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Abstract

We develop a competitive equilibrium theory of a market for votes. Before voting on a binary issue, individuals may buy and sell their votes with each other. We define *ex ante vote-trading equilibrium*, identify weak sufficient conditions for existence, and construct one such equilibrium. We show that this equilibrium must *always* result in dictatorship and the market generates welfare losses, relative to simple majority voting, if the committee is large enough. We test the theoretical implications by implementing a competitive vote market in the laboratory using a continuous open-book multi-unit double auction.

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1 Introduction

When confronted with the choice between two alternatives, groups, committees, and legislatures typically rely on majority rule. They do so for good reasons: as shown by May (1952), in binary choices, majority rule is the unique fair, decisive and monotonic rule. In addition, majority rule creates incentives for sincere voting: in environments with private values, it does so regardless of the information that voters have about others' preferences. A long tradition in political theory analyzes conditions under which majority voting yields optimal public decisions or has other desirable properties.[1]

An obvious weakness of majority rule is its failure to reflect intensity of preferences: an almost indifferent majority will always prevail over an intense minority. However, if votes can be freely traded as if they were commodities, then preference intensities could be reflected in the final vote. A natural intuition comes from the general theory of competitive equilibrium. Just as markets allocate goods in a way that reflects preferences, vote markets may allow voters who care more about the decision to buy more votes (and hence more influence), compensating other voters with money transfers (see, e.g., Buchanan and Tullock, 1962, Coleman, 1966, Haefele, 1971, Mueller, 1973, and Philipson and Snyder, 1996). However, to date there is no adequate model of decentralized trade for vote markets and so general questions about equilibrium allocations when voters can exchange votes with each other remain unanswered.

In particular this article seeks to answer, in the context of a relatively simple environment, two basic questions about vote trading in committees operating under majority rule. First, from a positive standpoint, what allocations and outcomes will arise in equilibrium if votes can be freely exchanged for a numeraire commotidy in a competitive market? Second, what are the welfare implications of these equilibrium outcomes, compared to a purely democratic majority rule institution where buying and selling of votes is not possible?

To answer these questions, we develop a competitive equilibrium model of vote markets where members of a committee buy and sell votes among themselves in exchange for money. The committee decides on a binary issue in two stages. In the first stage, members participate in a perfectly competitive vote market; in the second stage, all members cast their vote(s) for their favorite alternative, and the committee decision is taken by majority rule.

[1]Condorcet (1787) remains the classic reference. Among modern formal approaches, see for example Austen-Smith and Banks (1996), Ledyard and Palfrey (2002) or Dasgupta and Maskin (2008).

The main critique against vote markets arises from the externalities that bilateral vote trading imposes on third parties. Brams and Riker (1973), for instance, present examples in which exchanges of votes across issues are profitable to the pair of traders involved, and yet the committee obtains a Pareto inferior outcome. McKelvey and Ordeshook (1980) test the hypothesis in experimental data and conclude that the examples are not just theoretical curiosities, but can actually be observed in the laboratory. But there is no clear theoretical work that identifies exactly when we should expect such vote trading inefficiencies to arise in general, and when instead more positive results might emerge.[2]

A major theoretical obstacle to understanding how vote trading works in general is that in the standard competitive model of exchange, equilibrium (as well as other standard concepts such as the core) often fail to exist (Park, 1967; Kadane, 1972; Bernholtz, 1973, 1974; Ferejohn, 1974; Schwartz 1977, 1981; Shubik and Van der Heyden, 1978; Weiss, 1988, Philipson and Snyder, 1996, Piketty, 1994). Philipson and Snyder (1996) illustrate the point with an example along the lines of the following: Suppose there are three voters, Lynn and Lucy, on one side of an issue, and Mary, with more intense preferences than either Lynn or Lucy, on the other. Any positive price supporting a vote allocation where either side has more than two votes cannot be an equilibrium: one vote is redundant, and for any positive price it will be offered for sale. Any positive price supporting Mary's purchase of one vote cannot be an equilibrium: if Mary buys one vote, say Lynn's, then Lucy's vote is worthless, so Lucy would be willing to sell it for any positive price - again there is excess supply. But any positive price supporting no trade cannot be an equilibrium either: if the price is at least as high as Mary's high valuation, both Lynn and Lucy prefer to sell, and again there is excess supply; if the price is lower than Mary's valuation, Mary prefers to buy and there is excess demand. Finally, at zero price, the losing side always demands a vote, and again there must be excess demand. Philipson and Snyder (1996) and Koford (1982) circumvent the problem of nonexistence by formulating models with *centralized* markets and a market-maker. Both papers argue that vote markets are generally beneficial.

Other researchers have conjectured, plausibly, that nonexistence arises in this example because the direction of preferences is known, and hence losing votes are easily identified and worthless (Piketty, 1994). According to this view, the problem should not occur if voters are uncertain about others voters' preferences. But in fact nonexistence is still a problem. In our example, suppose that Mary, Lynn, and Lucy each know their own preferences but

[2] There are of course other possible critiques, on distributional and philosophical grounds.

otherwise only know that the other two are equally likely to prefer either alternative. A positive price supporting an allocation of votes such that all are concentrated in the hands of one of them cannot be an equilibrium: as in the discussion above, one vote is redundant, and the voter would prefer to sell it. But any positive price supporting an allocation where one individual holds two votes cannot be an equilibrium either: that individual holds the majority of votes and thus dictates the outcome; the remaining vote is worthless and would be put up for sale. Finally, a price supporting an allocation where Mary, Lynn and Lucy each hold one vote cannot be an equilibrium. By buying an extra vote, each of them can increase the probability of obtaining the desired alternative from 3/4 (the probability that at least one of the other two agrees with her) to 1; by selling their vote, each decreases such a probability from 3/4 to 1/2 (the probability that the 2-vote individual agrees). Recall that Mary's preferences are the most intense, and suppose Lynn's are weakest. Thus if the price is lower than 1/4 Mary's high valuation, Mary prefers to buy, and if the price is higher than 1/4 Lynn's low valuation, Lynn prefers to sell. The result must be either excess demand, or excess supply, or both, in all cases an imbalance. The conclusion is that there is still no general model of decentralized vote markets for which a competitive equilibrium exists.

A market for votes has several characteristics that distinguish it from neoclassical environments. First, the commodities being traded (votes) are indivisible.[3] Second, these commodities have no intrinsic value. Third, because the votes held by one voter can affect the payoffs to other voters, vote markets bear some similarity to markets for commodities with externalities: demands are interdependent, in the sense that an agent's own demand is a function of not only the prices, but also the demands of other traders. Fourth, payoffs are discontinuous at the points in which majority changes, and at this point many voters may be pivotal simultaneously.

In this paper, we respond to these challenges by modifying the concept of equilibrium. Specifically, we propose a notion of equilibrium that we call *Ex Ante Equilibrium*. As in the competitive equilibrium of an exchange economy with externalities, we require voters to best reply both to the equilibrium price and to other voters' demands. In addition, to solve the non-convexity problem, we allow for mixed –i.e., probabilistic– demands. This introduces the possibility that markets will not clear. Thus, instead of requiring that supply equals

[3] For discussions of nonexistence problems in general equilibrium models with indivisible goods, see Bevia et al (1999), Bikhchandani & Mamer (1997), Danilov et al (2001), Fujishige and Yang (2002), Gul & Stacchetti (1999) or Yang (2000).

demand in equilibrium with probability one, in the ex ante vote trading equilibrium we require market clearing only in expectation. The ex post clearing of the market is obtained through a rationing rule.[4]

In our model individuals have privately known preferences, and we prove existence of an ex ante vote trading equilibrium for a large set of parameters under a particular rationing rule. The proof is constructive. One striking property of the equilibrium is that there is always a dictator: i.e., when the market closes, one voter owns $(n+1)/2$ votes. The dictator must be one of the two individuals with most intense preferences. When the number of voters is small and the discrepancy between the highest valuations and all others is large, the market for votes may increase expected welfare. But the required condition for efficiency gains is highly restrictive, increasingly so for larger numbers of voters. We show that if the number of voters is sufficiently large, the market must be *less* efficient than simple majority voting without a vote market.

We test our theoretical results in an experiment by implementing the market in a laboratory with a continuous open-book multi-unit double auction.. Observed transaction prices support the the comparative statics properties of the ex ante competitive equilibrium. Price levels are generally higher than the risk-neutral equilibrium price. The prices fall over time with experience, but in most cases remain significantly higher than the risk neutral prices. We show that equilibrium price levels will be higher with risk-averse traders, while the comparative statics results remain unchanged, therby providing one possible explanation for experimental findings. Vote allocations are close to the ex ante equilibrium allocations, and the empirical efficiency of our laboratory markets closely tracks the theory.

Two other strands of literature are not directly related to the present article, but should be mentioned. First, there is the important but different literature on vote markets where candidates or lobbies buy voters' or legislators' votes: for example, Myerson (1993), Groseclose & Snyder (1996), Dal Bo (2007), Dekel, Jackson and Wolinsky (2008) and (2009). These papers differ from the problem we study because in our case vote trading happens *within* the committee (or the electorate). The individuals buying votes are members, not external subjects, groups or parties. Second, vote markets are not the only remedy advocated for majority rule's failure to recognize intensity of preferences in binary decisions. The mechanism

[4] Mixed demands have been used elsewhere in general equilibrium analysis, for example in Prescott and Townsend (1982). In their model, markets clear exactly, even with mixed demands, because there is a continuum of agents and no aggregate uncertainty. In our markets the number of agents is finite and there is aggregate uncertainty.

design literature has proposed mechanisms with side payments, building on Groves-Clarke taxes (e.g., d'Apremont and Gerard-Varet 1979). However, these mechanisms have problems with bankruptcy, individual rationality, and/or budget balance (Green and Laffont 1980, Mailath and Postlewaite 1990). A more recent literature has suggested alternative voting rules without transfers. Casella (2005), Jackson & Sonnenschein (2007) and Hortala-Vallve (2007) propose mechanisms whereby agents can effectively reflect their relative intensities and improve over majority rule, by linking decisions across issues. Casella, Gelman & Palfrey (2007), Casella, Palfrey and Riezman (2008), Engelmann and Grimm (2008), and Hortala-Vallve & Llorente-Saguer (2009) test the performance of these mechanisms experimentally and find that efficiency levels are very close to theoretical equilibrium predictions, even in the presence of some deviations from theoretical equilibrium strategies.

The rest of the paper is organized as follows. Section 2.1 defines the basic setup of our model. Section 2.2 introduces the notion of ex ante equilibrium. Section 2.3 presents the rationing rule. Section 2.4 shows existence by characterizing an equilibrium. Section 2.5 compares welfare obtained in equilibrium to simple majority rule. We then turn to the experimental part. Section 3 describes the design of the experiment, and section 4 describes the experimental results. Finally, section 5 discusses possible extensions. Section 6 concludes, and the Appendix collects all proofs.

2 The Model

2.1 Setup

Because we define a new equilibrium concept, we present it in a general setting. Some of the parameters will be specialized in our subsequent analysis and in the experiment. Consider a committee N of voters, $N = \{1, 2, ...n\}$, n odd, deciding on a single binary issue through a two-stage procedure. Each voter i is endowed with an amount m_i of the numeraire, and with $w_i \in \mathbb{Z}$ indivisible votes, where $\mathbb{Z}$ is the set of integers. Both $m = (m_1, ..., m_n)$ and $w = (w_1, ..., w_n)$ are common knowledge. In the first stage, voters can buy votes from each other using the numeraire; in the second stage, voters cast their vote(s), if any, for one of the two alternatives, and a committee decision, C, is taken according to the majority of votes cast. Ties are resolved by a coin flip. The model of exchange naturally focuses on the first stage and we simply assume that in the second stage voters vote for their favorite alternative.[5]

The two alternatives are denoted by $A = \{\alpha, \beta\}$, and voter i's favorite alternative, $a_i \in A$, is privately known. For each i the probability that $a_i = \alpha$ is equal to η_i and $\eta = (\eta_1, ..., \eta_n)$ is common knowledge. Let $S_i = \{s \in \mathbb{Z} \geq -w_i\}$ be the set of possible demands of each agent.[6] That is, agent i can offer to sell some or all of his votes, do nothing, or demand any positive amount of votes. The set of actions of voter i is the set of probability measures on S_i, denoted Σ_i. We write $S = S_1 \times ... \times S_n$ and let $\Sigma = \Sigma_1 \times ... \times \Sigma_n$. Elements of Σ are of the form $q_\sigma : S \to \mathbb{R}$ where $\sum_{s \in S} q_\sigma(s) = 1$ and $q_\sigma(s) \geq 0$ for all $s \in S$.

We allow for an equilibrium in mixed strategies where, ex post, the aggregate amounts of votes demanded and of votes offered need not coincide. A *rationing rule* R is an institution that maps the profile of voters' demands to a feasible allocation of votes. We denote the set of feasible vote allocations by $X = \left\{x \in \mathbb{Z}_+^n \middle| \sum x_i = \sum w_i\right\}$. Formally, a rationing rule R is a function from realized demand profiles to the set of probability distributions over vote allocations: $R : S \to \Delta X$ where $x_i \in]w_i, w_i + s_i[\ \forall i$ and $R(s) = w + s$ if $\sum s_i = 0$. Hence, a rationing rule must fulfill several conditions: a) R cannot assign less (more) votes than the initial endowment if the demand is positive (negative), b) R cannot assign more (less) votes than the initial endowment plus the demand if the demand is positive (negative) and c) if aggregate demand and aggregate supply of votes coincide, then all agents' demands are satisfied.

The particular (mixed) action profile, $\sigma \in \Sigma$, and the rationing rule, R, jointly imply a probability distribution over the set of final vote allocations that we denote as $r_{\sigma,R}(x)$. In addition, for every possible allocation we define the probability that the committee decision coincides with voter i's favorite alternative, a probability we denote by $\varphi_{x,a_i,\eta} := \Pr(C = a_i | x, \eta)$ - where $x \in X$ is the vote allocation and $a \in A$.

Finally, we define voters' preferences. The preferences of voter i are represented by a von Neuman Morgenstern utility function u_i, a concave function of the argument $v_i 1_{C=a_i} + m_i - (x_i - w_i)p$, where $v_i \in [0, 1]$ is a privately known valuation earned if the committee decision C coincides with the voter's preferred alternative a_i, 1_x is the indicator function, m_i is i's endowment of the numeraire, $(x_i - w_i)$ is i's net demand for votes, and p is the transaction price per vote.

We can now define $U_i(\sigma, R, p)$, the ex ante utility of voter i given some action profile,

[5] Equivalently, we could model a two stage game and focus on weakly undominated strategies.
[6] Negative demands correspond to supply.

the rationing rule, and a vote price p:

$$U_i(\sigma, R, p) = \sum_{x \in X} r_{\sigma,\Re}(x) \left[\begin{array}{c} \varphi_{x,a_i,\eta} \cdot u_i(v_i + m_i - (x_i - w_i)p) \\ +(1 - \varphi_{x,a_i,\eta}) \cdot u_i(m_i - (x_i - w_i)p) \end{array} \right]$$

One can see in the formula that the uncertainty about the final outcome depends on three factors: a) the action profile, b) the rationing rule and c) the preferences of other voters.

2.2 Ex Ante Vote Trading Equilibrium

Definition 1 *The set of actions σ^* and the price p^* constitute an* ***Ex Ante Vote Trading Equilibrium*** *(or, simply,* ***Ex Ante Equilibrium)*** *relative to rationing rule R if the following conditions are satisfied:*

1. *Utility maximization: For each agent i, σ_i^* satisfies*

$$\sigma_i^* \in \underset{\sigma_i \in \Sigma_i}{\arg} Max U_i\left(\sigma_i, \sigma_{-i}^*, R, p^*\right)$$

2. *Expected market clearing: In expectation, the market clears, i.e.,*

$$\sum_{s \in S} q_{\sigma^*}(s) \sum_{i=1}^{n} s_i = 0$$

The definition of the equilibrium shares some features of competitive equilibrium with externalities (e.g., Arrow and Hahn, 1971, pp. 132-6). Optimal demands are interrelated, and thus equilibrium requires voters to best reply to the demands of other voters. In contrast, the standard notion of competitive equilibrium for good markets requires agents to best-respond only to the price. The difference between the Ex Ante Equilibrium and the competitive equilibrium with externalities is that the former notion requires market clearing only in expected terms. The fact that demand and supply do not necessarily balance is the reason for the introduction of the rationing rule.[7]

[7] There always exists a trivial equilibrium, where, independently of the particular rationing rule and endowments, all voters neither demand nor offer any vote: the market clears and, since the probability of being rationed is 1, all voters are maximizing utility. This paper focuses on a nontrivial equilibrium where some trade occurs.

2.3 Rationing by voter

In general, the specification of the rationing rule can affect the existence (or not) of the equilibrium, and if an equilibrium exists, its properties. In this paper, we mainly focus on one specific rule, *rationing-by-voter* ($R1$), whereby each voter either fulfills his demand (supply) completely or is excluded from trade. After voters submit their orders, demanders and suppliers of votes are randomly ranked in a list, all rankings having the same probability. Then demands are satisfied in turn: the demand of the first voter on the list is satisfied with the first supplier(s) on the list; then the demand of the second voter on the list is satisfied with the first supplier(s) on the list with offers still outstanding, and so on. In case someone's demand cannot be satisfied, the voter is left with his initial endowment, and the process goes on with the next of the list.

$R1$ fulfills the conditions given in section 2.1. However, one aspect of this rationing rule is that it is possible for both sides of the market to be rationed. For example:

Example 1 *Suppose that voters 1, 2 and 3 offer to sell one vote and that voters 4 and 5 demand 2 votes each. The total supply (3 votes) is less than the total demand (4 votes).* $R1$ *will result in one supplier and one demander left with their initial endowments.*

We show in section 5 that our results continue to hold with little change under an alternative rationing rule we call *rationing-by-vote (*$R2$), where individual votes supplied are randomly allocated to each voter with an unfulfilled demand. With $R2$, only the long side of the market is rationed, but individuals must accept, and pay for, orders that are only partially filled.

2.4 Equilibrium

For the rest of the paper we assume that each voter prefers either alternative with probability 1/2 ($\eta_i = 0.5$) and is initially endowed with one vote ($w_i = 1$). In addition, we assume for now that all individuals are risk-neutral; in section 5 we show that the results extend to risk averse preferences. For simplicity, we set the initial endowment of money to zero ($m_i = 0$ for all i)–the value of m_i plays no role with risk-neutrality, and thus the restriction is with no loss of generality; depending on preferences, it could play a role with risk aversion. In this section we prove by construction the existence of an ex ante equilibrium when the rationing rule is $R1$ and a weak sufficient condition is satisfied.

Voters are ordered according to increasing valuation: $v_1 < v_2 < ... < v_n$.

Theorem 1 *Suppose $\eta_i = \frac{1}{2}$, $w_i = 1$, and $m_i = 0$ $\forall i$, agents are risk neutral, and $R1$ is the rationing rule. Then for all n there exists a finite threshold $\mu_n \geq 1$ such that if $v_n \geq \mu_n v_{n-1}$, the following set of price and actions constitute an Ex Ante Vote-Trading Equilibrium:*

1. *Price $p^* = \frac{v_{n-1}}{n+1}$.*
2. *Voters 1 to $n-2$ offer to sell their vote with probability 1.*
3. *Voter n demands $\frac{n-1}{2}$ votes with probability 1.*
4. *Voter $n-1$ offers to sell his vote with probability $\frac{2}{n+1}$, and demands $\frac{n-1}{2}$ votes with probability $\frac{n-1}{n+1}$.*

Proof. See the Appendix. ■

In this equilibrium, voters 1 to $n-2$ always offer to sell their vote, and voter n always demands the minimum number of votes to ensure himself a simple majority. The price makes voter $n-1$ indifferent between demanding the number of votes that would give him a majority and offering his own vote for sale. Finally, ex ante market clearing pins down the probability with which voter $n-1$ randomizes between offering to buy and offering to sell.

The equilibrium has four key features. First, and most striking, after rationing either voter n or voter $n-1$ will have a majority of votes: in equilibrium *there always is a dictator*. A market for votes does not distribute votes somewhat equally among high valuation individuals: in our ex ante vote trading equilibrium, it concentrates *all* decision-power in the hands of a single voter. The dictator will be voter n with probability $\frac{n+3}{2(n+1)}$, and voter $n-1$ with complementary probability $\frac{n-1}{2(n+1)}$; in either case, the dictator pays $\frac{n-1}{2}$ voters for their votes.

Second, the price of a vote is linked to the second highest valuation. The market can be loosely thought of as analogous to auctioning off the right to be a dictator on the issue, and thus it is not surprising that the equilibrium price is determined by the second highest valuation. In fact, the equilibrium price is precisely the price at which the second highest valuation voter is indifferent between "winning" the dictatorship or selling his vote and letting someone else be the dictator. Note that the value added from being dictator is bounded below 1/2 (1, the upper boundary of the support of v_i, times 1/2, the probability that the voter who has become dictator in one's stead favors the same alternative), while in the limit, as n approaches ∞, more and more votes are needed to become dictator. Thus the equilibrium price per vote must converge to zero. On the other hand, the total cost or "price of dictatorship" in the market is $\frac{n-1}{2} \cdot \frac{v_{n-1}}{n+1}$, which converges to $\frac{v_{n-1}}{2}$, the value of dictatorship to the second highest valuation voter.

Third, rationing occurs with probability 1: there is always either excess demand or excess supply of votes. The larger the committee size, the higher the probability that voter $n-1$ demands votes, and thus the higher the probability of positive excess demand. As n approaches ∞, with probability approaching 1, demand exceeds supply by one vote. Relative to the amounts traded, the imbalance is of order $O(1/n)$, and thus negligible in volume, recalling analyses of competitive equilibria with non-convexities (in particular the notion of Approximate Equilibrium, where in large economies allocations approach demands.[8]) But, contrary to private goods markets, in our market the imbalance is never negligible in its impact on welfare: it always triggers rationing and shuts $(n-1)/2$ voters out of the market.

Fourth, the existence of this equilibrium requires a sufficient "gap" between the highest and the second highest valuation. The gap is needed it to ensure that voter n is not better off selling his vote than demanding $\frac{n-1}{2}$ at the equilibrium price, $p^* = \frac{v_{n-1}}{n+1}$. The assumption, however, is very weak, in the sense that the required gap is extremely small. In the proof, an exact bound is derived[9]: the condition reduces to a minimum required percentage gap between v_n and v_{n-1} of 3 percent when $n = 7$, declining thereafter, and rapidly converging to 0 as n gets large. Figure 1 plots the required percentage gap (i.e., the minimum value of $v_n - v_{n-1}$, normalized by $v_{n-1} = 1$) as a function of n.

2.5 Welfare

In this section we compare the welfare obtained in the ex ante equilibrium to a situation without market, where voters simply cast their votes for their favorite alternative.

As remarked earlier, in the equilibrium characterized in Theorem 1 the vote market always results in dictatorship, with the dictator being either the voter with highest valuation, or, slightly less often, the voter with the second highest valuation. Even taking into account the dictator's high valuation, ignoring the will of all voters but one seems unlikely to be a

[8] See for example Starr (1969), and Arrow and Hahn (1971).

[9] $\mu_n = \frac{(n-1)(n+5)}{(n+1)\left(n+3-\binom{n-1}{\frac{n-1}{2}}2^{-(n-3)}\right)}$

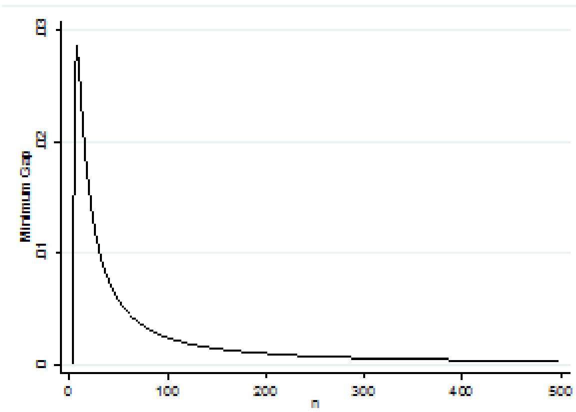

Figure 1: Minimum gap as a function of the committee size n.

desirable rule. Indeed, our main welfare result is that if n is sufficiently large, the vote market is inefficient relative to the majority rule outcome with no trade. At smaller committee sizes, the market can be welfare improving only if the profile of valuations is such that there is a large difference between the highest valuation (or two highest valuations) and the rest.

Ex ante welfare analysis is complicated by the requirement that valuations satisfy the condition identified in Theorem 1. One alternative is to specify a joint distribution of valuations $F(v)$ with $v = (v_1, ..., v_n)$ such that all draws of valuation profiles admit the equilibrium of Theorem 1. A simpler alternative, which we follow in this section, is to fix n, and a profile of valuations v that satisfies the condition in Theorem 1, and study the expected welfare associated with such profile in an infinite replication of the market and the vote. Focusing on an infinite replication means abstracting from specific realizations of the direction of preferences, of the realized demand of voter $n-1$, and of the resolution of the rationing rule, relying instead on the theoretical probabilities.

For given n and v, with $v_1 < ... < v_n$, expected welfare with the market, W_{VM}, and with majority rule, W_{MR}, are given by

$$W_{MR} = \left[v_n + v_{n-1} + (n-2)\,\overline{v}_{n-2}\right]\left(\frac{1}{2} + \binom{n-1}{\frac{n-1}{2}}2^{-n}\right) \tag{1}$$

$$W_{VM} = \left[\frac{n-2}{2}\overline{v}_{n-2} + \frac{3n+1}{4\,(n+1)}v_{n-1} + \frac{3n+5}{4\,(n+1)}v_n\right] \tag{2}$$

Note that the welfare comparison depends on four variables only: the highest valuation,

v_n, the second highest valuation, v_{n-1}, the average of the remaining lower valuations, $\overline{v}_{n-2} = \frac{\sum_{j=1}^{n-2} v_j}{n-2}$, and the size of the committee, n. We can state:

Proposition 1 *Fix n and a profile of valuations v with some v_n, v_{n-1}, and $\overline{v}_{n-2} > 0$, and such that $v_n \geq \mu_n v_{n-1}$. Then there always exists a finite $\overline{n}(v_n, v_{n-1}, \overline{v}_{n-2})$ such that if $n > \overline{n}$, $W_{VM} < W_{MR}$.*

Proof. See the Appendix. ■

The larger the number of voters, the larger is the expected cost of dictatorship, and the more skewed the distribution of valuations must be for the market to be efficiency enhancing. At sufficiently large n, as long as $v_n / \overline{v}_{n-2}$ is finite, the market must be inefficient.

The result is illustrated in Figure 2. The figure plots the area in which the vote market dominates majority rule, the area in which the reverse is true, and the very small area in which the equilibrium of Theorem 1 does not exist. The vertical axis is the ratio of the second to the highest valuation, $\frac{v_{n-1}}{v_n}$, and the horizontal axis is the ratio of the average of valuations v_1 to v_{n-2} to the highest valuation, $\frac{\overline{v}_{n-2}}{v_n}$, for the cases of $n = 5$, 9, 51, and 501. The two cases $n = 5$ and $n = 9$ are the committee sizes we study in the experiment, and the symbols in the figures correspond to the experimental valuations' profiles. At $n = 501$, the region of possible realizations violating the condition in Theorem 1 cannot be detected in the figure, and the vote market dominates majority rule only if the highest valuation is at a minimum about twenty times as high as the average of valuations v_1 to v_{n-2}.

The conclusion, logically straightforward given Theorem 1, contradicts common intuitions about vote markets. In the absence of budget inequalities and common values, a market for votes is often believed to dominate simple majority rule because it is expected to redistribute voting power from low intensity voters to high intensity voters.[10] Although such a redistribution is confirmed in our analysis, in equilibrium it occurs in extreme fashion: the efficiency conjecture generally fails because *all* decision power is concentrated in the hands of a single individual.

3 Experimental Design

A model of vote markets is difficult if not impossible to be tested with existingdata: actual vote trading is generally not available in the public record, and in many cases is prohibited

[10] Piketty (1994).

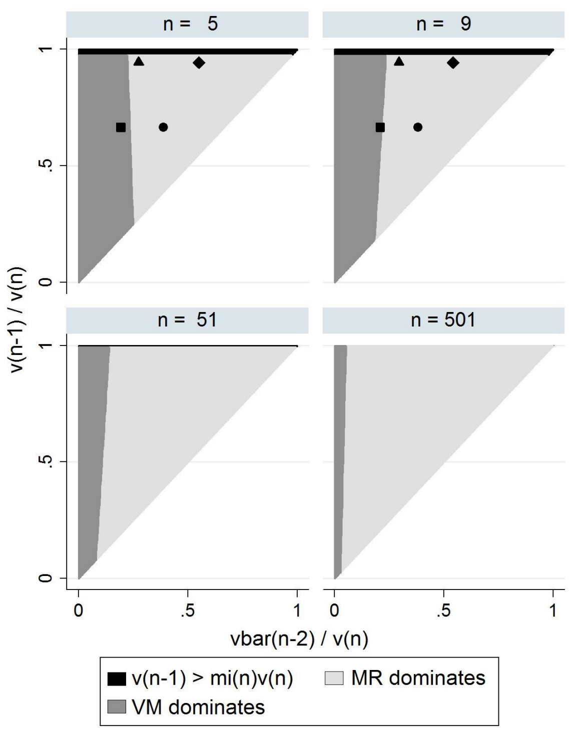

Figure 2: Welfare graphs. The graphs show the area in which the equilibrium of Theorem 1 does not exist (black); the area in which majority rule dominates vote markets (light grey), and the area in which vote markets dominate majority rule (dark grey). The symbols corresponds to different experimental parameters. Triangle: HB, Diamond: HT, Square: LB and Circle: LT.

by law. We must turn to the economics laboratory. However, exactly how to do this is not obvious. Like most competitive equilibrium theories, our modeling approach abstracts from the exact details of a trading mechanism. Rather than specifying an exact game form, the model is premised on the less precise assumption that under sufficiently competitive forces the equilibrium price will emerge following the law of supply and demand. But because of the nature of votes, a vote market differs substantially from traditional competitive markets, and our equilibrium concept is non-standard in the sense that markets clear only in expectation. Does this new competitive equilibrium concept applied to the different voting environment have any predictive value? Will a laboratory experiment organized in a similar way to standard laboratory markets (Smith 1965) lead to prices, allocations and comparative statics in accord with ex ante competitive equilibrium? These are the main question we address in this and next section.

Experiments were conducted at the Social Sciences and Economics Laboratory at Caltech during June 2009, with Caltech undergraduate students from different disciplines. Eight sessions were run in total, four of them with five subjects and four with nine. No subject participated in more than one session. All interactions among subjects were computerized, using an extension of the open source software package, Multistage Games.[11]

The voters in an experimental session constituted a committee whose charge was to decide on a binary outcome, X or Y. Each subject was randomly assigned to be either in favor of X or in favor of Y with equal probability and was given a valuation that s/he would earn if the subject's preferred outcome was the committee decision. Subjects knew that others would also prefer either X or Y with equal probability and that they were assigned valuations, different for each subject, belonging to the range [1,1000], but did not know either others' preferred outcome or the realizations of valuations, nor were they given any information on the distribution of valuations.

All subjects were endowed with one vote. After being told their own private valuation and their own preferred outcome, but before voting, there was a 2 minute trading stage, during which subjects had the opportunity to buy or sell votes. After the trading stage, the process moved to the voting stage, where the decision was made by majority rule. At this stage, voters simply cast all their votes which were automatically counted in favor of their preferred outcome. Once all subjects had voted, the results were reported back to everyone

[11] Documentation and instructions for downloading the software can be found at http://multistage.ssel.caltech.edu.

in the committee, the information was displayed in a history table on their computer screens, viewable throughout the experiment.

We designed the trading mechanism as a continuous double auction, following closely the experimental studies of competitive markets for private goods and assets (see for example Smith, 1982, Forsythe, Palfrey and Plott 1982, Gray and Plott, 1990, and Davis and Holt, 1992). At any time during the trading period, any subject could post a bid or an offer for one or multiple votes. Bid and offer prices (per vote) could be any integer in the range from 1 to 1000. New bids or offers did not cancel any outstanding ones, if there were any. All active bids or offers could be accepted and this information was immediately updated on the computer screens of all voters. A bid or offer for more than one unit was not transacted until the entire order had been filled. Active bids or offers that had not been fully transacted could be cancelled at any time by the voter who placed the order. The number of votes that different voters of the committee held was displayed in real time on each voter's computer screen and updated with every transaction. There were two additional trading rules. At the beginning of the experiment, subjects were loaned an initial amount of cash of 10,000 points, and their cash holdings were updated after each transaction and at the end of the voting stage. If their cash holdings ever became 0 or negative, they could not place any bid nor accept any offer until their balance became positive again.[12] Second, subjects could not sell votes if they did not have any or if all the votes they owned were committed.

Once the voting stage was concluded, the procedure was repeated with the direction of preference shuffled: subjects were again endowed with a single vote, valuation assignments remained unchanged but the direction of preferences was reassigned randomly and independently, and a new 2-minute trading stage started, followed by voting. We call each repetition, for a given assignment of valuations, a *round*. After 5 rounds were completed, a different set of valuations was assigned, and the game was again repeated for 5 rounds. We call each set of 5 rounds with fixed valuations a *match*. Each experimental session consisted of 4 matches, that is, in each session subjects were assigned 4 different sets of valuations. Thus in total a session consisted of 20 rounds.

The sets of valuations were designed according to two criteria. First, we wanted to compare market behavior and pricing with valuations that were on average low (L), or on average high (H); second, we wanted to compare results with valuations concentrated at

[12] The liquidity constraint was rarely binding, and bankruptcy never occurred. By the end of the last market, all subjects had positive cash holdings, even after loan repayment.

	Valuation Number								
Market	1	**2**	3	**4**	5	**6**	7	**8**	**9**
HT	190	**319**	433	**537**	635	**728**	784	**903**	**957**
LB	8	**31**	70	**125**	196	**282**	384	**501**	**753**
HB	14	**56**	127	**226**	353	**508**	691	**903**	**957**
LT	105	**177**	240	**298**	352	**404**	434	**501**	**753**

Table 1: Valuations of the different markets. In the case of n=5, only valuations in bold were used.

the bottom of the distribution (B), and with valuations concentrated at the top (T). This second feature was designed to test the theoretical welfare prediction: when valuations are concentrated at the bottom, the wedge between the top valuations and all others is larger, and thus the vote market should perform best, relative to majority voting. The B treatments correspond to the triangle (HB) and square (LB) symbols in Figure 2

For either $n = 5$ or $n = 9$, each of the 4 combinations, LB, LT, HB and HT, thus corresponds to a specific set of valuations. We call each a *market*. The exact values are reproduced in Table 1 and plotted in Figure 3 [13]

Sessions with the same number of subjects differed in the order of the different markets, as described in Table 2. Because, for given n, the equilibrium price depends only on the second highest valuation, HT and HB markets (and LT and LB markets) have the same equilibrium price. Thus in each session we alternated H and L markets. In addition, because we conjectured that behavior in the experiment could be sensitive to the dispersion in valuations, we alternated L and T markets. With these constraints, 4 experimental sessions for each number of subjects were sufficient to implement all possible orders of markets.

At the beginning of each session, instructions were read by the experimenter standing on a stage in the front of the experiment room.[14] After the instructions were finished, the experiment began. Subjects were paid the sum of their earnings over all 20 rounds multiplied by a pre-determined exchange rate and a show-up fee of $10, in cash, in private, immediately following the session. Sessions lasted on average one hour and fifteen minutes, and subjects' average final earnings were $29.

[13] Thus the experiment has 8 markets (4 markets for each of $n = 5$ and $n = 9$). We obtained the exact numbers by choosing high values with no focal properties ($v_n = 957$ and $v_{n-1} = 903$ for H, and $v_n = 753$ and $v_{n-1} = 501$ for L) and deriving the remaining valuations through the rule: $v_i = v_{n-1}\left(\frac{i}{n-1}\right)^r$ (with some rounding) with $r = 0.75$ for T and $r = 2$ for B. .

[14] A sample copy of the instructions for the case of nine players is attached as an Appendix.

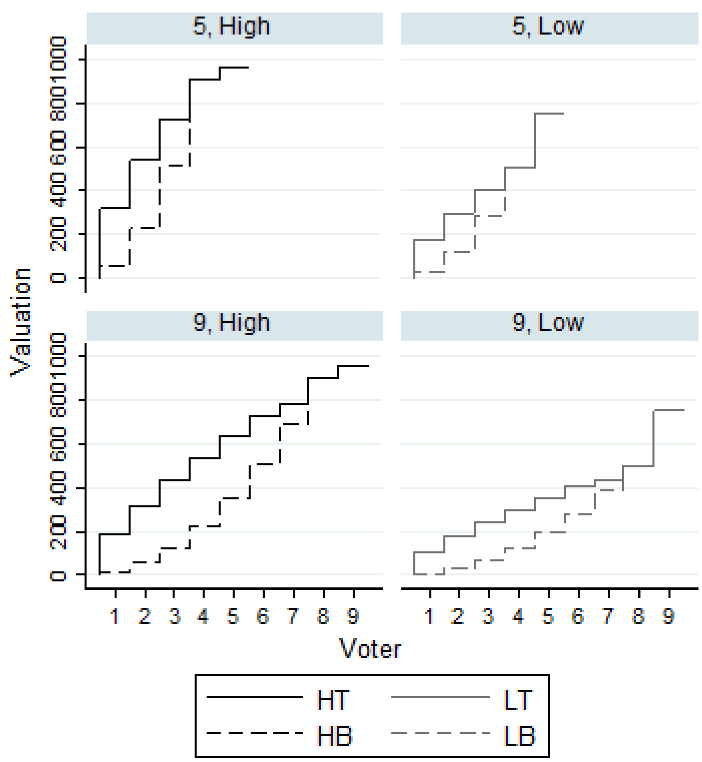

Figure 3: Experimental valuations. Graphs on the top/bottom correspond to treatments with committee size 5/9. Black/Gray graph correspond to treatments with high/low valuations (H/L). Solid/Dashed lines correspond to treatments with valuations concentrated on top/bottom (T/B).

	Match			
Session	1	2	3	4
1	HT	LB	HB	LT
2	LT	HB	LB	HT
3	HB	LT	HT	LB
4	LB	HT	LT	HB

Table 2: Orders of different markets.

4 Experimental Results

We organize our discussion of the experimental results by focusing, in turn, on prices, final vote allocations, and efficiency.

4.1 Prices

Figure 4 shows the transaction prices observed in the different markets, and the equilibrium prices. It is clear from the graph that there is significant overpricing: the percentage of transaction prices above equilibrium is 84% and 99% for committees of sizes 5 and 9 respectively. Risk aversion can explain part of the overpricing: equilibrium prices increase with the degree of risk-aversion, as we show formally in section 5. However, part of the overpricing is also due to inexperience, and transaction prices generally decline with experience.

Median prices are summarized in Table 3; the upper half of the table refers to $n = 5$ sessions, the lower half to $n = 9$ sessions. In both half tables, the equilibrium prices for each market, rounded to the nearest integer, are in the first row ("Theory"). The second row ("All") reports the median price for all transacted votes in each market, over all experimental sessions (the number of transacted votes is in parenthesis). The third and fourth row report median prices of transacted votes when subjects have gained some experience: in the third row are median transacted prices when the given market is implemented as the fourth and last match of the session ("Last match"); in the fourth row are median transacted prices focusing only on the last 2 rounds (out of 5) of each match, again with the relevant number of transacted votes in parenthesis. Thus "Last match" prices reflect trading activity after subjects have participated in three different markets for a total of fifteen rounds, but have no experience with the specific parameters for the given parameters of the fourth match. "Last 2 rounds" prices reflect trading activity after subjects have experienced three rounds with the specific parameters of the given treatment, but have not necessarily had experience

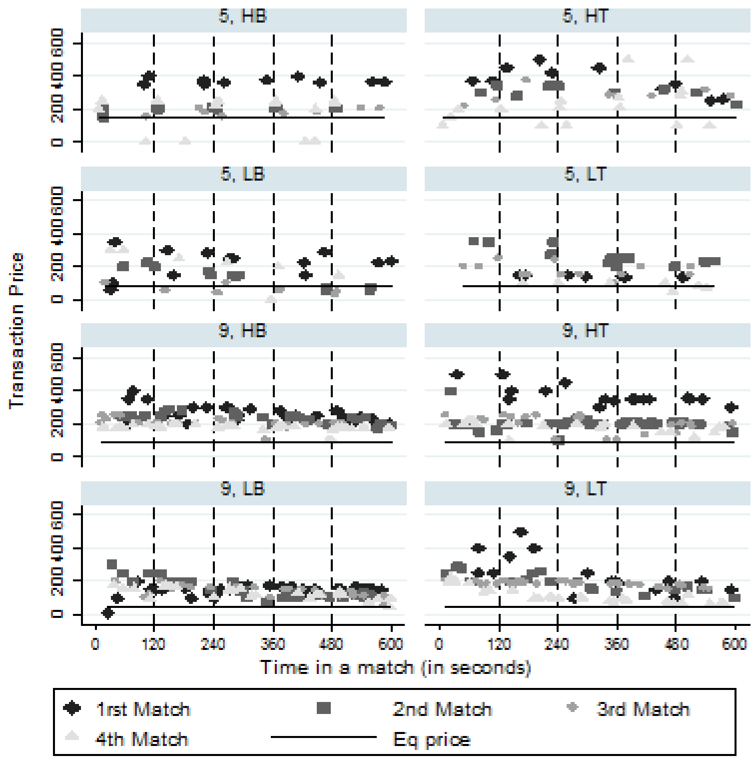

Figure 4: Prices of traded votes in different markets. Horizontal lines indicate the equilibrium prices. Vertical dashed lines indicate different rounds in a match.

n		LB	LT	HB	HT
5	Theory	84	84	151	151
	All	150 (47)	150 (43)	205 (47)	280 (46)
	Last match	225 (11)	100 (7)	230 (16)	200 (18)
	Last 2 rounds	150 (17)	150 (20)	203 (20)	275 (19)
9	Theory	50	50	90	90
	All	150 (116)	180 (100)	200 (118)	200 (101)
	Last match	130 (26)	100 (27)	180 (40)	190 (21)
	Last 2 rounds	128 (52)	150 (34)	200 (51)	200 (40)

Table 3: Median Prices of Traded Votes. Number of transactions in parentheses.

with different market parameters.

The overpricing is clear in the table, particularly in markets with 9 participants. However, the comparative statics implied by ex ante equilibrium are supported by the data. The theory implies that the market price should be higher in H markets than in L markets, and higher in markets with 5 participants than in markets with 9.[15] There are in total 24 relevant comparisons between H and L market prices in the table, and in 23 out of 24 cases the median price in the H markets is indeed higher.[16] There are 12 relevant comparisons between $n = 5$ and $n = 9$ markets, and again only in one case is the theoretical prediction violated. The support for the committee size comparison is weakest in LT markets, where the one violation occurs when the full data set is considered, and where experience leads to equality in median prices but fails to deliver the disparity implied by the theory. In addition, all else equal, the equilibrium price does *not* change between T and B markets. In the data there are some minor differences, but no systematic pattern.

As shown in Figure 4, overpricing is typically reduced by experience. Relative to median prices in the full data set, median prices in the last 2 rounds of each match are either strictly lower (in four treatments), or equal (in the remaining four treatments); median prices in the last match are strictly lower than the overall median in six of eight treatments, the exceptions being the two B treatments with $n = 5$.

These observations are statistically confirmed in a regression of all transaction prices on dummies for market characteristics and time variables (match, round, and time within a round). Prices are significantly higher in H markets, while a null hypothesis of no difference

[15] In fact, exactly twice as high, since $p = v_{n-1}/(n+1)$ and v_{n-1} is fixed at the same value.

[16] The only exception is $n = 5$, median price in the last match only, comparing HT and LB markets.

	$n = 5$	$n = 9$
Match	-37.37^{*}	-28.35^{*}
Round	-7.07	-13.29^{*}
Time	0.04	0.44^{**}
Dummies: H	92.26^{**}	57.36^{**}
T	45.11	22.12
Constant	214.11^{*}	247.75^{***}
R^2	0.38	0.52
Observations	183	435

Table 4: Linear regression for transaction prices on the listed variables. Data are clustered by session and standard errors are robust. Time is measured in seconds. * significant at 10%; ** significant at 5%; *** significant at 1%.

between B and T markets cannot be rejected at the 10 percent level. Our two measures of experience are both negatively correlated with the price for both 5 and 9-subject markets, and in three out of four cases the measure is statistically significant, if only at the 10 percent level. The results are summarized in Table 4.

4.2 Vote Allocations

Table 5 summarizes the observed trades. We distinguish between a *transaction* –a realized trade between two subjects–and an *order*–an offer to sell or a bid to buy votes that may or may not be realized. Note that both orders and transactions in principle may concern multiple votes: on the purchasing side, it is clearly feasible to demand and buy multiple units; on the selling side, although subjects enter the market endowed with a single vote, they could resell in bulk votes they have purchased. For each committee size, the first row is the average number of transactions per two-minute trading round, in the different markets. The transactions can be read as net trades because the percentage of reselling was in all cases lower than 5%.[17] As the table shows, the number of transactions is quite constant across markets, and slightly higher than the theoretical prediction of 2 for $n = 5$, and 4 for $n = 9$. Most transactions concerned individual votes (row 2) and indeed so did most orders in general, not only those that were accepted (row 5). Most transactions also originated from accepted offers (row 3), and again most orders, whether accepted or not, were offers to

[17] We define *speculation* as the total number of votes that were both bought and sold by the same player. Averaging across markets, it is 1 percent when $n = 5$ and 3 percent when $n = 9$ (there is no systematic effect across markets).

		LB	LT	HB	HT	Average
$n = 5$	**No. Transactions**	2.3	2.1	2.3	2.3	2.3
	% Unitary	83	95	96	96	92
	% From Offers	77	58	66	72	68
	No. Orders	5.8	5.8	4.4	6.8	5.7
	% Unitary	94	94	92	92	93
	% Offers	90	74	65	52	70
$n = 9$	**No. Transactions**	5.8	5.0	5.9	5.0	5.4
	% Unitary	100	85	97	100	96
	% From Offers	77	59	83	77	74
	No. Orders	16.0	15.1	16.6	14.6	15.6
	% Unitary	98	93	95	93	95
	% Offers	83	67	62	62	69

Table 5: Transactions' Summary

sell, as opposed to bids to buy (row 6).

In order to describe how close the final allocations of votes are to equilibrium, we define a distance measure d: for each market, d equals the minimal number of votes that would have to be exchanged between subjects to move from the observed final vote allocation to an equilibrium vote allocation, averaged over different rounds and different sessions.[18] To evaluate the meaning of specific d values, we compare them to the expected value of d when the allocation of votes is random - i.e. when each individual vote is allocated to each subject with equal probability. Calling such a measure rd, we construct a normalized distance measure δ, defined as $1 - d/rd$, and ranging between 0 and 1. δ equals 0 if $d = rd$, and 1 if $d = 0$; thus δ captures how much of the gap between the equilibrium allocation and the random allocation is closed by the experimental data.

Table 6 summarizes the results. *Range* reports the minimum and maximum distance observed across different rounds; and *Upper Bound* the maximal possible distance for each market and committee size. The table shows two main regularities. First, data from markets with valuations concentrated towards the bottom of the range tend to be closer to the theoretical predictions; the concentration of votes in the hands of the two subjects with

[18] For example, consider a committee with five members: $n = 5$. In equilibrium, members 4 and 5's possible vote holdings are: 0 and 3; or 1 and 3; or 3 and 1. The remaining voters must have either 0 or 1 vote. Suppose that the final vector of votes observed in the experiment were $(0, 1, 0, 2, 2)$. In this case $d = 1$, since we only need to transfer one vote from member 4 to member 5, or from member 5 to member 4. Suppose instead that the vector were $(0, 1, 2, 2, 0)$. Now $d = 2$: we need to transfer 1 vote from member 3 to member 4 and another vote, either from member 2 or 3 to member 5.

	$n = 5$				$n = 9$			
	LB	LT	HB	HT	LB	LT	HB	HT
Average d	.4	1.05	.75	1.2	2.25	3.25	2.55	3.05
St Dev	.49	.93	.54	.68	.89	.83	.67	1.29
Range	[0,1]	[0,3]	[0,2]	[0,3]	[0,4]	[1,5]	[1,3]	[2,7]
Upper Bound	4	4	4	4	8	8	8	8
Random: rd	1.77	1.77	1.77	1.77	3.75	3.75	3.75	3.75
Normalized δ	.77	.41	.58	.32	.40	.13	.32	.19

Table 6: Summary statistics of distance from final allocation of votes to equilibrium. Distance between final votes' allocations and equilibrium prediction, its standard deviation and its range (minimum and maximum observed). Normalized is equal to 1-(d/(rd)) where d is the average distance to equilibrium and rd the distance to equilibrium from a random allocation of votes. Upper Bound is the maximal possible distance.

highest valuations is naturally easier to achieve when the remaining voters are relatively more willing to sell. Second, the smaller committee approaches the theoretical allocation more closely than the larger committee–the value of δ is consistently higher. But it is not clear that this reflects more awareness of equilibrium strategies: concentrating votes into few hands is more easily achieved when the number of voters is small.

Contrary to prices, distance data show no evidence of learning: in ordered logit regressions of d (or δ) on dummies for market characteristics (H and T), and time variables (match and round), for both committee sizes, none of the time dummies is statistically significant, even at the 10 percent level.[19]

Figure 5 plots the actual distribution of d (in white) in the data, and of rd (in black). The horizontal axis reports possible distance values, from 0 to the theoretical maximum; the vertical axis is the fraction of experimental rounds, per committee size and type of market, whose final allocation is at 0, 1, 2, etc. distance from the equilibrium. The distribution of rd, the expected distance from equilibrium of the random vote allocation, is the average distribution obtained from 1,000,000 simulations and is invariant across markets. The distribution of d is shifted to the left, relatively to the distribution of rd. Most of the d distribution is concentrated on values 0 and 1 in the case of $n = 5$, and on values 2 and 3 in the case of

[19] Note that, for given committee size, rd is invariant across market types. Thus, for given n, d and δ differ by a constant only. In the logit regression, the market characteristics dummies are positive and significant at the 10 percent level if $n = 5$; if $n = 9$, the only significant variable is the market T dummy, positive and significant at the 5 percent level. We report the regression in the Appendix, Table 10.

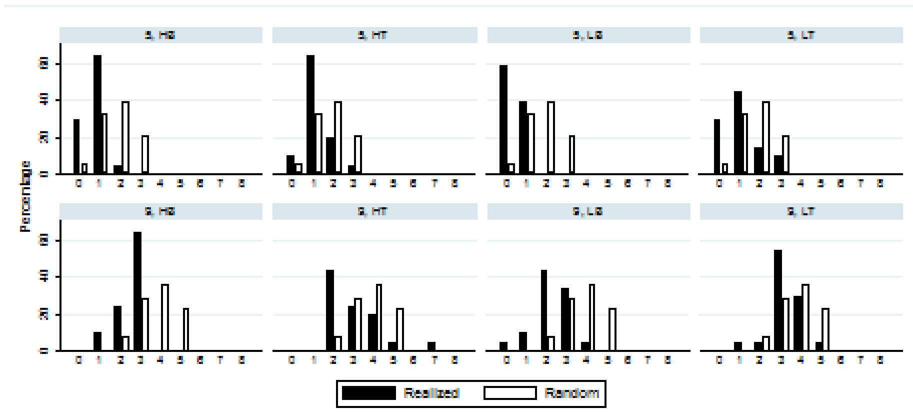

Figure 5: Distribution of distances d to equilibrium from experimental and random vote allocations.

$n = 9$.[20]

So far we have described the summary measure d without being explicit about the allocation of votes across individual subjects. But different vote allocations can have identical d, and yet quite different impacts on welfare. For instance, according to the theory the behavior of voters 1 to $n - 2$ should be identical. Therefore, the distance to equilibrium of $(3, 0, 0, 1, 1)$ and $(0, 0, 3, 1, 1)$ is the same–but the expected welfare of the first allocation is substantially inferior to the second.

Figure 6 shows the average number of votes held by subjects at the end of each round, compared to the equilibrium prediction. Subjects are ordered on the horizontal axis, from lowest to highest valuation. Again, markets with valuations concentrated at the bottom of the distributions (B) appear to conform to the theory relatively well: the highest valuation subjects end the round with a large fraction of the votes. In particular, if the market is LB, the distribution of votes across subjects does increase sharply for the highest valuation subjects, exactly as the theory suggests, and this is true for both $n = 9$ and $n = 5$. But in markets with valuations concentrated at the top of the distribution (T), the results deviate from the theory: the highest valuation subjects demand fewer votes on average than their equilibrium demand, and the number of votes held increases smoothly as the valuations increase.

[20] Assuming independence, the chi-squared Goodness-of-Fit test rejects the hypothesis that the experimental data can be obtained from the random distribution at a significance of at least 5% in all cases.

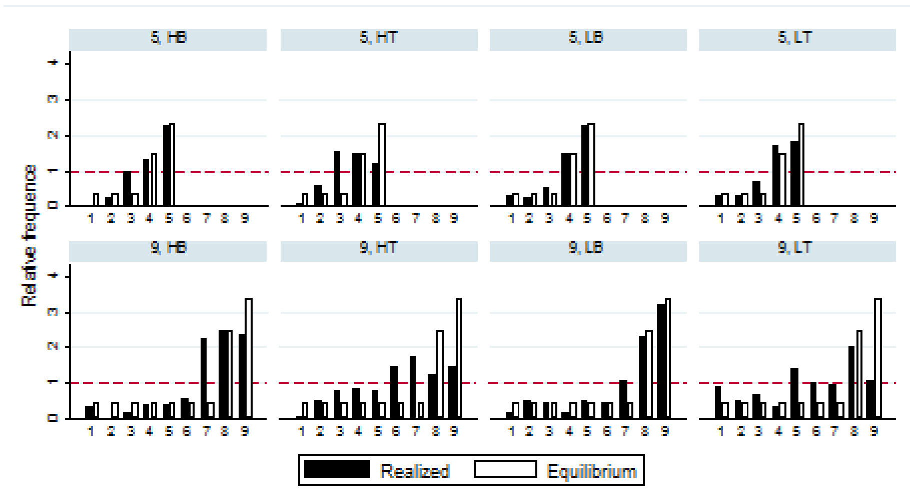

Figure 6: Average amount of votes held by subjects at the end of the trading stage and equilibrium (expected) allocation, ordered by valuation. The dotted line indicates no trade.

	Market				
n	HB	HT	LB	LT	Average
5	55	45	100	65	66.25
9	25	5	20	0	12.50

Table 7: Observed precentage of rounds in which there was a dictator.

Finally, Table 7 shows the observed frequency of dictatorship for different markets and different committee sizes. In $n = 5$ committees, dictatorship emerges two thirds of the time, a frequency that although lower than the theoretical prediction of one hundred percent, we still find remarkable. In the LB market, the data do match the theory perfectly in this regard: out of 20 rounds, in four different experimental sessions, all 20 result in dictatorship. In $n = 9$ committees, where the purchase of four, as opposed to two, votes is required for dictatorship, the results are much weaker, with an average frequency of dictatorship of 12 percent. In T markets, where valuations are concentrated at the top of the distribution, competition among the higher-value subjects clearly works against the concentration of five votes in the hand of the same subject; in B markets, dictatorship does in fact occur, between one fifth and one fourth of the times, a weak result compared to the theory but still a non-negligible frequency.

The probability of dictatorship increases with learning. A probit regression of the prob-

	Variable		Coef		
$n = 5$	Match		0.172	Obs	80
	Round		0.195^{***}	Pr > chi2	.
	Dummies:	H	-1.074^{**}	PseudoR^2	0.195
		T	-0.795^{***}	LpL	-41.19
$n = 9$	Match		0.406^{***}	Obs	80
	Round		0.175	Pr > chi2	.
	Dummies:	H	0.320	PseudoR^2	0.242
		T	-1.303^{***}	LpL	-22.84

Table 8: Probit regression of the probability of a dictator as a function of match number, round number, H and T dummies. Data are clustered by independent groups and standard errors are robust. * significant at 10%; ** significant at 5%; *** significant at 1%.

ability of a dictator as function of market characteristics (H and T dummies), and time dummies for round and match shows that the probability is significantly higher in the last round of each match if $n = 5$, or in the last match of each session if $n = 9$, with significance at the 1 percent level. In addition, as Table 7 shows, the probability is significantly lower in T markets, and, if $n = 5$, in H markets. The results are reported in Table 8.

4.3 Welfare

In this section we analyze the payoffs obtained by our committees and compare them to the equilibrium predictions and to the majority rule benchmark.

In theory, the equilibrium strategy is invariant to the direction of a voter's preferences. In the experiment, subjects participated in the market and submitted their orders without information about others' realized preferred alternative (or indeed about others' valuations) — all they knew was that any subject was assigned either alternative as preferred with probability 1/2. Thus a voter's trading behavior should be independent of both its own and other voters' realized direction of preferences. When evaluating the welfare implications of a specific vote allocation, the exact realization of the direction of preferences matters, but, as we argued in section 2.5 the interesting welfare measure is the *average* welfare associated with such an allocation, for all possible realizations of the directions of preferences. It is such a measure that we calculate on the basis of the experimental data and present in Table 9.

For each profile of valuations and for each realized allocation of votes, we compute the average aggregate payoff for all possible profiles of preferences' directions, weighted by the probability of their realization. For each profile of valuations, we then average the result again, over all realized allocations of votes, and obtain W_{VM}, our measure of experimental payoffs for each market. We compute the equivalent measure with majority voting, W_{MR} — that is, for each profile of valuations, for all possible realizations of preferences' directions, we resolve the disagreement in favor of the more numerous side; taking into account the probability of each realization, we then compute the average aggregate payoff. To ease the comparison of payoffs across the two institutions, the different markets, and the different committee sizes, we express both W_{VM} and W_{MR} as share of the average maximum payoff that the subjects could appropriate, normalized by a floor given by the average minimum possible payoff. The average maximum payoff, $\overline{W}$, is calculated by selecting the alternative favored by the side with higher aggregate valuation, for each realization of preferences; the average minimum payoff, $\underline{W}$, corresponds to selecting the alternative favored by the side with lower aggregate valuation. The welfare score then is: $(W_{VM} - \underline{W})/(\overline{W} - \underline{W})$, and correspondingly for majority voting.

Table 9 displays the experimental welfare scores, aggregated over all rounds and all matches, for each market and for either committee size, together with the theoretical predictions, and the corresponding welfare scores with majority rule.[21] The table highlights several interesting regularities. First, realized welfare mimics equilibrium welfare more closely in markets with low valuations, indeed very closely in three out of four cases. In markets with high valuations, realized welfare is consistently higher than predicted welfare. Second, Proposition 1 predicts that vote markets should perform better, relative to majority, when valuations are concentrated at the bottom of the distributions. In particular, vote markets should dominate majority in market LB, be slightly worse in market HB, and substantially worse in markets LT and HT, for both committee sizes. In our data, vote markets had higher average payoffs than majority in both B treatments, although only barely in market HB. In T treatments, where valuations are concentrated at the top of the distribution, majority rule outperforms our experimental results, a conclusion that remains true even though, as we saw, subjects deviated from equilibrium towards a more equalitarian allocation of votes, and thus were closer to majority rule than theory predicts. Finally, realized welfare is always (weakly) higher than equilibrium welfare, reflecting a lower frequency of dictatorship than

[21] We consider all rounds and all matches because, as we saw in previous section, we do not find significant temporal evolution in final vote allocations.

		Market			
		LB	LT	HB	HT
$n = 5$	Realized	0.93	0.88	0.93	0.90
	Majority Rule	0.90	0.96	0.92	0.99
	Equilibrium	0.92	0.88	0.90	0.85
$n = 9$	Realized	0.92	0.84	0.90	0.88
	Majority Rule	0.87	0.95	0.89	0.97
	Equilibrium	0.88	0.83	0.85	0.79

Table 9: Welfare scores in the experiment; in theory, and with simple majority rule.

predicted by theory.

5 Extensions

5.1 An Alternative Rationing Rule

The rationing rule plays a central role in guaranteeing existence and characterizing the ex ante vote trading equilibrium. A reasonable concern then is that our theoretical results could be quite special and disappear when using alternative rules. We show here that the analysis is robust to the most plausible alternative. Consider the following rule, which we call *rationing-by-vote*, or $R2$: If voters' orders result in excess demand, any vote supplied is randomly allocated to one of the individuals with outstanding purchasing orders, with equal probability. An order remains outstanding until it has been completely filled. When all supply is allocated, each individual who put in an order must purchase all units that have been directed to him, even if the order is only partially filled. If there is excess supply, the votes to be sold are chosen randomly from each seller, with equal probability.

As mentioned earlier, contrary to rationing-by-voter, or $R1$, rationing-by-vote guarantees that the short side of the market is never rationed, but forces individuals to accept partially filled orders. To illustrate the difference, recall example 1 where three voters supply one vote each, and two voters demand two votes each. Here, $R1$ rations one demander and one supplier; with $R2$, no supplier is rationed, and the two voters demanding two votes each are settled with two and one vote. Buying and paying for one's partial order when a different voter exits the market holding a majority of votes is clearly suboptimal.

Proposition 2 *Suppose* $\eta_i = \frac{1}{2}$, $w_i = 1$, *and* $m_i = 0$ $\forall i$, *agents are risk neutral, and* $R2$ *is the rationing rule. Then there exist* $\overline{n}$ *and a finite threshold* $k(n) \geq 1$ *such that for all* $n \leq \overline{n}$ *and* $v_{n-1} \geq k(n)v_{n-2}$ *the following set of price and actions constitute an Ex Ante Vote-Trading Equilibrium:*

1. *Price* $p^* = \frac{v_{n-1}}{2(n-1)}$.
2. *Voters* 1 *to* $n-2$ *offer to sell their vote with probability* 1.
3. *Voter* n *demands* $\frac{n-1}{2}$ *votes with probability* 1.
4. *Voter* $n-1$ *offers his vote with probability* $\frac{2}{n+1}$, *and demands* $\frac{n-1}{2}$ *votes with probability* $\frac{n-1}{n+1}$.

Proof. See appendix. ■

In particular, 1.–4. constitute an equilibrium if $n \leq 9$ and $v_{n-1} \geq 1.15v_{n-2}$, conditions satisfied in our experimental treatments. Note that with risk neutrality the constraint $m_i = 0$ $\forall i$ is again irrelevant, and imposed for simplicity of notation only.

The equilibrium is almost identical to the equilibrium with $R1$, with two differences. First, because orders can be filled partially, rationing can be particularly costly. In equilibrium, if voter $n-1$ attempts to buy, either he or voter n will be required to pay for votes that are strictly useless (since the other voter will hold a majority stake). To support voter $n-1$'s mixed strategy, the equilibrium price is lower than with the original rationing rule: $\frac{v_{n-1}}{2(n-1)}$ instead of $\frac{v_{n-1}}{n+1}$. Second, and again because of the possibility of filling partial orders, lower valuation voters can use their own orders strategically, refraining from selling to increase the probability that no dictator emerges. To rule out the possibility of such a deviation, the price must be high enough, relative to their valuation. This is the reason for requiring $v_{n-1} \geq k(n)v_{n-2}$.

With these two caveats, the change in rationing rule has little effect. In particular, equilibrium vote trading results in dictatorship, and the welfare implications are similar.

5.2 Risk Aversion

So far we have assumed risk neutrality. A natural extension is to analyze how risk aversion changes our results. The next proposition shows that the demands presented in Theorem 1 can be supported as an equilibrium when all agents have CARA utility functions with the same risk aversion parameter.

Proposition 3 *Suppose $\eta_i = \frac{1}{2}$, $w_i = 1$, and $m_i = 0$ $\forall i$, $u(\cdot) = -e^{-\rho(\cdot)}$ with $\rho > 0$, and R1 is the rationing rule. Then for all n there exists a finite threshold $\mu_n \geq 1$ such that if $v_n \geq \mu_n v_{n-1}$, the set of actions presented in Theorem 1 together with the price $p = \frac{2}{r(n+1)} \ln\left(\frac{1}{2} + \frac{1}{2}e^{rv_{n-1}}\right)$ constitute an Ex Ante Vote-Trading Equilibrium.*

Proof. See appendix. ■

The logic is virtually identical to the logic of Theorem 1. As in the case of risk neutrality, the equilibrium price makes voter $n-1$ indifferent between selling and demanding a majority of votes. Hence again the price is a function of v_{n-1}. The effect of higher risk aversion can be seen in the implicit equation that defines p. The equation is valid for any utility function and can be written as[22]: $\frac{1}{2}u\left(p\right) + \frac{1}{2}u\left(v_{n-1} + p\right) = u\left(v_{n-1} - \frac{n-1}{2}p\right)$. As before, in equilibrium voter $n-1$ is rationed with probability 1/2 whether his demand is -1 or $\frac{n-1}{2}$, and if he is rationed his utility is identical regardless of his demand. Thus the price p must be such that the voter is indifferent between demanding -1 or $\frac{n-1}{2}$ when not rationed. The left-hand side of the equation is the expected utility of demanding -1, conditional on not being rationed: it is a lottery in which the voter receives p and $v_{n-1}+p$ with equal probability, depending on whether voter n, who is dictator, agrees with voter $n-1$'s preferred alternative. The right hand side is the expected utility of demanding $\frac{n-1}{2}$, conditional on not being rationed, and thus equals $u(v_{n-1} - \frac{n-1}{2}p)$ with certainty. Higher risk aversion– more concave $u\left(\cdot\right)$–must lead to an increase in the equilibrium price. The intuition is clear: fixing the price, an increase in risk aversion makes the riskier lottery of selling less attractive relatively to demanding a majority of votes. In order to make voter $n-1$ indifferent, the price must be higher.

As in the case of Theorem 1, the equilibrium is supported if there is a sufficient gap between the highest and the second highest valuation. But once again the required gap is very small, indeed smaller than with risk neutrality in all numerical cases we studied.[23] Figure 7 shows the minimum required gap with CARA utility, for the two cases of $\rho = 1$ (the dashed line) and $\rho = 2$ (the dotted line), and as reference for risk neutrality (the solid line), as function of n. The minimum gap is always smaller than 1 percent if $\rho = 1$ and

[22] With CARA utility, the equation is solved by the equilibrium price in Proposition 3.

[23] A natural conjecture is that the gap condition is always easier to satisfy with risk aversion than with risk neutrality.

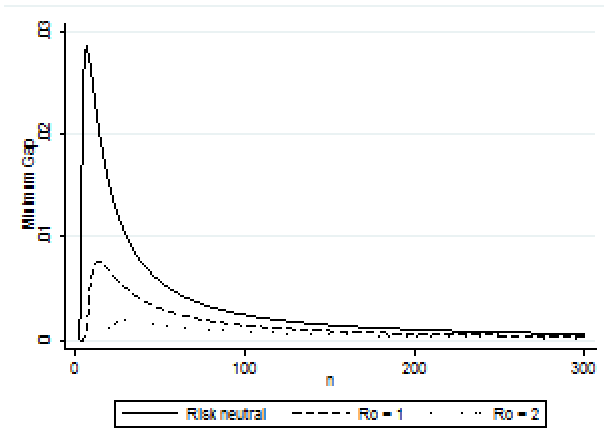

Figure 7: Minimum percentage gap with risk neutrality and with CARA utility funcion: $u(x) = -e^{-\rho x}$, with $\rho = 1$ and $\rho = 2$.

smaller than half a percent if $\rho = 2$, and disappears asymptotically as n increases. We have verified numerically that the condition is satisfied by our experimental parameters for all $\rho \in (0, 1000]$.

Evidence for risk aversion has been documented in experiments across a broad spectrum of environments, ranging from auctions to abstract games.[24] We conjecture that it may be a significant factor in explaining the high prices we observe in our data. How much can the price increase with risk aversion? The upper bound on the price increase is given by an "infinitely" risk averse agent, an agent uniquely motivated by the lowest payoff of the lottery. Hence, the infinitely risk averse agent will be indifferent whenever $p = 2\frac{v_{n-1}}{n+1}$: the price with risk aversion lies between the risk neutral price and twice the risk neutral price.[25] In our experiments prices generally fall at the upper end of this range. Given the broad extent of overpricing in our data (with some transacted prices above $2\frac{v_{n-1}}{n+1}$), risk aversion is likely to play a role but other factors must be present too, suggesting an interesting question for future research.

Figure 8 plots the equilibrium price with CARA relative to the price with risk neutrality for $n = 5$ and $n = 9$. As the level of risk aversion increases, the ratio of the price to the risk neutral price converges to 2. The figure suggests two further regularities. First, fixing

[24] See for example Cox et al. (1988), Goeree et al. (2002), and Goeree et al. (2003).

[25] Indeed, $\lim_{\rho\to\infty} \frac{2}{\rho(n+1)} \ln\left(\frac{1+e^{\rho v_{n-1}}}{2}\right) = 2\frac{v_{n-1}}{n+1}$. But the result holds not only with CARA, but for all concave $u(\cdot)$.

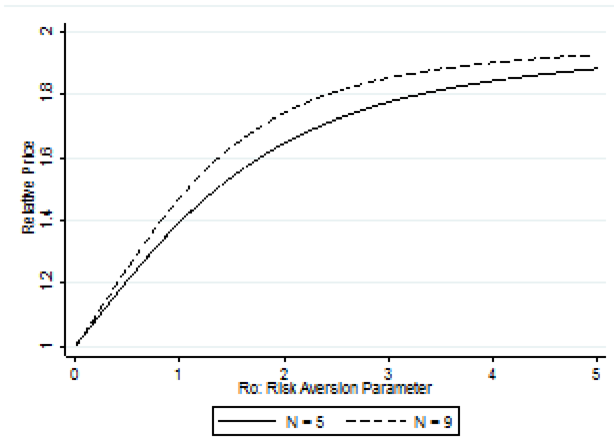

Figure 8: Equilibrium price with CARA utility function, relative to the equilibrium price with risk neutrality. The two lines correspond to different number of voters.

the level of risk aversion, an increase in committee size induces an increase in the relative equilibrium price. Second, relatively low levels of risk aversion translate into large increases in the price. In the case of $n = 5$, $\rho = 1$ $(= 2)$ implies a 40% (60%) increment of the equilibrium price with respect to the risk neutral benchmark.

6 Conclusions

This paper proposes and tests a competitive equilibrium to study allocations resulting from a market for votes. Vote markets are unique and have properties that require a non-standard approach. Votes have no intrinsic value, they are lumpy, and acquiring votes simply increases the probability of creating a good public outcome from the standpoint of the buyer. And because individuals disagree about what a "good" public outcome is, buying and selling votes create externalities. We define the concept of *Ex Ante Vote-Trading Equilibrium*, a concept that combines the standard price-taking assumption of competitive equilibrium in a market for goods with less standard assumptions: demands are best responses to the demands of the other traders, mixed demands are allowed, and market clearing is achieved by a rationing rule. Using a constructive proof, we establish sufficient conditions for existence of an ex ante equilibrium, in a vote market where voters have incomplete information about other members' preferences and a single binary decision has to be made, and we characterize the properties of this equilibrium.

A striking feature of the equilibrium vote allocation is that there always is a dictator: with probability one a single trader acquires a majority position. This feature pins down the welfare properties of the vote market. Because the market results in a dictator, and because the dictator must be one of the two highest valuation (or intensity) voters, the welfare effects depend on the distribution of valuations and on the size of the electorate or committee. When the number of voters is large, the market creates inefficiencies relative to simple majority voting. On average, efficiency is higher if the outcome reflects that median preference rather than the preference of the dictator, even if he is the voter with highest intensity. Put another way, as n gets larger, the externality problem becomes worse – in particular it creates more inefficiency than the failure of majority rule to reflect intensity of preferences. In small committees, the efficiency result depends on the distribution of valuations: the externality problem is relatively small, and a market for votes may enhance efficiency, as long as there is a sufficiently large difference between the highest valuation and the average valuation of the $n-2$ lowest-valuation voters.

The theoretical findings are examined using data from laboratory market experiments with five and nine person committees, where the vote market was conducted as a continuous multi-unit double auction. By varying the distribution of valuations and the number of traders/voters, we controlled the predicted outcomes. In line with the ex ante competitive equilibrium, prices were higher in smaller committees and in treatments where valuations were higher. Furthermore, prices did not vary systematically with valuations, once the top two valuations were fixed. With one exception, experimental prices were significantly higher than risk neutral equilibrium prices, albeit with significant downward convergence. We show that one explanation of the observed overpricing is risk aversion. If traders are risk averse, risk neutral pricing is only a lower bound, and the equilibrium price can be as high as double the risk neutral equilibrium price.

Observed vote allocations were close to equilibrium allocations, particularly in the B treatments where the two top valuations were significantly higher than the remainder. In smaller committees, such treatments resulted in dictatorship more than three quarters of the times, on average. In larger committees, where the purchase of four votes is required for dictatorship, the frequency of dictatorship was lower, but dictatorship still arose between one fourth and one fifth of the times when the discrepancy between the top two valuations and the others was large. The welfare results matched the theory: average experimental payoffs were higher than majority voting payoffs would have been, with the set of experimental valuations, when the discrepancy between top valuations and all others was large, and were lower otherwise.

There are still many open questions about vote markets, and in our experiment we explore only the simplest environment. A key direction of future research is to extend the model to multiple issues. Here we consider only a one-issue vote market, but in principle the same approach can be applied to the more general case of committees that vote on multiple issues, such as legislatures, boards, and standing committees. With multiple issues, the model will then be able to address questions of *vote trading* where the market effectively becomes a means for a voter to accumulate votes on issues he cares most about in exchange for his vote on issues he cares less about. The welfare properties of multi-issue vote markets are likely to be more complicated to analyze, and unclear. Extrapolating from our findings suggests significant inefficiencies, as found in other models of vote exchange that focus on simple bartering examples (Riker and Brams, 1971), rather than taking a general equilibrium approach as we do here. On the other hand, inefficiencies might be somewhat mitigated by the presence of multiple issues, and the more numerous possibilities for gains from trade, as in recent models of mechanisms that link decisions across multiple dimensions (Jackson and Sonnenschein, 2007, Casella, 2005, Hortala-Vallve, 2007).

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Appendix I

Proofs

Theorem 1. *Suppose $\eta_i = \frac{1}{2}$, $w_i = 1$, and $m_i = 0\ \forall i$, agents are risk neutral, and $R1$ is the rationing rule. Then for all n there exists a finite threshold $\mu_n \geq 1$ such that if $v_n \geq \mu_n v_{n-1}$, the following set of price and actions constitute an Ex Ante Vote-Trading Equilibrium:*

1. Price $p^* = \frac{v_{n-1}}{n+1}$.
2. Voters 1 to $n-2$ offer to sell their vote with probability 1.
3. Voter n demands $\frac{n-1}{2}$ votes with probability 1.
4. Voter $n-1$ offers to sell his vote with probability $\frac{2}{n+1}$, and demands $\frac{n-1}{2}$ votes with probability $\frac{n-1}{n+1}$.

Proof. Voter $\mathbf{n-1}$. If voter $n-1$ sells, the total supply of votes is $n-1$ votes. Voter n demands $\frac{n-1}{2}$ votes. Thus total demand equals $\frac{n-1}{2}$, and voter $n-1$'s probability of being rationed is equal to $\frac{1}{2}$. Therefore, $U^{n-1}(-1) = \frac{1}{2}v_{n-1} + \frac{1}{2}p$. On the other hand, if voter $n-1$ demands $\frac{n-1}{2}$, he is again rationed with probability $\frac{1}{2}$, and his expected utility is $U^{n-1}(\frac{n-1}{2}) = \frac{3}{4}v_{n-1} - \frac{n-1}{4}p$. The price that makes him indifferent is exactly $p = \frac{v_{n-1}}{n+1}$. Demanding other quantities is strictly dominated: a smaller quantity can be accommodated with no rationing, but voter $n-1$ would pay the units demanded, and voter n, with a majority, would always decide; a larger quantity is equivalent if $n-1$ is rationed, but is costly and redundant if $n-1$ is not rationed.

Voters $\mathbf{1, 2, ..., n-2}$. Buying votes can only be advantageous if it can prevent both voter $n-1$ and voter n from becoming dictator, but a demand of more than $\frac{n-1}{2}$ is always dominated. Thus the only positive demands to consider are $\frac{n-1}{2} - 1$ or $\frac{n-1}{2}$ votes. In the proposed equilibrium, for any $i \in \{1, ..., n-2\}$

$$U^i(-1) = \frac{1}{2}v_i + \frac{n^2-3}{2\,(n-2)\,(n+1)}p \tag{3}$$

$$U^i(\frac{n-1}{2}) = \begin{cases} \frac{4n+5}{6(n+1)}v_i - \frac{n^2+n-2}{6(n+1)}p & \text{if } n > 3 \\ \frac{3}{4}v_1 - \frac{1}{4}p & \text{if } n = 3 \end{cases} \tag{4}$$

$$U^i(\frac{n-3}{2}) = \begin{cases} \frac{n+2+(n-1)\phi^n}{3(n+1)}v_i - \frac{(n-3)(n+5)}{6(n+1)}p & \text{if } n > 3 \\ \frac{5}{8}v_1 & \text{if } n = 3 \end{cases} \tag{5}$$

where $\phi^n = 1 - \left(\frac{1}{2}\right)^{\frac{n+1}{2}}$. It is easy to see that selling dominates in the case of $n = 3$. In the case of $n > 3$, 3 is bigger than 4 whenever $\frac{v_{n-1}}{v_i} \geq \frac{n^2-4}{n^2+n-5}$, which holds for any positive n. On the other hand 3 is bigger than 5 whenever $\frac{v_{n-1}}{v_i} \geq \frac{(n^2-1)(n-2)(2\phi^n-1)}{n^3+3n^2-19n+21}$, which also holds for any positive n (see that $2\phi^n - 1 < 1$).

Voter n. In the proposed equilibrium, the expected utilities to voter n from demanding a majority of votes or from deviating and ordering g votes less are given by:

$$U^n(\frac{n-1}{2}) = \frac{3n+5}{4(n+1)}v_n - \frac{n^2+2n-3}{4(n+1)}p \tag{6}$$

$$U^n(\frac{n-1}{2} - g) = \frac{n-1+4\phi^n(g)}{2(n+1)}v_n - \left(\frac{n-1}{2} - g\right)p \tag{7}$$

where $\phi^n(g) = \sum_{i \geq g}^{\frac{n-1}{2}+g} \binom{\frac{n-1}{2}+g}{i} \cdot \left(\frac{1}{2}\right)^{\frac{n-1}{2}+g}$. 6 will be bigger than 7 whenever the following inequality holds:

$$g + 2 \cdot \phi^n(g) \leq \frac{n^2+3n+4}{2(n+1)}$$

The left hand side is increasing in g, and therefore reaches its maximum at $g = \frac{n-1}{2}$, that is, when voter n does not buy a vote at all. We need to show that

$$\phi^n\left(\frac{n-1}{2}\right) \leq \frac{1}{4}\frac{3n+5}{n+1}$$

The inequality holds for any n since $\phi^n\left(\frac{n-1}{2}\right) \leq \frac{3}{4}$.

Finally, the expected utility to voter n from selling his vote is given by:

$$U^n(-1) = \frac{n-1}{n+1}\frac{1}{2}[v_n + p] + \frac{2}{n+1}\left[\frac{1}{2} + \binom{n-1}{\frac{n-1}{2}}2^{-n}\right].$$

Thus $U^n(-1) \leq U^n(\frac{n-1}{2})$ whenever

$$\frac{v_n}{v_{n-1}} \geq \frac{(n-1)(n+5)}{(n+1)\left(n+3-\binom{n-1}{\frac{n-1}{2}}2^{-(n-3)}\right)}$$

Thus the Theorem holds with $\mu(n) = \frac{(n-1)(n+5)}{(n+1)\left(n+3-\binom{n-1}{\frac{n-1}{2}}2^{-(n-3)}\right)}$. ■

Proposition 1. Fix n and a profile of valuations v with some v_n, v_{n-1}, and $\overline{v}_{n-2} > 0$, and such that $v_n \geq \mu_n v_{n-1}$. Then there always exists a finite $\overline{n}(v_n, v_{n-1}, \overline{v}_{n-2})$ such that if $n > \overline{n}$, $W_{VM} < W_{MR}$.

Proof. Comparing equations 1 and 2, we can write:

$$W_{MR} \geq W_{VM} \Longleftrightarrow \frac{\overline{v}_{n-2}}{v_n} \leq \psi_n + \omega_n \frac{v_{n-1}}{v_n}$$

where $\psi_n = \frac{n+3}{4(n+1)(n-2)\chi_n} - \frac{1}{n-2}$, $\omega_n = \frac{n-1}{4(n+1)(n-2)\chi_n} - \frac{1}{n-2}$ and $\chi_n = \binom{n-1}{\frac{n-1}{2}} 2^{-n}$. The coefficient ψ_n reaches its maximum at $n = 3$, decreases at higher n, and converges to zero as $n \to \infty$. The coefficient ω_n is negative for $n \leq 5$, reaches its maximum at $n = 17$, decreases at higher n, and converges to zero as $n \to \infty$. Thus $\psi_n + \omega_n \frac{v_{n-1}}{v_n}$converges to 0 in n, and as long as $\overline{v}_{n-2}/v_n > 0$, the Proposition follows. ■

Proposition 2. *Suppose $\eta_i = \frac{1}{2}$, $w_i = 1$, and $m_i = 0$ $\forall i$, agents are risk neutral, and $R2$ is the rationing rule. Then there exist $\overline{n}$ and a finite threshold $k(n) \geq 1$ such that for all $n \leq \overline{n}$ and $v_{n-1} \geq k(n) v_{n-2}$ the following set of price and actions constitute an Ex Ante Vote-Trading Equilibrium:*

1. *Price $p^* = \frac{v_{n-1}}{2(n-1)}$.*

2. *Voters 1 to $n-2$ offer to sell their vote with probability 1.*

3. *Voter n demands $\frac{n-1}{2}$ votes with probability 1.*

4. *Voter $n-1$ offers his vote with probability $\frac{2}{n+1}$, and demands $\frac{n-1}{2}$ votes with probability $\frac{n-1}{n+1}$.*

In particular, 1–4 constitute an equilibrium if $n \leq 9$ and $v_{n-1} \geq 1.15 v_{n-2}$. The latter condition is satisfied in our experimental treatments; it is a sufficient condition for $n < 9$, and it is necessary and sufficient for $n = 9$.

Proof. If $n = 3$, in equilibrium $R2$ is identical to $R1$, and Theorem 1 1 applies here. immediately. The proof considers $n > 3$.

Voter $\mathbf{n-1}$. In the candidate equilibrium, he has expected utility $U^{n-1}(-1) = 1/2(p + v_{n-1}/2) + 1/2(v_{n-1}/2) = v_{n-1}/2 + p/2$. (a) Demanding a number of votes $x \in (0, (n-1)/2)$ cannot be a profitable deviation. Any such demand is satisfied with probability 1, causing an expenditure of $px > 0$ while leaving the probability of obtaining the desired outcome at

$1/2$, and thus results in expected utility $U^{n-1}(x) = v_{n-1}/2 - px$. (b) Demanding more than $(n-1)/2$ votes cannot be a profitable deviation: as long as voter $n-1$ has received less than $(n-1)/2$ votes, $n-1$'s order is outstanding whether his demand is $(n-1)/2$ or higher - and thus the deviation does not affect the probability of individual n being rationed; once voter $n-1$ has received $(n-1)/2$ votes, he controls the final outcome, and any further expenditure is wasted. (c) Finally, doing nothing ($U^{n-1}(0) = v_{n-1}/2$) is dominated by offering to sell.

Voter n. (a) Doing nothing is again dominated by selling: it is identical to selling if $n-1$ sells, and it is strictly dominated if $n-1$ buys. (b) Selling is dominated by demanding $(n-1)/2$ votes. If $n-1$ offers to buy, then n must prefer demanding $(n-1)/2$ to selling because in the identical circumstance, $n-1$, with smaller valuation, is indifferent between the two options. If $n-1$ offers to sell, again n must prefer to buy $(n-1)/2$: when $n-1$ sells, buying yields expected utility $v_n - (n-1)/2p$, while offering to sell means that no trade takes place (all voters try to sell) and n wins with probability $\mu \equiv \sum_{k=(n-1)/2}^{n-1} \binom{n-1}{k} (1/2)^{n-1}$ (the probability that at least $(n-1)/2$ of the other voters agree with him). But μ is declining in n, and thus is maximal at $n=3$, where it equals $3/4$. Hence when $n-1$ sells, n's expected utility from offering to sell has upper bound $(3/4)v_n$. But $v_n - (n-1)/2p = v_n - v_{n-1}/4 > (3/4)v_n$ for all $v_n > v_{n-1}$. Hence the only deviation to consider is demanding a quantity of votes x different from $(n-1)/2$. (c) Demanding a quantity x larger than $(n-1)/2$ cannot be profitable. If voter $n-1$ is selling, the order will be filled and is less profitable than demanding $(n-1)/2$; if voter $n-1$ is buying, the argument is identical to point 1b above. (d) Demanding a quantity x smaller than $(n-1)/2$ is not a profitable deviation either. In the candidate equilibrium n has expected utility equal to:

$$
\begin{aligned}
U^n\left(\frac{n-1}{2}\right) &= \left(\frac{2}{n+1}\right)(v_n - p\frac{n-1}{2}) + \\
&\quad + \left(\frac{n-1}{n+1}\right)\left[\frac{1}{2}\left(v_n/2 - p\frac{n-3}{2}\right) + \frac{1}{2}\left(v_n - p\frac{n-1}{2}\right)\right] = \\
&= \frac{v_n(5+3n) - nv_{n-1}}{4(n+1)}
\end{aligned}
$$

where the second expression is obtained by substituting for p. If n offers to buy $x < (n-1)/2$, his demand is always satisfied. His expected utility is $v_n/2 - px$ if $n-1$ is buying, and $\gamma(x)v_n - px$ if $n-1$ is offering to sell, where $\gamma(x) \equiv \sum_{k=(n-1)/2-x}^{n-1-x} \binom{n-1-x}{k} (1/2)^{n-1-x}$ is the probability that n obtains his preferred outcome when owning $x+1$ votes (while everyone else has one vote). Thus:

$$U^n(x) = \left(\frac{2}{n+1}\right)(\gamma(x)v_n - px) + \left(\frac{n-1}{n+1}\right)(v_n/2 - px)$$

With $x < (n-1)/2$, the difference $U^n\left(\frac{n-1}{2}\right) - U^n(x)$ is minimal when p is highest, i.e when $v_{n-1} = v_n$. But:

$$\begin{aligned} U^n\left(\frac{n-1}{2}\right)|_{v_{n-1}=v_n} &= \frac{5+2n}{4(n+1)} \\ U^n(x)|_{v_{n-1}=v_n} &= (1/2)\left(\frac{n-1+4\gamma(x)}{n+1} - \frac{x}{n-1}\right) \end{aligned}$$

It then follows immediately that $U^n\left(\frac{n-1}{2}\right)|_{v_{n-1}=v_n} > U^n(x)|_{v_{n-1}=v_n}$ for all $x > 0$ and all $\gamma(x) \leq 1$. Hence $U^n\left(\frac{n-1}{2}\right) > U^n(x)$: deviation is not advantageous.

Voters $\mathbf{1, 2, ..., n-2}$. (a) One possible deviation is for voter $i \in \{1, .., n-2\}$ to do nothing. When voter $n-1$ demands $(n-1)/2$ votes, supply is $n-3$ and it is then possible for neither $n-1$ nor n to be dictator. Call $r_e(n)$ the probability that votes supplied are allocated equally to n and $n-1$, when $n-1$ demands $(n-1)/2$ votes. I.e.:

$$r_e(n) = \binom{n-3}{(n-3)/2}(1/2)^{n-3}. \tag{8}$$

Then i's expected utility from doing nothing, U_i^0, is given by:

$$U^i(0) = \frac{2}{n+1}\left(\frac{v_i}{2}\right) + \frac{n-1}{n+1}\left[(1-r_e(n))\frac{v_i}{2} + r_e(n)\frac{3}{4}v_i\right]. \tag{9}$$

Voter i's expected utility from selling, $U^i(-1)$, is:

$$U^i(-1) = \frac{2}{n+1}\left(\frac{v_i}{2} + \frac{p}{2}\right) + \frac{n-1}{n+1}\left(\frac{v_i}{2} + p\right) \tag{10}$$

Comparing (9) to (10) and substituting (8), we derive;

$$\begin{aligned} U^i(-1) &\geq U^i(0) \Longleftrightarrow \\ \frac{v_{n-1}}{v_i} &\geq \frac{(n-1)^2}{n}\binom{n-3}{(n-3)/2}(1/2)^{n-2} \end{aligned} \tag{11}$$

The right hand side of equation (11) is smaller than 1 for all $n < 9$, and equals 10/9 at $n = 9$. Thus deviation to doing nothing is never advantageous for any i at $n = 5$ or 7; at $n = 9$, we need to impose $v_{n-1} \geq (10/9)v_{n-2}$. It will be shown below that this is not the binding restriction. However:

$$\lim_{n->\infty} \frac{(n-1)^2}{n} \binom{n-3}{(n-3)/2} (1/2)^{n-2} = \infty$$

Thus unless $v_i = 0$ for all $i < n-1$, there must always exist a number $\overline{n}$ such that for all $n > \overline{n}$ voter $n-2$ prefers to do nothing than selling. The actions and price described in the Proposition can only be an equilibrium for $n \leq \overline{n}$.

(b) The other possible deviation for voter i is demanding a positive number of votes x, with $x \in \{1, 2, .., (n-1)/2\}$. As before, demanding more than $(n-1)/2$ votes is never advantageous. Consider first i's expected utility from demanding $(n-1)/2$ votes. Call $\delta_i(n, x_{n-1})$ the probability that i becomes the dictator, i.e. the probability that he obtains $(n-1)/2$ votes, as function of n and of voter $n-1$'s demand, x_{n-1}. When $n-1$ demands $(n-1)/2$ votes, the total supply of votes is $n-3$, and $\delta_i(n, (n-1)/2)$ is the probability that at least $(n-1)/2$ votes are randomly allocated to voter i:

$$\delta_i(n, (n-1)/2) = \sum_{i=(n-1)/2}^{n-3} \sum_{z=0}^{n-3-i} \frac{(n-3)!}{i!z!(n-3-z-i)!} (1/3)^{n-3}$$

Similarly, $\delta_{-i}(n, (n-1)/2))$ is the probability that either n or $n-1$ become dictator, i.e. the probability that at least $(n-1)/2$ votes are randomly allocated to one of them:

$$\delta_{-i}(n, (n-1)/2) = 2 \sum_{z=(N-1)/2}^{n-3} \sum_{y=0}^{n-3-z} \frac{(n-3)!}{z!y!(n-3-z-y)!} (1/3)^{n-3}$$

Thus i's expected utility, when $n-1$ demands $(n-1)/2$ votes, either n or $n-1$ become dictator and i demands $x_i = (n-1)/2$ votes is given by:

$$Z^i_{d(-i)}\left(\frac{n-1}{2}\right) = 2\sum_{z=(n-1)/2}^{n-3}\sum_{y=0}^{n-3-z}\frac{(n-3)!}{z!y!(n-3-z-y)!}(1/3)^{n-3}$$
$$\left[\frac{v}{2} - p\left(n-3-z-y+\sum_{i=1}^{z-(n-1)/2}\binom{z-(n-1)/2}{i} i(1/2)^{z-(n-1)/2}\right)\right]$$

Finally, there is the probability that no dictator arises:

$$1-\delta_i-\delta_{-i} = \sum_{z=0}^{(n-3)/2}\sum_{y=(n-3)/2-z}^{(n-3)/2}\frac{(n-3)!}{z!y!(n-3-z-y)!}(1/3)^{n-3}$$

and the corresponding expected utility:

$$Z^i_{nod}\left(\frac{n-1}{2}\right) = \sum_{z=0}^{(n-3)/2}\sum_{y=(n-3)/2-z}^{(n-3)/2}\frac{(n-3)!}{z!y!(n-3-z-y)!}(1/3)^{n-3}\left[\frac{3v}{4} - p(n-3-z-y)\right]$$

We can then write:

$$\begin{aligned} U^i\left(\frac{n-1}{2}\right) &= \frac{n-1}{n+1}\left[Z^i_{d(-i)}\left(\frac{n-1}{2}\right) + Z^i_{nod}\left(\frac{n-1}{2}\right) + \delta_i\left(v - \frac{n-1}{2}p\right)\right] + \\ &\quad \frac{2}{n+1}\left[\frac{1}{2}\left(v - \frac{n-1}{2}p\right) + \frac{1}{2}\left(\frac{v}{2} - \frac{n-3}{2}p\right)\right] \end{aligned}$$

Comparing $U^i\left(\frac{n-1}{2}\right)$ to i's expected utility from selling, and substituting p, we find that $U^i(-1) \geq U^i\left(\frac{n-1}{2}\right)$ for all $i < n-1$ if and only if: $v_{n-1} \geq (25/24)v_{n-2}$, if $n=5$; $v_{n-1} \geq (11/10)v_{n-2}$, if $n=7$; and $v_{n-1} \geq 1.15v_{n-2}$, if $n=9$.

(c) For $n > 3$, demanding less than $(n-1)/2$ votes can in principle be advantageous if $n-1$ demands $(n-1)/2$ votes, and neither $n-1$ nor n emerge as dictators. The calculations are somewhat cumbersome, but follow the logic just described, and we do not report them here (they are available from the authors upon demand). They show: (1) $U^i(-1) \geq U^i\left(\frac{n-3}{2}\right)$ for all i as long as $v_{n-1} \geq v_{n-2}$, satisfied by definition. For $n=5$, this concludes the proof. (2) $U^i(-1) \geq U^i\left(\frac{n-5}{2}\right)$ for all i as long as $v_{n-1} \geq v_{n-2}$, satisfied by definition. For $n=7$, this completes the proof. (3) For $n=9$, $U^i(-1) \geq U^i\left(\frac{n-7}{2}\right)$ for all i as long as $v_{n-1} \geq 1.04v_{n-2}$.

The condition is not binding, because $v_{n-1} \geq 1.15 v_{n-2}$ is required to prevent $n-2$ to deviate to doing nothing.

Summarizing the conditions derived in (a), (b) and (c), we conclude that the actions and price described in the proposition are an equilibrium for $n = 5$ if and only if $v_{n-1} \geq (25/24) v_{n-2}$; for $n = 7$, if and only if $v_{n-1} \geq (11/10) v_{n-2}$; and for $n = 9$, if and only if $v_{n-1} \geq 1.15 v_{n-2}$. With $1.15 > 11/10 > 25/24$, the latter condition is sufficient for $n = 3, 5, 7$ and necessary and sufficient for $n = 9$. This is the statement in the Proposition. ■

Proposition 3. *Suppose* $\eta_i = \frac{1}{2}$, $w_i = 1$, *and* $m_i = 0$ $\forall i$, $u(\cdot) = -e^{-\rho(\cdot)}$ with $\rho > 0$, *and* $R1$ *is the rationing rule. Then for all* n *there exists a finite threshold* $\mu_n \geq 1$ *such that if* $v_n \geq \mu_n v_{n-1}$, *the set of actions presented in Theorem 1 together with the price* $p = \frac{2}{r(n+1)} \ln\left(\frac{1}{2} + \frac{1}{2} e^{r v_{n-1}}\right)$ *constitute an Ex Ante Vote-Trading Equilibrium.*

Proof. Voter $\mathbf{n-1}$. As in Theorem 1, p must be such that individual $n-1$ is indifferent between selling his vote or demanding a majority of votes. If voter $n-1$ offers to sell his vote, he is rationed with probability 1/2; whether he is rationed or not, the decision is made by voter n, who owns a majority of votes and agrees with voter $n-1$ with probability 1/2. Thus:

$$U^{n-1}(-1) = \frac{1}{4}\left(u(0) + u(v_{n-1}) + u(p) + u(v_{n-1} + p)\right)$$

If voter $n-1$ demands $\frac{n-1}{2}$ votes, he is again rationed withe probability 1/2; if he is not rationed, he is dictator, if he is rationed, the dictator is voter n who agrees with $n-1$ with probability 1/2. Hence:

$$U^{n-1}\left(\frac{n-1}{2}\right) = \frac{1}{4}\left(u(0) + u(v_{n-1})\right) + \frac{1}{2} u\left(v_{n-1} - \frac{n-1}{2} p\right).$$

Thus the price at which $n-1$ is indifferent must solve:

$$u(p) + u(v_{n-1} + p) = 2 \cdot u\left(v_{n-1} - \frac{n-1}{2} p\right) \tag{12}$$

In the case of a CARA utility, the price that makes voter $n-1$ indifferent is computable and equal to $p = \frac{2}{\rho(n+1)} \ln\left(\frac{1}{2} + \frac{1}{2} e^{\rho v_{n-1}}\right)$.

As in Theorem 1, demanding other quantities is strictly dominated because it is either equivalent to demanding $\left(\frac{n-1}{2}\right)$ if voter $n-1$ is rationed, or strictly worse, if he is not.

Voter n. In equilibrium, voter n's expected utility from demanding $\left(\frac{n-1}{2}\right)$ votes, is given by:

$$\begin{aligned} U^n\left(\frac{n-1}{2}\right) &= \frac{2}{n+1}u\left(v_n - \frac{n-1}{2}p\right) + \\ &+\frac{n-1}{2(n+1)}\left[\frac{1}{2}u(0) + \frac{1}{2}u(v_n) + u\left(v_n - \frac{n-1}{2}p\right)\right] \end{aligned}$$

If voter n deviates and offers his vote for sale, his expected utility is

$$\begin{aligned} U^n(-1) &= \frac{2}{n+1}\left[\phi^n\left(\frac{n-1}{2}\right)u(v_n) + \left(1-\phi^n\left(\frac{n-1}{2}\right)\right)u(0)\right] + \\ &+\frac{n-1}{2(n+1)}\left[\frac{1}{2}u(0) + \frac{1}{2}u(v_n) + \frac{1}{2}u(p) + u(v_n+p)\right] \end{aligned}$$

where, as defined earlier, $\phi^n\left(\frac{n-1}{2}\right) = \sum_{i=(n-1)/2}^{n-1}\binom{n-1}{i}\cdot\left(\frac{1}{2}\right)^{n-1}$ is the probability that at least $\frac{n-1}{2}$ other voters agree with him, in the event that no trade has occurred.

Finally, if voter n deviates and demands $\left(\frac{n-1}{2}-g\right)$ votes, his expected utility is:

$$\begin{aligned} U^n\left(\frac{n-1}{2}-g\right) &= \left(\frac{2}{n+1}\phi^n(g) + \frac{n-1}{2(n+1)}\right)u\left(v_n - \left(\frac{n-1}{2}-g\right)p\right) + \\ &+\left(\frac{2}{n+1}(1-\phi^n(g)) + \frac{n-1}{2(n+1)}\right)u\left(-\left(\frac{n-1}{2}-g\right)p\right) \end{aligned}$$

where again $\phi^n(g) = \sum_{i\geq g}^{\frac{n-1}{2}+g}\binom{\frac{n-1}{2}+g}{i}\cdot\left(\frac{1}{2}\right)^{\frac{n-1}{2}+g}$ is the probability that at least g other voters agree with him, when voter $n-1$ has offered his vote for sale and $\frac{n-1}{2}+g$ voters in all retain their vote.

All three expected utilities are continuous in p; $U^n\left(\frac{n-1}{2}\right)$ and $U^n\left(\frac{n-1}{2}-g\right)$ are everywhere strictly decreasing in p, $U^n(-1)$ is everywhere strictly increasing in p Notice that at $p=0$ (and thus $v_{n-1}=0$), $U^n\left(\frac{n-1}{2}\right)|_{p=0} > U^n(-1)|_{p=0}$, because $\phi^n\left(\frac{n-1}{2}\right) < 1$, and $U^n\left(\frac{n-1}{2}\right)|_{p=0} > U^n\left(\frac{n-1}{2}-g\right)|_{p=0}$, because $U^n\left(\frac{n-1}{2}-g\right)|_{p=0}$ is increasing in $\phi^n(g)$ and $\phi^n(g)$ is maximal at $g=0$. Thus for any v_n $U^n\left(\frac{n-1}{2}\right) > U^n(-1)$ and $U^n\left(\frac{n-1}{2}\right) > U^n\left(\frac{n-1}{2}-g\right)$ if v_{n-1} (and thus p) is sufficiently low. Equivalently there must exist a value μ_n such that if $v_n \geq \mu_n v_{n-1}$, $U^n\left(\frac{n-1}{2}\right) > U^n(-1)$ and $U^n\left(\frac{n-1}{2}\right) > U^n\left(\frac{n-1}{2}-g\right)$. This is the gap identified in the Proposition.

Voters $\mathbf{1,2,...,n-2}$. As in Theorem 1, for voter $i \in \{1,...,n-2\}$ deviation can be profitable only if he demands $\frac{n-1}{2}-1$ or $\frac{n-1}{2}$ votes. (1) Consider first deviation to demanding $\frac{n-1}{2}$ votes. If $n > 3$, the possible outcomes and their probabilities are represented in the following Table:

Offer		**Demand $\frac{n-1}{2}$ Votes**	
Outcome	Prob	Outcome	Prob
0	$\frac{1-\delta}{2}$	0	$\frac{1-\varepsilon}{2}$
p	$\frac{1}{2}\delta$		
v_i	$\frac{1-\delta}{2}$	$v_i - \frac{n-1}{2}p$	ε
$v_i + p$	$\frac{1}{2}\delta$	v_i	$\frac{1-\varepsilon}{2}$

where we define $\delta = \frac{1}{n+1} + \frac{(n-1)^2}{2(n+1)(n-2)}$ and $\varepsilon = \frac{n+2}{3(n+1)}$. Thus:

$$\begin{aligned} U^i(-1) &> U^i\left(\frac{n-1}{2}\right) \Longleftrightarrow \\ \delta \cdot [u(p) + u(v_i+p)] &\geq 2\varepsilon \cdot u\left(v_i - \frac{n-1}{2}p\right) \\ &\quad + (\delta - \varepsilon) \cdot [u(v_i) + u(0)] \end{aligned} \tag{13}$$

Note that $v_i \leq v_{n-1}$. At $v_i = v_{n-1}$, given equation (12), the fact that $u(\cdot)$ is increasing and the fact that $\delta > \varepsilon$, equation (13) holds with strict inequality. We can show that if equation (13) holds at $v_i = v_{n-1}$, it must hold at all $v_i < v_{n-1}$. Denote:

$$\Delta = \delta \cdot [u(p) + u(v_i+p)] - 2\varepsilon \cdot u\left(v_i - \frac{n-1}{2}p\right) - (\delta - \varepsilon) \cdot [u(v_i) + u(0)]$$

Then

$$\frac{\partial \Delta}{\partial v} = \delta \cdot [u'(v_i+p) - u'(v_i)] - \varepsilon \cdot \left[2 \cdot u'\left(v_i - \frac{n-1}{2}p\right) - u'(v_i)\right]$$

But the concavity of u then implies $\frac{\partial \Delta}{\partial v} < 0$, and the result is established.

If $n = 3$, selling is preferred to buying one vote if $2u(p) + 2u(v+p) \geq 3u(v) + u(0)$. Note first that because $2u'(v+p) - 3u'(v) < 0$ by concavity, we only need to check the condition at $v = v_{n-1}$. Using the specific functional form of CARA utility simplifies the rest of the proof. Recall that the price is given by $p = \frac{1}{2\rho} ln\left(\frac{1+e^{\rho v_{n-1}}}{2}\right)$. Hence, $e^{-\rho p} = \left(\frac{2}{1+e^{\rho v_{n-1}}}\right)^{\frac{1}{2}}$ and

$e^{-\rho(v+p)} = \left(\frac{2}{1+e^{\rho v_{n-1}}}\right)^{\frac{1}{2}} e^{-\rho v}$. The inequality that we need to verify reduces to

$$2\sqrt{2}\left(1+e^{\rho v_{n-1}}\right)^{\frac{1}{2}} \leq e^{\rho v_{n-1}} + 3$$

Define $x = 1 + e^{\rho v_{n-1}}$. Then, we want to show that $2\sqrt{2}\sqrt{x} \leq 2 + x$. But $2 + x - 2\sqrt{2}\sqrt{x}$ has a minimum at $x = 2$, which is 0. Hence, the condition is always satisfied.

(2) We now show that if $n > 3$, selling one's vote dominates demanding $\frac{n-3}{2}$ votes. If $n > 3$, the difference of utilities is given by:

$$\begin{aligned} U^k(-1) - U^k(\frac{n-3}{2}) &= -\frac{n^2-6n+11}{12(n+1)(n-2)}(u(v_k)+u(0)) \\ &+\frac{n^2-3}{4(n+1)(n-2)}(u(p)+u(v_k+p)) \\ &- \left[\frac{n-1}{3(n+1)}\left(\frac{1}{2}\right)^{\frac{n+1}{2}}\right] u\left(-\frac{n-3}{2}p\right) \\ &- \left[\frac{n-1}{3(n+1)}\left(1-\left(\frac{1}{2}\right)^{\frac{n+1}{2}}\right)\right] u\left(v_k - \frac{n-3}{2}p\right) \\ &- \frac{1}{n+1}\left[u\left(v_k-\frac{n-3}{2}p\right)+u\left(-\frac{n-3}{2}p\right)\right] \end{aligned} \tag{14}$$

It is somewhat cumbersome but not difficult to show that the expression is decreasing in v.[26] Hence it is minimal at $v = v_{n-1}$; if 14 is positive then, it is positive for all $v_k \leq v_{n-1}$. Again, we make use of the CARA functional form. Define $\lambda = \frac{1+e^{\rho v_{n-1}}}{2}$ so that $p = \frac{2}{(n+1)\rho} ln(\lambda)$. Thus:

$$\begin{array}{rr} e^{-\rho p} = \lambda^{-\frac{2}{n+1}} & e^{-\rho(v+p)} = e^{-\rho v}\lambda^{-\frac{2}{n+1}} \\ e^{\rho\frac{n-3}{2}p} = \lambda^{\frac{n-3}{n+1}} & e^{-\rho\left(v-\frac{n-3}{2}p\right)} = e^{-\rho v}\lambda^{\frac{n-3}{n+1}} \end{array}$$

[26] The proof is available upon request.

Substituting in equation 14, we can write:

$$\begin{aligned}
e^{\rho v_{n-1}}(U^k(-1) - U^k(\frac{n-3}{2})) &= \frac{n^2-6n+11}{12(n+1)(n-2)}(1+e^{\rho v_{n-1}}) \\
&- \frac{n^2-3}{4(n+1)(n-2)}\lambda^{-\frac{2}{n+1}}(1+e^{\rho v_{n-1}}) \\
&+ \left[\frac{n-1}{3(n+1)}\left(\frac{1}{2}\right)^{\frac{n+1}{2}}\right] e^{\rho v_{n-1}}\lambda^{\frac{n-3}{n+1}} \\
&+ \left[\frac{n-1}{3(n+1)}\left(1-\left(\frac{1}{2}\right)^{\frac{n+1}{2}}\right)\right]\lambda^{\frac{n-3}{n+1}} \\
&+ \frac{1}{n+1}\lambda^{\frac{n-3}{n+1}}(1+e^{\rho v_{n-1}})
\end{aligned}$$

First, because $e^{\rho v_{n-1}} \geq 1$, it is sufficient to show that

$$\begin{aligned}
\gamma &= \frac{n^2-6n+11}{12(n+1)(n-2)}(1+e^{\rho v_{n-1}}) - \frac{n^2-3}{4(n+1)(n-2)}\lambda^{-\frac{2}{n+1}}(1+e^{\rho v_{n-1}}) \\
&+ \frac{n-1}{3(n+1)}\lambda^{\frac{n-3}{n+1}} \\
&+ \frac{1}{n+1}\lambda^{\frac{n-3}{n+1}}(1+e^{\rho v_{n-1}})
\end{aligned}$$

is positive. Note that $1 + e^{\rho v_{n-1}} = 2\lambda$. Hence, we can write

$$\gamma = \frac{\lambda^{\frac{n-3}{n+1}}}{n+1}\left[\frac{n^2-6n+11}{6(n-2)}\lambda^{\frac{4}{n+1}} - \frac{n^2-3}{2(n-2)}\lambda^{\frac{2}{n+1}} + \frac{n-1}{3} + 2\lambda\right]$$

Denote $\Gamma = \frac{n^2-6n+11}{6(n-2)}\lambda^{\frac{4}{n+1}} - \frac{n^2-3}{2(n-2)}\lambda^{\frac{2}{n+1}} + \frac{n-1}{3} + 2\lambda$. We want to show that $\Gamma \geq 0$. Note that

$$\begin{aligned}
\frac{\partial\Gamma}{\partial\lambda} &= 2(\frac{n^2-6n+11}{3(n+1)(n-2)})\lambda^{\frac{3-n}{n+1}} - \frac{n^2-3}{(n+1)(n-2)}\lambda^{\frac{1-n}{n+1}} + 2 \\
\frac{\partial^2\Gamma}{\partial\lambda^2} &= 2(\frac{(3-n)(n^2-6n+11)}{3(n+1)^2(n-2)})\lambda^{\frac{2(1-n)}{n+1}} - \frac{(1-n)(n^2-3)}{(n+1)^2(n-2)}\lambda^{\frac{-2n}{n+1}}
\end{aligned}$$

Therefore,

$$\frac{\partial^2\Gamma}{\partial\lambda^2} \geq 0 \quad \Leftrightarrow \quad 2(n^2-6n+11)\lambda^{\frac{2}{n+1}} \leq 3(n-1)(n+3)$$
$$\Leftrightarrow \quad \lambda \leq \lambda^* = \left(\frac{3(n-1)(n+3)}{2(n^2-6n+11)}\right)^{\frac{n+1}{2}} \quad (>1)$$

Note that by construction, $\lambda \in [1,+\infty]$. Hence, $\frac{\partial\Gamma}{\partial\lambda}$ has a maximum at λ^*. But:

$$\frac{\partial\Gamma}{\partial\lambda}_{|\lambda=1} = \frac{5n^2-18n+19}{3(n+1)(n-2)}$$

which is always positive for $n \geq 3$. Moreover, as $\lambda \to \infty$, we can see that for $n > 3$,

$$\frac{\partial\Gamma}{\partial\lambda} \sim_{\lambda\to\infty} \frac{2}{n+1} > 0$$

Therefore, $\frac{\partial\Gamma}{\partial\lambda} \geq 0$ for any n and ρ. Hence, we only need to show that at $\lambda = 1$, $\Gamma \geq 0$. But $\Gamma_{|\lambda=1} = 0$. Thus: $\Gamma \geq 0$, which implies $\gamma \geq 0$, which implies $U^k(-1) - U^k(\frac{n-3}{2}) \geq 0$, concluding the proof for $n > 3$.

Finally, we need to show that demanding $\frac{n-3}{2}$ is also dominated when $n = 3$, a condition that amounts to showing $3u(v+p) - 4u(v) - 2u(0) + 3u(p) \geq 0$. But $3u'(v+p) - 4u'(v) \leq 0$, and thus we only need to check the inequality at $v = v_{n-1}$. Redefining $\lambda = \frac{1+e^{rv_{n-1}}}{2}$, the condition becomes $-6\lambda + 2\sqrt{\lambda} + 4\lambda\sqrt{\lambda} \geq 0$. The RHS is increasing in λ and is 0 at $\lambda = 1$. Hence, it is always satisfied. ■

Statistical Analysis of the Final Vote Allocation' s Distance to Equilibrium

	Variable		Coef		
$n = 5$	Match		-0.025	Obs	80
	Round		-0.184	Pr > chi2	0.029
	Dummies:	H	0.501^{*}	PseudoR^2	0.114
		T	0.948^{*}	LpL	-74.64
$n = 9$	Match		-0.141	Obs	80
	Round		0.010	Pr > chi2	0.065
	Dummies:	H	-0.006	PseudoR^2	0.062
		T	0.862^{**}	LpL	-102.69

Table 10: Ordered probit regression of distance to equilibrium d as a function of the number of a match, the number of a round, a dummy that takes value one if there are high values, H, and a dummy that takes value one if values are concentrated on the top, T. Data is clustered by session and standard errors are robust. * significant at 10%; ** significant at 5%; *** significant at 1%.

Appendix II

Sample Instructions

Thank you for agreeing to participate in this decision making experiment. During the experiment we require your complete, undistracted attention, and ask that you follow instructions carefully. You may not open other applications on your computer, chat with other students, or engage in other distracting activities, such as using your phone, reading books, etc.

You will be paid for your participation in cash, at the end of the experiment. Different participants may earn different amounts. What you earn depends partly on your decisions, partly on the decisions of others, and partly on chance.

The entire experiment will take place through computer terminals, and all interaction between you will take place through the computers. It is important that you not talk or in any way try to communicate with other participants during the experiments.

During the instruction period, you will be given a complete description of the experiment and will be shown how to use the computers. If you have any questions during the instruction period, raise your hand and your question will be answered out loud so everyone can hear. If you have any questions after the experiment has begun, raise your hand, and an experimenter will come and assist you.

The experiment you are participating in is a committee voting experiment, where you will have an opportunity to buy and sell votes before voting on an outcome.

At the end of the experiment you will be paid the sum of what you have earned, plus a show-up fee of $10.00. Everyone will be paid in private and you are under no obligation to tell others how much you earned. Your earnings during the experiment are denominated in POINTS. Your DOLLAR earnings are determined by dividing your earnings in POINTS by 350, that is, for every 350 POINTS you get 1 DOLLAR.

In this experiment you will be in a 5 member committee to decide on an outcome, X or Y. Each of you will be randomly assigned with probability 1/2 to be either in favor of X or in favor of Y. You will be told which outcome you favor, but will not be told the outcome favored by anyone else. You will also be assigned a Value, which you will earn if and only if your preferred outcome is the committee decision. If the opposite outcome is the committee decision you do not earn your value. Values will be different for different members. All values are integers between 1 and 1000 points.

Committee decisions are made by majority rule. Outcome X is the committee decision if there are more votes for X than for Y and vice versa.

Every round consists of two stages. Each committee member starts the round with one vote. After being told your value, but before voting, there will be a 2 minute trading stage, during which you and the other members of your committee will have an opportunity to buy or sell votes. We will describe how trading occurs momentarily.

After the trading stage ends, we proceed to the voting stage. In this stage you do not really have any choice. You will simply be asked to click a button to cast all your votes, if you have any, for your preferred outcome.

We will repeat this procedure for a set of 5 rounds, each consisting of the same two stages, trading and voting, described above. This set of five rounds is called a match. During each round of the match each of you keep the same Value you were assigned in round 1 of the match, but you will be randomly assigned to be in favor of X or Y (with each equally likely). Therefore, your preferred outcome can change from round to round. At the end of the fifth round of the match, a second match of 5 rounds will begin. In this new match, and you will be assigned a different Value, which you will keep for each of the 5 rounds in the second match. The experiment consists of 4 matches of 5 rounds each.

[SCREEN 1]

When we begin the experiment, you will see a screen like this. Your Subject ID# is printed at the very top left of your screen, and remains the same throughout the whole experiment.

The current match number, round number, your value, and your preferred outcome are displayed below your subject ID in the left part of the screen. The match number and round number are both equal to 1 now, indicating that this is the first election in your committee. Notice that this is an example where member 3's preferred outcome is X and his value 13. The committee number will identify you during the trading stage, and will be the same for the different rounds of a same match, and different between matches.

The middle panel is the trading window. Just above the panel, there is your cash holdings. At the beginning of the experiment, you will be loaned an initial amount of cash of 10.000 points, which will not be included in your final earnings. In the right part of the panel there is a table that clarifies how many votes each member of the committee currently has. Your information is highlighted and the other members' information is not. Notice that you do not see the values of the other members.

As the experiment proceeds, your cash holdings will be updated to reflect any earnings you make. It increases when you sell votes or when you earn your value as a result of the voting. It decreases when you buy votes. At the top of the panel, there is a countdown timer that tells you how much time is left in the trading period. The timer will turn red when there are 10 seconds left in the trading period, as you can see in the screen. There is a history panel in the lower part of the screen which will keep track of the history of the current and all past rounds and matches.

Trading occurs in the following way. At any time during this trading period, any member may post a bid to buy or an offer to sell one or multiple votes. At the bottom of the middle trading panel there is an area where you can type in your bid or your offer. When you do so, it will look like this: [SCREEN 2, 999 entered]. You also have to choose the amount of units you want to buy or sell [SCREEN 3]. Your bids or offers must always be between 1 and 1000. After you type in a price and a quantity, click the "bid" or "offer" button just to the right, and your bid or offer (price and quantity) will be posted on the trading board on the computer screens of all committee members, as you can see in this screen [SCREEN 4]. In this case, the column Bidder ID indicates that the member who made the bid was member 3; the bid price indicates that the price is 999. In the Bidder's Fulfilled columns you can see two numbers: the one on the right indicates the number of units he bid for, and the first number indicates the number of partial acceptances. In our example, he made a bid for one unit and nobody accepted so far. Whenever a new bid or offer is entered, it is added to the board, and does not cancel any outstanding bids or offer if there are any. When other members make bids or offers, you will also see the additions to the table as you can see now in the screen [SCREEN 5]. In this case member 2 made an offer for 201 for one unit, and member 1 made a bid for two units at price 3 (the price indicates the price paid per unit). All members in your committee see this information. The numbers on this slide are for illustration only.

If another member has an active bid or offer, then you may accept it. In order to accept a bid or an offer you just have to click on it and it will become highlighted in yellow [SCREEN 6]. In this case member 3 clicked the offer. At that point, a button below the table becomes active. If there is only one unit to be transacted, as in this case, by clicking the button the unit will be transacted and the transaction is highlighted in green [SCREEN 7].

If you accept an offer, as in this case, you will have an extra vote, and in exchange you will pay the other member the price of his offer. This information is immediately updated on your screens. See that the table on the right has been updated: now member 3 has 2 votes and member 2 has none and the cash holdings have also been updated. Similarly, if you have an active offer for exactly one unit, and another member accepts it, he will own your vote and he will pay you the amount of your offer. The same goes for bids that are accepted, except the transaction is a buy, by the person who posted the bid, rather than a sell. It's very important to remember that you post a Bid if you want to buy and post an offer if you want to sell!

If you accept a bid or offer and the order is for more than one unit, the transaction does not take place until the whole order has been filled. Thus, if someone submitted a bid to buy two votes, and you accept their bid, nothing happens yet because their order has not yet been filled. [SCREEN 8] As you can see now, some member accepted member 1's bid. It will be filled only after a second acceptance has been made, at which time both transactions will be executed simultaneously and the transaction is highlighted in green [SCREEN 9].

If you have an active bid or offer that has not been transacted you can cancel it. To do so, you need to click on it. [SCREEN 10] By doing so, the bid or offer will be highlighted in yellow and cancel button will become active. Clicking the cancel button you will cancel the bid or offer that you clicked on. It will then disappear from the screen [SCREEN 11]. If you have accepted a bid or offer for multiple units which has not been transacted, you can also cancel your partial acceptance. In that case, the bid or offer will remain in the screen but the number of partial acceptances will be updated. You may also cancel all your untransacted market activities at any time by clicking "Cancel All" button, located on the right hand side of the panel, below the table. See that, as the remaining time is less than 10 seconds, the remaining time is red.

The trading period ends after 2 minutes. There are two additional trading rules. First, if your cash holdings ever become 0 or negative, you may not place any bid nor accept any offer until it becomes positive again. Second, you may not sell votes if you do not have any or if all the votes you currently own are committed.

After 2 minutes, the trading stage of the round is over and we proceed to the voting stage. Your screen would now look like [SCREEN 12]. At this stage, you simply cast your votes by clicking on the vote button. These votes are automatically cast as votes for X if your preferred outcome is X, and are automatically cast as votes for Y if your preferred outcome is Y.

After you and the other members of the committee have voted, the results are displayed in the right hand panel, and summarized in the history screen. [SCREEN 13] We will then proceed to the next round. [SCREEN 14] In the next round, as you can see in the right table, all members' votes will be reinitialized to one and your preferred outcome will be randomly assigned. Because this round belongs to the same match, you will be able to see the bids offers and transactions of the previous rounds of the same match. [SCREEN 15].