by $\frac{e^{-\delta x}}{x^m}$, $\delta > 0$, many of the same results as above are obtained. The reason the throughput is not a strong function of the distance decay law is because of the way we perform scheduling. All of our transmissions take place over the i.i.d. links between nodes that are less than a distance of r apart and therefore the distance decay law affects only the interference term and does not produce a first order effect in the throughput scaling law.

VI. Conclusion and Future Directions

We have proposed a two-scale network model in which local connections are drawn at random and global connections depend on a distance-based decay law. We have analysed the throughput for this network and found that depending on the chosen parameters the throughput can vary widely.

In the case when r is so large that the entire network falls within the range of i.i.d. connections, we obtain the model of [2]. It can be shown that the achievable throughput result of [2] can also be obtained in this case. A model similar to that of [18] is possible if r is so small that no two nodes are within a range r of each other. As explained in Section III-D, our strategy of scheduling cannot be applied in the case when r (or $\frac{r}{R}$) becomes small enough to obtain the model of [18]. This is because our scheduling scheme relies on there being infinitely many nodes having i.i.d. connections, which is precisely what is not allowed in the the purely distance-dependent model. Investigating scheduling algorithms that work in both types of network models is an interesting direction for future work.

Another interesting question is that of upperbounds. In theory, it is possible to obtain min-cut upperbounds for this network. If we consider an arbitrary partition of the network with the K sources on one side and the K destinations on the other, an upperbound on the cut capacity is given by the MIMO capacity of this system. This MIMO capacity is well understood in the case of all links being Gaussian [20] and also in the case of all links being distance-dependent [14]. However, for the two-scale network, we will have a combination of i.i.d. and distance-dependent links. The authors expect that the capacity of such a MIMO system cannot scale faster than N, but determining what that capacity is and its exact dependence on $f(\gamma)$ and the distance decay law are challenging questions.

Finally we propose two more models, as anticipated in Section I-A. In the two-scale network, the model for the channel strength changes abruptly when the distance between nodes exceeds r. More generally, we can consider models where this transition takes place more smoothly. If $p_x(\gamma)$ denotes the distribution from which channel strengths between two nodes that are separated by a distance of x, are drawn, a multiscale model in which this transition takes place between distances r_1 and r_2 would be described as in the equation at the top of the following page.

Another model, which we might call the mixture-model, where the channel strength always has some probability of being purely random or being distance-dependent would be described as follows:

$$p_x(\gamma) = \frac{R-x}{R} f(\gamma) + \frac{x}{R}\frac{1}{x^m}\exp(-\gamma x^m)$$

Here, the probability that the channel strength is i.i.d. decreases with distance as $\frac{R-x}{R}$. The analysis of these models presents some interesting challenges. For instance, the scheduling ideas used in this paper can be repeated for the first model above, by simply using r_1 in place of r. For the latter model, some new ideas for finding a non-colliding schedule may be called for. For both models, we expect the SINR analysis to be similar to that in this paper. The two-scale model and these models represent some avenues for future work.

References

[1] A. Z. Broder, A. M Frieze, S. Suen, E. Upfal, "An Efficient Algorithm for the Vertex-Disjoint Paths Problem in Random Graphs," *Proc 7th Symp. Discrete Algorithms*, Atlanta, 1996, pp 261– 268.
[2] R. Gowaikar, B. Hochwald, B. Hassibi, "Communication over a Wireless Network with Random Connections," *IEEE Trans. Inform. Theory*, vol. 52, pp. 2857–2871 , Jul. 2006.
[3] A. Özgür, O. Léveque and D. Tse, "Hierarchical Cooperation Achieves Optimal Capacity Scaling in *Ad Hoc* Networks," *IEEE Trans. Inform. Theory*, vol. 53, pp. 3549–3572, Oct. 2007.
[4] F. Baccelli, M. Klein, M. Lebourges and S. Zuyev, "Stochastic geometry and architecture of communication networks," *J. Telecommunication Systems*, vol. 7, pp. 209–227, 1997.
[5] B. Bollobás, *Random Graphs*, 2nd ed., Cambridge: University Press, 2001.
[6] H. Bertoni, *Radio Propagation for Modern Wireless Systems*, Prentice-Hall PTR, 2000.
[7] T. Rappaport, *Wireless Communications: Principles and Practice*, Prentice-Hall, 2002.
[8] Digital Mobile Radio Towards Future Generation Systems: COST 231 Final Report, *http://www.lx.it.pt/cost231/final_report.htm*
[9] O. Dousse and P. Thiran, "Connectivity versus capacity in dense ad hoc networks," *Proc. 23rd INFOCOM*, Hong Kong, Mar. 2004.
[10] G. J. Foschini, "Layered space-time architecture for wireless communication in a fading environment when using multi-element antennas," *Bell Labs. Tech. J.*, vol. 1, no. 2, pp. 41–59, 1996.
[11] M. Gastpar and M. Vetterli, "On the capacity of wireless networks: the relay case," *Proc. 21st INFOCOM*, New York, Jun. 2002, pp. 1577–1586.
[12] M. Grossglauser and D. Tse, "Mobility increases the capacity of ad-hoc wireless networks," *IEEE/ACM Trans. Networking*, vol. 10, pp. 477–486, Aug. 2002.
[13] R. Hekmat and P. van Mieghem, "Study of connectivity in wireless ad-hoc networks with an improved radio model," *Proc. 2nd Workshop on Model. and Optim. in Mobile, Ad Hoc and Wireless Networks*, Cambridge, UK, Mar. 2004.
[14] O. Léveque and E. Telatar, "Information theoretic upper bounds on the capacity of large extended ad hoc wireless networks," *IEEE Trans. Inform. Theory* vol 51, pp. 858–865, Mar. 2005.
[15] M. Franceschetti, O. Dousse, D. Tse and P. Thiran, "On the throughput capacity of random wireless networks," *IEEE Trans. Info. Theory*, vol 52, pp. 2756–2761, Jun. 2006.
[16] S. Toumpis and A. Goldsmith, "Capacity bounds for large wireless networks under fading and node mobility," *Proc. 41st Allerton Conf. on Comm. Cont. and Comp.*, Monticello, Oct. 2003, pp. 1369–1378.
[17] S. Weber, X. Yang, J. Andrews and G. de Veciana, "Transmission capacity of wireless ad hoc networks with outage constraints," submitted to *IEEE Trans. Inform. Theory*, vol. 51, pp. 4091–4102, Dec. 2005.
[18] P. Gupta and P. R. Kumar "The capacity of wireless networks," *IEEE Trans. Inform. Theory*, vol. 46, pp. 388–404, Mar. 2000.
[19] P. Gupta and P. R. Kumar, "Towards an information theory of large networks: an achievable rate region," *IEEE Trans. Info. Theory*, vol. 49, pp. 1877–1894, Aug. 2003.
[20] I. E. Telatar, "Capacity of multi-antenna Gaussian channels," *Eur. Trans. Telecom.*, vol. 10, pp. 585–595, Nov. 1999.
[21] O. Tonguz and G. Ferrari, "Connectivity and transport capacity in ad hoc wireless networks," *Proc. 32nd IEEE Comm. Theory Workshop*, Mesa, AZ, Apr. 2003.
[22] L.-L. Xie and P. R. Kumar, "A network information theory for wireless communication: Scaling laws and optimal operation," *IEEE Trans. Info. Theory*, vol. 50, pp. 748–767, May 2004.
[23] R. Gowaikar, B. Hochwald, B. Hassibi, "An Achievability Result for Random Networks," *Proc. IEEE ISIT 2005*, Adelaide, Australia, pp 946-950.

$$p_x(\gamma) = \begin{cases} f(\gamma) & \text{if } x \le r_1 \\ \frac{r_2-x}{r_2-r_1} f(\gamma) + \frac{x-r_1}{r_2-r_1} \frac{\mu_\gamma r_2^m}{x^m} \exp(-\gamma \frac{x^m}{\mu_\gamma r_2^m}) & \text{if } r_1 < x \le r_2 \\ \frac{\mu_\gamma r_2^m}{x^m} \exp(-\gamma \frac{x^m}{\mu_\gamma r_2^m}) & \text{if } x > r_2 \end{cases} .$$

Radhika Gowaikar (S'03) received the B.Tech. degree from the Indian Institute of Technology, Bombay, in 2001 and the M.S. and Ph.D. degrees from the California Institute of Technology, Pasadena, CA, in 2002 and 2006 respectively, all in electrical engineering. She has been working at Qualcomm Inc since 2006.

Her research interests include sensor and ad hoc networks, network coding for wireless networks, and decoding in multiple antenna systems.

Babak Hassibi was born in Tehran, Iran, in 1967. He received the B.S. degree from the University of Tehran in 1989, and the M.S. and Ph.D. degrees from Stanford University in 1993 and 1996, respectively, all in electrical engineering.

He has been with the California Institute of Technology since January 2001, where he is currently Professor and Executive Officer of Elecrical Engineering. From October 1996 to October 1998 he was a research associate at the Information Systems Laboratory, Stanford University, and from November 1998 to December 2000 he was a Member of the Technical Staff in the Mathematical Sciences Research Center at Bell Laboratories, Murray Hill, NJ. He has also held short-tem appointments at Ricoh California Research Center, the Indian Institute of Science, and Linkoping University, Sweden. His research interests include wireless communications and networks, robust estimation and control, adaptive signal processing and linear algebra. He is the coauthor of the books (both with A.H. Sayed and T. Kailath) *Indefinite Quadratic Estimation and Control: A Unified Approach to H^2 and H^∞ Theories* (New York: SIAM, 1999) and *Linear Estimation* (Englewood Cliffs, NJ: Prentice Hall, 2000). He is a recipient of an Alborz Foundation Fellowship, the 1999 O. Hugo Schuck best paper award of the American Automatic Control Council (with H. Hindi and S.P. Boyd), the 2002 National Science Foundation Career Award, the 2002 Okawa Foundation Research Grant for Information and Telecommunications, the 2003 David and Lucille Packard Fellowship for Science and Engineering and the 2003 Presidential Early Career Award for Scientists and Engineers (PECASE), and was a participant in the 2004 National Academy of Engineering "Frontiers in Engineering" program.

He has been a Guest Editor for the IEEE Transactions on Information Theory special issue on "space-time transmission, reception, coding and signal processing" was an Associate Editor for Communications of the IEEE Transactions on Information Theory during 2004-2006, and is currently an Editor for the Journal "Foundations and Trends in Information and Communication".