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Nomogram for the Evaluation of Blackbody Radiancy and of Peak and Total Intensities for Spectral Lines with Doppler Contour*

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A nomogram has been constructed for the determination of blackbody radiancy and of peak and total intensities for spectral lines with Doppler contour. The basic equations used for the construction of the nomogram and the use of the nomogram are described briefly. A method is outlined for determining absolute values of total intensities for spectral lines with combined Doppler and resonance contour by using the nomogram in conjunction with the "curves of growth."

I. INTRODUCTION

FOR the analysis of spectra obtained from low pressure combustion flames, as well as in certain astrophysical applications, it may be of interest to determine peak and total intensities for spectral lines with Doppler contour. Since these calculations are somewhat laborious, we have constructed a nomogram (see Fig. 1) to facilitate the determination of the desired quantities. The nomogram is useful for temperatures (T) from 1000°K to 20 000°K, molecular weights (M) from 2 to 80 grams/mole, wavelengths (λ) from 0.10 to 0.80 micron, and values of SX/c from 10^{-4} to 10 cm^{-1}. Here S/c represents the integrated intensity of the line under study in cm^{-2}-$atmos^{-1}$ and is related to the f value at the temperature T through the expression

$$S/c=2.3789\times10^{7}\times(273.1/T)f\ \mathrm{cm}^{-2}\ \mathrm{atmos}^{-1}.$$

The optical density X is expressed in cm-atmos.

The blackbody radiancy R_0 is determined by the

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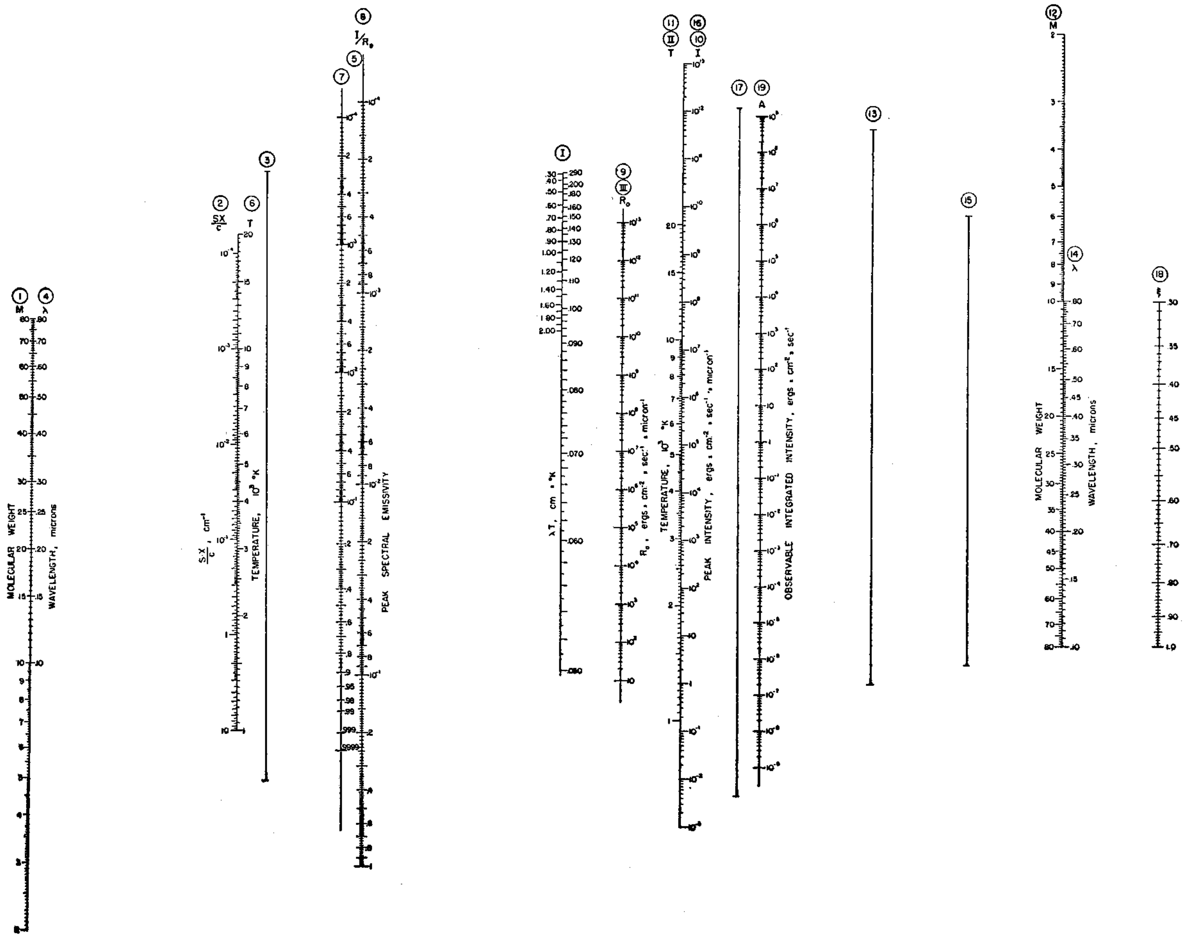

FIG. 1. Nomogram for the determination of blackbody radiancy, and peak and total emitted intensities of spectral lines with Doppler contour.

Planck distribution law according to the expression

$$R_0 = (2\pi hc^2/\lambda^5)[\exp(hc/k\lambda T)-1]^{-1}, \qquad (1)$$

where h is Planck's constant, c equals the velocity of light, λ is the wavelength, and k represents the Boltzmann constant. The nomogram is easier to use than available compilations[1] for the determination of R_0 as a function of λ and T.

The peak intensity of radiation $I \equiv I_{\max}$ emitted by a spectral line with Doppler contour is given by the relation

$$I \equiv I_{\max} = R_0\{1-\exp-[(SX/c)\lambda(mc^2/2\pi kT)^{\frac{1}{2}}]\}, \qquad (2)$$

where m equals the mass per molecule.

The total intensity A of radiation emitted from a spectral line with Doppler contour is obtained conveniently from the expression

$$A = (\lambda/\zeta)I_{\max}(2\pi kT/mc^2)^{\frac{1}{2}}, \qquad (3)$$

where the self-absorption parameter ζ see Fig. 2 is defined by the relation[2]

$$\zeta = \{(P_{\max}X)\sum_{n=0}^{\infty}[(n+1)^{\frac{1}{2}}(n+1)!]^{-1}(-P_{\max}X)^n\}^{-1} \times[1-\exp(-P_{\max}X)] \qquad (4)$$

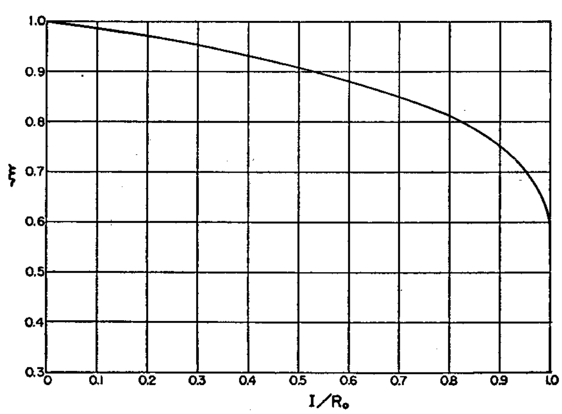

FIG. 2. The self-absorption parameter ξ as a function of I/R_0.

[1] See, for example, "Planck Radiation Functions and Electronic Functions" prepared by W.P.A. for City of New York, under sponsorship of the National Bureau of Standards, 1941.

[2] R. Ladenburg, Z. Physik 65, 200 (1930). The quantity ζ is plotted as a function of I/R_0 in Fig. 2.

and

$$P_{max}=S(m/2\pi kT\lambda^{-2})^{\frac{1}{2}}. \quad (5)$$

II. USE OF THE NOMOGRAM

The nomogram shown in Fig. 1 is useful for the determination of R_0, $I=I_{max}$, and A. The scales are numbered in order of their use either with Roman or with Arabic numerals. Scales 3, 5, 13, 15, and 17 represent intermediate steps and are not graduated. Without using excessive care in interpolation on a small nomogram ($8\frac{1}{2}$ in.$\times$11 in.), it should be possible to obtain estimates which are accurate to ten percent or better. Copies of large nomograms can be obtained on request.

A. Determination of Blackbody Radiancy

The blackbody radiancy R_0 may be obtained as a function of T and λ by drawing a straight line between scale I (λT in cm$\times$°K) and scale II (T in °K) to obtain R_0(in ergs$\times$cm$^{-2}\times$sec$^{-1}\times$micron^{-1}) on scale III.

B. Determination of Peak Intensities ($I=I_{max}$) Emitted by Spectral Lines with Doppler Contour

A straight edge placed to intersect scales 1 and 2 at the values of M and SX/c, respectively, intersects line 3 at a point representing $(SX/c)m^{\frac{1}{2}}$. This point and a point on scale 4 at the known value of λ locate a point on line 5, which, with a point on scale 6 at the value of T, determines a point on scale 7 at the value of I/R_0. The value of R_0 is found on scale III as in (A) above. The quantity I/R_0 on scale 8 (transferred from scale 7) and R_0 on scale 9 determine $I=I_{max}$ on scale 10.

C. Determination of Total Intensities Emitted by Spectral Lines with Doppler Contour

The observable integrated intensity A is obtained on scale 19 by starting with scale 11 and following the numbers in succession. The peak intensity $I=I_{max}$ is now considered to be on scale 16.

III. OTHER USES

The nomogram can, of course, be used to obtain, for example, the quantity SX/c if A or I have been determined experimentally and T, λ, and M are known. Similarly, if intensity *ratios* are known empirically, *ratios* of SX/c can be obtained without difficulty. These, in turn, may be used to calculate concentration ratios by utilizing appropriate theoretical expressions for relative numerical values of S/c.

The nomogram is not suitable for the determination of total intensities if the spectral line shape is described by the combined effects of Doppler, natural and collision broadening. For the calculation of absolute intensities in the general case use may be made of the "curves of growth" which are well known from astrophysical studies.[3] The procedure is outlined briefly in the following paragraph.

(1) Add the known values of the natural and collision half-width to obtain the quantity γ(cm^{-1}). Calculate the Doppler half-width $\Delta\omega_D$ and determine the parameter

$$a=\gamma/\Delta\omega_D. \quad (6)$$

(2) Determine the quantity

$$(SX/c)\lambda(mc^2/2\pi kT)^{\frac{1}{2}}=-\ln[1-(I/R_0)] \quad (7)$$

either independently or else by use of the nomogram. Next calculate the quantity $\log[10.6(SX/c)\lambda(mc^2/2\pi kT)^{\frac{1}{2}}]$, which represents the abscissa used in conventional "curves of growth."

(3) The ordinate $A_\omega/2\Delta\omega_D$ of the "curves of growth" is related to our total absorption A in such a way that

$$(A_\omega/2\Delta\omega_D)_a/(A_\omega/2\Delta\omega_D)_{a=0}=A_a/A, \quad (8)$$

where A is the total intensity for lines with pure Doppler contour (i.e., $a=0$) and A_a is the corresponding quantity for any given value of a. Hence, in order to determine absolute values of A_a it is only necessary to obtain from the "curves of growth" the quantity $A_\omega/2\Delta\omega_D$ for $a=0$ and for any given value of a at the known value of the abscissa. By using the nomogram to determine A it is then a simple matter to calculate A_a from Eq. (8).

[3] See, for example, A. Unsöld, *Physik der Sternatmosphären* (Edwards Brothers, Inc., Ann Arbor, 1948), p. 168.