Frequency Offset Tracking for OFDM Systems via Scattered Pilots and Virtual Carriers

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***Abstract*— In this paper, we propose a new carrier frequency offset (CFO) tracking algorithm for orthogonal frequency division multiplexing (OFDM) systems. Assuming that the channel remains constant during two consecutive OFDM blocks, a CFO estimation algorithm is proposed based on the limited number of pilots in each OFDM block. Identifiability of this pilot based algorithm is fully discussed under the noise free environment. A weighted algorithm is then developed by considering both pilot carriers and virtual carriers. The asymptotic mean square error (MSE) of the proposed algorithm is provided, and simulation results clearly show the performance improvement of the proposed algorithm over the existing methods.**

I. Introduction

Orthogonal frequency-division multiplexing (OFDM) [1], is a promising candidate for next generation high-speed wireless communication systems. In OFDM systems, it is well known that a CFO, caused by oscillators' mismatch or Doppler effects, destroys the subcarriers orthogonality and results in a substantial bit error rate (BER) degradation [2].

Frequency synchronization for OFDM systems usually contains the acquisition stage and the tracking stage. Several CFO acquisition methods have been proposed in [3]- [7] by using the periodic nature of the time domain signal. However, using the periodic nature greatly reduces the CFO estimation region. Furthermore, all these methods, except [3]- [4], are only applicable in CFO acquisition stage because consecutive training blocks are required.

In the tracking stage, the CP based method [3], [4], can still be used for residue CFO estimation. This method will be referred as Beek's method in this paper. However, the performance of Beek's method depends critically on the difference between the length of the CP and the channel length. Therefore, an alternative way is to implement CFO tracking based on the scattered pilots available in existing OFDM standards. However, all the above mentioned CFO estimation methods [3]- [7] fail to work with the limited number of pilots. To the best of the authors' knowledge, only an algorithm in [8] considers the scattered pilot symbols. This method is referred as Classen&Meyr's method in this paper. There are two drawbacks of Classen&Meyr's method: 1) It is only applicable to a sufficiently small CFO; 2) An error floor appears at high SNR.

In this paper, we first develop a new CFO tracking algorithm by using the scattered pilot carriers. Identifiability of this pilot aided algorithm is studied for the noise free case. We then consider further utilizing the virtual carriers existing in practical OFDM standards. A weighted algorithm, called pv-algorithm is then proposed by exploiting both scattered pilots and virtual carriers. We show that in the pv-algorithm, the p-algorithm part increases the estimation accuracy, while the v-algorithm part reduces the ambiguity effect. Moreover, we derive the asymptotic mean square error (MSE) of our proposed algorithm, and the optimal weight in the pv-algorithm is given in a closed-form.

II. Problem Formulation

Let K denote the number of subcarriers in one OFDM block. The index sets for pilot carriers and virtual carriers are denoted as $\mathcal{P}$ and $\mathcal{V}$, respectively.The transmitted symbol on the kth subcarrier in the mth OFDM block is

$$s_k(m) = \begin{cases} p_k(m) \in \mathcal{C}_p, & k \in \mathcal{P}, \\ 0, & k \in \mathcal{V}, \\ d_k(m), \in \mathcal{C}_d & \text{otherwise}, \end{cases} \quad (1)$$

where $d_k(m)$ is the information symbol from the signal constellation $\mathcal{C}_d$, and $p_k(m)$ is the pilot symbol from the signal constellation $\mathcal{C}_p$. The power of the pilot symbols is normalized, i.e., $|p_k(m)| = 1$. Suppose the channel length is upper bounded by LT_s. The equivalent discrete channel vector can then be represented as $\mathbf{h} = [h_0, ..., h_L]^T$, with normalized power $\|\mathbf{h}\|^2 = 1$. For DFT-based OFDM, a CP with length P is added in the front of each transmitted block. If $P \geq L$, the mth received block after the removal of the CP is given by

$$\mathbf{y}(m) = e^{j2\pi\phi((m-1)K_s+P)}\mathbf{\Omega}(\phi)\mathbf{F}\mathbf{H}\mathbf{s}(m) + \mathbf{n}(m), \quad (2)$$

where K_s represents $K + P$, for simplicity in notations; $\mathbf{s}(m) = [s_0(m), ..., s_{K-1}(m)]^T$ is the mth OFDM block transmitted on all subcarriers; $\mathbf{F}$ is the $K \times K$ normalized IDFT matrix with the (a, b)th entry given by $\frac{1}{\sqrt{K}} e^{\frac{j2\pi(a-1)(b-1)}{K}}$; $\mathbf{\Omega}(\phi) = \text{diag}\{1, e^{j2\pi\phi}, ..., e^{j2\pi\phi(K-1)}\}$; ϕ is the normalized CFO by $1/T_s$; $\mathbf{H}$ is the diagonal matrix with its (k, k)th element given by $\mathbf{H}(k, k) = H_{k-1} \triangleq \sum_{l=0}^{L} h_l e^{-\frac{j2\pi(k-1)l}{K}}$, and each elements of $\mathbf{n}(m)$ is the sample of zero-mean white Gaussian noise with the variance σ^2.

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III. Frequency Offset Tracking Method

A. Pilot Based Tracking: p-Algorithm

Let $\tilde{\mathbf{s}}(m) = [\tilde{s}_0(m), \tilde{s}_1(m), ..., \tilde{s}_{K-1}(m)]^T$ denote the K-point DFT of $\mathbf{y}(m)$. Then,

$$\begin{aligned}&\tilde{s}_k(m)e^{-j2\pi\phi((m-1)K_s+P)}\\ &= \frac{s_k(m)H_k e^{j(K-1)\pi\phi}\sin(\pi K\phi)}{K\sin(\pi\phi)} + \mathrm{ICI}(m) + \tilde{n}_k(m), \quad (3)\end{aligned}$$

where

$$\mathrm{ICI}(m) = \sum_{v=0,\, v\neq k}^{K-1} \frac{H_v s_v(m) e^{\frac{j(K-1)\pi(v+K\phi-k)}{K}} \sin\pi(v+K\phi-k))}{K\sin\left(\frac{\pi(v+K\phi-k)}{K}\right)}. \quad (4)$$

For noise free case, $\tilde{s}_k(m) = H_k s_k(m)$ if $\phi = 0$. A non-zero ϕ both introduces ICI and reduces the effective signal to noise ratio (SNR) to a factor of $\frac{e^{j(K-1)\pi\phi}\sin(\pi K\phi)}{K\sin(\pi K\phi)}$.

For a slow fading channel, the channel impulse response (CIR) in two consecutive blocks can be assumed static. Based on this fact, the Classen&Meyr's method is developed by using a few number of pilots. Meanwhile, Classen&Meyr's method assumes a sufficiently small ϕ and a not high SNR, so that the ICI is much smaller than the noise and can, thus, be ignored. The CFO is then estimated as

$$\hat{\phi} = \frac{1}{2\pi K_s} \times \tan^{-1}\frac{\sum_{k\in\mathcal{P}}\Im\{\tilde{s}_k^*(m)\tilde{s}_k(m+1)/\left(s_k^*(m)s_k(m+1)\right)\}}{\sum_{k\in\mathcal{P}}\Re\{\tilde{s}_k^*(m)\tilde{s}_k(m+1)/\left(s_k^*(m)s_k(m+1)\right)\}} \quad (5)$$

Obviously, (5) is valid only when $\phi \ll \frac{1}{K}$, and even a small CFO variation during the tracking may cause the failure of the algorithm. There also exist other problems: 1) the estimation accuracy of (5) is limited by ignoring the ICI term. 2) At high SNR, since the ICI term is comparable to or even larger than the noise, the approximation in (5) is not valid any more.

In order to overcome all these drawbacks, we propose a new CFO tracking algorithm. Let ε be the unknown CFO. The CFO estimator is proposed as

$$\hat{\phi} = \arg\min_{\varepsilon} g_p(\varepsilon), \quad (6)$$

where $g_p(\varepsilon)$ is shown in (7) at the bottom of this page, and $\mathbf{f}_k$ is the kth column of $\mathbf{F}$.

In the absence of noise, if $\varepsilon = \phi$, both the terms in (7) become H_k and the estimator is minimized to zero. Note that, at least one of H_k, $k \in \mathcal{P}$ is not zero, which is also assumed by Classen&Meyr's method.

Remarks: The exact knowledge of the channel model and the channel length are critical for the channel statistics based algorithms [9], and the channel length based algorithm, e.g. Beek's method [3]. Unfortunately, the exact form of both these two terms are hard to obtain in practical transmissions. In contrast, neither of these two factors is important for the proposed p-algorithm. The only assumption is that the CIR is constant for two consecutive OFDM blocks, which is relatively easier to be satisfied, especially in a slow fading channel.

B. Identifiability For p-Algorithm

We study the uniqueness of the estimator (6) under the noise free environment. The unknown CFO is assumed to be within the full region (-0.5, 0.5], and the trivial ambiguity $\hat{\phi} = \phi \pm b$, $b \in \mathcal{I}$ is excluded from the consideration. For the noise free case, (6) reduces to

$$\hat{\phi} = \{\varepsilon | g_p(\varepsilon) = 0\}. \quad (8)$$

Obviously, the true CFO ϕ is the solution to (8). The ambiguity appears if $\exists\ \bar{\phi} \neq \phi$ such that $g_p(\bar{\phi}) = 0$, which is equivalent to

$$\begin{aligned}0 &= \sum_{v=0}^{K-1} H_v \underbrace{\frac{e^{\frac{j(K-1)\pi(v+K\Delta\phi-k)}{K}}\sin(\pi(v+K\Delta\phi-k)}{K\sin(\frac{\pi(v+K\Delta\phi-k}{K})}}_{\alpha_{vk}} \\ &\quad \times \underbrace{\left(s_v(m)/s_k(m) - s_v(m+1)/s_k(m+1)e^{j2\pi\Delta\phi K_s}\right)}_{\beta_{vk}} \\ &= \sum_{v=0}^{K-1} H_v\alpha_{vk}\beta_{vk}, \quad (9)\end{aligned}$$

for all $k \in \mathcal{P}$ and $\Delta\phi \neq 0$, where $\Delta\phi \triangleq \phi - \bar{\phi}$.

Case 1: $K\Delta\phi \notin \mathcal{I}$: In this case, $\alpha_{vk} \neq 0$ for all v. The discussion is further divided into two subcases.

1) Not all $\beta_{vk} = 0$: The ambiguity happens when $\sum_v H_v\alpha_{vk}\beta_{vk} = 0$. This type of ambiguity will be referred as h-ambiguity in this paper. Since H_v is a linear combination of continuous complex random variables h_l, the probability for h-ambiguity is zero. Therefore, the h-ambiguity can be ignored.

2) All $\beta_{vk} = 0$: We call this kind of ambiguity as d-ambiguity. In order to avoid this type of ambiguity, the value on pilots can be properly designed such β_{vk} is not zero for some $k \in \mathcal{P}$. For example, we can take $s_{k_1}(m) = 1$, $s_{k_1}(m+1) = 1$, while choose $s_{k_2}(m) = 1$, $s_{k_2}(m+1) = -1$. Then, $\beta_{k_1k_1}$, $\beta_{k_1k_2}$ cannot be zero, simultaneously.

Case 2: $K\Delta\phi \in \mathcal{I}$ or more specifically, $K\Delta\phi \in \mathcal{I}_{K-1} \triangleq \{1, ..., K-1\}$[1]. Let $\tilde{v}_k = \left((k - K\Delta\phi) \mod K\right)$. Obviously, $\tilde{v}_k \neq k$ when $\Delta\phi \neq 0$. In this case, $\alpha_{\tilde{v}_k k} = 1$, and $\alpha_{vk} = 0$, $\forall v \neq \tilde{v}_k$. The ambiguity happens if $\beta_{\tilde{v}_k k} = 0$ or $H_{\tilde{v}_k} = 0$. Since the latter can be equivalently considered as if the $\tilde{v}_k$th

[1] We only need to consider this subset since $K\Delta\phi' = K\Delta\phi + bK$ only provides a trivial ambiguity in $\Delta\phi' = \Delta\phi + b$.

$$g_p(\varepsilon) = \sum_{k\in\mathcal{P}} \|\mathbf{f}_k^H\mathbf{\Omega}(-\varepsilon)\mathbf{y}(m)/s_k(m) - e^{-j2\pi\varepsilon K_s}\mathbf{f}_k^H\mathbf{\Omega}(-\varepsilon)\mathbf{y}(m+1)/s_k(m+1)\|^2. \quad (7)$$

carrier is a virtual carrier, we incorporate the discussion on $H_{\tilde{v}_k} = 0$ into the discussion on $\beta_{\tilde{v}_k k} = 0$, i.e.

$$s_{\tilde{v}_k}(m)/s_k(m) - s_{\tilde{v}_k}(m+1)/s_k(m+1)e^{j2\pi\Delta\phi K_s} = 0, \quad (10)$$

for all $k \in \mathcal{P}$. The discussion is divided into three subcases.

1) All $\tilde{v}_k \in \mathcal{P}$: The ambiguity under this subcase is called p-ambiguity. To avoid this type of ambiguity, one can design the pilot symbols such that equation (10) does not hold for some $\tilde{v}_k \in \mathcal{P}$. For example, if $\mathcal{P} = \{k_1, k_2, k_3\}$, we can set $s_{k_1}(m) = s_{k_1}(m+1) = s_{k_3}(m) = s_{k_3}(m+1) = 1$, while taking $s_{k_2}(m) = 1$, $s_{k_2}(m+1) = -1$. Then, (10) does not hold simultaneously for k_1, k_2 and k_3.

2) All $\tilde{v}_k \notin \mathcal{P}$: This subcase can be further divided into two sub-subcases.

- At least one $\tilde{v}_k, k \in \mathcal{P}$ does not belong to $\mathcal{V} \bigcup \mathcal{N}$, where $\mathcal{N}$ denotes the subcarrier index set for channel nulls. Without loss of generality, we denote this specific k and $\tilde{v}_k$ as k_1 and $\tilde{v}_{k_1}$, respectively. The ambiguity under this sub-subcase is called c-ambiguity. Since the values of $s_{\tilde{v}_{k_1}}(m)$, $s_{\tilde{v}_{k_1}}(m+1)$ are selected from a finite alphabet, all the possible values of $\Delta\phi$ in (10) should belong to the set

$$\Psi = \left\{ \frac{1}{2\pi K_s} \arg\left(\frac{s_1}{s_2}\right) + \frac{\omega}{K_s} \pm \frac{\iota}{K_s} \middle| \forall s_1, s_2 \in \mathcal{C}_d, \iota \in \mathcal{I} \right\}, \quad (11)$$

where

$$\omega = \frac{1}{2\pi} \arg\left(\frac{s_{k_1}(m+1)}{s_{k_1}(m)}\right). \quad (12)$$

Let

$$\mu \in \mathcal{A} \triangleq \left\{ \frac{1}{2\pi} \arg\left(\frac{s_1}{s_2}\right) \middle| \forall s_1, s_2 \in \mathcal{C}_d \right\} \quad (13)$$

represent all the possible phase differences for a certain signal constellation $\mathcal{C}_d$. The c-ambiguity can be excluded if

$$\frac{(\mu + \omega + \iota)K}{K_s} \notin \mathcal{I}_{K-1}, \quad \forall \mu \in \mathcal{A}. \quad (14)$$

Note that, if pilots $s_{k_1}(m)$ and $s_{k_1}(m+1)$ could be chosen arbitrarily, (14) can be easily satisfied. However, this is not the case in practical transmissions, where the values of pilots are usually obtained from a constellation $\mathcal{C}_p$. Here, we discuss when pilots can only be chosen as ± 1, which is consistent with IEEE 802.11a standard [10]. Due to the symmetry, it is sufficient to consider only the case when $s_{k_1}(m) = s_{k_1}(m+1) = 1$, which means ω is 0. Then, the c-ambiguity can be excluded if

$$\frac{(\mu + \iota)K}{K_s} \notin \mathcal{I}_{K-1}, \quad \forall \mu \in \mathcal{A}. \quad (15)$$

Instead of designing the pilots values, we need to properly choose the signal constellation $\mathcal{C}_d$, and set the values of K, K_s. For example, since one value of μ must be zero, then $\frac{\iota K}{K_s} \notin \mathcal{I}_{K-1}$ is required. Therefore, K and K_s should at least be coprime numbers. A detailed discussion on designing the signal constellation will be proved in the next subsection.

- All $\tilde{v}_k, k \in \mathcal{P}$ belong to $\mathcal{V} \bigcup \mathcal{N}$, where $\mathcal{N}$ denotes the subcarrier index set for channel nulls. We call this type of ambiguity as n-ambiguity. Since the index set for $\tilde{v}_k$ is actually a $K\Delta\phi$ cyclic shift from the set $\mathcal{P}$, we can properly design the number and the positions of pilot carriers such that at least one of $\tilde{v}_k$ does not belong to $\mathcal{V} \bigcup \mathcal{N}$ for any cyclic integer shift $K\Delta\phi \in \mathcal{I}_k$. A simple way is to choose $|\mathcal{P}| > |\mathcal{V}| + |\mathcal{N}|$, where $|\cdot|$ denotes the cardinality of a set. However, one may find better choices, if the CIR or at least the positions of channel nulls are known.

3) Otherwise: The ambiguity for this subcase can be avoided by the methods in either of the previous two subcases.

Conclusion: Under the noise free condition, the CFO in the region (-0.5, 0.5] can be uniquely estimated[2] from the p-algorithm by properly designing system parameters, i.e., $\mathcal{P}$, $p_k(m)$, K, K_s and $\mathcal{C}_d$.

Remarks: The p-algorithm can be applied not only for CFO tracking but also for CFO acquisition because: 1) It could provide a full range estimation; 2) It can be applied before the CIR is estimated.

C. Constellation Rotation: A Case Study for 802.11a WLAN

From the previous discussion, it is known that system parameters should be properly designed to eliminate the CFO estimation ambiguity. However, there exist several constraints that may bring inflexibility when designing some of these parameters. For example, K is generally taken as 2^p, so that the fast fourier transform (FFT) operation can be implemented. Meanwhile, symmetric signal constellations are normally adopted, e.g., PSK, QAM.

An example here follows the IEEE 802.11a standard, where the parameters are chosen as $K = 64, P = 16, K_s = 80$, $\mathcal{V} = \{0, 27, ..., 37\}$, and $\mathcal{P} = \{8, 22, 44, 58\}$. Every pilot takes the value of $+1$ or -1. Obviously, the d-ambiguity can be readily removed by assigning ± 1 to different pilots. Meanwhile p-ambiguity and n-ambiguity do not exist due to the position of pilot carriers[3]. We only need to deal with the c-ambiguity that happens when

$$\frac{64}{80}(\mu + \iota) = \frac{4}{5}(\mu + \iota) \in \{1, ..., 63\}. \quad (16)$$

It is readily seen that $\mu \in \{0, 0.25, 0.5, 0.75\}$ or equivalently $\arg\left(\frac{s_1}{s_2}\right) \in \{0, \pi/2, \pi, 3\pi/2\}$ are the only cases that may introduce c-ambiguity. Moreover, since K, K_s are not coprime numbers, any signal constellation $\mathcal{C}_d$ may cause c-ambiguity. A way to resolve the c-ambiguity is to take double sets of modulations; namely, for OFDM block with odd index, we use signal constellation $\mathcal{C}_{d1}$, whereas for OFDM blocks with even index, we use signal constellation $\mathcal{C}_{d2}$. Let $\bar{s}_1$ be an arbitrary symbol in $\mathcal{C}_{d1}$ and $\bar{s}_2$ be an arbitrary symbol in $\mathcal{C}_{d2}$. Then, $\mathcal{C}_{d1}$, $\mathcal{C}_{d2}$ should be designed such that $\arg\left(\frac{\bar{s}_1}{\bar{s}_2}\right) \notin \{0, \pi/2, \pi, 3\pi/2\}$.

[2] h-ambiguity that happens with probability zero is ignored here.
[3] Channel nulls on subcarriers are not considered here.

To keep the system BER performance unaffected, we suggest a constellation rotation scheme, i.e. $\mathcal{C}_{d2}$ is a rotation from $\mathcal{C}_{d1}$ by a proper angle θ. For example, $\mathcal{C}_{d1}$ is taken as QPSK, while $\mathcal{C}_{d2}$ is taken as $\pi/4$-QPSK. Then, $\arg\left(\frac{\bar{s}_1}{\bar{s}_2}\right)$ only belongs to $\{\pi/4, 3\pi/4, 5\pi/2, 7\pi/4\}$. Hence, the c-ambiguity can be totally removed for noise-free case. Furthermore, the rotation $\theta = \pi/4$ can also remove the c-ambiguity for higher order constellations, e.g. 16-QAM or 64-QAM.

D. Virtual Carriers Based Tracking: v-Algorithm

The algorithms purely relying on virtual carriers have been studied in [11]- [12]. In this paper, we only quote their results and provide some necessary modifications. The CFO estimator based on virtual carriers is written as

$$\hat{\phi} = \arg\min_{\varepsilon} \sum_{q=m}^{m+1} \sum_{k\in\mathcal{V}} \|\mathbf{f}_k^H \Omega(-\varepsilon)\mathbf{y}(q)\|^2 = \arg\min_{\varepsilon} g_v(\varepsilon). \quad (17)$$

In (17) we consider both the m and $(m+1)$th received blocks to be consistent with the p-algorithm. The identifiability study of v-algorithm has been fully exploited in [13] and will not be restated here.

E. Co-Consideration: pv-Algorithm

From previous subsections, we know that p-algorithm and v-algorithm consider two parallel ways on CFO estimation. Since, in many practical OFDM standards, pilot carriers and virtual carriers coexist, then a joint consideration may bring several benefits. A reasonable combination of both p-algorithm and v-algorithm can be expressed as the weighted sum of the two corresponding cost functions. The combined estimator is given by

$$g_{pv}(\varepsilon) = g_p(\varepsilon) + \gamma g_v(\varepsilon). \quad (18)$$

Besides improving the performance accuracy, an important advance of pv algorithm is its robustness to CFO ambiguity. Suppose $\exists\bar{\phi} \neq \phi$ such that $g_p(\bar{\phi}) = 0$. From intuition, since $g_p(\bar{\phi})$ and $g_v(\bar{\phi})$ are obtained through different approaches and possess different structures, the probability for $\bar{\phi}$ to be also the null point for $g_v(\varepsilon)$ is small. However, the true CFO ϕ must be the null point for both $g_p(\varepsilon)$ and $g_v(\varepsilon)$. Therefore, after the addition, the false null in either estimator will be compensated by the other estimator, leaving only the true ϕ being the null of $g_{pv}(\varepsilon)$. Note that, the ambiguity is still possible to happen once ϕ is the common null of both $g_p(\varepsilon)$ and $g_v(\varepsilon)$. However, the probability may be greatly reduced compared with using either estimator.

IV. Performance Analysis

For the ease of analysis, we assume that all pilots are taken as $+1$. Assuming SNR$\gg 1$, the expectation and the variance of the proposed estimator can be approximated by

$$\begin{aligned} \mathrm{E}_{pv}\{\hat{\phi}\} &\cong \phi, && (19)\\ \mathrm{Var}_{pv}\{\hat{\phi}\} &\cong \frac{\sigma^2}{8\pi^2}\frac{2Z_p + \gamma^2 Z_v}{(Z_p+\gamma Z_v)^2}, && (20)\end{aligned}$$

where

$$\begin{aligned} Z_p &= \|\mathbf{F}_p\mathbf{F}_p^H(\mathbf{D}\Delta\boldsymbol{\eta} - K_s\boldsymbol{\eta}(m+1))\|^2, && (21)\\ Z_v &= \sum_{q=m}^{m+1}\boldsymbol{\eta}^H(q)\mathbf{D}\mathbf{F}_v\mathbf{F}_v^H\mathbf{D}\boldsymbol{\eta}(q), && (22)\\ \mathbf{D} &= \mathrm{diag}\{0,1,...,K-1\}, && (23)\\ \boldsymbol{\eta}(m) &= \mathbf{FHs}(m), && (24)\\ \Delta\boldsymbol{\eta} &= \boldsymbol{\eta}(m) - \boldsymbol{\eta}(m+1), && (25)\\ \mathbf{F}_p &= [\mathbf{f}_{p_1}, \mathbf{f}_{p_2}, ..., \mathbf{f}_{p_{|\mathcal{P}|}}], \quad p_i\in\mathcal{P}, && (26)\\ \mathbf{F}_v &= [\mathbf{f}_{v_1}, \mathbf{f}_{v_2}, ..., \mathbf{f}_{v_{|\mathcal{V}|}}], \quad k_i\in\mathcal{V}, && (27)\end{aligned}$$

Meanwhile, the CFO estimation variances by using either $g_p(\varepsilon)$ or $g_v(\varepsilon)$ are

$$\begin{aligned} \mathrm{Var}_p\{\hat{\phi}\} &\cong \frac{\sigma^2}{4\pi^2 Z_p}, && (28)\\ \mathrm{Var}_v\{\hat{\phi}\} &\cong \frac{\sigma^2}{8\pi^2 Z_v}, && (29)\end{aligned}$$

respectively. Detailed derivation can be found in [14]. The closed form of the optimal weight γ can be obtained regardless of all other parameters. Taking the derivative of (20) with respect to γ, the minimum value of $\mathrm{Var}_{pv}\{\hat{\phi}\}$ is always achieved at $\gamma = 2$.

V. Numerical Results for CFO Estimation

In this section, we examine the performance of the proposed estimators under various scenarios. All parameters are taken from IEEE 802.11a standard. The 4-ray channel model with an exponential power delay profile

$$E\{|h_l|^2\} = \rho\exp(-l/10), \quad l = 0,...,L \quad (30)$$

is used where ρ is the coefficient to normalize the overall channel gain. Each channel path is complex Gaussian. The normalized estimation mean square errors (NMSE) is defined as

$$\mathrm{NMSE} = \frac{1}{N}\sum_{i=1}^{N}\frac{(\hat{\phi}_i - \phi)^2}{\phi^2}, \quad (31)$$

where $N = 100$ Monte-Carlo runs are taken for average.

A. CFO Less than Subcarrier Spacing

In this example, different CFOs are taken from the region (-0.5/64,0.5/64]. The performance of p-algorithm, v-algorithm, pv-algorithm, Classen&Meyr's method and Beek's method are compared. We assume that the estimated channel length is $\hat{L} = 12$ in order to give a fair comparison between the Beek's method and our proposed algorithms (see reasons in [4]). QPSK constellation is used for all OFDM blocks. The NMSEs versus SNR for different algorithms are shown in Fig. 1, and the theoretical results for p-algorithm, v-algorithm, and pv-algorithms are given as well. As seen from the figure, Classen&Meyr's method can give a relatively satisfying performance at lower SNR with a normalized CFO 0.1. However, in high SNR region, the Classen&Meyr's method has an error floor. Meanwhile, even when CFO is as small as 0.25

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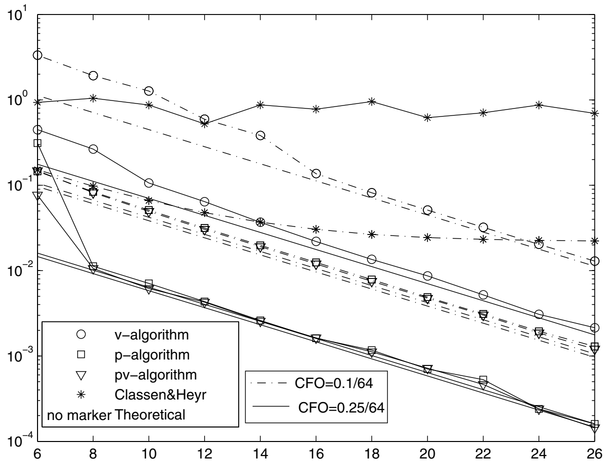

Fig. 1. NMSEs versus SNR for different CFO estimation algorithm: CFO smaller than subcarrier spacing.

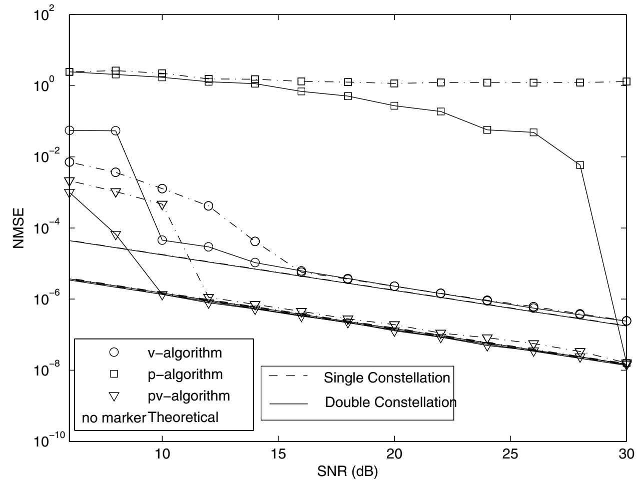

Fig. 2. NMSEs versus SNR for different CFO estimation algorithm: CFO larger than subcarrier spacing.

subcarrier spacing, the Classen&Meyr's method fails, because the ICI term cannot be ignored any more. On the contrary, since our p-algorithm does not make any approximation, it does not have an error floor and is also valid for a large CFO value. It is also noted that, Beek's method, p-algorithm and pv-algorithm give comparable performance. However, our major concern is that the performance of Beek's method is greatly affected by the channel length, or the estimate of the channel length. For example, if $L = 16$, or if $L < 16$ but the estimate $\hat{L} = 16$ due to the power leakage, then the Beek's method can not even be applied. For v-algorithm, although no error floor is met, the performance is much worse than either the p-algorithm or the pv-algorithm. This is because that v-algorithm only consider the orthogonality between subcarriers and is actually a blind type CFO estimation method. From intuition, pilot aided algorithm outperforms v-algorithm. Meanwhile, we find that the numerical performance of p-algorithm and pv-algorithm agree with theoretical analyzes very well, which verifies our analytical studies.

B. CFO Larger than Subcarrier Spacing

One important contribution of our proposed algorithm is its applicability for CFO greater than subcarrier spacing. In this example, we consider the performance of p-algorithm, v-algorithm, pv algorithm. Note that, Classen&Meyr's method and Beek's method are not included here because they are not applicable for this scenario. The constellation schemes with and without rotation are compared. For the former scheme, $\mathcal{C}_{d1}$ is QPSK and $\mathcal{C}_{d2}$ is $\pi/4$-QPSK, while for the latter scheme, QPSK constellation is used for all OFDM blocks. The CFO is taken as large as 0.25 of total bandwidth, which is 16 subcarrier spacings. NMSEs versus SNR are shown in Fig. 2. It is seen from Fig. 2 that, the pv-algorithm is about 12 dB better than v-algorithm and gives accurate estimation over all SNRs. However, it is also noted that the p-algorithm

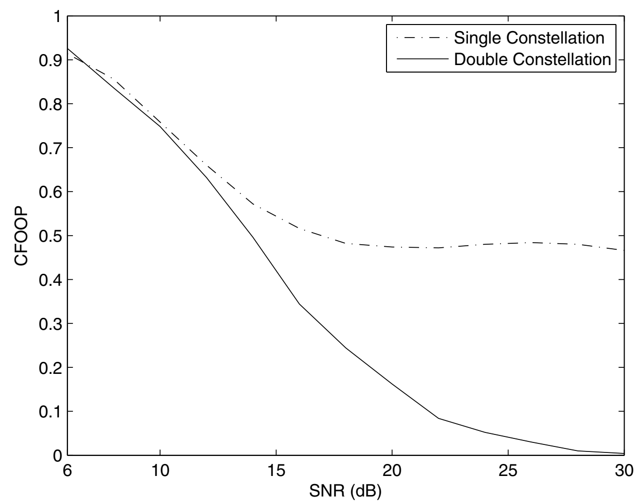

Fig. 3. CFOOP versus SNR for p-algorithm: Comparison of two modulation schemes.

with constellation rotation cannot yield good performance for SNR$<$ 30 dB, and the p-algorithm without constellation rotation fails at all SNR. In the simulations, we have observed that several Monte-Carlo runs give wrong estimation. Remember in section III-B, we only provide the discussion on ambiguity elimination for noise free environment. If the noise presents, the outlier may happen. Nevertheless, our pv-algorithm benefits from both algorithms. The p-algorithm part increases the estimation accuracy while the v-algorithm part reduce the outlier probability.

A reasonable way to evaluate the advantages of the constellation rotation scheme is to consider the CFO outlier probability (CFOOP), which is defined as

$$\text{CFOOP} = \frac{\text{The number of runs with outlier}}{\text{The total number of Monte Carlo runs}}. \tag{32}$$

where the outlier in the presence of the noise is considered to happen when the estimated $\hat{\phi}$ stays outside the region $[\phi - 0.5/K, \phi + 0.5/K]$.

The comparison of CFOOP for p-algorithm with different constellation schemes is shown in Fig. 3. Clearly, the CFOOP is reduced to zero at high SNR using the constellation rotation scheme. However, as analyzed in subsection III-B, the CFOOP for non-rotation scheme can never be zero.

VI. Conclusions

In this paper, a novel CFO tracking method is developed for practical OFDM systems. The proposed algorithm considers both pilot carriers and virtual carriers, hence is compatible to most practical standards. The ambiguity for pilot based algorithm is studied. A constellation rotation scheme is suggested to reduce the ambiguity effect. Performance of our proposed algorithms are analyzed, and numerous simulation results are conducted to validate the theoretical results.

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