Local quantum measurement and relativity imply quantum correlations: supplementary information

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We include a derivation of the POPT states for completeness. We follow the more general version in [1]. The outline is the following: using no-signalling, we apply Gleason's theorem on both sides, Alice and Bob. This implies that the no-signaling POPT state is bilinear on Alice and Bob measurements, which gives its form.

We denote the local POVMs by $M_A = \{Q_a\}_a$ and $M_B = \{R_b\}_b$. The joint probability distribution is given by a function ω acting on POVM elements

$$p(a,b|M_A,M_B) = \omega(Q_a,R_b)\ . \tag{1}$$

Notice that for any pair of POVMs

$$\sum_{a,b}\omega(Q_a,R_b) = 1\ , \tag{2}$$

but ω is not assumed to be bilinear at this point. No-signalling implies that for all M_B

$$\begin{aligned}\sum_b \omega(Q_a,R_b) &= \sum_b p(a,b|M_A,M_B) \\ &= p(a|M_A,M_B) = p(a|M_A) = \omega(Q_a)\ .\end{aligned} \tag{3}$$

That is, the marginal distribution is well defined.

For any POVM element Q_a on Alice's side we can define a corresponding function ω_a which acts on Bob's POVM elements. The function ω_a is defined by its action on any POVM element R_b with the equation

$$\omega_a(R_b) = \omega(Q_a,R_b)\ . \tag{4}$$

Notice that, for every POVM M_B on Bob's side, no-signalling from Bob to Alice implies that

$$\sum_b \omega_a(R_b) = \sum_b \omega(Q_a,R_b) = \omega(Q_a)\ . \tag{5}$$

Because ω_a adds to the constant value $\omega(Q_a)$ when it is summed over any POVM, we can use Gleason's theorem [2, 3, 4] to identify ω_a with an *unnormalized* quantum state $\tilde\sigma_a$ on Bob's side. Specifically, for any POVM element R_b, we have

$$\omega_a(R_b) = \omega(Q_a,R_b) = \mathsf{tr}(\tilde\sigma_a R_b)\ . \tag{6}$$

The previous equation allows us to define, for any given POPT ω, a map $\hat\omega$ from POVM elements Q_a on Alice's side to unnormalized quantum states on Bob's side

$$\hat\omega(Q_a) = \tilde\sigma_a\ . \tag{7}$$

Now choose an informationally complete POVM $M_B = \{R_b\}$ on Bob's side. Then $\hat\omega$ is given by the functions ω^b defined by

$$\omega^b(Q_a) = \omega(Q_a,R_b) = \mathsf{tr}(\tilde\sigma_a R_b)\ . \tag{8}$$

We use no-signalling from Alice to Bob to apply Gleason's theorem to each function ω^b from the informationally complete POVM, as we did before with no-signalling in the other direction. The action of ω^b is then given by an unnormalized quantum state, which implies that it is linear. This proves that $\hat\omega$ is linear.

Once we have established the linearity of $\hat\omega$ we can identify it with the operator $\mathcal{W}$ introduced in the text according to

$$\hat\omega(Q_a) = \frac{1}{d}\mathcal{W}(Q_a^T)\ . \tag{9}$$

Finally, we can write

$$\begin{aligned}\omega(Q_a,R_b) = \mathsf{tr}(\hat\omega(Q_a)R_b) &= \frac{1}{d}\mathsf{tr}(\mathcal{W}(Q_a^T)R_b) \\ &= \mathsf{tr}((Q_a\otimes R_b)W_{AB})\ .\end{aligned} \tag{10}$$

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