Quantum noise and dynamics in quantum well and quantum wire lasers

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We calculate the relaxation oscillation corner frequency f_r and the linewidth enhancement factor α for both a quantum well and a quantum wire semiconductor laser. A comparison of the results to those of a conventional double heterostructure device indicates that f_r can be enhanced by $2\times$ in the quantum well case and $3\times$ in the quantum wire case while α is reduced in both cases.

The application of semiconductor lasers in optical communication systems requires both broad band modulation and low noise characteristics. Two important parameters which determine, in part, such properties are the relaxation oscillation corner frequency f_r, which sets the useful direct modulation bandwidth,[1] and the linewidth enhancement factor α (or amplitude phase coupling factor), which determines the relation of AM to FM modulation indices[2] as well as the degree to which spectral purity is degraded by the amplitude phase coupling.[3–6] Although these properties have received considerable attention both theoretically and experimentally for conventional devices (i.e., three-dimensional active layer; 3D AL), little effort has been directed towards measurement or calculation of these properties in lasers with one-or two-dimensional active layers (Burt[7] has recently considered α in a quantum well laser. Our result for this case is included here for completeness.) In this letter, we will calculate these parameters for both a quantum well (2D AL) and a quantum wire (1D AL) semiconductor laser.[8] The results indicate that modulation performance can be significantly improved (f_r enhanced) with simultaneous reduction of phase noise and parasitic FM (α reduced) as compared to conventional devices.

The expressions for f_r and α are of the form[1,6]

$$f_r = \frac{1}{2\pi}\left(\frac{P\omega}{2\tau\mu_0}\frac{d\chi_I(n)}{dn}\right)^{1/2}, \tag{1}$$

$$\alpha = \frac{d\chi_R(n)/dn}{d\chi_I(n)/dn}, \tag{2}$$

where P, ω, τ are the photon density, frequency, and passive cavity lifetime of the lasing mode; μ_0 is the nonresonant value of the refractive index; n is the carrier density. $\chi_R(n)$ and $\chi_I(n)$ are the real and imaginary parts of the complex susceptibilities of the active medium. Their derivatives with respect to the carrier density are given, respectively, by[9]

$$\frac{d\chi_I(n)}{dn} = \int dE\frac{dg(n,E)}{dn}\frac{\hbar/T_2}{(E-E_1)^2+(\hbar/T_2)^2} \tag{3}$$

$$\frac{d\chi_R(n)}{dn} = \int dE\frac{dg(n,E)}{dn}\frac{E-E_1}{(E-E_1)^2+(\hbar/T_2)^2}, \tag{4}$$

where E_1 and T_2 are the lasing photon energy and the collisional broadening time due to carrier-carrier and carrier-phonon interaction, $g(n,E)$ is the gain envelope function which is given by

$$g(n,E) = CM\rho_{\rm red}\,[f_c(n,E) - f_v(n,E)], \tag{5}$$

where $\rho_{\rm red}$ is the reduced density of states, $f_c(f_v)$ is the conduction band (valence band) state occupation (Fermi) function which is a function of carrier concentration n through the quasi-Fermi energy, M is the square of the dipole matrix element, and C contains constants which are independent of the active layer geometry and material properties. All geometry dependences in f_r and α enter mainly through the gain envelope dependence of $\chi_R(n)$ and $\chi_I(n)$. In the gain envelope function $g(n,E)$ the geometry dependent factors are the reduced density of states $\rho_{\rm red}$ and the matrix element M. Note that $\rho_{\rm red}$ and M should be calculated for each subband then summed up when the system has subband structures.

In a quantum well structure, having well width L_z and sufficient number of high barrier layers, the reduced density of states with respect to heavy (or light) holes in the $i_{\rm th}$ subband is expressed as follows[10]:

$$\rho_{\rm red}(E) = \frac{m_j^*}{\pi\hbar^2 L_z}H\left[E - \frac{\hbar^2}{2m_j^*}\left(\frac{\pi i}{L_z}\right)^2\right], \tag{6}$$

where m_j^* ($j = l,h$) is the reduced mass with respect to heavy holes (h) or light holes (l) and $H(E)$ is the Heaviside function. In conventional double heterostructures, the value of M can be determined from Kane's model.[11] In the case of GaAs, $M = 1.33m_0E_g$ ($\equiv M_0$), where m_0 is the electron mass and E_g is the band-gap energy. In the three-dimensional bulk crystal the interaction between light and electrons is considered to be approximately isotropic. In a quantum well structure, however, the interaction between light and electrons is nonisotropic, owing to the discreteness of the electronic wave number normal to the active layer. Hence, M depends on the polarization direction of the light. In fact, the dependence of gain on polarization direction has been observed; the gain of the TE mode is observed to be about $2\times$ larger than that for the TM mode.[12] The TE mode M of the optical transitions between electrons and heavy holes in the $i_{\rm th}$ subband, calculated for a quantum well using Kane's model considering only k-conservative transition, is given by[13]

$$M = \frac{3}{4}\left[1 + \frac{1}{E}\frac{\hbar^2}{2m_h^*}\left(\frac{\pi i}{L_z}\right)^2\right]M_0. \tag{7}$$

Form this equation M is about 1.5 M_0 at near equivalent band edge.

Figure 1 shows the differential gain envelope $g'(n,E)$ [i.e., $dg(n,E)/dn$] in a GaAs/$Ga_{0.3}Al_{0.7}$As multiple quantum well laser with $L_z = 100$ Å (we call this laser as a quantum well laser for simplicity, hereafter) and a GaAs double heterostructure (DH) laser when the peak gain is taken equal to a total loss of 100 cm^{-1}. The number of quantum wells and

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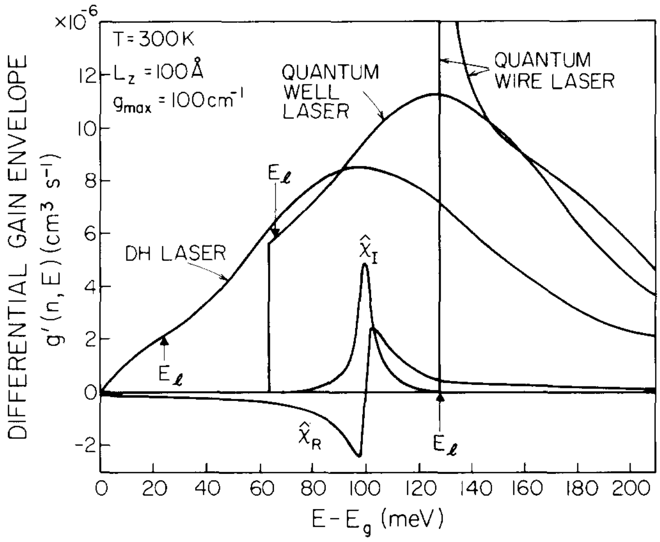

FIG. 1. Differential gain envelope $g'(n,E)$ of a DH laser, a quantum well laser, and a quantum wire laser under the condition that peak gain is equal to 100 cm^{-1}. The lasing photon energy (i.e., the location of peak gain) E_l, $\hat{\chi}_I(E)$ and $\hat{\chi}_R(E)$ are also illustrated.

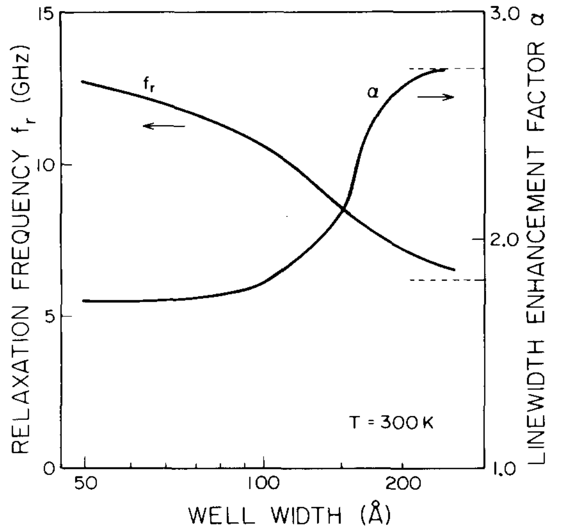

FIG. 2. α and f_R of a quantum well laser as a function of well width L_z.

the thickness of the barrier layers are determined so that the optical confinement factor is the same as that of the DH laser. E_l indicates the lasing photon energy (i.e., the location of peak gain). The functions $\hat{\chi}_I(E) = (\hbar/T_2)/[(E-E_l)^2 + (\hbar/T_2)^2]$ and $\hat{\chi}_R(E) = (E-E_l)/[(E-E_l)^2 + (\hbar/T_2)^2]$ which appear in Eqs. (3) and (4) are also illustrated (E_l is set at $E_g + 100$ meV and T_2 is assumed to be 0.2 ps). For simplicity the contribution of the light hole to $g'(n,E)$ is neglected in this figure. The $g'(n,E)$ in a $GaAs/Ga_{0.3}Al_{0.7}As$ multiple quantum wire laser whose quantum dimensions (L_z and L_y) are equal to 100 Å is also included for later discussion (we call this laser a quantum wire laser for simplicity, hereafter). f_r and α involve quantities which are convolutions of $g'(n,E)$ and $\hat{\chi}_I$ and $\hat{\chi}_R$. The curves in Fig. 1 show $g'(n,E)$ to be non-negative at all energies. Therefore, all contributions to $d\chi_I/dn$ will be non-negative. Hence the numerator of Eq. (1) and denominator of Eq. (2) in a quantum well laser increase compared with that of a DH laser owing to the large $g'(n,E)$ as shown in Fig. 1. As a result, increases in f_r over conventional values are expected for the quantum well case. Contributions to $d\chi_R/dn$ [the numerator of Eq. (2)], however, will be positive for transitions above E_l and negative for transitions below E_l because $\hat{\chi}_I(E)$ is negative below E_l. Therefore, the asymmetry of $g'(n,E)$ about E_l is reflected in the size and sign of α. Since E_l of quantum well laser is near equivalent band gap, the asymmetry of $g'(n,E)$ is large, leading to an increase of $d\chi_R/dn$. Hence both numerator and denominator in Eq. (2) increase in a quantum well laser, which makes it difficult to determine, without a numerical calculation, whether α is reduced or increased in a quantum well laser. Note that the position of E_l is a little higher than the equivalent band gap owing to the energy broadening effects (the finite value of T_2), which relaxes the asymmetric contribution of $g'(n,E)$.

Figure 2 shows the calculated results of α and f_r as a function of well width L_z in GaAs. In this calculation, the maximum internal gain which is necessary for laser oscillation is assumed to be 100 cm^{-1}. The broken line gives the values for a conventional DH laser. In the calculation of f_r we have assumed $\tau = 2.6$ ps, $T_2 = 0.2$ ps, and $P = 3.8\times 10^{13}$ cm^{-3}. As shown in the figure, it should be possible to double f_r in a quantum well over its value in conventional (3D) lasers using $L_z < 80$ Å. For the range of L_z, α is also reduced. This latter result was also found by Burt,[7] who, however, did not estimate the value of α at E_l. It should be noted that α also contains a free-carrier plasma dispersion contribution which we have neglected in this calculation.

Next, we discuss the possibility of the further improvement in f_r and α in quantum wire structures, in which the electrons are confined in both transverse dimensions. We consider a quantum wire structure whose quantum dimensions are L_z and L_y. In this case the reduced density of states with respect to electrons and heavy (or light) holes in the (i,k) subband is[8]

$$\rho_{\rm red} = \left(\frac{2m_j^*}{\hbar^2}\right)^{1/2} \frac{1}{2\pi L_z L_y} \times \frac{1}{\sqrt{(E - \hbar^2\pi^2/2m_j^*[(i/L_z)^2 + (k/L_y)^2])}}. \quad (8)$$

On the other hand, we found that M of the transition for (i,k) sideband is maximum when the polarization of the electrical field is parallel with the quantum wire direction. The expression of M of the transition between electrons and heavy holes in this case is given by

$$M = \left(\frac{1}{E}\frac{\hbar^2\pi^2}{2m_h^*}\left[\left(\frac{i}{L_z}\right)^2 + \left(\frac{k}{L_y}\right)^2\right]\right)\frac{3}{2}M_0. \quad (9)$$

This equation indicates that M is nearly 1.5 M_0 at equivalent band-gap edge. Even though the value of M is nearly equal to the M of the quantum well laser, increases in $g'(n,E)$, due to a narrower spectral distribution of electronic states as shown in Fig. 2, lead to improvement over the quantum well case. Figure 3 shows the calculated values for f_r and α as function of $L_z(=L_y)$. These results indicate that f_r can be made about three times larger than that of a DH laser and α can be substantially reduced. Thus, the calculated results suggest that a quantum wire structure should prove effective for improving the quantum noise characteristics and dynamics.

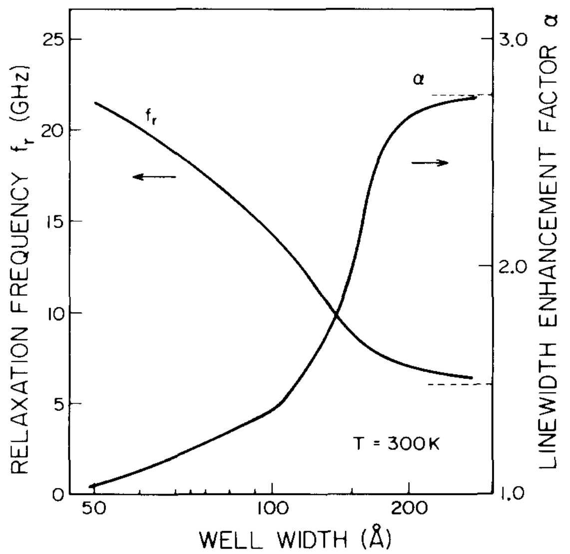

FIG. 3. α and f_R of a quantum wire laser as a function of well width L_z.

These effects can be demonstrated using high magnetic fields as discussed in a separate paper.[14]

In conclusion, we have considered the noise characteristics and dynamics in quantum well and quantum wire semiconductor lasers by calculating the relaxation oscillation corner frequency and the linewidth enhancement factor. The results indicate that f_r can be doubled with a concomitant reduction of α in quantum well structures. In the quantum wire lasers, f_r can be enhanced by $3\times$ with further reduction of α. It is thus expected that realization of quantum wire lasers would lead to significant improvements in dynamic and spectral properties over conventional devices.

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