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D-Sphalerons and the Topology of String Configuration Space

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We show that unstable D-branes play the role of "D-sphalerons" in string theory. Their existence implies that the configuration space of Type II string theory has a complicated homotopy structure, similar to that of an infinite Grassmannian. In particular, the configuration space of Type IIA (IIB) string theory on $\mathbf{R}^{10}$ has non-trivial homotopy groups π_k for all k even (odd).

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1. Introduction

Most of the recent progress in non-perturbative string theory has been facilitated by the powerful constraints imposed by supersymmetry. It seems vital, for both theoretical and phenomenological reasons, to extend our understanding to configurations where some of these constraints have been relaxed. Non-supersymmetric string vacua are of course notoriously difficult to study. It seems reasonable, therefore, to first analyze non-supersymmetric excitations in the supersymmetric vacua of the theory.

During the last year it has been realized that the spectrum in some vacua of string theory contains not only BPS D-branes, but also stable non-BPS D-branes [1,2,3]. A very useful perspective for the study of these non-supersymmetric D-brane configurations has been developed by Sen [1]. In this framework, one views stable D-branes as bound states on the worldvolume of an unstable system composed of BPS D-branes and anti-D-branes with a higher worldvolume dimension. This construction has been further generalized [3,4], leading to a systematic framework which implies that D-brane charges on a compactification manifold X are classified by a generalized cohomology theory of X known as K-theory as suggested by previous work on Ramond-Ramond charges [5].

A crucial role in this framework is played by unstable Dp-brane systems. In dimensions where RR-charged Dp-branes exist, one can construct unstable systems by considering Dp-D$\overline{p}$ pairs. For the "wrong" values of p, where stable RR-charged Dp-branes do not exist, it was realized [6,4] that one can still construct an *unstable* Dp-brane.[1] Thus, in addition to the RR-charged BPS D-branes, there are unstable Dp-branes for p odd in Type IIA theory, and p even in Type IIB theory.

Such unstable D-branes can be directly constructed in the boundary-state formalism. Consider Type IIA or IIB theory in $\mathbf{R}^{10}$.[2] The boundary state describing a Dp-brane can have a contribution from the closed string NS-NS sector and RR sector. For each p, there is a unique boundary state in the NS-NS sector that implements the correct boundary conditions, and survives the corresponding GSO projection. The RR sector, on the other hand, contains a unique boundary state only for those Dp-branes that can couple to a RR

[1] Such unstable D-branes (in particular, the spacetime-filling D9-branes) are indeed crucial in the systematic classification of D-brane charges in Type IIA theory and its relation to $K^{-1}(X)$ groups of spacetime [4].

[2] In this paper, we focus on Type IIA and Type IIB string theory in $\mathbf{R}^{10}$, making only occasional comments about orientifolds and compactifications.

form C_{p+1}; for all other values of p, the GSO projection kills all possible boundary states in the RR sector.

The supersymmetric RR-charged Dp-brane is described by

$$|\mathrm{D}p\rangle_{\mathrm{BPS}} = \frac{1}{\sqrt{2}} \left(|B\rangle_{\mathrm{NS\,NS}} \pm |B\rangle_{\mathrm{R\,R}} \right), \tag{1.1}$$

where the sign in front of the RR component of the boundary state determines the RR charge of the brane.

In contrast, the boundary state describing the Dp-brane for the "wrong" values of p – where C_{p+1} is absent – contains only a NS-NS component,

$$|\mathrm{D}p\rangle = |B\rangle_{\mathrm{NS\,NS}}. \tag{1.2}$$

The relative factor of $\sqrt{2}$ and the consistency of this set of boundary states follows from constructing the cylinder amplitudes with all possible pairs of boundary states and imposing the condition that the cylinder amplitude has a consistent open string interpretation [7].

The spectrum of open strings ending on unstable D-branes is non-supersymmetric and contains a tachyon. To see this, note that the absence of the RR sector in the boundary state implies the absence of the GSO projection in the open-string loop channel, and as a result, the open-string spectrum contains *both* the lowest tachyonic mode T *and* the gauge field A_M. For N coincident unstable D-branes in Type II theory, the gauge symmetry is $U(N)$, and the tachyon T is in the adjoint representation of $U(N)$.

The unstable Dp-branes with worldvolumes of the "wrong" dimension represent legitimate classical solutions of open string theory, despite the fact that they are non-supersymmetric, unstable, and carry no charge. And, as for BPS D-branes, one expects these solutions to be a good approximation to solutions of the full closed and open string theory for small string coupling. The present paper is devoted to clarifying the physical interpretation of the unstable D-branes in string theory.

To whet the reader's appetite we offer the following observation. In Type IIA theory, unstable Dp-branes exist for p odd and, in particular, there is a Type IIA D-instanton. This D-instanton represents a Euclidean solution of the theory with a fluctuation spectrum containing one negative eigenvalue. Instantons with exactly one negative eigenvalue often represent a "bounce," or false vacuum decay; the square root of the fluctuation determinant is imaginary due to the single negative eigenvalue and the imaginary part of the vacuum amplitude gives the vacuum decay rate. For a review see [8]. In higher-dimensional theories with gravity, such vacuum decay often has disastrous consequences, leading to a complete annihilation of spacetime that starts by nucleation of a hole which then expands with a speed approaching the speed of light [9].

These observations lead to an intriguing question: does the existence of a D-instanton with one negative eigenvalue in Type IIA theory signal that its supersymmetric vacuum is false, and therefore unstable to decay? Before jumping to conclusions, declaring that the supersymmetric Type IIA vacuum (and, by duality, perhaps all other supersymmetric vacua) is unstable, and interpreting this as a string phenomenologist's dream, one needs to carefully examine whether the Type IIA D-instanton represents a bounce for false vacuum decay.

In an attempt to answer this question, we will clarify the role of all the unstable Dp-branes. In particular, we will see that the Type IIA D-instanton does not represent a bounce signaling an instability of the supersymmetric Type IIA vacuum. Instead, the D-instanton is tied to a completely different physical phenomenon, also with a precedent in field theory. We will find that the unstable D-branes in superstring theory are intimately related to the surprisingly complicated topological structure of the configuration space of string theory. In field theory, classical solutions with a negative mode that are mandated by non-trivial homotopy of the configuration space are called sphalerons. The main observation of this paper is that the unstable D-branes are precise string-theoretical analogs of sphalerons of field theory; we hope to convince the reader that it makes sense to call them D-sphalerons. Thus, the spectrum of D-branes in Type IIA theory consists of D$(2p+1)$-brane sphalerons ("D$(2p+1)$-sphalerons" for short) and BPS D$2p$-branes, while the Type IIB spectrum contains D$2p$-sphalerons and BPS D$(2p+1)$-branes. We will see that the existence of the D-sphalerons follows from the fact that the configuration space of IIA (IIB) string theory in $\mathbf{R}^{10}$ has nontrivial homotopy groups π_k for all k even (odd), and is thus homotopically at least as complicated as an infinite Grassmannian (the infinite unitary group).

2. Unstable D-branes

In our discussion, it will be convenient to use interchangeably several different representations of Dp-branes, which we first review.

(i) The traditional representation, as a hypersurface Σ_{p+1} in spacetime where fundamental strings can end. In string perturbation theory, D-brane dynamics is described by open strings ending on the brane; the boundary conditions are summarized by the closed-string boundary states (1.1) and (1.2). The unstable D-brane carries no charge, and the only long-distance fields associated with it are the dilaton and graviton; the theory is in its supersymmetric vacuum in the regions far away from Σ_{p+1}.

(ii) The topological defect representation, as a bound state extended along a submanifold Σ_{p+1} inside the worldvolume Σ_{q+1} of an unstable Dq-brane system with $q > p$.

(iii) The spacetime representation in terms of a solution to the closed string equations of motion. This is well understood for BPS D-branes [10,11] and will be partially developed in what follows for non-BPS D-branes.

There are two unstable D-brane systems relevant for the construction in point (ii). When q is such that RR-charged Dq-branes exist, the unstable system is given by N Dq-D$\overline{q}$ pairs. For the complementary values of q, the unstable system is simply the set of $2N$ unstable Dq-branes of (1.2).

In both cases, the worldvolume theory on Σ_{q+1} contains a tachyon field T which behaves as a Higgs field, rolling down to the minimum of its potential and Higgsing the gauge symmetry on Σ_{q+1}. The structure of the gauge symmetries and the symmetry breaking patterns are summarized in the following table:

unstable system:	gauge symmetry:	tachyon:	vacuum manifold:
N Dq-D$\overline{q}$ pairs	$U(N)\times U(N)$	$(N,\overline{N})$	$U(N)$
$2N$ unstable Dq's	$U(2N)$	adjoint	$U(2N)/U(N)\times U(N)$

(2.1)

Notice that in the case of the unstable Dq-branes, the correct spectrum of stable Dp-branes as defects in flat spacetime is reproduced by the symmetric Higgs pattern, with $U(2N)$ broken to $U(N)\times U(N)$ [4]. The role of configurations with an odd number of unstable Dq-branes, as well as asymmetric Higgs patterns, will be discussed in section 6.

The Higgs mechanism, whereby the tachyon uniformly condenses to the minimum of its potential, can be thought of as the worldvolume representation of how the unstable brane system decays to the vacuum. This interpretation of the Higgs mechanism leaves one obvious puzzle: the existence of the residual gauge symmetry, which should be absent in the true supersymmetric vacuum of the theory. Various attempts to resolve this puzzle have been proposed in the literature [12,13]. In this paper, we will not address this issue, and will simply *assume* that the unstable D-brane system with the tachyon uniformly condensed to the minimum of its potential is nothing but a somewhat awkward representation of the supersymmetric vacuum of the theory.[3]

The unstable D-brane systems (2.1) support a host of topological defects, which are interpreted as lower-dimensional D-branes. Stable defects are classified by non-trivial elements in the homotopy groups of the vacuum manifolds,

$$\begin{aligned} \pi_{2k+1}(U(N)) &= \mathbf{Z}, \\ \pi_{2k}(U(N)) &= 0, \end{aligned} \tag{2.2}$$

and

$$\begin{aligned} \pi_{2k+1}(U(2N)/U(N)\times U(N)) &= 0, \\ \pi_{2k}(U(2N)/U(N)\times U(N)) &= \mathbf{Z}. \end{aligned} \tag{2.3}$$

(These formulas hold in the stable regime, of N sufficiently large for fixed k.) These homotopy groups are directly related to K-theory groups of spacetime [4,3]; therefore, D-brane charges are naturally described in K-theory.

First, consider a BPS Dp-brane. This brane can be represented as a codimension p' defect along $x^i = 0, i = 1, \dots p'$, in an unstable system of Dq-branes with $q = p + p'$. For a codimension p' defect, the tachyon field T maps the sphere $S^{p'-1}$ at infinity in the transverse dimensions to the vacuum manifold $\mathcal{V}$, thus defining an element of $\pi_{p'-1}(\mathcal{V})$. In even codimension $p' = 2k$, the unstable system consists of $N = 2^{k-1}$ Dq-D$\overline{q}$ pairs, and in odd codimension $p' = 2k-1$, of $N = 2^{k-1}$ unstable Dq-branes. The corresponding tachyon condensate is given explicitly by

$$T = f(r)\Gamma_i x^i, \tag{2.4}$$

where Γ_i are the gamma matrices of the rotation group $SO(p')$ in the transverse dimensions x^i.[4] The gauge field A_M on the Dq-brane system is also non-zero, such that the energy of the whole configuration is finite.

For even p', we will have occasion to use two distinct definitions of the gamma matrices. Let $\mathcal{S}_+$ and $\mathcal{S}_-$ be the two 2^{n-1} dimensional irreducible spinor representations of $SO(2n)$.

[3] Strong evidence supporting this assumption has been recently obtained, with the use of string field theory, by Sen and Zwiebach [14] in the closely related case of an unstable D-brane in the bosonic string.

[4] The convergence factor $f(r)$ only depends on the radial coordinate, and asymptotes to T_0/r as $r \to \infty$, with T_0 one of the eigenvalues of T at the minimum of its potential; $f(0) = 1$. This convergence factor will be systematically omitted throughout the paper.

We can either define Γ_i to be $2^{n-1} \times 2^{n-1}$ matrices mapping $\mathcal{S}_+$ to $\mathcal{S}_-$, or to be $2^n \times 2^n$ matrices mapping $\mathcal{S}_+ \oplus \mathcal{S}_-$ to itself. Which definition is being used will be always be implied by the stated dimensionality of the matrices.

Now, consider an unstable Dp-brane, described by the boundary state (1.2). The tachyon field now defines a homotopically trivial map to the vacuum manifold which is reflected by the instability of the Dp-brane. However we still expect a solution with the core carrying a finite energy density along the worldvolume Σ_{p+1}. We will argue that this brane is also described by the same formula (2.4), as a defect of codimension p' in a corresponding unstable brane system, even though there is no direct topological argument as there is for BPS D-branes.

Consider as an example a Dp-brane with p even. In IIA theory this is a stable BPS brane and may be represented in various ways as a topological defect in higher dimensional unstable brane systems. The simplest is as a kink in the real tachyon field of the unstable D$(p+1)$-brane of IIA. Now we compare this to the unstable Dp-brane of IIB, taking as our starting point the unstable D$(p+1)$-D$\overline{(p+1)}$ system with a complex tachyon and a "mexican hat" potential. If we can establish that a cross-section of this potential gives the double-well potential of the IIA Dp system, then it is clear that the previous kink solution is again a solution, but now with an instability due to the possibility of pulling the kink off the top of the potential.

That the potential has this property follows from the rules developed in [1]. Open strings ending on an unstable Dp-brane are assigned 2×2 Chan-Paton matrices. The $U(1)$ gauge field is assigned to 1 and the real tachyon is assigned to σ_1. Open strings of the Dp-D$\overline{p}$ system also have 2×2 Chan-Paton matrices: the $U(1) \times U(1)$ gauge fields are assigned to 1 and σ_3, and the complex tachyon is assigned to σ_1 and σ_2. Then at the level of disk diagrams, the action for the tachyon of the unstable Dp-brane is the same as for the σ_1 component of the Dp-D$\overline{p}$ tachyon.

This line of argument actually establishes that any solution on the unstable Dp-brane yields a solution on the Dp-D$\overline{p}$ system, once the real tachyon is mapped to the σ_1 component of the complex tachyon, and the $U(1)$ gauge field is mapped to the 1 component of the $U(1) \times U(1)$ gauge field. The remaining fields associated to σ_2, σ_3 will only appear at least quadratically in fluctuations, since the trace over Chan-Paton matrices eliminates any linear terms. The quadratic fluctuations may destabilize a solution constructed in this fashion, but won't change the fact that it is a solution. One can also translate solutions in the opposite direction; a similar argument establishes that solutions on two coincident unstable Dp-branes can be mapped to solutions on a Dp-D$\overline{p}$ system. Again, the additional fields on the Dp-D$\overline{p}$ system appear at least quadratically in fluctuations, and so at worst destabilize the solution. We thus conclude that the unstable Dp-brane is also described by (2.4) as a defect on the worldvolume of an unstable Dq-brane system, despite the fact that this configuration is topologically unstable.

In the following we will make frequent use of the ability to translate stable and unstable solutions in the above manner, although we will not know the explicit solutions beyond their asymptotic behavior.

2.1. Type IIA D-instanton

Equipped with these different representations of unstable D-branes, let us return to the Type IIA D-instanton. For a classical Euclidean solution with one negative eigenvalue to represent a bounce, it has to satisfy several conditions.

(a) Tunnelling from the false vacuum.

(b) The Euclidean bounce solution.

Fig. 1: False vacuum decay and the Euclidean instanton (the "bounce") with one negative eigenvalue that dominates the path integral.

First of all, it has to be asymptotic to the false vacuum in all directions. The fate of the false vacuum after the tunneling can be read off from the bounce, by identifying its turning point, and evolving the configuration classically in the Minkowski signature. For a solution to admit such a procedure, it has to have a reflection symmetry along a codimension-one surface, which we can identify with the surface of constant Euclidean time $\tau_E = 0$. Moreover, we must be able to Wick-rotate the solution to the Minkowski regime. The turning point is defined to be a point on the trajectory where the kinetic energy vanishes, so by (Euclidean) energy conservation the potential energy at the turning point equals the potential energy of the false vacuum. Other points on the bounce trajectory have higher potential energy, they are under the barrier. If $\Phi(\vec{x}, \tau_E)$ represents the bounce, and $\Phi(\vec{x}, -\tau_E) = \Phi(\vec{x}, \tau_E)$ then the Euclidean kinetic energy vanishes at the turning point $\tau_E = 0$.

Consider the Type IIA D-instanton, first in the supergravity approximation. The supergravity solution that represents our D-instanton should respect the $SO(10)$ rotation symmetry, and be asymptotic to the supersymmetric vacuum of Type IIA theory. The only fields that can be excited are the metric and the dilaton; unlike in Type IIB theory, there is no "axion" that could be excited. It is useful to interpret Type IIA theory as M-theory on S^1. The Type IIA dilaton is related to the 11-11 component of the eleven-dimensional metric. Therefore, the only field excited in the D-instanton background is the eleven-dimensional metric. The equations of motion for this metric are just the vacuum Einstein equations, constrained by the requirement of $SO(10)$ rotation symmetry and $U(1)$ translation symmetry along the eleventh dimension. By the eleven-dimensional analog of the Birkhoff theorem, the solution – at least away from the D-instanton core – has to be given by the Euclidean Schwarzschild metric, which in appropriate coordinates takes the form

$$ds^2 = \left(1 - \frac{M}{r^8}\right)(dx^{11})^2 + \frac{dr^2}{1 - M/r^8} + r^2 d\Omega_9^2. \tag{2.5}$$

Furthermore, it is clear that this solution of M-theory represents a D-brane in the sense of a possible end point for strings. To see this, note that a membrane wrapped on the "cigar" of Euclidean Schwarzschild represents a fundamental string far from the core, and this string clearly ends at the core of the solution.

In spite of all this circumstantial evidence, the usual smooth Euclidean Schwarzschild solution does not correctly represent the IIA D-instanton. In the solution (2.5) there are two parameters: the "mass" parameter M, and the value of the radius of the eleventh dimension. This is as it should be, because we expect two parameters in the IIA D-instanton system in the supergravity approximation: the string coupling constant at infinity, and the number N of D-instantons. Supergravity cannot distinguish the discreteness of the second quantum number N, and sees it as a smooth parameter M. The string coupling is related in the usual way to the radius of the eleventh dimension x^{11}, and can be adjusted arbitrarily. This of course leaves a conical singularity at $r = r_0 \equiv M^{1/8}$, corresponding to the location of the D-instanton(s) at $r = r_0$.

As we will now explain, such a singularity is inevitably present in the supergravity solution describing the D-instanton. If we impose the additional requirement of smoothness at $r = r_0$ on (2.5), we obtain the Euclidean Schwarzschild black hole, with the radius

R_{11} of S^1 uniquely determined by the parameter M, $R_{11} = M^{1/8}/4$. So far we have ignored the presence of fermions in the theory. The D-instanton is a non-supersymmetric solution in a supersymmetric theory, asymptotic at infinity to the supersymmetric vacuum. Therefore, the spin structure it carries has to preserve supersymmetry asymptotically at infinity. In the eleven-dimensional representation, the spin structure on the eleven-manifold (2.5) describing the D-instanton has to correspond to periodic boundary conditions on the fermions around the S^1. In contrast, the smoothness of the Euclidean black hole implies that it can carry only one spin structure, with fermions antiperiodic around the S^1. This in turn implies that the metric of the D-instanton always has to have a singularity at $r = r_0$, in order to carry the correct, that is periodic, spin structure.

As we have just seen, the singularity of the metric at the location of the D-instanton cannot be resolved by supergravity; in particular, one cannot count the number of negative modes of the solution in the supergravity approximation.

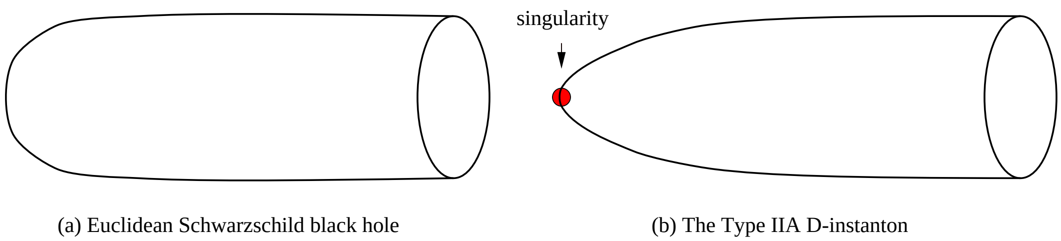

Fig. 2: Two supergravity solutions: (a) The Euclidean Schwarzschild black hole in eleven dimensions; (b) the Type IIA D-instanton.

On the other hand, the Euclidean Schwarzschild is a smooth solution, with only one free parameter – the value of the string coupling at infinity, and will have exactly one negative mode. Since its spin structure is that of antiperiodic fermions around S^1 at infinity, the Euclidean Schwarzschild represents a bounce relevant for the fate of the vacuum in a large class of compactifications, related to M-theory (or string theory) on S^1 with the non-supersymmetric spin structure [15,16], and also describes black hole nucleation in M-theory at finite temperature, as in [17].

The singularity found at the tip of the supergravity solution is resolved in string theory by the presence of the D-branes. In this representation, one can count the number of negative modes of this configuration. N coincident D-instantons will have N^2 negative modes, from the open-string tachyon in the adjoint of $U(N)$. Clearly, it is only the single-instanton configuration that can in principle represent a bounce. Notice that $N = 1$ will correspond to a small value of M, and therefore the supergravity approximation will be invalid for this system.

We will now analyze the turning point configuration for the Type IIA D-instanton by using the representation of the IIA D-instanton as a topological defect of the familiar form

$$T = \Gamma \cdot x, \tag{2.6}$$

on 32 unstable D9-branes. Here Γ_i are 32×32 SO(10) gamma matrices.

Consider a $9+1$ split of coordinates, $x^i = (\vec{x}, x^{10})$. Using two equivalent representations of the Γ matrices of $SO(10)$, we can write (2.6) in two forms leading to two different physical interpretations of the D-instanton (2.6). First, (2.6) can be written as

$$T = \begin{pmatrix} x^{10}\, 1_{16} & \vec{\Gamma} \cdot \vec{x} \\ \vec{\Gamma} \cdot \vec{x} & -x^{10}\, 1_{16} \end{pmatrix}, \tag{2.7}$$

where $\vec{\Gamma}$ are the gamma matrices of $SO(9)$. This corresponds to first forming sixteen D8–branes and sixteen D$\overline{8}$-branes as kinks localized at $x^{10} = 0$ on 32 D9-branes, represented in (2.7) by the terms along the diagonal. The D-instanton then appears as the bound state $\vec{\Gamma} \cdot \vec{x}$ of sixteen D8-D$\overline{8}$ pairs.

Alternatively, one can write (2.6) as

$$T = \begin{pmatrix} \vec{\Gamma} \cdot \vec{x} & x^{10}\, 1_{16} \\ x^{10}\, 1_{16} & -\vec{\Gamma} \cdot \vec{x} \end{pmatrix}. \tag{2.8}$$

In this picture, we use the D9-branes to first prepare a D0-D$\overline{0}$ pair (represented by the diagonal terms in (2.8)), with their worldline along x^{10}, and then form a kink along x^{10} on the worldline of the D0-D$\overline{0}$ system. It is this latter representation (2.8) of (2.6) that is useful for determining the physical meaning of the "halfway point" of the D-instanton. Setting $x^{10} = 0$ in (2.8) leaves the configuration consisting of a D0-D$\overline{0}$ pair at Euclidean time $x^{10} = 0$.

The D-instanton does indeed possess a reflection symmetry in x_{10}, which in the form (2.8) is given by $T(\vec{x}, -x^{10}) = \sigma_3 T(\vec{x}, x^{10}) \sigma_3^{-1}$. However, because of the gauge transformation which accompanies the reflection, this symmetry does not imply vanishing of the kinetic energy[5] at the symmetry point $x^{10} = 0$, which therefore is not a proper turning

[5] Strictly speaking, we should study the vanishing of the gauge covariant kinetic energy, but turning on a non-zero gauge field to cancel the time derivative of T will simply generate a non-zero electric field leading to non-zero gauge kinetic energy.

point. Alternatively, we can use the decomposition (2.8) of the D-instanton to see that an energy condition is being violated at the halfway point, and the instanton therefore cannot represent a legitimate bounce. We have seen that the halfway point consists of a D0-$\overline{\rm D0}$ pair on top of the supersymmetric vacuum; however, such a configuration carries positive energy with respect to the supersymmetric vacuum, and its nucleation is forbidden by energy conservation. We conclude that the Type IIA D-instanton does not lead to false vacuum decay of the supersymmetric Type IIA vacuum.

3. Unstable D-Branes as D-Sphalerons

We have argued that the Type IIA D-instanton does not represent a bounce for false vacuum decay of the supersymmetric vacuum in Type IIA theory. In this section, we start collecting evidence leading to a different physical interpretation of all the unstable D-branes, and the Type IIA D-instanton in particular.

3.1. Sphalerons in field theory

In field theory, sphalerons [18] are static solutions of the classical equations of motion with a single negative mode, whose existence is implied by a non-contractible loop in the configuration space of the theory.

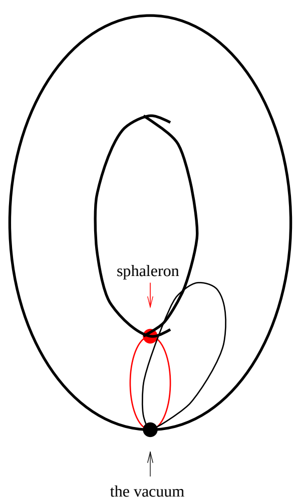

Fig. 3: The topological argument tying the existence of a non-contractible loop in the configuration space with the existence of a static solution with one negative eigenvalue (the sphaleron). The vertical axis corresponds to the energy.

The argument [19,18] goes as follows. Consider Yang-Mills gauge theory with matter in $D+1$ spacetime dimensions. This theory has a configuration space $\mathcal{A}$, of all physically inequivalent, finite energy configurations on the D-dimensional space. Assume now that $\mathcal{A}$ contains a non-contractible loop, i.e., that $\pi_1(\mathcal{A}) \neq 0$. If $\mathcal{A}$ is sufficiently compact, the situation can be visualized as in Figure 3. Choose an arbitrary non-contractible loop ℓ in $\mathcal{A}$ which begins and ends in the vacuum, and parameterize this loop by $t \in [0, 2\pi]$. Without any loss of generality, assume that the energy along ℓ grows monotonically as we move away from the vacuum, and reaches its absolute maximum $E(\ell)$ at the half-point $t = \pi$. Since ℓ is non-contractible, there is a loop ℓ_0 homotopically equivalent to ℓ and such that

$$E_0 \equiv E(\ell_0) \leq E(\ell') \tag{3.1}$$

for all loops ℓ' that are homotopically equivalent to ℓ. The point in the configuration space $\mathcal{A}$ that corresponds to $t = \pi$ along such a minimal loop ℓ_0 is guaranteed to be a static, finite-energy solution of the theory, called *the sphaleron*. The spectrum of fluctuations around the sphaleron will contain precisely one negative eigenvalue, corresponding to the two directions in which the sphaleron can slide down to the true vacuum along the loop ℓ_0 in the configuration space.

A loop ℓ in the configuration space $\mathcal{A}$ represents a one-parameter set of D-dimensional configurations, and can be viewed as a $D+1$-dimensional Euclidean configuration with the Euclidean time given by the loop parameter t. The loop ℓ will be non-contractible if this $D+1$ dimensional configuration is topologically stable. At infinity in all Euclidean dimensions, this configuration is mapped to the vacuum manifold $\mathcal{V}$ of the theory. Thus, the non-contractible loop determines a non-trivial element of $\pi_D(\mathcal{V})$.

Notice that the sphaleron in a D-dimensional space carries no conserved topological quantum numbers, since it can be continuously connected to the vacuum. In other words, the sphaleron can be unwrapped at S^{D-1} at infinity, and corresponds to the trivial element in $\pi_{D-1}(\mathcal{V})$. However, it is the non-contractible loop in the configuration space that is supported by the non-trivial element in $\pi_D(\mathcal{V})$. In this sense, there is a certain similarity between instantons and the Euclidean configuration representing the non-contractible loop, as it is the same quantum number that is responsible for both. However, even though the topology is similar, the energetics is different. In the case of an instanton, we impose a single condition of finite action in $D+1$ dimensions, while in the case of a loop in configuration space, we impose the finite-energy condition in D dimensions for each value of the loop parameter t.

We now illustrate this general construction with a few simple examples:

(1) The original sphaleron of [18] was found in a simplified version of the standard model, given by the $SU(2)$ theory with a doublet Higgs in $3+1$ dimensions. The vacuum manifold is a three-sphere S^3. A non-contractible loop exists, and corresponds to a point-like topological defect in four Euclidean dimensions that non-trivially wraps around the S^3, and corresponds to the generator in $\pi_3(S^3) = \mathbf{Z}$. The sphaleron is an unstable static solution in the vacuum sector.

(2) As an even simpler example consider the Abelian Higgs model,

$$S = \int d^2x \left\{ -\frac{1}{4} F_{\mu\nu}^2 + |(\partial_\mu - iA_\mu)\phi|^2 - \frac{1}{4}\lambda(|\phi|^2 - 1)^2 \right\}. \tag{3.2}$$

The configuration space of this theory also has a non-contractible loop, given by a point-like vortex in two Euclidean dimensions, stable because the Higgs field at infinity corresponds to the generator of $\pi_1(S^1)$. The corresponding sphaleron [20] is given by

$$\phi = \tanh\left[\frac{1}{2}\sqrt{\lambda}(x - x_0)\right] e^{i\beta(x)}, \quad A_0 = 0, \quad A_1 = \partial_x \beta(x), \tag{3.3}$$

where x_0 is arbitrary and the only condition on β is

$$\beta(\infty) - \beta(-\infty) = \pi. \tag{3.4}$$

3.2. Unstable D0-brane as a D-sphaleron in Type IIB theory

Consider Type IIB string theory on $\mathbf{R}^{10}$. This theory has an unstable D-particle with a single real tachyon. The system of N such coincident D-particles has a tachyon in the adjoint of $U(N)$ on the worldline. Upon orientifold projection to Type I, the unstable Type IIB D-particle becomes the $\mathbf{Z}_2$-charged stable non-BPS D0-brane of Type I string theory [1,2,3]. This is because only the antisymmetric part of the adjoint tachyon survives the Ω projection, leaving an instability for $N > 1$ but making the $N = 1$ system stable. Here, however, we are interested in the unstable D0-brane of Type IIB theory in its own right.

The D0-brane of Type IIB theory can be viewed as a defect, represented by $\Gamma{\cdot}x$ of (2.4), on sixteen D9-$\overline{\text{D9}}$ pairs, where Γ_i are the $SO(9)$ gamma matrices of the rotation group in the nine transverse dimensions x^i. This configuration is topologically unstable: the tachyon maps the 8-sphere at infinity to the vacuum manifold, but the relevant homotopy group $\pi_8(U(16)) = 0$ is trivial. The $SO(9)$ group has only one spinor representation $\mathcal{S}$, and the gamma matrices represent a map $\mathcal{S} \to \mathcal{S}$. Since the $\Gamma \cdot x$ configuration carries no D-brane charge, it corresponds to the trivial element in K-theory. Thus, the Chan-Paton bundle supported by the D9-branes is isomorphic to the Chan-Paton bundle of the D$\overline{9}$-branes, and both are identified with $\mathcal{S}$ (extended to the whole spacetime manifold $\mathbf{R}^{10}$).

We now claim that the D0-brane is a D-sphaleron, i.e., it is a static solution of the equations of motion of Type IIB string theory that has one negative mode, and represents the top of the potential barrier along a non-contractible loop in the configuration space of Type IIB string theory on the non-compact space $\mathbf{R}^9$. We will prove this directly by constructing the corresponding non-contractible loop in the configuration space, i.e., a one-parameter set of configurations on $\mathbf{R}^9$ (parametrized by $t' \in [0, 2\pi]$) which begins and ends in the supersymmetric vacuum, and at $t' = \pi$ passes through the configuration describing the unstable D0-brane.

In our construction, we use the defect representation of the D0-brane, as $\Gamma \cdot x$ on sixteen D9-D$\overline{9}$ pairs. A one-parameter family of configurations on $\mathbf{R}^9$ can be viewed as a Euclidean configuration on $\mathbf{R}^9 \times \mathbf{R}$, parametrized by $y^I = (x^i, t)$. Using these coordinates, consider

$$T(y) = \Gamma_I \cdot y^I \tag{3.5}$$

(where Γ_I are now the 16×16 gamma matrices thought of as maps between the two inequivalent irreducible spinor representations of the $SO(10)$ rotation group, $\Gamma_I : \mathcal{S}_+ \to \mathcal{S}_-$). This loop in the space of configurations indeed satisfies our requirements. It is topologically stable, because now the tachyon wraps S^9 at infinity once around the non-contractible S^9 in the vacuum manifold $U(16)$ (recall again that $\pi_9(U(16)) = \mathbf{Z}$). Thus, despite our ignorance about the overall normalization factor, the family (3.5) will indeed flow to a certain topologically non-trivial family of configurations. This family is asymptotic to the supersymmetric vacuum at $t \to \pm\infty$, and by construction passes through the D0-brane configuration at $t = 0$.

In fact, the proper framework for understanding the non-contractible loop (3.5) in the configuration space is K-theory. Even though at each t the Chan-Paton bundles of D9-branes and D$\overline{9}$-branes are isomorphic (and given by the $SO(9)$ spinor bundle $\mathcal{S}$), they wrap the extra dimension t in a topologically nontrivial way, and span the non-isomorphic $SO(10)$ spinor bundles $\mathcal{S}_+$ and $\mathcal{S}_-$ (in accord with the fact that the 16×16 gamma matrices of $SO(10)$ in (3.5) provide a map $\mathcal{S}_+ \to \mathcal{S}_-$). Thus, the Chan-Paton bundles of the whole one-parameter family of D9-D$\overline{9}$ pairs represent a non-trivial element in the K-theory group of the extended manifold parametrized by (x^i, t). As an element of K-theory, the topological charge that stabilizes (3.5) can be physically identified as one of the RR charges of Type IIB theory (namely, the D-instanton charge).

Hence, we conclude that

(1) the configuration space of Type IIB string theory has a non-contractible loop (supported by a topological charge that takes values in K-theory), and

(2) the D0-brane of Type IIB string represents the D-sphaleron at the top of the potential barrier traversed by the loop.

It should be pointed out that two important assumptions enter into this conclusion. First, we have not defined from first principles, such as string field theory, what we understand by the configuration space of Type II string theory. Instead, we are using the explicit construction of the D-sphalerons, in conjunction with the existence of RR charges as implied by K-theory, to *deduce* that the appropriately defined configuration space supports a non-contractible loop. This configuration space contains all perturbative string configurations, plus the configurations of all possible sets of D-brane configurations (and possibly more). A priori, we cannot rule out the possibility that there is some yet to be understood part of the configuration space which makes the above loop contractible. However, this possibility seems unlikely, since the existence of a non-contractible loop in the configuration space follows from a topological argument: the loop is non-contractible because it carries a non-trivial K-theory class (essentially, one unit of the D-instanton charge). As long as RR charges are conserved in the theory, it will not be possible to shrink the loop to a point.

Second, the presence of a non-contractible loop only implies the existence of a sphaleron solution if the configuration space is compact. Pure Yang-Mills theory has non-contractible loops, but the non-compactness of configuration space generated by scale transformations forbids the existence of finite size sphaleron solutions. We are assuming that in string theory, the string scale cuts off this source of noncompactness, and that the resulting object is the same as found by quantizing open strings with Dirichlet boundary conditions.

Our conclusions can be easily generalized to the configuration space of extended configurations that fall off at infinity in directions normal to an extended hypersurface in space. Just as in the case of the Type IIB D0-brane, one can interpret all the unstable Type IIB D$2p$-branes with $p > 0$ as D-sphalerons, and deduce the the existence of a non-contractible loop in the corresponding configuration spaces of extended configurations.

4. D-Sphalerons in Type IIA Theory

We now turn to a discussion of the interpretation of the Type IIA D-instanton.

Just as a non-contractible loop in the space of finite energy IIB configurations implied the existence of the D0-sphaleron, a non-contractible loop in the space of finite action IIA Euclidean histories gives rise to a D-instanton with a single negative mode. To exhibit the non-contractible loop, we proceed in parallel to the IIB discussion, now starting with 32 unstable D9-branes. Introducing the parameter t and SO(10) gamma matrices Γ_i, the non-contractible loop is given by

$$T = \sum_{i=1}^{10} \Gamma_i x^i + \Gamma_{11} t. \tag{4.1}$$

The loop gives a nontrivial element of $\pi_{10}(U(32)/U(16)\times U(16))$. We identify the halfway point of the loop at $t = 0$ with the IIA D-instanton.

Thus, the reason for the existence of the IIA D-instanton is not instability of the vacuum, rather it is required by the nontrivial topology of the space of histories in IIA string theory. The topological charge that makes (4.1) stable corresponds to a non-trivial element of K-theory, with a very interesting physical interpretation: in K-theory, this topological charge can be identified as the RR D(-2)-brane charge. Recall that in Type IIA string theory, there is a RR ten-form F_{10} (related to the cosmological constant in massive Type IIA theory), which couples to the D8-brane; formally, the magnetic dual of the D8-brane should be a D(-2)-brane, a concept that is indeed very hard to understand in physical terms. Here we have found a natural physical role of the D(-2)-brane charge (if not the D(-2)-brane), as the topological charge responsible for the non-contractible loop in the space of Type IIA histories. In a formal sense, one can think of the D(-2)-brane as an "object" localized in the extra dimension of the one-parameter family of histories traversing this non-contractible loop.

So far, our discussion of Type IIA theory has been focused on interpreting the Type IIA D-instanton, and therefore we were looking at the space of Euclidean histories. Similar arguments can also be used to analyze the configuration space of Type IIA theory. Interpreting *nine* of the eleven dimensions in (4.1) as space dimensions, and the remaining two as extra parameters, we can view (4.1) as a two-parameter family of string configurations that corresponds to a non-contractible two-sphere in the configuration space of Type IIA string theory. The corresponding sphaleron at the far pole of this non-contractible S^2 is easy to find by setting the two parameters representing the S^2 in (4.1) equal to zero. The sphaleron configuration that we obtain,

$$T = \begin{pmatrix} \vec{\Gamma}\cdot\vec{x} & 0 \\ 0 & -\vec{\Gamma}\cdot\vec{x} \end{pmatrix} \tag{4.2}$$

(with $\vec{x}$ describing the nine space dimensions and $\vec{\Gamma}$ the Gamma matrices of $SO(9)$), was already encountered in a different context in (2.8), and describes a coincident D0-D$\overline{0}$ pair.

The identification of the sphaleron in Type IIA configuration space as a D0-D$\overline{0}$ pair nicely agrees with the expected counting of negative modes. The D0-D$\overline{0}$ system has a complex tachyon, from the open string stretching between the D0 and the D$\overline{0}$-brane. This tachyon gives two real negative modes, precisely as expected from the sphaleron on the far pole of an S^2.

5. Topology of Configuration Space in String Theory

We have seen how to relate a single D-sphaleron to a non-contractible loop in configuration space. This loop is non-contractible because the corresponding one-parameter family of string configurations carries a topological charge in K-theory, even though each individual configuration carries zero charge. This structure clearly generalizes to multi-parameter families of string configurations. Looking back at (2.2) and (2.3) (or, more abstractly, invoking Bott periodicity in K-theory), we can generalize the construction of the section 3, and demonstrate that the string configuration space of Type IIB (IIA) string theory contains non-contractible spheres S^k of arbitrarily large odd (even) dimension k. In turn, each non-contractible S^k implies the existence of a sphaleron solution (with exactly k negative modes), at the pole of S^k opposite to the vacuum. What is the physical interpretation of such higher sphaleron solutions?

In this section we show that these higher sphalerons do not represent novel solutions; rather, they can be interpreted as multiple coincident D0-sphalerons of the previous section. We will demonstrate explicitly that we recover the correct counting of negative modes on k D-sphalerons.

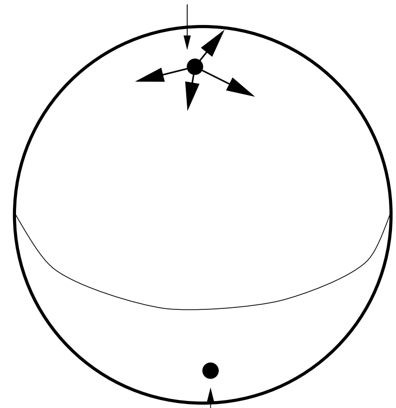

Fig. 4: Non-contractible $2n-1$-sphere in the configuration space and the corresponding n sphaleron configuration.

5.1. Higher non-contractible spheres in the IIB configuration space

We begin with a concrete example relating two coincident D0-branes in IIB to a non-contractible S^3 in the space of finite energy nine dimensional field configurations. We have seen that a single D0-brane can be represented as the point $t=0$ on the loop $T=\Gamma_i x^i+\Gamma_{10}t$, where $i=1\ldots 9$, and Γ_i are 16×16 $SO(10)$ gamma matrices. To represent two D0-branes, we introduce three parameters t_1,t_2,t_3, and define a non-contractible S^3 in terms of $SO(12)$ gamma matrices by

$$T=\tilde{\Gamma}_i x^i+\tilde{\Gamma}_{10}t_1+\tilde{\Gamma}_{11}t_2+\tilde{\Gamma}_{12}t_3. \tag{5.1}$$

Choosing a convenient representation for $\tilde{\Gamma}_i$, this becomes

$$T=\begin{pmatrix} \Gamma_i x^i+\Gamma_{10}t_1 & (t_2-t_3)\,1_{16} \\ (t_2+t_3)\,1_{16} & -(\Gamma_i x^i+\Gamma_{10}t_1) \end{pmatrix}. \tag{5.2}$$

It is evident that the "far pole" of the S^3 at $t_1=t_2=t_3=0$, , as depicted in fig. 4, represents two coincident D0-branes.

On the two D0-branes we expect to find $2^2=4$ negative modes. Three negative modes arise from motion on the S^3, *i.e.* $\delta T=\tilde{\Gamma}_{9+i}\delta t_i$ $(i=1,2,3)$. The final negative mode arises from motion on the non-contractible S^1 as for a single D0-brane:

$$T+\delta T=\begin{pmatrix} \Gamma_i x^i+\Gamma_{10}\delta t & 0 \\ 0 & -\Gamma_i x^i+\Gamma_{10}\delta t \end{pmatrix}. \tag{5.3}$$

So we indeed correctly reproduce the 4 negative modes known to exist from the quantization of open strings.

This procedure can be directly generalized to construct a non-contractible S^n for all odd values of n, whose existence is suggested by Bott periodicity of the homotopy groups (2.2). The generalization involves an interesting subtlety, which is best illuminated as follows. To simplify the argument, consider the unstable D0-brane as a real kink on the worldsheet of a coincident D1-D$\overline{1}$ pair along its space-like dimension x. The non-contractible S^1 in the configuration space is described by the stable vortex on the two-manifold spanned by (x,t), where t is the parameter along the loop. At each fixed t, we have one D1-D$\overline{1}$ pair. Similarly, the non-contractible S^3 discussed above corresponds to a point-like defect on a four-manifold spanned by (x,t^1,t^2,t^3); to construct such a defect, we need a family consisting of *two* D1-D$\overline{1}$ pairs at each t^i. This procedure can be iterated; in each step, as we add two more parameters t^{2k},t^{2k+1}, the $\Gamma\cdot y$ representation of the non-contractible S^{2k+1} requires doubling the number of D1-D$\overline{1}$ pairs. Thus, the non-contractible S^{2k+1} requires a family of 2^k D1-D$\overline{1}$ pairs parametrized by $t^1,\ldots t^{2k+1}$. Notice that the number of D1-D$\overline{1}$ pairs grows exponentially with growing k.

This construction certainly leads to a non-contractible S^{2k+1} in the configuration space, and one might be tempted to identify the configuration at $t^i=0, i=1,\ldots 2k+1$, as the corresponding sphaleron. However, a small puzzle immediately appears. While it is easy to show that the configuration at $t^i=0$ is given by

$$T = x\cdot 1_{2^k} \tag{5.4}$$

and consists therefore of 2^k coincident D0-sphalerons, it is also straightforward to see that for $k>1$ such a configuration has too many negative modes to represent the sphaleron at the far pole of S^{2k+1}, whose number of negative modes should grow linearly and not exponentially with k.

This puzzle is resolved by the following observation. One can certainly use the $\Gamma\cdot y$ construction to conveniently construct the non-trivial element of $\pi_{2k+1}(U(N))$, but the number $N=2^k$ of D1-D$\overline{1}$ pairs needed in this construction is not the smallest one possible; in fact, it is deeply inside the stability regime. In order to identify the sphaleron, we have to minimize the energy of the configuration at the far pole of the S^{2k+1}, and for that we need to use the smallest possible number of D1-D$\overline{1}$ pairs allowed by the stability bound. This bound requires $N\geq k+1$ pairs to properly accommodate π_{2k+1}! On this minimal number $k+1$ of D1-D$\overline{1}$ pairs, the sphaleron at $t^i = 0$ corresponds to $k+1$ coincident D0-branes.

Thus, we claim that the sphaleron on the far pole of the non-contractible S^{2n-1} is given by n coincident unstable D0-branes. It is now easy to see that the count of the number of negative modes indeed works as expected. The configuration of n coincident D0-branes exhibits n^2 negative modes, corresponding to the open-string tachyon in the adjoint of $U(n)$. Just like in the case of $n = 2$ discussed explicitly above, it is important to realize that the system of n D0-branes contains subsystems of $p < n$ D0-branes that sit at the far pole of S^{2p-1} for all $p = 1, \ldots n-1$. Motion on each S^p is associated with p negative modes. Thus, the total number of negative modes is

$$1 + 3 + 5 + \cdots + 2n - 1 = n^2, \tag{5.5}$$

as expected.

An analogous counting of negative modes goes through for configurations of coincident D$2p$-branes, including configurations which include branes of different dimensionalities. It is a satisfying consistency check that in all these cases, we reproduce the same spectrum of negative modes as arises from the quantization of open strings on non-BPS D-branes.

We are therefore led to conclude that

(1) the configuration space of Type IIB string theory has a homotopy structure which is at least as complicated as that of the infinite unitary group $U(N)$, $N \to \infty$: π_k of the configuration space is non-trivial for all odd k;

(2) similarly, the configuration space of Type IIA string theory has a homotopy structure at least as complicated as that of an infinite Grassmannian, $U(2N)/U(N) \times U(N)$, with all π_{2k} nontrivial.

5.2. Connection to K-theory

Our discussion so far has involved specific examples of D-brane sphalerons and non-trivial homotopy groups of the configuration space of Type II string theory in flat $\mathbf{R}^{10}$. It is perhaps worth stressing that the connection between D-sphalerons, K-theory, and the non-trivial homotopy groups of the string configuration space is quite universal, and our results naturally generalize to more complicated cases, including compactifications and orientifolds.

Consider any compactification of Type II or Type I theory. For simplicity, we will discuss the case of Type IIB theory compactification on X, but the generalization to other theories is straightforward. Stable D-branes on X are classified by elements of the (reduced) K-theory group $K(X)$, which in turn can be identified as the group of equivalence classes of pairs of Chan-Paton bundles (E,F) on a number of spacetime-filling D9-D$\overline{9}$ pairs wrapping X. The equivalence relation corresponds to creation and annihilation of pairs from/to the vacuum.

Imagine now an n-parameter family of D9-D$\overline{9}$ pairs, with Chan-Paton bundles $(E(t),F(t))$. In our discussion so far, the parameters $t=(t^1,\ldots t^n)$ were coordinates on an S^n, but one can consider a general n-manifold Y of parameters. For any fixed t, $(E(t),F(t))$ defines an element $\alpha(t)$ of $K(X)$, and the whole family defines an element of $K(X\times Y)$. Even if $\alpha(t)$ is trivial for each t, the element of $K(X\times Y)$ defined by the whole family can be non-trivial. When this is so, the family represents a non-contractible manifold Y in the configuration space of the theory in the vacuum sector.

Thus, there is an intimate relation between the homotopy structure of the string configuration space on X and K-theory groups of $K(X\times Y)$ for various Y. Since the latter are related to the spectrum of D-brane charges on X, the homotopy structure of the configuration space is closely related to the stable D-brane spectrum on X. Assuming that the configuration space is sufficiently compact, the non-trivial elements of the homotopy groups in turn imply the existence of corresponding D-sphalerons.

6. Tachyon Condensation and Massive Type IIA Vacua

As mentioned in section 2, we have been imposing certain restrictions in our study of tachyon configurations on unstable D9-branes in IIA. We chose to start with an even number $2N$ of D9-branes, and assumed that tachyon condensation Higgsed the gauge group according to $U(2N)\to U(N)\times U(N)$. This symmetry breaking pattern with an even number of unstable D9-branes arose in [4], where it was found to be directly related to K-theory and the classification of all D-brane charges in Type IIA theory. However, the role of other Higgs patterns, and configurations with an odd number of unstable D9-branes was left somewhat mysterious in the analysis of [4].

In this section our conditions will be relaxed: we allow an arbitrary number N of D9-branes, as well as the general Higgsing pattern $U(N)\to U(k)\times U(N-k)$. We will be led to an interpretation of these configurations in terms of vacua with non-vanishing flux for the RR 10-form F_{10}.

Let us recall some aspects of vacua with F_{10} flux [21,22]. Including the non-propagating field F_{10} in type IIA supergravity leads to so-called massive IIA supergravity [23]. The field equations of this theory admit solutions with constant F_{10}, and which preserve all 32 supersymmetries. In string theory it has been argued [22,24,25] that such vacua exist only for a discrete set of fluxes, $\nu \equiv {}^*F_{10} = n\mu_8$, where μ_8 is the tension of a BPS D8-brane. D8-branes play the role of domain walls between distinct vacua, with ν jumping by μ_8 upon crossing a D8-brane. We also remark that the massive IIA theory has a cosmological constant via $S \sim \int d^{10}x\sqrt{-g}\,\nu^2$, and that the theory cannot be obtained from the dimensional reduction of any known eleven-dimensional theory.

To connect the above facts to our discussion, we first examine the simple case of a single unstable D9-brane. On the worldvolume of the D9-brane there is a neutral tachyon T, whose potential $V(T)$ is assumed to be of the standard double-well form, with a local maximum at $T = 0$ and minima at $T = \pm T_0$. As in [4], it is conjectured that a BPS D8-brane is represented by a kink configuration; *i.e.* $T = f(x_9)x_9$ describes a D8-brane at $x_9 = 0$, where $f(x_9)$ is a smooth function behaving as $T_0/|x_9|$ for large $|x_9|$. The kink will carries the RR charge of a D8-brane given that on the D9-brane there exists a coupling to the RR 9-form potential C_9 of the form [4,1,26]

$$S = \frac{\mu_8}{2T_0}\int dT \wedge C_9. \tag{6.1}$$

There is no straightforward way to directly compute the coefficient of this term, since the presence of T_0 in the denominator shows that it depends on unknown details of the tachyon potential. We have chosen the coefficient so that the kink carries the charge of a single D8-brane as in [26].

Now consider the homogeneous tachyon configurations $T = 0$ and $T = \pm T_0$, and imagine an adiabatic process in which the tachyon is taken from one such solution to another. The quadratic term for F_{10} along with the coupling (6.1) yield the field equation

$$d^*F_{10} = \frac{\mu_8}{2T_0}dT. \tag{6.2}$$

Hence in taking the tachyon from one minimum, $T = -T_0$, to the other, $T = +T_0$, we find that F_{10} changes by $\Delta\nu = \mu_8$. Given the previous quantization condition for ν in the massive IIA vacua, it is natural to conclude that in the process of shifting the tachyon we have moved from one massive IIA vacuum to an adjacent one. In this interpretation, a D8-brane, described as a kink, indeed represents a domain wall between distinct massive

IIA vacua. On the other hand, if we adiabatically take the tachyon from $T=-T_0$ to the unstable local maximum at $T=0$, we find $\Delta\nu=\mu_8/2$ which, perhaps surprisingly, forces us to admit values of ν not included among the massive IIA vacua. That is, we learn that in order to respect the quantization of ν after tachyon condensation, we must have that a single D9-brane with vanishing T can only exist in the presence of half odd integer units of flux: $\nu=(n+1/2)\mu_8$.

The foregoing analysis is easily generalized to the case of N unstable D9-branes. We assume that $V(T)$ has minima of the form

$$T=T_0\begin{pmatrix}1_k & 0\\ 0 & -1_{N-k}\end{pmatrix}, \tag{6.3}$$

and that on the D9-branes there exists a coupling

$$S=\frac{\mu_8}{2T_0}\int \mathrm{Tr}(dT)\wedge C_9. \tag{6.4}$$

Adiabatic variation of T then gives $\Delta\nu=\frac{1}{2}\frac{\mu_8}{T_0}\Delta\mathrm{Tr}(T)$. By moving between different minima, one can reach values for ν corresponding to any given massive IIA vacuum. For N even, it is consistent to take $\nu=0$ at $T=0$, and also after tachyon condensation to the traceless configuration $k=N/2$. This is what has been assumed in the bulk of this paper. But for N odd, consistency with the quantization condition requires one to include half odd integer units of flux at $T=0$.

One might be suspicious of the need to introduce half odd integer units of flux, given what was said about the difficulty in computing the coefficient of the term (6.1). Perhaps the assumed coefficient is incorrect by a factor of two, so that a kink truly represents two D8-branes. To allay such suspicions, we will compute the spectrum of fermion zero modes on the kink, and see that we obtain a single $8+1$ dimensional Majorana fermion, modulo one assumption, as we should if the kink represents a single D8-brane.

The computation is closely related to one performed in [27], which yielded the fermion zero modes on a Type I D0-brane regarded as a kink on a D1-D$\overline{1}$ pair. On an unstable D9-brane are two Majorana-Weyl fermions of opposite chiralities, $\psi_\pm$. We take these to couple to the tachyon at quadratic order through an action of the form

$$S=\int d^{10}x\left\{\frac{i}{2}f_1(T)[\psi_+^T\Gamma^0\Gamma^\mu\partial_\mu\psi_+ +\psi_-^T\Gamma^0\Gamma^\mu\partial_\mu\psi_-]+f_2(T)\psi_+^T\Gamma^0\psi_-\right\}. \tag{6.5}$$

Γ^μ are purely imaginary $SO(9,1)$ gamma matrices. $f_{1,2}(T)$ are functions of T and its derivatives. The action is restricted by a $\mathbf{Z}_2$ symmetry [1] which flips the sign of T along with one of the fermions; the symmetry requires f_1 to be an even function of T, and f_2 to be an odd function of T. The couplings are also restricted by a non-linearly realized supersymmetry acting on the fermion fields as discussed in [13,28]. It is not clear whether this fact is compatible with the last term in (6.5), or with the analogous term in [27], although the equations of motion which follow from (6.5) appear to be compatible with those in [28] to lowest order. This question deserves closer scrutiny, for now we will assume that (6.5) is correct and proceed.

For the tachyon background we take a kink located at $x_9 = 0$. As with the tachyon potential $V(T)$, there is no systematic way to calculate the functions $f_{1,2}$. Our main assumption is that for a kink background f_2/f_1 goes to a nonzero constant — which can be taken to be positive — for large x_9, and hence to a negative constant for large $-x_9$ as the tachyon moves from one minimum of its potential to the other. Fermion zero modes are obtained from normalizable solutions to the Dirac equation which depend only on x_9. Defining the linear combinations

$$\chi_\pm = \psi_+ \pm \psi_-, \tag{6.6}$$

the Dirac equation is found to be

$$\partial_9 \chi_\pm = -\left[\frac{1}{2}\frac{\partial_9 f_1}{f_1} \pm i\frac{f_2}{f_1}\Gamma^9\right]\chi_\pm. \tag{6.7}$$

The solutions are

$$\chi_\pm = f_1^{-1/2} \exp\left[\mp i \int_0^{x_9} dx_9' \, \frac{f_2}{f_1}\Gamma^9\right]\chi_\pm^{(0)}, \tag{6.8}$$

where $\chi_\pm^{(0)}$ are constant spinors. Given the assumed behavior of f_2/f_1, normalizability requires

$$\Gamma^9 \chi_+^{(0)} = -i\chi_+^{(0)}, \qquad \Gamma^9 \chi_-^{(0)} = +i\chi_-^{(0)}. \tag{6.9}$$

With these projections, the spectrum of fermion zero modes is that of a single $8+1$ dimensional Majorana fermion. Thus we have verified that a kink on an unstable D9-brane represents a single BPS D8-brane, which in turn requires that an odd number of unstable D9-branes be accompanied by half odd integral units of 10-form flux.

In closing this section, we point out that according to [29,30] an $8+1$ dimensional theory with an odd number of Majorana fermions potentially suffers from a global gravitational anomaly. In the present case, the $8+1$ dimensional theory on the kink was obtained by starting from an anomaly free $9+1$ dimensional theory, which indicates that the anomaly should cancel through some global version of anomaly inflow. This anomaly problem has recently been addressed in [31].

7. Conclusions and Outlook

In this paper, we have established the existence of finite-energy sphalerons in perturbative string theory, and identified them with the previously studied unstable D-branes. Thus, the unstable D-branes are legitimate objects in string theory, tied to the existence of a complicated homotopy structure of the configuration space of the theory and the existence of RR charges (or, more generally, charges in K-theory) in the "right" dimensions. As mentioned earlier, it is clear from the connection to K-theory that the structure uncovered in this paper is very universal, and a much richer spectrum of D-sphalerons is to be expected upon compactification. It will be interesting to unravel the implications of such D-sphalerons in more complicated situations.

Our construction of D-sphalerons was perturbative in g_s. Unlike their RR-charged BPS counterparts, D-sphalerons do not carry any conserved quantum numbers, and there is no a priori reason to expect that they survive as pronounced objects beyond the regime of weak string coupling. Therefore, our conclusions about the structure of the configuration space are strictly valid at small g_s only. Nonetheless, since the existence of D-sphalerons is protected by the existence of BPS RR-charges (and is therefore topological in nature, related to K-theory), it seems natural to expect that at least some aspects of the sphalerons will survive even at large g_s. In principle, one can ask whether the homotopy structure of the string configuration space can be recovered in a dual description of a given theory. It is amusing that infinite Grassmannians appeared previously in the string theory literature in early attempts to go beyond perturbation theory, where they played the role of the universal moduli space of all Riemann surfaces (including surfaces of infinite genus) [32].

Our construction sheds light on the existence of the elusive D(-2)-brane of Type IIA string theory, which couples to F_{10} and therefore is important for issues that have to do with the cosmological constant. The D(-2)-brane charge was found responsible for the existence of a non-contractible loop in the space of Type IIA histories in $\mathbf{R}^{10}$.

Although the Type IIA D-instanton – being an example of a D-sphaleron – does not cause false vacuum decay, of the supersymmetric vacuum of IIA theory, the closely related Euclidean Schwarzschild instanton will lead to false vacuum decay of M theory on

$\mathbf{R}^{10} \times S^1$ with the anti-periodic choice of spin structure on the S^1 following the analysis of [9]. This process has interesting generalizations to other non-supersymmetric string compactifications [16].

Finally, we have not yet explored the physical implications of D-sphalerons in string theory. In field theory, sphalerons represent solutions at the top of a finite-energy barrier that can be classically overcome under favorable circumstances. In certain regimes they provide the leading semi-classical contribution to certain processes such as baryon number violation in the standard model.

At finite temperature, one can create field-theory sphalerons because they are soft and large objects, relatively easy to create by a large number of soft quanta in the thermal ensemble. In high-energy scattering processes, on the other hand, it might be difficult to create a soft large sphaleron by scattering a few very energetic quanta, and it has been argued in field theory that baryon-mediated processes are not enhanced [33].

In string theory, D-sphalerons are objects that have a hard core under a stringy halo. Therefore, one can expect that – unlike in field theory – the stringy D-sphalerons could play an important role in high-energy scattering processes. On the other hand, their possible role at finite temperatures seems more obscure. At small values of the string coupling, the mass of the sphalerons is proportional to $\sqrt{\alpha'}/g_s$, and before we reach that energy regime in the thermal ensemble, we encounter the Hagedorn transition.

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References

[1] A. Sen, "Stable non-BPS bound states of BPS D-branes," *JHEP* **9808** (1998) 010, hep-th/9805019; "SO(32) spinors of type I and other solitons on brane-antibrane pair," *JHEP* **9809** (1998) 023, hep-th/9808141; "Type I D-particle and its interactions," *JHEP* **9810** (1998) 021, hep-th/9809111; "Non-BPS states and branes in string theory," hep-th/9904207, and references therein.

[2] O. Bergman and M. R. Gaberdiel, "Stable non-BPS D-particles," *Phys. Lett.* **B441** (1998) 133, hep-th/9806155.

[3] E. Witten, "D-Branes and K-Theory," *JHEP* **9812** (1998) 019; hep-th/9810188.

[4] P. Hořava, "Type IIA D-Branes, K-Theory, and Matrix Theory," *Adv. Theor. Math. Phys.* **2** (1999) 1373, hep-th/9812135.

[5] R. Minasian and G. Moore, "K-Theory and Ramond-Ramond Charge," *JHEP* **9711** (1997) 002, hep-th/9710230.

[6] A. Sen, "BPS D-branes on non-supersymmetric cycles," *JHEP* **9812** (1998) 021, hep-th/9812031.

[7] J. Cardy, "Boundary Conditions, Fusion Rules, and the Verlinde Formula," *Nucl. Phys.* **B324** (1989) 581;
P. Hořava, "Strings on World-Sheet Orbifolds," *Nucl. Phys.* **B327** (1989) 461;
"Open Strings from Three Dimensions: Chern-Simons-Witten Theory on Orbifolds," (Prague, 1990), *J. Geom. Phys.* **21** (1996) 1, hep-th/9404101.

[8] S. Coleman, "The Uses of Instantons," in *Aspects of Symmetry* (Cambridge University Press, 1985).

[9] E. Witten, "Instability of the Kaluza-Klein Vacuum," *Nucl. Phys.* **B195** (1982) 481.

[10] G. T. Horowitz and A. Strominger, "Black Strings and p-Branes," *Nucl. Phys.* **B360** (1991) 197.

[11] P. Di Vecchia, M. Frau, I. Pesando, S. Sciuto, A. Lerda and R. Russo, "Classical p-Branes from Boundary State," *Nucl. Phys.* **B507** (1997) 259.

[12] P. Yi, "Membranes from Five-Branes and Fundamental Strings from Dp-Branes," *Nucl. Phys.* **B550** (1999) 214; hep-th/9901159.

[13] A. Sen, "Supersymmetric World-volume Action for Non-BPS D-branes," hep-th/9909062.

[14] A. Sen and B. Zwiebach, "Tachyon Condensation in String Field Theory," hep-th/9912249.

[15] D. Brill and G. T. Horowitz, "Negative energy in string theory," *Phys. Lett.* **B262** (1991) 437.

[16] J.A. Harvey, P. Hořava and P. Kraus, work in progress.

[17] D.J. Gross, M.J. Perry and L.G. Yaffe, "Instability of Flat Space at Finite Temperature," *Phys. Rev.* **D25** (1982) 330.

[18] N.S. Manton, "Topology in the Weinberg-Salam Theory," *Phys. Rev.* **DD28** (1983) 2019; F.R. Klinkhamer and N.S. Manton, "A Saddle-Point Solution in the Weinberg-Salam Theory," *Phys. Rev.* **D30** (1984) 2212.

[19] C.H. Taubes, "The Existence of a Nonminimal Solution to the $SU(2)$ Yang-Mills-Higgs Equations on R^3," *Commun. Math. Phys.* **86** (1982) 257; **86** (1982) 299.

[20] A. I. Bochkarev and M. E. Shaposhnikov, "Anomalous Fermion Number Nonconservation At High Temperatures: Two-Dimensional Example," *Mod. Phys. Lett.* **A2** (1987) 991; D. Y. Grigorev and V. A. Rubakov, "Soliton Pair Creation At Finite Temperatures. Numerical Study In (1+1)-Dimensions," *Nucl. Phys.* **B299** (1988) 67.

[21] J. Polchinski, "Dirichlet-Branes and Ramond-Ramond Charges," *Phys. Rev. Lett.* **75** (1995) 4724, hep-th/9510017.

[22] J. Polchinski and A. Strominger, "New Vacua for Type II String Theory," *Phys. Lett.* **B388** (1996) 736, hep-th/9510227.

[23] L. J. Romans, "Massive N=2a Supergravity In Ten Dimensions," *Phys. Lett.* **B169** (1986) 374.

[24] E. Bergshoeff, M. de Roo, M.B. Green, G. Papadopoulos and P.K. Townsend, "Duality of Type II 7-branes and 8-branes," *Nucl. Phys.* **B470** (1996) 113, hep-th/9601150.

[25] M.B. Green, C.M. Hull and P.K. Townsend, "D-Brane Wess-Zumino Actions, T-Duality and the Cosmological Constant," *Phys. Lett.* **B382** (1996) 65, hep-th/9604119.

[26] M. Billó, B. Craps and F.Roose, "Ramond-Ramond couplings of non-BPS D-branes," *JHEP* **9906** (1999) 033; hep-th/9905157.

[27] A. Sen, "$SO(32)$ Spinors of Type I and Other Solitons on Brane-Antibrane Pair," *JHEP* **9809** (1998) 023, hep-th/9808141.

[28] T.Yoneya, "Spontaneously Broken Space-Time Supersymmetry in Open String Theory Without GSO Projection," hep-th/9912255.

[29] L. Alvarez-Gaumé and E. Witten, "Gravitational Anomalies," *Nucl. Phys.* **B234** (1984) 269.

[30] E. Witten, "Global Gravitational Anomalies," *Commun. Math. Phys.* **100** (1985) 197.

[31] G. Moore and E. Witten, "Self-Duality, Ramond-Ramond Fields, and K-Theory," hep-th/9912279.

[32] G. Segal and G. Wilson, "Loop groups and equations of KdV type," *IHES Publ. Math.* **61** (1985) 5; D. Friedan and S.H. Shenker, "The integrable analytic geometry of quantum string," *Phys. Lett.* **B175** (1986) 287; L. Alvarez-Gaumé, C. Gomez and C.Reina, "Loop groups, Grassmannians and String Theory," *Phys. Lett.* **B190** (1987) 55; C. Vafa, "Operator formalism on Riemann surfaces," *Phys. Lett.* **B190** (1987) 47; A. Schwarz, "Fermionic String and Universal Moduli Space," *Nucl. Phys.* **B317** (1989) 323; "Grassmannian and String Theory," *Commun. Math. Phys.* **199** (1998) 1, hep-th/9610122.

[33] M. Dine, O. Lechtenfeld, B. Sakita, W. Fischler and J. Polchinski, "Baryon Number Violation at High Temperature in the Standard Model," *Nucl. Phys.* **B342** (1990) 381.