Chapter 26

Long Distance Quantum Communication with Atomic Ensembles

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1. Introduction

The ability to distribute quantum resources over long distances is a key element for the development of several practical applications in the field of quantum information.¹ In this way, major efforts have been made in the past decade to extend the maximum distances for the observation of entanglement or for the realization of quantum cryptography. Using optical fibers as the quantum channel, entanglement has already been achieved between photons separated by 50 km of fiber,² and quantum cryptography is now a commercial technology³ that allows quantum key distribution over distances up to around 100 km.⁴⁻⁶ Progress has also been made towards entanglement distribution in free space, with the achieved distances of about 10 km on ground atmosphere^{7,8} being a substantial step toward the development of satellite-based quantum communication.⁹⁻¹¹

However, in spite of all these developments, the quantum communication protocols that have been tested up to now suffer from some fundamental limitations on the maximum achievable distance.¹² Quantum key distribution setups, for example, seem to be approaching their limits. The problem is the absorption of photons in the quantum channel, which grows exponentially with the length of the channel. In classical communication, the attenuation problem is solved in a straightforward way by amplifica-

tion of the signal. It is not possible, however, to generate perfect copies of a quantum state,¹³ which prevents the amplification techniques to be used for quantum communication.

A first step to solve this problem was the development of techniques for entanglement purification.^{14,15} These techniques were inspired by quantum algorithms for error correction, and are designed to obtain a set of transmitted states of high fidelity (*i.e.*, high overlap with a maximally entangled state) out of a larger set with lower fidelity, if this fidelity is still above a certain value F_{min} . Based on these ideas, a solution for the long-distance-problem in quantum communication was then formulated with the development of the concept of quantum repeater.^{16,17} The idea is to divide the quantum channel in several segments with lengths that allow the transmission of information with fidelity $F > F_{min}$, and then apply the entanglement purification algorithms in a hierarchical way, beginning in the smaller portions and going up to larger regions until a high fidelity is obtained for the entanglement of pairs of particles located on the extremes of the communication channel. The connection between the different segments is done by entanglement swapping.^{18,19} The essential advantage of the quantum repeater protocol is that the number of trials to entangle a pair of particles in this case grows only polynomially with the distance, and not exponentially as in the protocols usually used for quantum communication.

The development of quantum repeaters, however, introduces new challenges which need to be addressed. It is necessary to entangle distant qubits, store them over operational time scales, and perform collective local operations on several of these qubits. Quantum memory is essential, since all purification protocols have a probabilistic nature. If the system had no memory, the purification of all segments would have to succeed at the same time for the algorithm to work, which is improbable. The requirement of quantum memory implies, in practice, that the local qubits need to be stored in atomic internal states. Since the communication between distant sites still needs to be done by photons, a quantum repeater should rely on systems that present strong atom-photon coupling. Usually this strong coupling is obtained with the aid of high finesse cavities, in the field known as Cavity Quantum Electrodynamics (CQED). However, in spite of important recent advances,^{20–22} these techniques still require a reasonably complicated experimental setup, which could prevent the application on a larger scale of quantum communication and error correction protocols for CQED, as the ones described in Refs. 23 and 24, respectively.

This was the context in which Duan, Lukin, Cirac and Zoller (DLCZ)

proposed an alternative protocol²⁵ for the implementation of quantum repeaters with atomic ensembles, which would require a much simpler experimental apparatus. The DLCZ protocol introduces novel ideas in several levels, from the physical process that couples light to atoms, to the way entanglement is achieved between distant atomic ensembles, and also on the details of the entanglement purification process. The main objective of the present chapter is to give an overview of this protocol and of the developments made towards its experimental implementation. Emphasis will be given on the experimental investigation carried out by our group at Caltech,^{26–28} but different developments by other groups will also be discussed alongside.^{29–32}

Section 2 provides a description of the basic elements of the protocol, while the rest of the chapter is dedicated to the experimental progress. Sections 3 through 5 describe then the realization of the first step in the implementation of the DLCZ protocol, *i.e.*, the control of the coupling between photons and atomic ensemble. Section 3 describes the first observations of such coupling as required by the protocol, through spontaneous Raman scattering on the single-photon level. In Sec. 4, it is shown that this process can also be used as a conditional source of single photons, a new application that was not in the original DLCZ proposal. Section 5 describes an in-depth analysis of the temporal structure of the two-photon wavepacket emitted by the sample. This gives detailed information on readout and decoherence timescales, which are important to the characterization of the quantum memory in our experimental setup.

Sections 6 and 7 discuss further developments. In Sec. 6, we report recent improvements in the coherence time of our system, and discuss the prospects for further increases. In Sec. 7, we discuss the experimental strategies to take the next step in the implementation of the DLCZ protocol: the generation and characterization of entanglement between atomic ensembles by the detection of a single photon. Finally, in Sec. 8 we draw our conclusions, and perspectives for future developments.

2. DLCZ Protocol for Quantum Repeaters

The building block of the DLCZ protocol is an ensemble of N_a identical atoms with lambda-type energy level configuration as shown in Fig. 1. The metastable lower states $|g\rangle$ and $|s\rangle$ can be, *e.g.*, hyperfine or Zeeman sub-levels of the electronic ground state of alkali-metal atoms, thus ensuring a long coherence lifetime. First, all atoms are prepared in the state $|g\rangle$.

By sending in a weak, off-resonant laser pulse, one atom of the ensemble might be transferred from $|g\rangle$ to $|s\rangle$, thus emitting a photon at a frequency or polarization different from the original exciting field. A key element of the protocol is the collective enhancement of this spontaneous Raman scattering in a forward direction, which is determined by the spatial mode of the laser pulse and the geometry of the excitation region.³³ If the laser intensity is low enough so that two excitations are very unlikely, the detection of the photon generated in this process is a signature that the ensemble was excited to a symmetrical collective state,^{25,33} which can be explicitly written as

$$|1_a\rangle = \frac{1}{\sqrt{N_a}} \sum_{i=1}^{N_a} |g\rangle_1 \cdots |s\rangle_i \cdots |g\rangle_{N_a}, \quad (1)$$

where the sum goes over all atoms addressed by the laser pulse, and $|1_a\rangle$ indicates the state of the atomic ensemble with just one excitation. This is the “writing” step of the protocol (Fig. 1a).

Fig. 1. Relevant level structure of the atoms in the ensemble for (a) writing and (b) reading processes, with $|g\rangle$ the initial ground state and $|s\rangle$ the ground state for storing a qubit. $|e\rangle$ and $|e'\rangle$ are excited states. The transition $|g\rangle \rightarrow |e\rangle$ is initially coupled by a classical laser pulse (write beam) detuned from resonance, and the forward-scattered Stokes light (field 1) comes from the transition $|e\rangle \rightarrow |s\rangle$, which has different polarization or frequency to the write light. A classical read pulse then couples the transition $|s\rangle \rightarrow |e'\rangle$, leading to the emission of forward-scattered anti-Stokes light (field 2) from the transition $|e'\rangle \rightarrow |g\rangle$.

Since the excitation probability χ is very small, the whole state of the system consisting of atoms and forward-scattered mode of light is in the following form:

$$|\phi\rangle = |0_a\rangle|0_1\rangle + e^{i\beta}\sqrt{\chi}|1_a\rangle|1_1\rangle + O(\chi), \quad (2)$$

where $\chi \ll 1$, $|n_1\rangle$ stands for the state of the forward-propagating light with n excitations, β is an arbitrary phase, and $|0_a\rangle \equiv \bigotimes_i^{N_a} |g\rangle_i$. $O(\chi)$

represents all the other possible excitation processes, which in the ideal case occur with probabilities of order χ^2 . The system remains in this state for a time on the order of the lifetime of the ground states. The state of Eq. (2) shows already a certain amount of entanglement (small, because χ is small) in the excitation-number basis between the atomic ensemble and the forward-scattered light field. One can thus manipulate the state of the atoms by acting on the state of the emitted field. Note, however, that the state of Eq. (2) and in the following equations can be considered entangled only if one knows the phase β .

By sending in a second (“read”) pulse resonant with the $|s\rangle \rightarrow |e'\rangle$ transition, the state of the atomic ensemble can be transferred (read out) to another forward-propagating light field 2 at the $|e'\rangle \rightarrow |g\rangle$ transition (see Fig. 1b). In this way, it is possible to access the quantum state of the atoms. This reading process is then closely related to low-light-level Electromagnetically Induced Transparency.^{34–36} The case in which $|e\rangle = |e'\rangle$ is called, in the following, a three-level scheme of excitation, while at four-level excitation schemes $|e\rangle \neq |e'\rangle$. Note also that the resultant state of the two forward-scattered modes (1,2) can be written, in the ideal case, as

$$|\phi_{1,2}\rangle = |0_1\rangle|0_2\rangle + \sqrt{\chi}|1_1\rangle|1_2\rangle + \chi|2_1\rangle|2_2\rangle + O(\chi^{3/2}), \quad (3)$$

i.e., the photon numbers in the two modes are correlated, precisely as for parametric down conversion.³⁷

The scheme to create an entangled pair of ensembles is shown in Fig. 2. Two write pulses are sent into two ensembles, L and R . At the output of the ensembles, the scattered fields and the ensembles are in the state:

$$\begin{aligned} |\Phi_{LR}\rangle &= |\Phi_L\rangle \otimes |\Phi_R\rangle \\ &= [|0_a\rangle_L|0_1\rangle_L + e^{i\beta_L}\sqrt{\chi}|1_a\rangle_L|1_1\rangle_L + O_L(\chi)] \\ &\quad \otimes [|0_a\rangle_R|0_1\rangle_R + e^{i\beta_R}\sqrt{\chi}|1_a\rangle_R|1_1\rangle_R + O_R(\chi)]. \end{aligned} \quad (4)$$

The scattered fields are later combined on a 50-50 beam splitter, with outputs directed to two photodetectors. In the ideal case, neglecting terms of order χ describing two or more excitation, detection of a photon in either detector then projects the state of the ensembles to the entangled state

$$|\phi_{L,R}\rangle = \frac{1}{\sqrt{2}} (|0_a\rangle_L|1_a\rangle_R \pm e^{i\eta_1}|1_a\rangle_L|0_a\rangle_R), \quad (5)$$

where η_1 relates to the difference of phase shifts in the two channels connecting the ensembles to the beam splitter (see Sec. 8), and the initial phase difference $\beta_L - \beta_R$. The sign $\pm$ depends on which detector records the event.

Note that the presence of certain sources of noise, especially dark noise on the detectors, modifies the above pure state to

$$\hat{\rho}_{LR} = \frac{1}{c_0 + 1} (c_0 |0_a\rangle_L |0_a\rangle_R \langle 0_a|_R \langle 0_a|_L + |\phi_{L,R}\rangle \langle \phi_{L,R}|), \quad (6)$$

where c_0^2 gives the probability for a detection not related to excitations on the ensembles. This state is called an effective maximally entangled (EME) state.²⁵ The DLCZ protocol is designed to be resilient to this important kind of noise, since it always requires further detection events to proceed. The detection events without excitations in the relevant atomic modes have then a high probability of being discarded right after they occur. This is the main origin of the “build-in entanglement purification” characteristic of the protocol.²⁵ Note that in writing (6), we have neglected higher-order terms involving pairs of excitations (one in each ensemble) and two excitations in either ensemble, which are intrinsic to the protocol of DLCZ, as well as diverse imperfections relevant to actual experimental implementations.

Fig. 2. Setup for generating entanglement between two atomic ensembles L and R . The ensembles are pencil-shaped, and excited by synchronized writing pulses. After filtering, the Stokes pulses are collected and coupled to optical channels. The pulses after the transmission channels interfere at a 50%-50% beam splitter BS, with outputs directed towards two single-photon detectors D_1 and D_2 . Ideally, if D_1 or D_2 records a detection event, the process is finished and entanglement is successfully generated. Otherwise, the system is restored to its initial state and the process is repeated until D_1 or D_2 records a click.

The maximum distance between the above entangled ensembles is limited by the attenuation length of the communication channels of field 1. To extend the distance between entangled ensembles, one applies the entanglement swapping scheme, Fig. 3. In this way, two pairs of ensembles need first to be prepared in the entangled state described by Eq. (5). Note that, since the entanglement process is probabilistic, the two pairs of ensembles do not necessarily become entangled at the same time. Due to the system memory, however, once a pair is entangled, one can hold the en-

tangled state and wait for the other pair to attain entanglement. This is an important advantage of the quantum repeater idea, which is responsible for the polynomial growth in number of trials with the distance between the final entangled pair. Other quantum communication schemes, like the quantum relay³⁸ that require all parts to be entangled at the same time, present an exponential growth with distance.

Fig. 3. Setup for extending the distance between entangled ensembles (entanglement swapping).²⁵ Two pairs of ensembles - L and I_1 , and I_2 and R - are initially distributed at three sites L , I , and R . Each of the pairs is prepared in an EME state in the form of Eq. (6). (a) The stored atomic excitations of two nearby ensembles I_1 and I_2 are converted into light by two simultaneous read pulses. The emitted anti-Stokes fields interfere at a 50%-50% beam splitter, and then are detected by the single photon detectors D_1 and D_2 . (b) If either D_1 or D_2 clicks, the protocol is successful and an EME state is established between the ensembles L and R with twice the communication distance. Otherwise, the process fails, and the previous entanglement generation and swapping need to be repeated until a click is recorded in D_1 or D_2 .

Once the two entangled pairs are obtained, the distance of entanglement is then doubled through the entanglement connection scheme shown in Fig. 3. Two read pulses are sent into ensembles I_1 and I_2 located in the middle, and the scattered fields are combined on a 50-50 beam splitter, Fig. 3a. A single click in either the detectors prepares then ensembles L and R in an EME state like the one of Eq. (6) (Fig. 3b). The “vacuum” coefficient c_0 now also includes the probability of reading the two excitations from the pairs (I_1, I_2) at the same time, leaving no remaining excitation in L or R .

The EME states created between distant ensembles can then be used to implement several entanglement-based communication schemes, such as

quantum cryptography, teleportation, and Bell-inequality measurements. We refer the reader to the original DLCZ paper²⁵ for the details on how to implement each of these schemes. In order to appreciate the broad applicability of the DLCZ protocol, however, it is important to note that a pair of entangled ensembles can be used to represent a qubit, with the states $\{|0_a\rangle_L|1_a\rangle_R, |1_a\rangle_L|0_a\rangle_R\}$ mapping to the usual qubit basis, $\{|0\rangle, |1\rangle\}$, in the ideal case. An experiment in this context has been recently reported by Matsukevich and Kuzmich.³²

The probabilistic character of the DLCZ scheme, however, imposes stringent requirements for the coherence times for the quantum memories at the various sites in Fig. 3. Namely, the coherence time τ_{memory} for each memory is required to be much longer than the time Δt_c taken to establish an entanglement connection between two nodes in the network. Since Δt_c is given approximately by the inverse of the rate R_s of detection events for the photons 1 generated by the write pulse, we require roughly

$$\tau_{\text{memory}} \gg (P_1 R_1)^{-1}. \quad (7)$$

Here P_1 gives the probability of detection of a scattered photon from a write pulse in each trial of the experiment and R_1 gives the rate at which successive trials can be carried out. The maximum repetition rate is given by the inverse of the time light takes to travel from one site to the other, so that $R_1 < c/l$. Since $|\chi|^2 \ll 1$ and the propagation and detection efficiencies from an atomic ensemble to D_1, D_2 are also small, $P_1 \sim 10^{-2}$, which with $l \gtrsim 10$ km implies $\tau_{\text{memory}} \gg 10^{-3}$ seconds. Notice that τ_{memory} results, in practice, in a new limit for the maximum distance for communication with this protocol. The search for larger memory times is then one of the major goals in the actual implementation of the protocol, as discussed in more detail in Sec. 6.

3. Nonclassical Photon Pairs from an Atomic Ensemble

As the first step to realize the DLCZ protocol, our group investigated the correlation between the fields 1 and 2 at the single photon level.²⁶ Since photon 2 is generated by reading out the stored collective excitation, these correlation measurements probe directly the nature of the quantum memory in our experiments. We were particularly interested in testing the quantum character of the field correlations in the single-photon regime required by the protocol.

The experimental setup for this investigation is illustrated in Fig. 4. The atomic sample consists of a magneto-optical trap (MOT) of cesium

atoms, where the Cs hyperfine manifolds $\{|6S_{1/2}, F = 4\rangle; |6S_{1/2}, F = 3\rangle; |6P_{3/2}, F' = 4\rangle\}$ correspond to levels $\{|g\rangle; |s\rangle; |e\rangle\}$, respectively. The period for each trial is $4 \mu s$, in which the trap light is switched off during $1 \mu s$. In this “dark” period the write and read pulses are sent into the MOT to generate the two forward scattered fields 1 and 2. In the first experiment²⁶ these fields were then directed onto two single-photon detectors D1 and D2. In the following experiments^{27,28} four detectors were used, as illustrated in Fig. 4. The write pulse is made sufficiently weak so that the probability χ to scatter one Raman photon into field 1 is much less than unity for each pulse. The read pulse is about 100 times more intense than the write pulse to optimize the readout efficiency of the atomic excitation to field 2.

Fig. 4. Experimental setup. Write and read pulses propagate sequentially into a cloud of cold Cs atoms (MOT), generating pairs of correlated output photons 1 and 2. The write and read pulses have orthogonal polarizations, are combined into a single input at polarizing beam splitter PBS1, and are then focused in the Cs MOT with a waist of approximately $30 \mu m$. The output fields are split by PBS2, which also serves as a first stage of filtering the (write, read) beams from the (1,2) fields. For example, field 2 is transmitted by PBS2 to be subsequently registered by detector D_2 while the read pulse itself is reflected by 90° at PBS2. Further filtering is achieved by passing each of the outputs from PBS2 through separate frequency filters. SM stands for single mode fibers.

A challenging aspect of the experiment is to separate the classical pulses from the weak nonclassical fields, since they are temporally and spatially overlapped, and their frequencies are only 9 GHz apart. In the first experiment²⁶ the filtering had three stages. First, field 1 (2) was separated from the write (read) pulse in a polarizing beam splitter (PBS) right after the MOT chamber. Later, the leakage of the excitation pulses that still escapes the PBS in the wrong direction was spectrally filtered by optically-pumped vapor cells.³⁹ Finally, field 1(2) was distinguished from the read

(write) pulse by temporal gating of the detection. A further improvement was to use a four-level scheme of excitation,²⁷ in which write and read pulses are 42 nm apart. This allows a fourth filtering stage by narrow-bandwidth optical filters, and the study of correlations with temporally overlapped write and read pulses.^{27,28}

In order to investigate the quantum nature of the correlations, we use the fact that there exists a well-defined border between the classical and quantum domains for fields 1 and 2 that can be operationally accessed via coincidence detection, as was first demonstrated in the pioneering work by Clauser.⁴⁰ In this way, we measure the joint detection probability $p_{1,2}$ for both detectors (D1 and D2) to record an event in the same trial, and the singles probabilities p_1 and p_2 to register an event in detectors D1 and D2, respectively. By splitting field i with a 50-50 beam splitter and directing the output to the two detectors, the joint probabilities $p_{i,i}$ are also measured, where $i = 1$ or 2 . Fields for which the Glauber-Sudarshan phase-space function is well-behaved (*i.e.*, classical fields) are constrained by a Cauchy-Schwarz inequality for the various probabilities,⁴⁰⁻⁴² namely:

$$R = \frac{[\tilde{g}_{1,2}(t)]^2}{\tilde{g}_{1,1} \tilde{g}_{2,2}} \leq 1, \quad (8)$$

where $\tilde{g}_{1,1} \equiv p_{1,1}/p_1^2$, $\tilde{g}_{2,2} \equiv p_{2,2}/p_2^2$, $\tilde{g}_{1,2}(t) \equiv p_{1,2}/(p_1 p_2)$, and t denotes the time separation between the detection of photons 1 and 2.

In our first experiment,²⁶ we measured $\tilde{g}_{1,1} = 1.739 \pm 0.020$ and $\tilde{g}_{2,2} = 1.710 \pm 0.015$. This is in correspondence to the expectation that fields 1 and 2 should each exhibit Gaussian statistics with $\tilde{g}_{1,1} = \tilde{g}_{2,2} = 2$ for the DLCZ protocol in the ideal case, which is here degraded by diverse sources of background counts.⁴¹ By contrast, for the cross-correlations of fields 1 and 2, we record $\tilde{g}_{1,2}(\delta t) = 2.335 \pm 0.014$, with $\delta t = 415$ ns giving the time difference between the rising edges of write and read pulses. Hence the inequality (8) for classical fields is strongly violated, namely $R = 1.84 \pm 0.06 \not\leq 1$, where all errors indicate the statistical uncertainties. This violation of the Cauchy-Schwarz inequality (8) clearly demonstrates the nonclassical character of the correlations between photons 1 and 2. Moreover, the measured coincidence rates in this experiment explicitly documented the cooperative nature of the emission process.⁴¹

Several developments on these measurements of non-classical correlations followed from this first work. Jiang *et al.*,³⁰ at University of Science and Technology of China, used a magnetically-shielded vapor cell instead of the MOT of the previous work. In this configuration, they were

able to violate the non-classical inequality with a delay between write and read pulses of $2 \mu\text{s}$, for pulse durations of about $1 \mu\text{s}$, explicitly obtaining $R = 1.34 \pm 0.05$. In a setup in our group, the use of a 4-level scheme of excitation improved the filtering, and allowed measurements of large violations of the inequality (8) by decreasing the delay between the excitation pulses.²⁷ Further increase in the violation was observed in the analysis of the temporal structure of the correlations between photons 1 and 2 (see Sec. 5 and Ref. 28). Table 1 summarizes the measured values of R and $\tilde{g}_{1,2}$ for these several situations.

Table 1. Measurements of the violation of the classical inequality $R \leq 1$ and associated cross-correlation function $\tilde{g}_{1,2}$ for photons 1 and 2. δt denotes here the time between the rising edges of write and read pulses, and T_p gives their duration. τ is the temporal window for detection.

Experimental conditions	$\tilde{g}_{1,2}$	R
Cs MOT, 3-level scheme, $\delta t = 405 \text{ ns}$, $T_p \approx 40 \text{ ns}$, $\tau = 60 \text{ ns}$, Ref. 26	2.335 ± 0.014	1.84 ± 0.06
Rb vapor cell, 3-level scheme, $\delta t = 2 \mu\text{s}$, $T_p \approx 1 \mu\text{s}$, $\tau = 1 \mu\text{s}$, Ref. 30	2.043 ± 0.031	1.34 ± 0.05
Cs MOT, 4-level scheme, $\delta t = 200 \text{ ns}$, $T_p \approx 140 \text{ ns}$, $\tau = 200 \text{ ns}$, Ref. 27	7.30 ± 0.04	23 ± 1
Cs MOT, 4-level scheme, $\delta t = 50 \text{ ns}$, $T_p \approx 140 \text{ ns}$, $\tau = 200 \text{ ns}$, Ref. 27	10.20 ± 0.06	53 ± 4
Cs MOT, 4-level scheme, $\delta t = 200 \text{ ns}$, $T_p \approx 140 \text{ ns}$, Optimum-windows analysis with $\tau = 30 \text{ ns}$, Ref. 28	21.4 ± 0.5	202 ± 60
Cs MOT, 4-level scheme, $\delta t = 50 \text{ ns}$, $T_p \approx 140 \text{ ns}$, Optimum-windows analysis with $\tau = 30 \text{ ns}$, Ref. 28	23.0 ± 0.5	292 ± 57

Another closely related set of correlation measurements have been reported in Refs. 29 and 31 by the group of M. Lukin at Harvard University. In Ref. 29, strong intensity correlations were observed between macroscopic fields generated in the spontaneous Raman process, and their quantum nature was demonstrated by an analysis of the fluctuation spectral density with respect to the shot-noise, or vacuum-state limit. This experiment was carried out in the regime of large photon number for the (1,2) fields with $N \approx 10^3 - 10^4$ and the associated fluctuations on the order of $N^{1/2} \gg 1$. In Ref. 31, nonclassical correlations were observed in the regime $N \approx 1$, and the sub-Poissonian nature of the anti-Stokes field generated by the read pulse was demonstrated, as discussed in more detail in the following section.

4. Atomic Ensemble as Conditional Source of Single Photons

The large degree of quantum correlation between fields 1 and 2 observed in Ref. 27 (see Table 1) supports the presence of field correlations for single photons that are required by the DLCZ protocol. This observation, however, allows applications that go beyond this protocol. Most notably, it indicates that this system might work as a good conditional source of single photons.

Efforts to generate single-photon wave packets can be broadly divided into techniques that provide photons “on demand” (*e.g.* quantum dots^{43–45} or single atoms⁴⁶ coupled to microcavities) and those that produce photons as a result of conditional measurement on a correlated quantum system. For conditional generation, the detection of one photon from a correlated pair results in a one-photon state for the second field, as was first achieved using “twin” photons from atomic cascades^{40,47} and parametric down-conversion.⁴⁹ The photon pair production discussed so far in this chapter introduces then a new avenue for producing single photons via conditional measurement, *i.e.*, using optically thick atomic ensembles with quantum memory. The system memory adds the important capability of controlling the delay between the trigger detection and the single photon.

In order to investigate directly the single photon character of photon 2, we have measured the suppression of the two-photon component in the state of photon 2 conditioned on the detection of photon 1. This two-photon component is observed through the correlation function $w(1_2, 1_2|1_1)$ as: $w(1_2, 1_2|1_1) \equiv p^{(c)}(1_2, 1_2|1_1)/[p^{(c)}(1_2|1_1)]^2$. Here, $p^{(c)}(1_2, 1_2|1_1)$ is the conditional probability for detecting two photons from field 2 once an initial photon from field 1 was detected, and $p^{(c)}(1_2|1_1)$ is the probability for detection of one photon 1_2 given a detection event 1_1 . Bayes’ theorem allows the conditional probabilities to be written in terms of single and joint probabilities, so that^{47,48}

$$w \equiv w(1_2, 1_2|1_1) = \frac{p_1 p_{1,2,2}}{p_{1,2}^2}, \quad (9)$$

where $p_{1,2,2}$ gives the joint probability of detecting one photon in field 1 and two photons in field 2. Classical fields must satisfy the inequality $w \geq 1$. For independent coherent states, we should find $w = 1$, while for thermal beams, $w = 2$. However, for the state $|\phi_{12}\rangle$ of Eq. (3) we should have $w = 4\chi$, which is much smaller than 1 for small χ and approaches the ideal case $w \rightarrow 0$ for a “twin” Fock state $|1_1 1_2\rangle$. For the state represented by $\tilde{g}_{1,2}$,

$\tilde{g}_{1,1}$ and $\tilde{g}_{2,2}$ the value of the correlation function w can also be expressed as: $w = \tilde{g}_{1,1}\tilde{g}_{2,2}/\tilde{g}_{1,2}$.

To measure w directly, the record of photodetection events on all four detectors of Fig. 4 was used to calculate the probabilities appearing in Eq. 9, with the results of this analysis shown in Fig. 5. Significantly, as the degree of cross-correlation expressed by $\tilde{g}_{1,2}$ increases (*i.e.*, decreasing χ), w drops below the classical level of unity, indicative of the sub-Poissonian character of the conditional state of field 2. With $\delta t = 200$ ns, $w = 0.34 \pm 0.06$ for $\tilde{g}_{1,2} = 7.3$, while with $\delta t = 50$ ns, $w = 0.24 \pm 0.05$ for $\tilde{g}_{1,2} = 10.2$. The solid line in Fig. 5 is given by $w = A/\tilde{g}_{1,2}$, where $A = \langle \tilde{g}_{1,1} \rangle \langle \tilde{g}_{2,2} \rangle$ is a constant determined from the measurements of $\tilde{g}_{1,1}$ and $\tilde{g}_{2,2}$. No corrections for dark counts or other backgrounds have been applied to the data in Fig. 5. We conclude that the observations shown in Fig. 5 represent a sizable nonclassical effect in support of the conditional generation of single photons for field 2.

Fig. 5. Threefold correlation function w for a detection event 1_1 for field 1 followed by two events $(1_2, 1_2)$ for field 2 versus the normalized cross correlation $\tilde{g}_{1,2}$. $w < 1$ for sub-Poissonian fields in support of the single-photon character of field 2. Statistical uncertainties are indicated by the error bars. The full curve is given by $w = A/\tilde{g}_{1,2}$, where $A = \langle \tilde{g}_{1,1} \rangle \langle \tilde{g}_{2,2} \rangle$ is a constant independently determined from measurements of $\tilde{g}_{1,1}$ and $\tilde{g}_{2,2}$.

Another observation of the single photon character of field 2 was reported by Eisaman *et al.*³¹ Measurements of the second order intensity correlation function $g_{n_S}^{(2)}(AS)$ of field 2 as a function of the mean photon number in field 1 revealed a parameter region where $g_{n_S}^{(2)}(AS) < 1$ (see Fig. 6), indicating the nonclassical character of the anti-Stokes photon states. The Mandel Q parameter³⁷ for field 2 was calculated from the results of Fig. 6 to be $Q = -0.09 \pm 0.03$ for $n_S = 2$ ($Q \geq 0$ for classical states and $Q = -1$ for Fock states).

5. Temporal Structure of the Nonclassical Correlations

As discussed so far in this chapter, central to the DLCZ protocol is the ability to write and read collective spin excitations into and out of an atomic ensemble, with efficient conversion of discrete spin excitations to single-photon wave packets. A critical aspect of such wave packets is that they are emitted into well defined spatiotemporal modes to enable quantum interference between emissions from separate ensembles (*e.g.*, for entanglement based quantum cryptography.²⁵)

Fig. 6. Conditional nonclassical state generation.³¹ Diamonds show experimentally measured values of the second-order intensity correlation function $g_{n_S}^{(2)}(AS)$ for the anti-Stokes pulse conditioned on the detection of n_S photons in the Stokes channel. The inset shows the mean number of photons in the anti-Stokes pulse $\bar{n}_{n_S}^{AS}$ as a function of n_S . For classical light, $g^{(2)} \geq 1$. The solid and dashed lines come from theoretical models explained in Ref. 31.

Following initial measurements of pulse shapes for the fields (1,2) (Fig. 2 of Ref. 29), Eisaman *et al.* have reported a detailed investigation of the manipulation of the temporal profile for the Stokes and anti-Stokes fields³¹ in the few photon regime. As shown in Fig. 7, these authors achieved good correspondence between their measurements and the theoretically calculated photon fluxes. The observed dynamics provide evidence for the collective nature of the process, and indicates possible ways to control the shape of the single-photon wave packets.

The high efficiencies achieved in the work of Ref. 27 enabled us to investigate in detail the temporal properties of the nonclassical correlations between emitted photon pairs,²⁸ providing a direct look at different im-

Fig. 7. Stokes and anti-Stokes pulse shapes, from Ref. 31. (a) Experimentally measured and theoretically calculated values of the Stokes photon flux dn_S/dt . For each plot, n_S represents the total number of photons emitted from the cell. Write laser power was varied from 25 μ W to 100 μ W. (b) Experimentally measured (left) and theoretically calculated (right) values of the anti-Stokes photon flux dn_{AS}/dt . The experimental pulse shapes correspond to a Stokes pulse with $n_S \approx 3$ photons, and the theoretical curves assume an initial spin-wave with $n_{spin} = 3$ excitations and an optical depth of ≈ 20 . Each curve is labeled with the power of the retrieve laser. (b and inset) Theoretical calculation of the number of flipped spins per unit length dn_{spin}/dz for $n_{spin} = 3$. (c) Measured anti-Stokes pulse width (full-width at half-max) and total photon number as a function of the retrieve laser intensity.

portant features of the complete two-photon wave packet (field 1 + field 2) generated by the system. To address the photon statistics, we use four avalanche photodetectors, a pair for each field, as in the previous section. In the following analysis, our main quantity of interest is $p_\tau(t_1, t_2)$, the joint probability for photoelectric detection of photon 1 at time t_1 and photon 2 at time t_2 within a time window given by τ . The times for this quantity are counted starting from the beginning of the write pulse. Two other quantities necessary to measure R with temporal resolution are $p_\tau(t_1, t_1)$ and $p_\tau(t_2, t_2)$, the joint probabilities for two-photon events in fields 1 and 2, respectively. These quantities are determined from the record of time-stamped detections on all four photodetectors, allowing us to measure autocorrelations and cross correlations simultaneously. The detectors have a time resolution of 2 ns (minimum bin size), but usually we need to con-

sider larger bins to acquire enough events for the statistics. The specific expression that we use to calculate R from these quantities is

$$R_{\tau}(t_1, t_2) \equiv \frac{[p_{\tau}(t_1, t_2)]^2}{p_{\tau}(t_1, t_1)p_{\tau}(t_2, t_2)}. \quad (10)$$

Another important quantity to measure is the joint probability $q_{\tau}(t_1, t_2)$ for detections from fields 1 and 2 coming from different trials, *i.e.*, uncorrelated. In this case, t_1 and t_2 have different time origins, they begin on the rising edge of write pulses from different trials. This quantity is expected to be proportional to the product of intensities of the fields 1 and 2, in reasonable correspondence to the form shown in Figs. 8(b) and 8(d) for our roughly rectangular write and read pulses, but distinctively different from $p_{\tau}(t_1, t_2)$ in Figs. 8(a) and 8(c). The comparison of $p_{\tau}(t_1, t_2)$ and $q_{\tau}(t_1, t_2)$ quantifies the degree of two-photon correlation for the system.

In our experiment, we focus on two cases: (I) nearly simultaneous application of write and read pulses with offset $\delta t = 50$ ns less than the duration of either pulse, and (II) consecutive application of write and read pulses with $\delta t = 200$ ns. Results for $p_{\tau}(t_1, t_2)$ and $q_{\tau}(t_1, t_2)$ are presented in Fig. 8. For both $\delta t = 50$ and 200 ns, $p_{\tau}(t_1, t_2) \gg q_{\tau}(t_1, t_2)$, indicating the strong correlation between fields 1 and 2, with the maximal ratio $\tilde{g}_{1,2} = p_{\tau}(t_1, t_2)/q_{\tau}(t_1, t_2) \gtrsim 30$. In Fig. 8, $\tau = 4$ ns, leading to statistical errors of about 8% for the largest values shown.

In case (I), Fig. 8a shows that $p_{\tau}(t_1, t_2)$ peaks along the line $t_2 - t_1 = \delta t_{12} \simeq 50$ ns with a width $\Delta t_{12} \simeq 60$ ns, in correspondence to the delay δt_{12} and duration Δt_{12} for read-out associated with the transition $|b\rangle \rightarrow |e'\rangle \rightarrow |a\rangle$ given an initial transition $|a\rangle \rightarrow |e\rangle \rightarrow |b\rangle$.²⁹ In case (II) with the read pulse launched 200 ns after the write pulse, excitation is “stored” in the atomic ensemble until the readout. The production of correlated photon pairs should now be distributed along $t_2 \simeq \delta t + \delta t_{12}$ with width $\simeq \Delta t_{12}$. Instead, as shown in Fig. 8c, $p_{\tau}(t_1, t_2)$ peaks towards the end of the write pulse (*i.e.*, $t_1 \gtrsim 100$ ns), and near the beginning of the read pulse (*i.e.*, $200 \lesssim t_2 \lesssim 300$ ns). Early events for field 1 lead to fewer correlated events for field 2, as $p_{\tau}(t_1, t_2)$ decays rapidly beyond the line $t_2 - t_1 = \tau_d \simeq 175$ ns. The marked contrast between $p_{\tau}(t_1, t_2)$ for $\delta t = 50$ and 200 ns results in a diminished ability for the conditional generation of single photons from excitation stored within the atomic ensemble²⁷ and, more generally, for the implementation of the DLCZ protocol for increasing Δt . The underlying mechanism is decoherence within the ensemble, as will be discussed in the next section.

Fig. 8. Probability of joint detection scaled by 10^9 from fields (1,2) at times (t_1, t_2) with $t_1 = t_2 = 0$ at the beginning of the write pulse. $p_\tau(t_1, t_2)$ comes from joint detections within the same trial. $q_\tau(t_1, t_2)$ gives the probability of joint detection from different trials, *i.e.*, uncorrelated. (a) p_τ and (b) q_τ for overlapped write and read pulses, $\delta t = 50$ ns. The solid line corresponds to $t_2 = t_1$. (c) p_τ and (d) q_τ for consecutive non-overlapping write and read pulses, $\delta t = 200$ ns. Note that $p_\tau \gg q_\tau$ in both situations, indicating the strong correlation between fields 1 and 2.

The temporal dependence of R closely follows $p_\tau(t_1, t_2)$.²⁸ To deduce R from Eq. 10, we acquired the joint detection probabilities $p_\tau(t_1, t_1)$ for field 1 and $p_\tau(t_2, t_2)$ for field 2 from the same record of photoelectric events as for Figs. 8a and 8c. The maximal observed ratio R in a short temporal window of 30 ns is $R^{\max} = 292 \pm 57$ for $\Delta t = 50$ ns and $R^{\max} = 202 \pm 60$ for $\delta t = 200$ ns ($R = 198 \pm 33$ in the neighboring bin), which strongly violate the classical inequality $R \leq 1$. The relatively large errors in R arise predominantly from the uncertainties in $p_\tau(t_1, t_1)$ and $p_\tau(t_2, t_2)$, as the rate of detection of two photons in the same field is much lower than that of the correlated detection.

6. Decoherence in the Atomic Ensemble

As discussed in the previous section, the temporal structure of the two-photon wave packet emitted by the sample is strongly affected by decoherence effects, once the read and write pulses are delayed such that they do

not overlap. This decoherence results in a strong decrease of the correlations between photons 1 and 2. The timescale for this decrease is on the order of 100 ns, very short for any process that relies only on atomic ground-state coherence.

This decoherence timescale, however, is consistent with the inhomogeneous broadening of the ground state caused by the quadrupole magnetic field of the trap. In order to confirm that this is indeed the origin of the observed fast decoherence, we developed a model for the process taking into account the effect of the spatial variation of the MOT magnetic field and the resulting splitting of the atomic Zeeman states, and compared these results with measurements of $\tilde{g}_{1,2}$ as a function of the delay δt between read and write pulses. Figure 9a shows the result of this comparison.

In Fig. 9a, the normalized correlation function $\tilde{g}_{1,2}$ was determined from the ratio of integrated coincidence counts to singles counts over the entire detection window of 200 ns. The initial growth of $\tilde{g}_{1,2}$ for small δt comes from the fact that photon 2 can only be detected after the generation of a photon 1, since we initially optically pump all atoms to the $F = 4$ hyperfine state. In this way, if the read pulse arrives prior to the write pulse ($\delta t = -T_p$) we should observe no coincidence counts, and the number of coincidences should grow as the read pulse is delayed toward positive values. If the decoherence timescale is much longer than the pulse duration and the read pulse is weak, the integral number of coincidences should reach a maximum when the pulses are exactly back to back ($\delta t = T_p$), and then stay constant at this maximum for further delays. For strong read pulses, the maximum can be achieved at a shorter timescale, on the order of $\delta t \approx \delta t_{12}$ (see comments to Fig. 8a), staying constant later on. For the experiments described in the previous sections, the decoherence time is on the order of the pulse duration and the read pulse is strong, resulting in a maximum at $\delta t < T_p$ followed by a fast decay, as described by the open circles in Fig. 9a.

Since this strong decoherence seems to be related to the MOT quadrupole field, an immediate way to improve the coherence time is then to turn off this field. This needs to be done, however, as fast as possible in order to keep the high optical density of the atomic medium together with a high repetition rate for interrogation trials. The measurements of $\tilde{g}_{1,2}(\delta t)$ in this situation are shown in Fig. 9a (squares). Note that the coherence time improves significantly, but there is still a decay related to residual magnetic fields (≈ 30 mG) in the MOT region. When comparing the curves with and without magnetic field, it is important to have in mind that they were taken at different coupling conditions and atomic densities, so that the

change in their maxima should not be attributed only to a change in the magnetic field. The modification in the rate of loss of coherence, however, can only be observed by turning off the field.

Fig. 9. (a) Variation of $\tilde{g}_{1,2}$ with the delay δt between write and read pulses. The open circles (filled squares) give the experimental results in the presence (absence) of the MOT magnetic field. The solid (dashed) curve is the theoretical result for $K = 1.1$ MHz ($K = 40$ kHz). (b) Variation of $\tilde{p}_{1,2}$ with the delay δt between write and read pulses for (solid curve) $K = 1.1$ MHz and an unpolarized sample, (dashed curve) $K = 40$ kHz and an unpolarized sample, and (dash-dotted curve) $K = 40$ kHz and an initially spin polarized sample with all atoms in $|F = 4, m_F = 0\rangle$. The dotted curve corresponds to an initially spin polarized sample classically excited by fields with polarizations such that only a magnetic insensitive transition is allowed, see text for details. In (b), the same arbitrary scaling factor was used for all curves.

In order to understand in detail the shape of the experimental curves in Fig. 9a, we extended the theoretical treatment of Duan *et al.*³³ to include the reading process as well as the full set of Zeeman states for the $F = 3, 4$ hyperfine levels. The sample of Cs atoms is assumed to be initially unpolarized and distributed over the same range of magnetic fields as for the MOT. With write and read pulses that approximate those used in our experiment and separated by δt , we calculate the joint probability $p_{1,2}^{th}(\delta t)$ to generate a pair of photons in the fields (1, 2). We compare the quantity $\tilde{p}_{1,2} \equiv \xi p_{1,2}^{th}$ to the measured $\tilde{g}_{1,2}$ by way of a single overall scaling parameter ξ for all δt , as the rate of single counts in fields (1, 2) is measured not to depend on δt (to within 20%). The form of $\tilde{p}_{1,2}$ strongly depends upon the inhomogeneity of Zeeman splitting across the MOT, which is described by the parameter $K = \mu_B g_{F_g} L b / h$, where L is the MOT diameter, b is the gradient of the magnetic field for the MOT, and g_{F_g} is the Landé factor. The solid curve in Fig. 9a is the theoretical result for an initially unpolarized

sample with $K = 1.1$ MHz, as for the experiment with the quadrupole field on (where $L \approx 3.6$ mm and $b \approx 8.4$ G/cm). The fast decay of the correlations is then adequately described by the model.

The dashed curve in Fig. 9a gives the decay expected for a magnetic-field gradient such that $K = 40$ kHz. This gives a reasonable approximation to the behavior of $\tilde{g}_{1,2}$ under the action of the residual magnetic field, even though the spatial dependence of this field can be more complicated than a simple linear gradient. The change in K from 1.1 MHz to 40 kHz is consistent with the reduction of the ground state linewidth between the two cases, as measured directly by Raman spectroscopy⁵⁰ in our setup.

The theory developed to explain the data in Fig. 9a, on the other hand, can also be used to devise new ways to improve the system. The inclusion of the Zeeman structure in the theory, for example, allows the study of different polarization schemes for both classical excitation and photon detection. It also allows the investigation of the role of the atomic initial state on the measured correlations. In Fig. 9b we give two examples of possible ways to improve the system. The solid and dashed lines in the figure represent the two experimental conditions of Fig. 9a (initially unpolarized samples with $K = 1.1$ MHz and $K = 40$ kHz), but now with the same scaling factor. The dash-dotted curve shows how the $K = 40$ kHz curve changes if the system is initially spin polarized, with all atoms in the $|F = 4, m_F = 0\rangle$ state. Note that in this case the value of $\tilde{g}_{1,2}$ considerably increases, and the system develops a plateau coming from the predominant transition $|F = 4, m_F = 0\rangle \rightarrow |F = 3, m_F = 0\rangle \rightarrow |F = 4, m_F = 0\rangle$, which is magnetic-field insensitive. Furthermore, it is possible to devise a polarization scheme of excitation that allows only this specific transition for any δt , as *e.g.* when the write pulse and field-1 detection are σ^+ polarized, and the read pulse and field-2 detection are σ^- . This is the case for the dotted curve in Fig. 9b.

The idealized improvements described by the dotted and dash-dotted curves of Fig. 9b, however, will probably be limited by two effects which are not taken into account by the theory. First, in our experimental setup we should see a decay with a timescale on the order of $100 \mu\text{s}$ due to the average time the cold atoms take to cross the $30 \mu\text{m}$ beam waist of the classical write and read pulses. Second, the theory assumes the presence of a magnetic field predominantly in the z direction, which defines the quantization axis. This can be obtained by applying an extra DC magnetic field along that direction,^{51,52} but any residual transverse field should lead to some decay of the plateau. In spite of these restrictions, however, we believe that such

improvements could lead to an increase of perhaps two orders of magnitude over the largest experimental decoherence time of Fig. 9a. It is also clear that there is a benefit in the careful preparation of the initial state for the magnitude of the measured correlations. This is an important point that should also be taken into account when considering the implementation of the DLCZ protocol in vapor cells.

It is essential to have in mind, however, that all coherence-time values discussed above are still far from fulfilling the requirement imposed by Eq. (7) on τ_{memory} . The largest memory times that can be obtained in vapor cells or MOTs (of the order of milliseconds) should be enough to provide proof of principle demonstrations of all of the critical elements of the DLCZ protocol. However, in order for this protocol to become really an alternative to long distance quantum communication, new technologies must be introduced. An association of purely optical traps (*e.g.*, a FORT⁵³) with good magnetic shielding may eventually provide the required duration for the coherence time.

7. Prospect for Entanglement between Distant Ensembles

All previous sections discussed experiments related to the control of the photon generation process that is the basis of the DLCZ protocol. As explained in Sec. 2, however, this is just the first step. Entangling two spatially separated atomic ensembles is now the next essential step towards the realization of the protocol. In this section, we discuss the requirements to realize experimentally such entanglement and how to verify it.

As already mentioned in Sec. 2, to create entanglement between two spatially separated ensemble, L and R , we start by simultaneously exciting the two ensembles with a weak write pulse. At that point, the state of the system is given by Eq. (4). The photons emitted in the forward scattered-mode in each ensemble are then directed towards the input modes of a 50-50 beam splitter, as shown in Fig. 2. If the excitation probability is small enough and provided that the two fields entering the beam splitter are indistinguishable, the information concerning the origin of those fields is erased, and a detection event in the photodetector projects the two ensembles onto an entangled state. In an idealized setting, the resulting state is

$$|\Psi_{LR}\rangle = [\epsilon_L|1_a\rangle_L|0_a\rangle_R \pm e^{i\eta_1}\epsilon_R|0_a\rangle_L|1_a\rangle_R], \quad (11)$$

where $\epsilon_L(\epsilon_R)$ is the normalized amplitude of photon generation from ensemble $L(R)$, with $\epsilon_L^2 + \epsilon_R^2 = 1$. The phase η_1 is the sum of two contributions:

$\eta_1 = \Delta\beta + \Delta\gamma$, where $\Delta\beta$ is the phase difference of the classical write beams at the two ensembles L and R , and $\Delta\gamma$ is the difference of phase shift acquired by the field 1 during the propagation from the ensembles to the beam splitter. Note that this phase η_1 should be well-defined operationally and remain constant, in order to create a measurable entangled state.

To verify that an entangled state of the two ensembles has been created, the first step is to map the delocalized atomic excitation into a field state, by applying simultaneously a strong read beam at the two ensembles. Assuming that the mapping succeeds with a probability unity in an idealized case, the conditional overall state of the system becomes:

$$|\Psi_{LR}\rangle = [\epsilon_L|1_2\rangle_L|0_2\rangle_R \pm e^{i(\eta_1+\eta_2)}\epsilon_R|0_2\rangle_L|1_2\rangle_R], \quad (12)$$

where the phase η_2 is, similarly to η_1 , the sum of two contributions, taking into account the phase difference of the read beams at the ensembles and the propagation phase of field 2. In this idealized setting, the atomic entangled state has now been mapped onto a photonic state entangled in the photon number basis (one photon with the vacuum), in two different spatial modes L and R . To show entanglement, there are two possibilities. The first one is to use the scheme proposed by Tan *et al.*⁵⁴ to demonstrate non-locality of a single photon (Fig. 10a). The second one is to reconstruct the density matrix of the entangled state, a procedure known as quantum tomography (Fig. 10b). The former method has the advantage that it detects entanglement irrespective of assumptions about multiple (and zero) excitations. On the other hand, it does not determine how much entanglement one created. Tomography does give a quantitative answer but requires one to explicitly take into account multiple- and zero-excitation probabilities.

Of course, the states in Eqs. (3)-(6) and (11)-(12) are idealizations relative to an actual experiment. Specifically, Eq. (6) is not realistic in that (a) terms with 2 or more excitations in either ensemble are neglected, (b) it is assumed that all excitations are in the correct “modes”, both for the signal mode and the atomic ensembles, (c) it is assumed that each excitation of the atomic medium leads to a signal photon, and (d) diverse sources of uncorrelated background light are omitted. Hence any verification protocol must be expanded beyond the idealizations of Eqs. (11),(12) to provide a robust, model independent determination of entanglement.⁵⁵

An initial attempt to create entanglement between two ensembles located within the same MOT has been reported by Matsukevich and Kuzmich.³² Unfortunately, due to a verification protocol that does not address many of the system’s imperfections listed above, the interpretation

of their results is strongly model dependent.⁵⁵ Although the experiment demonstrated a substantial amount of coherence between the two ensembles, appropriate analysis and measurements are missing to determine conclusively the creation of entanglement. Particularly, we understand that the measurements reported in Ref. 32 can be reproduced by states of the two ensembles that are unentangled.⁵⁵

Fig. 10. Schematic setup for verification of entanglement between two atomic ensembles L and R , for entanglement generated as suggested by the DLCZ protocol. Conditioned on the detection of a first photon 1, the single excitation stored in the pair of ensembles is retrieved by read pulses that simultaneously excite each ensemble. (a) In the single photon non-locality scheme, the two output channels from the ensembles are directed to separate homodyne-detection setups. Correlations between the two detection regions are then used to verify if a Bell's inequality is violated, in which case entanglement between the two output channels is demonstrated. LO stands for local oscillator. (b) In the quantum tomography scheme, the two output channels from the ensembles are directed to the same beam splitter, and a phase ϕ can be introduced in one of the channels in a controlled way. The overall detection probabilities for single and double events are then obtained, together with the visibility of fringes observed with the variation of ϕ . These quantities are then used to reconstruct the density matrix ρ of the system, from which a measure of entanglement can be calculated.

7.1. Single photon non-locality

The scheme of Tan, Walls, and Collett⁵⁴ to demonstrate single photon non-locality is a double homodyne measurement, in which the signals from the two modes L and R are mixed with a local oscillator (LO) at a 50-50 beam splitter, as shown in Fig. 10a. The intensity of the LO must be very weak, such that when a photodetection occurs after the beam splitter, there is in

principle no way of telling if the photon came from the signal or from the LO. If the coincidence count rate between two detectors placed after the beam splitters is recorded as a function of the phase shift between the two LOs, an interference fringe is expected with a visibility:⁵⁴

$$V = \frac{1}{(\alpha^2 + 1)}, \quad (13)$$

where α is the amplitude of the LO. If α is very small, the visibility tends towards one. If V is higher than $1/\sqrt{2}$, a Bell inequality can be violated when four detectors are used.

This experiment requires the detection of triple coincidences, since the entanglement is always conditioned on the detection of a first photon. To date, only one experiment has demonstrated this scheme, using conditional single photons generated from parametric down conversion.⁵⁶ However, the background due to coincidences from either two signal photons or two photons from the LO had to be subtracted, in order to obtain enough visibility to violate a Bell inequality. Moreover, the standard analysis needs to be extended beyond the ideal state of Eq. (12) to include the various imperfections of any implementation of the DLCZ protocol.²⁵

7.2. Quantum tomography

Another possibility to demonstrate entanglement is to reconstruct the density matrix $\rho_{L,R}$ of the photonic state in the two modes L and R and to calculate the amount of entanglement using *e.g.* Wootters' formula.⁵⁷ As previously discussed, the state given by Eq. 12 is an ideal state. In a real experiment, the state will be a mixed state containing contaminations from vacuum and higher order photon states, due to inherent atomic excitation statistics and various background light sources. If one neglects the events where two excitations are created in the same ensemble,⁵⁸ the total photonic state is an entangled state of two qubits living in a Hilbert space spanned by: $|1_2\rangle_L|0_2\rangle_R$, $|0_2\rangle_L|1_2\rangle_R$, $|1_2\rangle_L|1_2\rangle_R$ and $|0_2\rangle_L|0_2\rangle_R$. The density matrix can be written (for simplicity, we ignore all coherence terms between states with different numbers of excitations):

$$\rho_{L,R} = \begin{pmatrix} p_{00} & 0 & 0 & 0 \\ 0 & p_{01} & D & 0 \\ 0 & D^* & p_{10} & 0 \\ 0 & 0 & 0 & p_{11} \end{pmatrix}, \quad (14)$$

where D satisfies $|D|^2 \leq p_{01}p_{10}$. The diagonal elements, which are the probabilities p_{ij} to find i photons in mode L and j photon in mode R , can

be determined by directly counting the number of photons in each mode. The coherence term D can be determined by adding a phase shift in one mode and then mixing the resulting state on a 50/50 beam splitter (see Fig. 10b). By recording the count rate after the beam splitter as function of the phase shift, one can measure an interference fringe with a visibility V . It can be shown that $V = 2|D|/(p_{10} + p_{01})$. Once the density matrix has been reconstructed, the amount of entanglement can be determined using Wootters' formula. The result for the concurrence C is:

$$C = \max(2|D| - 2\sqrt{p_{00}p_{11}}, 0). \quad (15)$$

The entanglement is given by:

$$E = h\left(\frac{1 + \sqrt{1 - C^2}}{2}\right) \quad (16)$$

with $h(p) = -p \log_2 p - (1-p) \log_2 (1-p)$. We see that the probabilities p_{00} and p_{11} to have no excitation and one excitation in each mode, respectively, are essential in order to determine the amount of entanglement. Advantages of this method are that it requires no LO and that the coherence terms (which requires interferometric stability) can be determined with the record of double coincidences.

Finally, it is important to have in mind that probabilities such as p_{00} and p_{11} must be deduced from corresponding photodetection probabilities, *e.g.*, Q_{00} and Q_{11} . The actual experimental values (p_{00}^{exp} , p_{11}^{exp} , ...) to be used as inputs for the previous expressions to evaluate the amount of entanglement in the system must always be obtained by an inversion procedure of the direct measurements of photodetection probabilities. This inversion procedure takes into account, *e.g.*, the propagation and detection efficiencies in the experimental setup.

8. Concluding Remarks

The DLCZ protocol represents today one of the most promising schemes to solve the problem of long distance quantum communication. It suggests new ways to store information in atomic ensembles, to manipulate it, and to generate entanglement between remote systems. In this chapter, we have presented a discussion of initial experimental steps towards the realization of this protocol²⁵ that are being pursued by groups worldwide.^{26–32} These experiments have already made important progress to generate and store single quanta in atomic ensembles.

However, much work remains to be done in order to demonstrate an elementary version of a quantum repeater using the DLCZ protocol. In particular, many experimental challenges must be solved in order to connect distant atomic ensembles. Nevertheless, the results presented in this chapter already indicate that the implementation of this protocol will lead to the development of many important tools for various tasks in quantum information science and quantum control.

Acknowledgments

The authors gratefully acknowledge fruitful discussions with W. Bowen, A. Boca, J. McKeever, and D. Boozer. This work is supported by ARDA, by the Caltech MURI Center for Quantum Networks, and by the NSF. D.F. is supported by a fellowship from CNPq (Brazilian agency). H.d.R. is supported by a fellowship from the Swiss National Science Foundation.

References

1. *The Physics of Quantum Information*, D. Bouwmeester, A. Ekert, and A. Zeilinger, Springer-Verlag, Berlin, Germany, 2001.
2. I. Marcikic, H. de Riedmatten, W. Tittel, H. Zbinden, M. Legré, and N. Gisin, *Phys. Rev. Lett.* **93**, 180502 (2004).
3. See the website of the companies *MagiQ* (www.magiqtech.com) and *id Quantique* (www.idquantique.com).
4. N. Gisin, G. Ribordy, W. Tittel, and H. Zbinden, *Rev. Mod. Phys.* **74**, 145 (2002).
5. D. Stucki, N. Gisin, O. Guinnard, G. Ribordy, and H. Zbinden, *New J. Phys.* **4**, 41 (2002).
6. C. Gobby, Z.L. Yuan, and A.J. Shields, *Appl. Phys. Lett.* **84**, 3762 (2004).
7. K.J. Resch, M. Lindenthal, B. Blauensteiner, H.R. Bhm, A. Fedrizzi, C. Kurtsiefer, A. Poppe, T. Schmitt-Manderbach, M. Taraba, R. Ursin, P. Walther, H. Weier, H. Weinfurter, and A. Zeilinger, *Opt. Exp.* **13**, 202 (2005).
8. C.-Z. Peng, T. Yang, X.-H. Bao, J.-Zhang, X.-M. Jin, F.-Y. Feng, B. Yang, J. Yang, J. Yin, Q. Zhang, N. Li, B.-L. Tian, and J.-W. Pan, *quant-ph/0412218* (2004).
9. B.C. Jacobs and J.D. Franson, *Opt. Lett.* **21**, 1854 (1996).
10. R.J. Hughes, J.E. Nordholt, D. Derkacs, and C.G. Peterson, *New J. Phys.* **4**, 43 (2002).
11. C. Kurtsiefer, P. Zarda, M. Halder, H. Weinfurter, P.M. Gorman, P.R. Tapster, and J.G. Rarity, *Nature* **419**, 450 (2002).
12. G. Brassard, N. Lütkenhaus, T. Mor, and B.C. Sanders, *Phys. Rev. Lett.* **85**, 1330 (2000).
13. W.K. Wootters, and W.H. Zurek, *Nature* **299**, 802 (1982).

14. C.H. Bennett, G. Brassard, S. Popescu, B. Schumacher, J.A. Smolin, and W.K. Wootters, *Phys. Rev. Lett.* **76**, 722 (1996).
15. D. Deutsch, A. Ekert, R. Jozsa, C. Macchiavello, S. Popescu, and A. Sanpera, *Phys. Rev. Lett.* **77**, 2818 (1996).
16. H.-J. Briegel, W. Dür, J.I. Cirac, and P. Zoller, *Phys. Rev. Lett.* **81**, 5932 (1998).
17. W. Dür, H.-J. Briegel, J.I. Cirac, and P. Zoller, *Phys. Rev. A* **59**, 169 (1999).
18. C.H. Bennett, G. Brassard, C. Crépeau, R. Jozsa, A. Peres, and W.K. Wootters, *Phys. Rev. Lett.* **70**, 1895 (1993).
19. M. Zukowski, A. Zeilinger, M.A. Horne, and A.K. Ekert, *Phys. Rev. Lett.* **71**, 4287 (1993).
20. *Cavity Quantum Electrodynamics*, P. Berman (Eds.), Academic Press, San Diego 1994.
21. H.J. Kimble, *Phys. Scr.*, T **76**, 127 (1998).
22. H. Mabuchi and A.C. Doherty, *Science* **298**, 1372 (2002).
23. J.I. Cirac, P. Zoller, H.J. Kimble, and H. Mabuchi, *Phys. Rev. Lett.* **78**, 3221 (1997).
24. S.J. van Enk, J.I. Cirac, and P. Zoller, *Science* **279**, 205 (1998).
25. L.-M. Duan, M.D. Lukin, J.I. Cirac, and P. Zoller, *Nature* **414**, 413 (2001).
26. A. Kuzmich, W.P. Bowen, A.D. Boozer, A. Boca, C.W. Chou, L.-M. Duan, and H.J. Kimble, *Nature* **423**, 731 (2003).
27. C.W. Chou, S.V. Polyakov, A. Kuzmich, and H.J. Kimble, *Phys. Rev. Lett.* **92**, 213601 (2004).
28. S.V. Polyakov, C.W. Chou, D. Felinto, and H.J. Kimble, *Phys. Rev. Lett.* **93**, 263601 (2004).
29. C.H. van der Wal, M.D. Eisaman, A. Andr, R.L. Walsworth, D.F. Phillips, A.S. Zibrov, and M.D. Lukin, *Science* **301**, 196 (2003).
30. W. Jiang, C. Han, P. Xue, L.-M. Duan, and G.-C. Guo, *Phys. Rev. A* **69**, 043819 (2004).
31. M.D. Eisaman, L. Childress, A. Andr, F. Massou, A.S. Zibrov, and M.D. Lukin, *Phys. Rev. Lett.* **93**, 233602 (2004).
32. D. N. Matsukevich and A. Kuzmich, *Science* **306**, 663 (2004).
33. L.-M. Duan, J.I. Cirac, and P. Zoller, *Phys. Rev. A* **66**, 023818 (2002).
34. M.D. Lukin, *Rev. Mod. Phys.* **75**, 457 (2003).
35. D.A. Braje, V. Balić, G.Y. Yin, and S.E. Harris, *Phys. Rev. A* **68**, 041801 (2003).
36. D.A. Braje, V. Balić, S. Goda, G.Y. Yin, and S.E. Harris, *Phys. Rev. Lett.* **93**, 183601 (2004).
37. L. Mandel and E. Wolf, *Optical Coherence and Quantum Optics*, Cambridge Univ. Press, Cambridge, UK, 1995.
38. B.C. Jacobs, T.B. Pittman, and J.D. Franson, *Phys. Rev. A* **66**, 052307 (2002).
39. E.B. Alexandrov, M.V. Baladas, D. Budker, D. English, D.F. Kimball, C.-H. Li, and V.V. Yashchuk, *Phys. Rev. A* **66**, 042903 (2002); *Phys. Rev. A* **70**, 049902 (2004).
40. J.F. Clauser, *Phys. Rev. D* **9**, 853 (1974).

41. Supplementary Information accompanying Ref. 26 at <http://www.nature.com/nature/journal/v423/n6941/supinfo/nature01714.html>.
42. *Optical Coherence and Quantum Optics*, L. Mandel and E. Wolf, Cambridge Univ. Press, Cambridge, UK, 1995.
43. P. Michler, A. Kiraz, C. Becher, W. V. Schoenfeld, P. M. Petroff, Lidong Zhang, E. Hu, and A. Imamoglu, *Science* **290**, 2282 (2000).
44. E. Moreau, I. Robert, J.M. Gérard, I. Abram, L. Manin, and V. Thierry-Mieg, *Appl. Phys. Lett.* **79**, 2865 (2001).
45. M. Pelton, C. Santori, J. Vuckovic, B. Zhang, G.S. Solomon, J. Plant, and Y. Yamamoto, *Phys. Rev. Lett.* **89**, 233602 (2002).
46. J. McKeever, A. Boca, A. D. Boozer, R. Miller, J. R. Buck, A. Kuzmich, and H. J. Kimble, *Science* **303**, 1992 (2004).
47. P. Grangier, G. Roger, and A. Aspect, *Europhys. Lett.* **1**, 173 (1986).
48. P.G. Kwiat and R.Y. Chiao, *Phys. Rev. Lett.* **66**, 588 (1991).
49. C.K. Hong and L. Mandel, *Phys. Rev. Lett.* **56**, 58 (1986).
50. J. Ringot, P. Szriftgiser, and J.C. Garreau, *Phys. Rev. A* **65**, 013403 (2001).
51. G. Avila, V. Giordano, V. Candelier, E. de Clercq, G. Theobald, and P. Cerez, *Phys. Rev. A* **36**, 3719 (1987).
52. S.-I. Ohshima, Y. Nakadan, and Y. Koga, *IEEE Trans. Instrum. Meas.* **37**, 409 (1988).
53. J.D. Miller, R.A. Cline, and D.J. Heinzen, *Phys. Rev. A* **47**, 4567 (1993).
54. S.M.Tan, D.F.Walls, and M.J.Collett, *Phys.Rev.Lett.* **66**, 252 (1991).
55. S.J. van Enk and H.J. Kimble, submitted to publication.
56. B. Hessmo, P. Usachev, H. Heydari, and G. Björk, *Phys.Rev.Lett.* **92**, 180401 (2004).
57. W.K. Wootters, *Phys.Rev.Lett* **80**, 2245 (1998).
58. This approximation is applied just to explain the basic elements in the idea of quantum tomography. In the actual experiment, components with larger photon number have necessarily to be taken into account. In particular, the probabilities p_{02} and p_{20} arise in the same order as does p_{11} . In this more general case, one cannot apply Wootters' formula, but instead one could calculate the negativity, as defined by G. Vidal and R.F. Werner [*Phys. Rev. A* **65**, 032314 (2002)], to determine the amount of entanglement.