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Stability of Hydraulic Systems with Focus on Cavitating Pumps

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ABSTRACT

Increasing use is being made of transmission matrices to characterize unsteady flows in hydraulic system components and to analyze the stability of such systems. This paper presents some general characteristics which should be examined in any experimentally measured transmission matrices and a methodology for the analysis of the stability of transmission matrices in hydraulic systems of order 2. These characteristics are then examined for cavitating pumps and the predicted instabilities (known as auto-oscillation) compared with experimental observations in a particular experimental system

RÉSUMÉ

L'identification et l'analyse des critères de stabilité des écoulements non-stationnaires dans les composantes hydrauliques se font de plus en plus au moyen des matrices de fonctions de transfert.

Ce papier présente quelques caractéristiques générales qui se doivent d'être considérées en vue d'analyser une matrice de fonctions de transfert obtenue expérimentalement de même qu'une méthodologie pour mettre en évidence les critères de stabilité associés a une matrice de deuxième ordre.

Ce schème est part la suite appliqué aux pompes cavitantes et la prédiction des régimes instables (connue sous le nom d'auto-oscillation) est comparée aux observations expérimentales pour un système particulier.

1. INTRODUCTION

Hydraulic systems involving components in which phase changes occur often encounter instabilities which lead to large pressure and mass flow rate excursions. The prediction of such instabilities and the design of ameliorative hardware are usually hindered by a lack of knowledge of the dynamic response of the components in which the cavitation, boiling or other phase change process occurs.

In the present paper we present a general methodology for such problems and consider its application to the common instability problems which are experienced in systems involving cavitating pumps. Instabilities in such systems are often termed "auto-oscillation" and have been the subject of a number of studies (Refs. 1 to 15). They have been demonstrated to be system instabilities caused by the "active" nature of the dynamic characteristics of a cavitating inducer. In the next sections we developed a characterization for such activity and criteria for evaluating instability.

2. DYNAMIC ANALYSES OF HYDRAULIC SYSTEMS

The traditional procedures for the dynamic analyses of hydraulic systems involve the integration of the equations of motion in the time domain particularly by the method of characteristics (Refs. 16 and 17). These have the advantages that non-linear terms can be incorporated but the methods are not readily adaptable to complicated flows of the kind that occur in many hydraulic devices such as pumps and turbines. The alternative approach of solution in the frequency domain has been used less often (e.g. Ref. 18); it has the disadvantage that it is usually necessary to confine the analysis to small linear perturbations. However, more complex hydraulic devices can be readily incorporated in such an approach; furthermore, experiments to measure the dynamic characteristics of such devices are most readily performed by introducing perturbations over a range of frequencies and the results are then presented as functions of perturbation frequency (e.g., Refs. 12,15,19).

Within the context of the frequency domain analyses which have proliferated in recent years (e.g., Refs. 18,20,15) the vast majority have been guided by electric network theory (e.g., Ref. 21) and have been confined to systems in which the flow is completely described by two state variables, usually pressure, p, and flow rate, q, though total pressure, h, has advantages over the former as will be demonstrated later. This corresponds to so-called four terminal network theory in the electrical context and transmission matrices for any component of the system are 2 X 2 matrices which are functions of frequency, Ω, and the mean or time averaged flows in the component. For example, if the linear perturbations in total pressure and mass flow rate are described by $Re\{\tilde{h}e^{j\Omega t}\}$ and $Re\{\tilde{m}e^{j\Omega t}\}$ where t is time, j is the imaginary unit, Re denotes real part of and $\tilde{h}$ and $\tilde{m}$ are complex in general then the transmission function, $[T]$, can be defined as

$$\begin{Bmatrix} \tilde{h}_2 \\ \tilde{m}_2 \end{Bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{Bmatrix} \tilde{h}_1 \\ \tilde{m}_1 \end{Bmatrix} \tag{1}$$

where subscripts 1 and 2 define values at inlet to and discharge from the component; $[T]$ is often referred to as the transfer function or matrix though it should strictly be termed the transmission function or matrix.

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It is important to note that such a description is confined to the special class of hydraulic systems which can be broken down into components such that the flow at each dividing point (not necessarily all points) is characterizable by only two state variables. This confines the analysis to either (a) incompressible fluid flows (b) compressible fluid flows in which the perturbations are barotropic : the perturbation density, $\tilde{\rho}$, is directly related to the perturbation pressure, $\tilde{p}$ and therefore is not an independent state variable (c) components whose inlet and discharge are single phase flows of the type (a) or (b) though not necessarily the same phase (examples are a cavitating pump with single phase liquid flow in and out, or an ideal evaporator or condenser) (d) two-phase flows represented by homogeneous flow models since they are usually equivalent to (b).

General liquid/gas two phase systems do not however fall in this restricted class since they usually require at least four state-variables (e.g., pressure, gas flow rate, liquid flow rate and void fraction) for complete characterization though some reduction of the order of the system can be achieved with certain two-phase flow models (e.g., the drift-flux model in which the relative velocity is a function only of the void fraction). Very limited data is available on transmission matrices of order greater than two. Brown (22) presents a unified approach to such problems but the material is confined to uniform systems in which the coefficients of the governing differential equations are independent of position; this eliminates all but the simplest fluid systems or components.

The analysis in this paper is similarly confined to systems of order 2. In the next section we present some of characteristics of these systems and a methodology for stability analysis.

3. SOME PROPERTIES OF TRANSMISSION MATRICES

It is clear that provided one can construct transmission matrices for each of the components into which the hydraulic system is broken then one has available a complete dynamic model of the system to use for stability and transient analyses. The major difficulty is usually a lack of knowledge of the transmission matrices for complex hydraulic components. In this respect one must rely on experimental measurements of the transmission functions, though sometimes such measurements may suggest analytical approaches as in the case of cavitating pumps (Refs. 12,23,24). When faced with experimentally measured transmission matrices it is often desirable to evaluate certain properties of those matrices so that one can anticipate how that hydraulic component might affect the dynamics of a complete system incorporating that component.

One such property is the determinant, D, of $[T]$. The matrix $[T]$ is said to be reciprocal if $D = 1$ and the overall transmission matrix for any parallel or series combination of reciprocal components is also reciprocal. In the context of hydraulic systems it is readily shown that incompressible flows within rigid boundaries ($T_{21} = 0$, $T_{22} = 1$) and with total head losses which are functions only of flow rate ($T_{11} = 1$) are reciprocal. Furthermore, an accumulator or surge tank envisaged as acting at a point ($T_{11} = 1$, $T_{12} = 0$, $T_{21} = -j\Omega C$, $T_{22} = 1$) and having a compliance C is reciprocal. Systems comprised of the above elements are analogous to L,R,C networks and have the same properties.

As an addenda to this it is well-known and readily shown that any uniform system of any order, N, has a determinant, D, given by

$$D = \exp\left(j\ell \sum_{m=1}^{N} \gamma_m\right) \tag{2}$$

where ℓ is the distance between stations 1 and 2 and $\sum_{m=1}^{N} \gamma_m$ is the sum of of the complex wave numbers corresponding to the N wave propagation speeds in that system. Consequently, $|D|$ is unity. Though series combinations of such components retain the same property, general parallel combinations do not. For convenience we term such systems quasi-reciprocal since they tend toward reciprocity at low frequencies.

The classifications passive or active transmission matrices are more immediately relevant to the stability of the systems. A component is considered active if there is a possible state in which there is a net output of fluctuation energy from that component and passive if no such state exists. It is clear that if all elements of a system are passive then the system will be stable. Furthermore, most hydraulic system elements are passive; indeed L,R,C systems are always passive. In contrast, pumps or turbines may be active since they represent possible sources of fluctuation energy; hence the focus in the present paper.

We consider next the conditions for net gain or loss of fluctuation energy in a component considering only those cases of incompressible inlet and discharge flows it follows that the time-averaged flux of fluctuation energy into a hydraulic component $\Delta\tilde{E}$ is given by

$$\Delta\tilde{E}=\tilde{E}_1-\tilde{E}_2=\frac{1}{4\rho_L}\left[\bar{\tilde{m}}_1\tilde{h}_1+\tilde{m}_1\bar{\tilde{h}}_1-\bar{\tilde{m}}_2\tilde{h}_2-\tilde{m}_2\bar{\tilde{h}}_2\right] \tag{3}$$

where ρ_L is the fluid density and the overbar denotes the complex conjugate. Substitution for $\tilde{m}_2,\tilde{h}_2$ from the transmission matrix yields the alternative form

$$\Delta\tilde{E}=\frac{|h_1|^2}{4\rho_L}\left[-A-B\left(\frac{\tilde{m}_1}{\tilde{h}_1}\right)\left(\frac{\bar{\tilde{m}}_1}{\bar{\tilde{h}}_1}\right)+(1-C)\frac{\tilde{m}_1}{\tilde{h}_1}+(1-\bar{C})\frac{\bar{\tilde{m}}_1}{\bar{\tilde{h}}_1}\right] \tag{4}$$

where

$$A = T_{11}\bar{T}_{21}+T_{21}\bar{T}_{11} = \text{purely real} \tag{5}$$

$$B = T_{22}\bar{T}_{12}+T_{12}\bar{T}_{22} = \text{purely real} \tag{6}$$

$$C = \bar{T}_{11}T_{22}+\bar{T}_{21}T_{12} = \text{complex in general} \quad . \tag{7}$$

Note that C is somewhat suggestive of the determinant D; in fact

$$|C|^2 = |D|^2 + AB \quad . \tag{8}$$

From this it is readily shown that the component is

(A) Conservative (i.e. $\Delta\tilde{E} = 0$) for all modes of excitation if and only if $A = B = 0$ and $C = 1$. Therefore not only must it be quasi-reciprocal ($|D| = 1$) but also

$$\frac{T_{11}}{\bar{T}_{11}} = -\frac{T_{12}}{\bar{T}_{12}} = -\frac{T_{21}}{\bar{T}_{21}} = \frac{T_{22}}{\bar{T}_{22}} = D \quad . \tag{9}$$

(B) Completely Passive ($\Delta\tilde{E}>0$) for all modes of excitation if and only if

$$A < 0 \tag{10}$$

$$|D|^2 + 1 = 2RE(C) < 0 \quad . \tag{11}$$

Note that these imply $B < 0$.