

signals are transmitted in such systems, and with results which point clearly and unequivocally to their transmission in an independent medium or coordinate system. If light signals are sent out simultaneously in the "fore" and "aft" directions from a source moving in a circular path, and the two signals are brought back to the source by a series of reflections, they do not arrive simultaneously; the source has moved forward to meet one signal and has moved away from the other. This is the situation in the Michelson-Gale and Sagnac experiments, which yield positive first-order results exactly in conformity with the idea that the light signals travel in a fixed ether. The contractions of length and clock frequencies which account for nul effects in uniform motion of translation, being of the second order, do not materially alter the rotational effects. The optical phenomena in both uniformly and rotationally moving systems are completely explainable by a fixed ether and the Fitzgerald-Lorentz-Larmor contractions.<sup>3</sup>

This survey of the background of the query "Is there an ether?" shows that the grounds for "abolishing" the ether were mistaken, and consequently Dirac's contribution would more properly have been entitled "Properties of the ether suggested by recent speculations."

<sup>1</sup> P. A. M. Dirac, *Nature* 168, 906 (1951).

<sup>2</sup> The above argument will be found in greater detail, with full references, in Herbert E. Ives, *Proc. Am. Phil. Soc.* 95, No. 2, 125 (April, 1951).

<sup>3</sup> See Herbert E. Ives, *Proc. Roy. Dublin Soc.* 26, 9 (May 1, 1952).

### The $X_n$ and $Y_n$ Functions of Hopkins, Occurring in the Theory of Diffraction

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IN a paper which concerns the disturbance near focus of waves of radially nonuniform amplitude, and which forms an extension of Lommel's classical analysis,<sup>1</sup> Hopkins<sup>2</sup> was led to the introduction of two new functions,

$$\left. \begin{aligned} X_n(y, z) &= y \frac{\partial U_n}{\partial y} = \sum_{s=0}^{\infty} (-1)^s (n+2s) \left(\frac{y}{z}\right)^{n+2s} J_{n+2s}(z), \\ Y_n(y, z) &= -y \frac{\partial V_n}{\partial y} = \sum_{s=0}^{\infty} (-1)^s (n+2s) \left(\frac{z}{y}\right)^{n+2s} J_{n+2s}(z). \end{aligned} \right\} \quad (1)$$

Here  $U_n$  and  $V_n$  denote the Lommel functions and  $J_n$  is a Bessel function of the first kind. Recently Boivin<sup>3</sup> derived interesting "multiplication" theorems which express  $U_n(\alpha^2 y, \alpha z)$ ,  $V_n(\alpha^2 y, \alpha z)$ ,  $X_n(\alpha^2 y, \alpha z)$ , and  $Y_n(\alpha^2 y, \alpha z)$  in series involving these functions with successive orders but with arguments  $(y, z)$ . These formulas are of importance when one studies diffraction by concentric arrays of ring-shaped apertures.

I pointed out elsewhere<sup>4</sup> that the  $Y_n$  functions can be expressed in terms of the Lommel  $V_n$  functions by means of the simple relation

$$\left. \begin{aligned} Y_n(y, z) &= \frac{1}{2} \left[ \frac{z^2}{y} V_{n-1}(y, z) + y V_{n+1}(y, z) \right], \\ \text{and similarly} \\ X_n(y, z) &= \frac{1}{2} \left[ \frac{z^2}{y} U_{n+1}(y, z) + y U_{n-1}(y, z) \right]. \end{aligned} \right\} \quad (2)$$

It is therefore possible to express the new formulas of Boivin for  $X_n(\alpha^2 y, \alpha z)$  and  $Y_n(\alpha^2 y, \alpha z)$  in terms of the ordinary Lommel functions with arguments  $(y, z)$ , giving\*

$$\left. \begin{aligned} X_n(\alpha^2 y, \alpha z) &= \sum_{s=0}^{\infty} \frac{(\alpha^2 - 1)^s y^s}{2^{s+1} s!} \left[ \frac{z^2}{y} U_{n-s+1}(y, z) \right. \\ &\quad \left. + 2s U_{n-s}(y, z) + y U_{n-s-1}(y, z) \right], \\ Y_n(\alpha^2 y, \alpha z) &= \sum_{s=0}^{\infty} (-1)^s \frac{(\alpha^2 - 1)^s y^s}{2^{s+1} s!} \left[ \frac{z^2}{y} V_{n+s-1}(y, z) \right. \\ &\quad \left. - 2s V_{n+s}(y, z) + y V_{n+s+1}(y, z) \right]. \end{aligned} \right\} \quad (3)$$

Alternatively, one can replace  $y$  by  $\alpha^2 y$  and  $z$  by  $\alpha z$  in (2) and apply Boivin's formulas for  $U_n(\alpha^2 y, \alpha z)$  and  $V_n(\alpha^2 y, \alpha z)$ . One then obtains the simpler relations

$$\left. \begin{aligned} X_n(\alpha^2 y, \alpha z) &= \sum_{s=0}^{\infty} \frac{(\alpha^2 - 1)^s y^s}{2^{s+1} s!} \\ &\quad \times \left[ \frac{z^2}{y} U_{n-s+1}(y, z) + \alpha^2 y U_{n-s-1}(y, z) \right], \\ Y_n(\alpha^2 y, \alpha z) &= \sum_{s=0}^{\infty} (-1)^s \frac{(\alpha^2 - 1)^s y^s}{2^{s+1} s!} \\ &\quad \times \left[ \frac{z^2}{y} V_{n+s+1}(y, z) + \alpha^2 y V_{n+s-1}(y, z) \right]. \end{aligned} \right\} \quad (4)$$

The  $U$  and  $V$  functions of the higher orders which occur in (3) and (4) can be easily calculated with the help of the recurrence relations

$$\left. \begin{aligned} U_n(y, z) + U_{n+2}(y, z) &= \left(\frac{y}{z}\right)^n J_n(z), \\ V_n(y, z) + V_{n+2}(y, z) &= \left(\frac{z}{y}\right)^n J_n(z). \end{aligned} \right\} \quad (5)$$

Equations (3) or (4), together with (5), make it possible to evaluate  $X_n(\alpha^2 y, \alpha z)$  and  $Y_n(\alpha^2 y, \alpha z)$  from tables of the four basic functions  $U_1(y, z)$ ,  $U_2(y, z)$ ,  $V_0(y, z)$ ,  $V_1(y, z)$  of Lommel's original problem. This will considerably simplify calculations on the basis of Boivin's theory of the effect of waves presenting a radial variation of amplitude.

<sup>1</sup> E. Lommel, *Abh. Bayer. Akad. Wiss.* 15, 229 (1885).

<sup>2</sup> H. H. Hopkins, *Proc. Phys. Soc. (London)* B62, 22 (1949).

<sup>3</sup> A. Boivin, *J. Opt. Soc. Am.* 42, 60 (1952).

<sup>4</sup> E. Wolf, *Proc. Roy. Soc. (London)* A204, 535 (1951).

\*An error in sign of the term  $sU_{n-s}(y, z)$  in Boivin's formula for  $X_n(\alpha^2 y, \alpha z)$  is corrected here.

### Determination of Absolute $f$ Values from Relative Intensity Measurements for Spectral Lines with Doppler Contour\*

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THE experimental determination of absolute intensities for isolated spectral lines or for entire vibration-rotation bands involves formidable experimental difficulties. For this reason it is highly desirable to consider the use of techniques which permit the determination of absolute  $f$  values from relative intensity measurements performed by the use of a low resolution spectrograph. It is the purpose of this note to call attention to a useful experimental procedure for spectral lines with Doppler contour and for optical densities which are sufficiently large to assure a nonlinear dependence of intensity on optical density. The method is a generalization of a two-path experiment proposed for emission studies on flames.<sup>1</sup>

For spectral lines with Doppler contour it is well known<sup>2</sup> that the total absorbed intensity of radiation  $A[\nu_{lu}(K)]$ , for the line identified by the index  $K$  and with center at the frequency  $\nu_{lu}$ , is

$$A[\nu_{lu}(K)] \approx R^0(\nu_{lu}) (mc^2/2\pi kT\nu_{lu}^2)^{-1} [P_{\max}(K)X] \times \left\{ \sum_{n=0}^{\infty} [(n+1)!(n+1)!]^{-1} [-P_{\max}(K)X]^n \right\}, \quad (1)$$

if  $R^0(\nu_{lu})$  is the intensity of the incident radiation at  $\nu_{lu}$ ,  $m$  is the mass of the absorber,  $c$  equals the velocity of light,  $k$  is the Boltzmann constant,  $T$  represents the absolute temperature,  $X$  is the optical density (in cm-atmos) of the absorber, and  $P_{\max}$  is the maximum value of the spectral absorption coefficient (in cm<sup>-1</sup>-atmos<sup>-1</sup>). The quantity  $P_{\max}$  is related to the integrated intensity  $S$  (in cm<sup>-2</sup>-atmos<sup>-1</sup>) through the relation

$$P_{\max} = S(mc^2/2\pi kT\nu_{lu}^2)^{\frac{1}{2}};$$

the quantity  $S$  (in  $\text{cm}^{-2}\text{-atmos}^{-1}$ ) is related to the dimensionless  $f$  value through the expression

$$S = 2.3789 \times 10^7 (273.1/T) f.$$

It is apparent from Eq. (1) that the dimensionless ratio  $R = A[\nu_{lu}(K)]/R^0(\nu_{lu})(mc^2/2\pi kT\nu_{lu}^2)^{-1/2}$  is a unique function<sup>2</sup> of  $P_{\max}X$ , i.e.,  $R = \varphi(P_{\max}X)$ . If  $R = R_1$  for  $X = X_1$  and  $R = R_2$  for  $X = X_2$ , then  $R_1/R_2 = \varphi(P_{\max}X_1)/\varphi(P_{\max}X_2) = \varphi^1(P_{\max}X_1, X_1/X_2)$ . The function  $\varphi^1$  can be determined without difficulty for arbitrary values of  $X_1/X_2$ . For  $X_1/X_2 = 2$  the results have been published elsewhere.<sup>1</sup> From the known values of  $X_1/X_2$  and the measured values of  $R_1/R_2$  it is a simple matter to obtain  $P_{\max}X_1$  and hence  $P_{\max}$  or  $S$ . In this manner absolute  $f$  values can be determined from relative intensity measurements for spectral lines with Doppler contour.

For studies in the visible and ultraviolet regions of molecular spectra, and at ordinary temperatures,  $h\nu_{lu} \gg kT$  and<sup>1</sup>

$$\frac{\partial \ln\{P_{\max}(K)X/[g_u(q_{lu})^2]K\}}{\partial(E_u - h\nu_{lu})} = -1/kT, \quad (2)$$

where  $g_u(q_{lu})^2$  represents the relative transition probability for the line with index  $K$ ,  $E_u$  is the energy of the upper state, and  $h$  is Planck's constant. At fixed values of  $T$ , Eq. (2) is useful for checking the consistency of experimentally determined values of  $P_{\max}(K)X$ .

The two-path absorption experiments can be generalized to spectral lines with combined Doppler- and collision-broadening if the line shape is known or is to be measured.

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<sup>1</sup> S. S. Penner, *J. Chem. Phys.* 20, 1341 (1952).

<sup>2</sup> R. Ladenburg, *Z. Physik* 65, 200 (1930).

decided that a real effort should be made during the late fall or early winter of 1952. The three spectroscopically minded sections were particularly eager to hear him, so the schedule was built around Detroit (Monday night, December 1), the Ohio Valley Section at Cincinnati on Tuesday night, and Chicago on Thursday night. At the suggestion of Bernard Boyd, Chairman of the Detroit Section, several spectroscopic organizations in the area were contacted. The newly organized Indiana Spectrographers Society was pleased to be included, and Mr. Scribner went there on Wednesday night from Cincinnati and proceeded thence to Chicago to speak before a joint meeting of our Chicago Section and the Chicago Chapter of the American Association of Spectrographers. The following night he addressed the Milwaukee Chapter of the Spectrographers Association.

This trip provided the speaker with an opportunity to visit several spectroscopic laboratories and discuss matters of mutual interest with many vigorous workers in this rapidly growing field. He visited laboratories in such plants as the Ford Motor Company in Dearborn, the A. O. Smith Corporation in Milwaukee, the U.S. Naval Ordnance Plant in Indianapolis, and the Argonne National Laboratory in Chicago. Local spectroscopists were eager to greet him, to help him with his transportation problems, and to make sure that he was very well fed! Officers of the local sections who contributed largely to working out the arrangements included H. D. Veldhuis of Detroit, Howard Bales of the Ohio Valley Section, and William Strickland of Chicago.

We should be happy that such capable and distinguished scientists as Mr. Scribner are willing to devote the time and energy to arduous speaking tours like this one. We express the hope that he and the National Bureau of Standards, where he is Chief of the Spectroscopy Section, will have been rewarded scientifically by the contacts that he made. It is certain that he has contributed in an important way to our local section program: to the members of the three sections and the others who had the good fortune to hear him speak.

## Local Section News

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### O.S.A. Traveling Lecturer in the Midwest

Lectures on each of five successive nights—that was the heavy schedule assigned to Bourdon F. Scribner during one hectic week early in December. He had agreed to visit our three midwestern sections, and the week was filled by scheduling him to address certain spectrographic groups also. But spectroscopists are made of staunch stuff, and he went through the routine very successfully and without complaint except perhaps at some of the early snow. The title "New Tools in Spectroscopy" is a proper indication of the fact that his talk covered many of the newer and more exciting phases of spectroscopic instrumentation. Photoelectric spectrophotometers of the integrating type are becoming well-established in analytical laboratories, but the rapid-scan instruments promise new applications in both fundamental and applied spectroscopy; Mr. Scribner described both laboratory and commercial models and gave examples of their application. The high frequency excitation of gases, another powerful use of electronic techniques in spectroscopy, provides the advantages of ease of control and freedom from gas clean-up in discharge tubes, and produces spectra with some unusual characteristics. The Echelle grating's commercial mounting and its applications both to fundamental and applied spectroscopy were described.

This particular lecture tour was a fruition of the desires expressed for two or three years by representatives of the Detroit Section for Mr. Scribner to "make the rounds." There have been scheduling difficulties in previous years, and it was

### Contributed Paper Session at the New England Section

The innovation of having a session devoted entirely to short contributed papers was introduced last year by the New England Section, as described in the November, 1952, issue of this Journal [*J. Opt. Soc. Am.* 42, 865-867 (1952)]. This type of program proved to be so popular with the members of the section that two such sessions were scheduled for the activity year 1952-1953. The first one was held on the evening of December 18, 1952, in the physics lecture room at Boston University. There were six 15-minute papers, with a break after the first three for cider, doughnuts, and conversation. Again there was a fine audience response to this opportunity to hear national-meeting-quality papers at a home-town gathering. Abstracts of the papers follow.

1. **An Electronic Method for Automatic Focusing.**\* ROY C. GUNTER, JR., *Boston University Optical Research Laboratory and Clark University.*—The maintenance of best focus throughout the variations of temperature, pressure, and altitude encountered in modern aerial photography has of recent years been receiving increased attention. It is shown in this talk that it is possible to utilize certain information and communication theories to establish a criterion for best focus based on the spatial frequency response of the optical system. These spatial frequencies are scanned with a photosensitive tube, and the ac output is used to operate a servo system such as to adjust the lens-focal plane distance for maximum frequency response within the band width in which there is maximum interest. Preliminary experiments showed that the electronic technique should be capable of control within