A NOTE ON TURBOPUMP BLADE CAVITATION COMPLIANCE

FOR THE POGO INSTABILITY

C. Brennen and A. J. Acosta

Engineering Science Department
California Institute of Technology
Pasadena, California

Introduction

During the first or booster stage of flight many liquid-propellant rockets have experienced severe longitudinal vibrations caused by a closed loop interaction between the first longitudinal structural mode and the dynamics of the propulsion system. This, "POGO" instability, reviewed in Reference 1, has been the subject of intensive research since it was first encountered. One of the most important transients in the dynamic modelling of the propulsion system is the "cavitation compliance" of the turbopumps[3] defined as the negative of the derivative of the cavity and bubble volume in the pump and its suction line with respect to the section pressure. Thus, it describes the occillatory source/sink behavior of the pump due to changes in the cavity volume. Past analyses[1,2] have suggested dividing this compliance into two components corresponding to the two major types of pump cavitation, namely blade cavitation and back-flow cavitation.

The purpose of this paper is to present some preliminary results of theoretical calculations of blade cavitation compliance. The most satisfactory starting point would be a theory for unsteady cavitating flow in a cascade. Whilst work on this is in progress at the present time, the low frequency or quasistatic approach based on existing steady flow theory is much simpler and in inself yields interesting results.

Linearized Theory for Cavitating Cascades

Free streamline potential flow models of cavitation on a cascade of foils have been employed extensively in the past to study blade cavitation in turbomachinery. Though there have been more exact analyses[5] most of the methods have been based on a linearized approach and many of the latter suffer from their neglect of finite blade thickness. The present computations are based on an adaption of the simple solution of Acosta and Hollander[6] for partial cavitites on a cascade of infinitely long foils (as shown in Fig. 1 where the symbols used below are defined). This geometry is then conformally mapped into the ζ-plane of Figure 2 by

$$z = e^{-i\beta}\, \ln\left(1 - \frac{\zeta}{\zeta_1}\right) + e^{-i\beta}\, \ln\left(1 - \frac{\zeta}{\bar{\zeta}_1}\right). \qquad (1)$$

With infinitely thin blades and the conventional linearization the solution for $w(\zeta) = u - iv$ (u,v are velocity components in the x,y directions) could then be written down by inspection. The present authors, however, considered the question of how the important effects of finite leading edge radius and blade thickness might be most easily incorporated into this solution. This was accomplished by the addition of the simple round-nose signular component $-iD/\zeta$ to the expression for $w(\zeta)$; then

$$w = -\frac{iD}{\zeta} + B\left(\frac{\zeta}{\zeta-\ell}\right)^{\frac{1}{2}} - A\left(\frac{\zeta-\ell}{\zeta}\right)^{\frac{1}{2}} + V_c$$

where the cavitation number, σ, is defined by $(1+\sigma)^{\frac{1}{2}} = V_c/V_1$, V_c being the cavity free streamline velocity. The real constants A and B and σ can be found in terms of D, ℓ by the application of the conditions at upstream and downstream infinity.

The coordinates of a point on the cavity/foil profile were then calculated in terms of the 'parameter' $\xi(\zeta = \xi + i\eta)$. Outside the interval $0 < \xi < \ell$, the foil profile becomes

$$Y_B(\xi) = -\frac{2D}{V_1}\tan^{-1}\left[\frac{\xi\cos\beta}{1-\xi\sin\beta}\right]$$

the absicca x being given by Eq. (1). Then assuming this to be the hidden foil profile within that interval as well, the leading edge redius is $2D^2\cos\beta/V_1^2$ and the ratio of downtream foil thickness/normal cascade spacing, d^*, is D/V_1. The cavity profile is described by an addition, $y_c(\xi)$, on top of the foil profile where

$$y_c(\xi) = 2\,\mathrm{Imag}\left(e^{-i\beta}\left\{\begin{array}{l}\left(\frac{V_c}{V_1} - \frac{V_2}{V_1}\right)\ell n\left(1-\left(\frac{\xi}{\xi-\ell}\right)^{\frac{1}{2}}\Big/ 1+\left(\frac{\xi}{\xi-\ell}\right)^{\frac{1}{2}}\right) \\ + \left(e^{-i\alpha} - \frac{V_c}{V_1} + \frac{iD}{V_1\zeta_1}\right)\ell n\left(\left(\frac{\zeta_1}{\zeta_1-\ell}\right)^{\frac{1}{2}} \right. \\ \left. - \left(\frac{\xi}{\xi-\ell}\right)^{\frac{1}{2}}\Big/\left(\frac{\zeta_1}{\zeta_1-\ell}\right)^{\frac{1}{2}} + \left(\frac{\xi}{\xi-\ell}\right)^{\frac{1}{2}}\right)\end{array}\right\}\right).$$

A dimensionless volume of the cavity/unit depth of the plane, V^*/h^2, was then calculated for various values of σ, α, β, d^*. Sample calculations of $K^* = (-\partial V^*/\partial\sigma)/h^2$ with $\alpha = 5^\circ$, $\beta = 75^\circ$ are shown in Figure 3. (Note that in the above analysis and Figure 1, h, the blade spacing, is set to 2π). Increase in the blade thickness, d^*, causes an increase in the choked cavitation number, σ_c. But Figure 3 shows that the compliance effect becomes reversed above a certain cavitation number, $\sigma \simeq 0.039$ in the present sample calculation.

Blade Compliance

From the definition of cavitation number the blade compliance K may be related to the calculated K^* by

$$K = K^*\frac{2h\,A_i}{Z\rho V_1^2}$$

where A_i is the inducer inlet area, Z the number of blades and ρ the liquid density. As an aside, note that K^* can be related to dimensionless compliance of Ghahremani and Rubin[4]. Most of the experimental data[2,4] has been obtained for pumps whose inducer is relatively highly loaded. Thus the backflow is large and its compliance dominates that due to blade cavitation so that the experimental values for total compliance generally lie to the right and above the curves in Figure 3. One might, however, expect better correlation between the present theory and experiment for lightly loaded inducers in which the backflow is small.

Acknowledgment

The work was partially and jointly supported by the Office of Naval Research Contract N00014-67-A-0094-0012 and MASA Contract NAS 8-28046.

References

1. Anon, 1970. Prevention of coupled structure-propulsion instability (POGO). NASA Space Vehicle design criteria (structures) Re. NASA SP-8055.
2. Ghahremani, F. G. 1970. Turbo-pump cavitation compliance. Aerospace Corporation Rep. No. TOR-0059 (6531-01)-2.
3. Rubin, S. 1966. Longitudinal instability of liquid rockets due to propulsion feedback (POGO). Journal Spacecraft and Rockets, 3,8,p. 1188.
4. Ghahremani, F. G. and Rubin, S. 1971. Empirical evaluation of pump inlet compliance (quarterly Status Report), Aerospace Corporation.
5. Stripling, L. B. and Acosta, A. J. 1962. Cavitation in turbo pumps -- Part I. AS,E Paper No. 61-WA-112.
6. Acosta, A. J. and Hollander, A. 1959. Remarks on cavitation in turbo-machines. California Institute of Technology Report E-79.3.

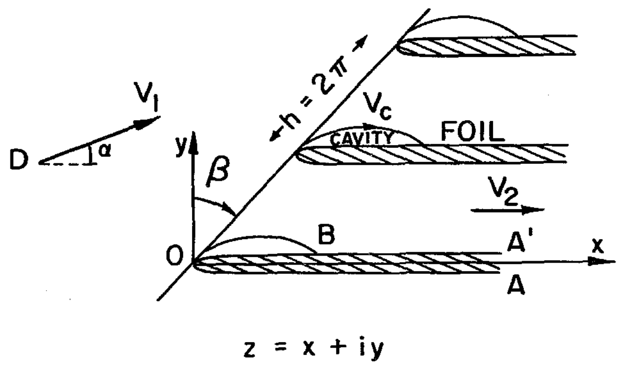

Figure 1 Partial Cavities on a Cascade of Foils

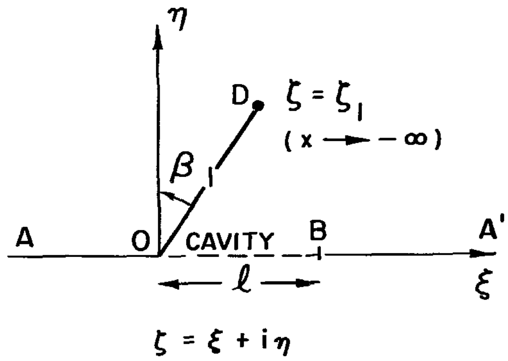

Figure 2 Conformal Mapping into Plane

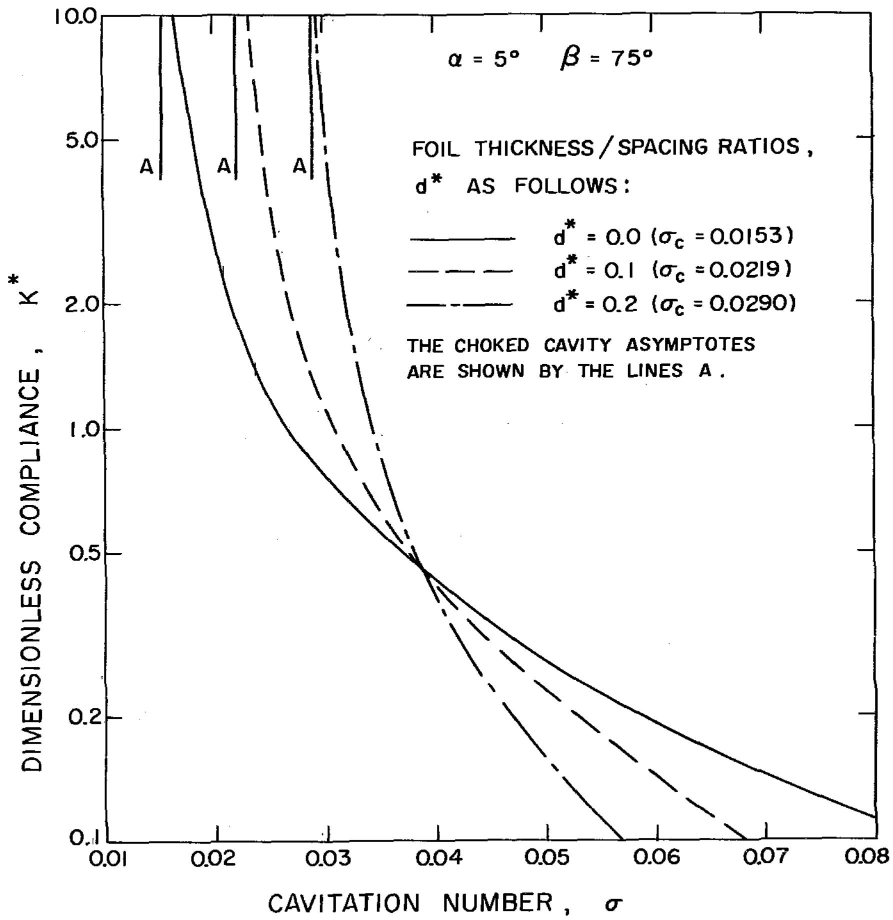

Figure 3 Blade Compliance as a Function of Cavitation Number for an Inducer