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COMBUSTION INSTABILITIES: MATING DANCE OF CHEMICAL, COMBUSTION, AND COMBUSTOR DYNAMICS
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Abstract

Combustion instabilities exist as consequences of interactions among three classes of phenomena: chemistry and chemical dynamics; combustion dynamics; and combustor dynamics. These dynamical processes take place simultaneously in widely different spatial scales characterized by lengths roughly in the ratios $(10^{-3} - 10^{-6}):1:(10^{3} - 10^{6})$. However, due to the wide differences in the associated characteristic velocities, the corresponding time scales are all close. The instabilities in question are observed as oscillations having a time scale in the range of natural acoustic oscillations. The apparent dominance of that single macroscopic time scale must not be permitted to obscure the fact that the relevant physical processes occur on three disparate length scales. Hence, understanding combustion instabilities at the practical level of design and successful operation is ultimately based on understanding three distinct sorts of dynamics.

Research under the Caltech MURI has been accordingly organized as three main tasks. The purpose of this paper is to summarize the essential content of the program generally and to interpret the work in terms of the relevant physical processes. This approach reveals the motivations and justifications for the individual research projects; their connections within the global context of combustion instabilities; will clarify successes, failures, differences, and future possibilities of the MURI program; and will summarize immediate opportunities for design and development. The paper is discursive, emphasizing the essential ideas, and the theoretical and physical framework within which combustion instabilities can be understood. Without that framework, problems arising in practice appear as individual cases rather than examples of a general phenomenon, and it would be virtually impossible to work out rules of thumb and guidelines for design.

A few specific results are given to illustrate certain aspects of general behavior. The most important new recent result may simultaneously answer some longstanding practical questions, and have significant far-reaching consequences for continuing research in this field. We have found that the characteristics of the dynamical behavior of a combustor seem to be immensely more sensitive to changes in the velocity-dependent combustion dynamics than to changes in the pressure-dependent dynamics. We believe that this conclusion sets the course we should follow to understand at last the reasons for significant changes of the dynamics of a combustor subsequent to seemingly inconsequential adjustments of propellant composition.

Chemical dynamics and **combustion dynamics** are general subjects arising as well in other contents. **Combustor dynamics** is the subject that particularly defines the context for combustion instabilities. Hence in this paper we emphasize combustor dynamics and its immediate connections with the other two subjects which are covered more thoroughly in companion papers.

1. Introduction

Combustion instabilities occupy a distinctive position in the development of rockets. They are entirely unwanted, but nevertheless are nearly inevitable in new designs of high performance systems; and not uncommonly they persist in operational motors. Unplanned for, they can cause large added costs if they suddenly appear late in development. The very fact that instabilities do occur often as surprises suggests what is true: there is a certain lack of basic understanding at all levels and, it seems also, failure occasionally to put into practice what is understood.

Attention and support have been given to research in this area for more than fifty years, albeit usually not in sustained fashion. Considerable progress has in fact been achieved. In addition to what is known and understood, important practical tools exist. At least as importantly, the classes of problems that must be solved

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to bring the subject to a satisfactory level for practical purposes are now well-defined. The major difficulties are associated chiefly with developing methods to solve those problems; almost all associated with experimental requirements.

There are several reasons for the somewhat erratic nature of research activities directed to combustion instabilities. First, despite the appealing fundamental aspects of the field, the most serious justification for research flows from practical problems. Thus the appearance of unacceptable oscillations in a motor intended for operational use is often soon followed by increased support for research, possibly only for a brief period. After the problem in question has been more-or-less solved, often in *ad hoc* fashion, support for research decays as anxieties subside. Second, while it has long been recognized that physical processes on several different time and length scales collaborate in some sense to produce the phenomena generically called combustion instabilities, successful collaborations among the corresponding communities of researchers have been rare. Finally, the problems that arise are in fact difficult to solve, to a large extent due to intrinsic limitations of current experimental methods that only slowly are being overcome. The difficulties arise chiefly for two reasons: measurements intended to clarify the basic mechanisms must be made at small length and time scales; and the non-uniform high temperature, chemically active environment presents forbidding conditions for both *in-situ* and non-intrusive methods. Thus the most extensive data available are for global properties, mainly pressure and accelerations. Even that information inevitably falls short of what one would prefer.

The MURI Programs, one directed by the University of Illinois at Urbana-Champagne (UIUC) and one by Caltech, have provided the first opportunities for true collaborations among researchers working with all of the disciplines relevant to combustion instabilities. In a broad sense, the two MURI programs differ chiefly in the following respects: the UIUC MURI had no efforts in combustor dynamics; and the Caltech MURI had no work in ingredient and propellant synthesis. Otherwise, there was coincidence of general areas of research, but not in emphasis. For further details, consult the Websites (2000).

Owing to his participation in the Caltech MURI, the author has organized this paper in correspondence with that program, which comprises as tasks: I, *Chemical Dynamics*; II, *Combustion Dynamics*; and III, *Combustor Dynamics*.

This paper is intended to provide a broad summary of the content of the MURI programs with particular attention to those aspects most closely related to the requirements of industrial and governmental organization. For that reason the following remarks are organized in a top-down fashion. Figure 1 is a diagram showing the scheme of the remarks here, essentially a flow chart for the disciplines and content of the MURI programs. The arrows connecting the various boxes indicate that the subjects indicated are mutually influential. In reality, the connections are fairly well understood at the research level but have yet to produce extensive consequences in practical applications. Part of the purpose of this and accompanying papers (Blomshield, 2000; Flanagan, 2000; Brewster, 2000; Beckstead, 2000; Yang, 2000) is to improve that situation. Much can be done early in development programs to reduce the probabilities of combustion instabilities appearing later in full-scale motors.

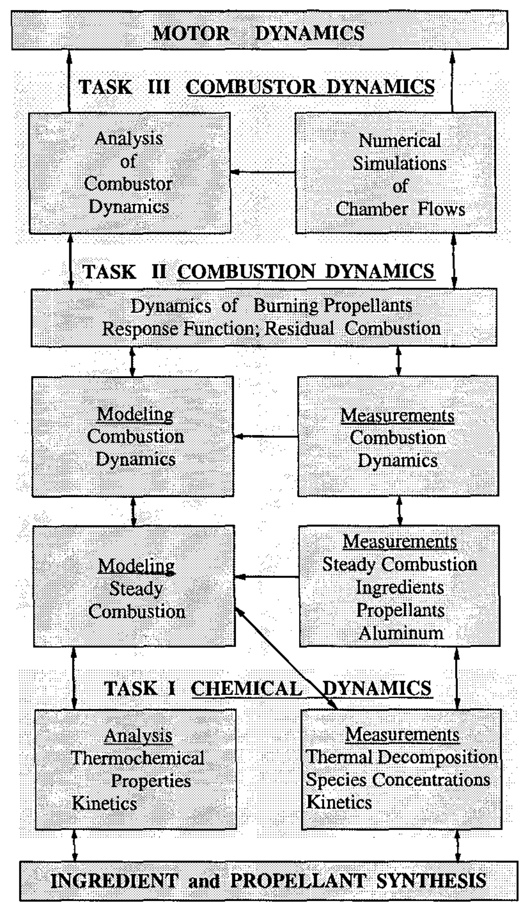


Figure 1

A brief summary of the main characteristics of behavior in full-scale motors helps define the kinds of problems that must be addressed by research in this area. Success achieved in the programs discussed here must then be measured partly by the extent to which the results of the research contribute to treating those problems. In any event, observations like those cited in Section 2 have motivated the form of the analyses comprising Task III concerned with combustor dynamics. The term 'combustor dynamics' refers generally to unsteady motions in a combustor. The various forms of combustion instabilities form a subset of combustor dynamics. Global analysis of combustor dynamics in particular has been constructed intentionally to be easily applicable to practical problems. Although still in early stages of development, numerical simulations also are based on formulations of realistic problems.

Numerical results for the flow field and dynamical behavior in a combustor cannot be obtained without quantitative specifications of the dynamical behavior of the combustion processes, the subject of Task II. In the present context, 'combustion dynamics' refers to the responsiveness of the combustion processes to changes in the local values of the flow properties, mainly pressure, velocity and temperature. That responsiveness provides a feedback path connecting the *combustion* dynamics with the *combustor* dynamics.

Whether they are associated with residual combustion within the volume or with combustion processes at the surface of a burning propellant, the *combustion* dynamics take place at length scales of the order of a millimeter or so and less, down to as small as a few microns. But the *combustor* dynamics evolve on the scale of the chamber, a factor of 10^3–10^6 larger than the scale of the combustion processes. This wide separation of scales implies that in some sense one should be able to average spatially over the smaller scales to produce models of the *combustion* dynamics more appropriate to macroscopic scales of the *combustor* dynamics.

The results of such averaging procedures (which are only implicit in most analyses) are usually represented as *response* or *admittance functions*. We must emphasize that spatial averaging in the manner suggested need *not* rid the model of dependence on heterogeneities on a small scale. For example, response functions should be functions of the average oxidizer size in a propellant and possibly higher order moments of the distribution as well; or of the aluminum droplet size in residual combustion.

Introduction of response functions therefore creates the essential and fundamental connection between combustor dynamics and combustion dynamics, the main subjects addressed in Task II. In fact, the work in Task II also includes many projects concerned only with matters of steady combustion. The reason is basic: however we may represent the combustion dynamics, the central idea is that we are trying to model the dynamical behavior, here its response to an imposed disturbance. That dynamical system, a burning propellant or an aluminum drop, say, is defined in the first instance by complete specification of its steady combustion behavior. Hence knowledge of the steady process—burn rate, distribution of temperature, dependence on particle size distribution, chemistry, and chemical kinetics—is prerequisite to analysis of the dynamics. It is at that level that the results of Task I become necessary.

Steady combustion of any material is the macroscopic consequence of the collective action of an enormous number of microscopic processes. Those include convective, conductive and radiative transfer of energy; and physical and chemical kinetics. Representation of the modes of energy transfer is accomplished as part of combustion analysis. The required information about the kinetic processes must be supplied independently; that is a chief purpose of the work in Task I.

In fundamental respects, then, we may view in reverse the overall scheme just described and illustrated in Figure 1. We can conceive of the ideal world in which for given ingredients and propellants the appropriate experimental and analytical methods could be directed from the bottom up within the three levels or tasks, and emerge with predictions of the dynamics of a specified motor design. We cannot presently carry out such a program. A reasonable question is: How far are we from the ideal success, and what are the obstructions? It is a fair assessment that the MURI programs have produced significant advances, but there have been failures with respect to expectations held five years ago, and serious deficiencies remain in this field.

There should be no doubt, in the author's opinion, that we have a much clearer understanding now of the problems that must be solved, and of the deficiencies of methods available, than we did when the MURIs began. Perhaps the best justification for this conclusion is that, with the benefit of what has been learned in these programs, we can now in retrospect quite easily recognize where the original programs were flawed. That is not intended so much as a criticism in hindsight, as a remark that several negative results (unobtainable other than by actually carrying out the research) enable us now to dismiss as useless several methods that appeared to some to have good possibilities at the time. We have learned much in five years.

2. Some Typical Cases of Combustion Instabilities in Full-Scale Motors

The widespread concern with combustion instabilities due to their appearance in all types of propulsion systems is apparent from the abbreviated chronology given in Figure 2. In addition to the generic influences and the particular specific examples cited in Figure 2, there are many tens of cases of unacceptable instabilities in solid rockets alone; e.g. see a partial compilation given by Blomshield in this meeting (Blomshield, 2000).

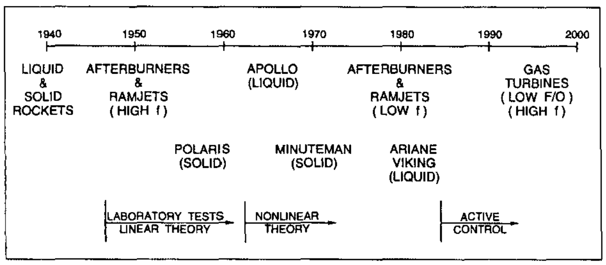


Figure 2

When certain kinds of anomalous behavior were first observed in rocket firings (e.g. enhanced heat transfer rates, unexplained increases of mean pressure, excessive burning rates and occasional failures) the causes were unknown. An important reason was the inability of early instrumentation to detect oscillations in the acoustic frequency range. That situation changed in the mid-forties and early fifties. It is from that era that the idea of unstable motions in a combustor date, and hence the term "combustion instability". The notion of 'instability' is commonly realized as a growing dynamical motion in a linear system. Thus, understanding combustion instabilities begins with the idea that they arise as the results of linearly unstable motions growing out of small disturbances. Figure 3 is a reproduction of a pressure trace in a motor illustrating well that sort of behavior. The average ('DC') part of the pressure has been removed by filtering. However, in many cases, it is essential to examine the total time-dependent pressure. The example in Figure 4 makes the point: a significant rise of mean pressure accompanies the growing oscillations.

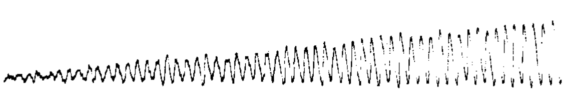


Figure 3

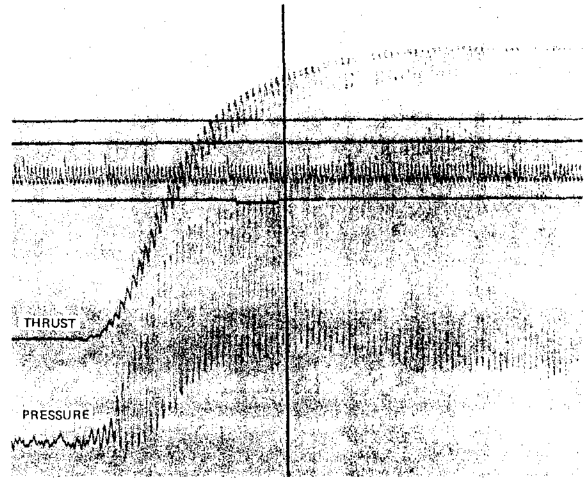


Figure 4

Perhaps the best known case of an unacceptable linear instability occurring in a large operational system, is the problem encountered in the Stage 3 motor of the Minuteman II vehicle, in the late 1960's. Most of the understanding of the problem was summarized in several papers given in a special session of the 7th Joint Propulsion Meeting (1971). A pressure record of flight data is reproduced in Figure 5. Owing to the low sampling rate and the compressed time scale, the oscillations are revealed as a swelling of the trace of mean pressure. The unsteady motions are true linear instabilities and were remarkably reproducible, both in flight tests and in ground tests.

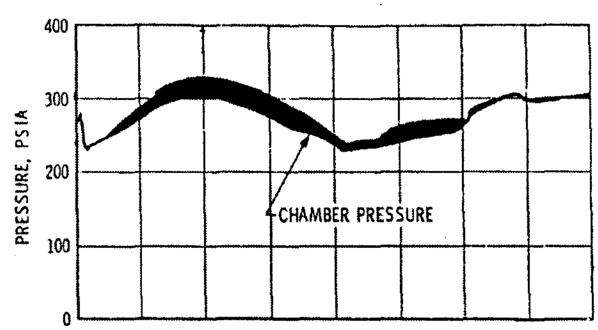


Figure 5

The preceding examples are all true linear instabilities formally classified as 'Hopf' or *supercritical bifurcations* in the field of dynamical systems. An unstable motion of this sort occurs when a small disturbance is unstable in the sense that however small it is, the energy flow to the motion depends on the motion itself and exceeds the energy loss, an example of feedback causing a system to become unstable. The instability exists when some parameter assumes a value greater than some 'critical' value. The term 'supercritical' derives from the phenomenon of unstable fluid motion when the Reynolds number exceeds its critical value for the particular flow in question. A second important class of unstable motions comprises *subcritical bifurcations*[1]. In this case a linearly stable system will nevertheless support an unstable motion if it is exposed to a sufficiently large initial disturbance. This behavior has been known for more than four decades to exist in liquid and solid propellant rockets. Subcritical bifurcations in rockets have commonly been called "triggered" or "pulsed" instabilities. Alternative labels, favored in the Russian literature are *soft excitations* (linear instabilities, supercritical excitations) and *hard excitations* (nonlinear or pulsed instabilities, subcritical bifurcations)

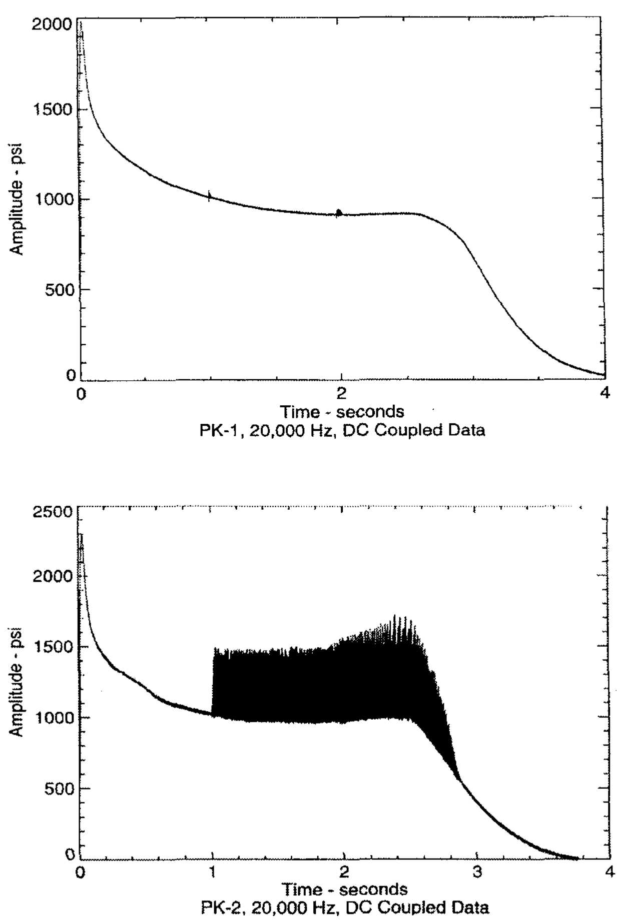


Figure 6

Experimentally, the simplest situation is that displayed in Figure 6 reproducing the results of two recent firings of identical motors (Blomshield, 2000). The motor is evidently stable to small disturbances (pulses) having amplitudes below 40 psi but it is unstable to pulses 100 psi and larger. There are no data to establish the precise limits. This behavior fits exactly the definition of a subcritical bifurcation quoted above.

Supercritical bifurcations arise in systems that behave linearly in the vicinity of the bifurcation; i.e. small amplitude motions either grow or decay exponentially in time. In contrast, subcritical bifurcations involve intrinsically nonlinear behavior. In both cases, if a disturbance is unstable, it may grow without limit, causing destructive failure or the motion will evolve into an oscillation having finite amplitude—a common occurrence. Both the amplitude and the frequency may vary with time, but in any event the motion is conventionally called a *limit cycle*. The fact that the motion may not be perfectly periodic seems usually to be a secondary effect associated with relatively slow changes of parameters characterizing the system.

The long-time behavior of combustion instabilities is always a consequence of nonlinear effects because the limit cycles arise either from intrinsically unstable linear motions or in a nonlinearly unstable system. A third possibility for more-or-less steady oscillations is that they are driven or forced. The mechanism for a forced motion must, by definition, be insensitive to the motion itself (*i.e.* independent of the unsteady velocity, pressure,...). One possibility is unilateral energy transfer to the unsteady motions from the mean flow of combustion products; or possibly from turbulence or noise. It has also been suggested but unsupported by any experimental evidence or analysis, that unsteadiness associated with heterogeneous combustion of the propellant may serve as a source of noise tending to drive acoustic oscillations in the chamber. All available evidence shows that combustion *instabilities* observed in practice are *not* forced motions. Moreover, there seems to be no mechanism known at this time that will generate large-amplitude *forced* motions in combustion chambers. It is certainly true that low level oscillations can be forced in a stable system, a matter discussed further in Section 6.

It is the grand puzzle of combustion instabilities to discover the reasons for observed behavior, the main types being those just described. There is practically universal agreement that the ultimate source of the instabilities is transformation of energy released in combustion processes to the potential and kinetic energies of mechanical motions in the compressible gases within the chamber. The central question is: how does that transfer or transformation of energy take place? In the scheme of the Caltech MURI (Figure 1) the theory and analysis of the motions is the subject of Task III. Faithful representation of the actual behavior

[1] Strictly, the existence of pulsed instabilities evolving to stable oscillations (stable limit cycles) requires a subcritical bifurcation accompanied by a turning point.

requires models of the transformation processes; a major objective of Task II is constructing those models, based on analysis and experimental results, including those of Task I. The arrows connecting the blocks covering the two tasks represent the connections between the three sorts of dynamics, that is, the processes effecting the energy transfer.

Entirely satisfactory results of Tasks II and III would consist in 1) thorough understanding of the dynamical behavior of motors, expressed in terms of the geometrical configuration and the detailed forms of the response function; and 2) thorough understanding of the dependence of the response functions on the composition and physical properties of the propellants in question; and on impressed pressure and velocity fluctuations. How the combustion dynamics, or response functions, depend on the pressure and velocity disturbances applied to a burning surface is conventionally indicated by the terms *pressure coupling* and *velocity coupling*. We take these terms to imply the influences of imposed pressure and velocity disturbances on the burning rate, the coupling being associated with dynamics of the combustion processes. Thus if there is no burning, there can be no pressure or velocity coupling in the sense intended here.

The word 'coupling' captures exactly the content of the connections between the types of physical behavior covered by Tasks II and III. It is that coupling, represented by the response function, that constitutes the feedback internal to a motor. And it is the feedback, acting according to familiar principles, that connects an amplifier (the acoustical system within the combustion chamber) to an oscillator. This important idea gives a useful point of view of combustion instabilities generally, showing clearly how the combustor and combustion dynamics are connected.

Ultimately, both for theoretical and for practical purposes, the overarching objective of research in this field must be to understand the connections between small changes in propellant composition and the dynamical behavior of a motor. The fundamental work covered by Task I is essential for that purpose, as well as for providing basic information required in the modeling carried out in Task II. Although there are observations of dynamical behavior in motors suggesting significant consequences of small changes in the propellant, no extensive compilation of practical experience exists. It is not possible now to carry out completely the process from to bottom to top suggested in Figure 1. Only when that is accomplished will we be justified to claim that we understand the way in which the dynamics on widely different length scales cooperate to produce combustion instabilities. One purpose of this paper is to explain what remains to be understood and to clarify where future effort should be placed.

The obstacles to progress towards the overarching objective defined above are best understood within a theoretical framework. Although the framework is constructed with the help of some mathematics, and hence given quantitative and predictive value, here it is the physical content and interpretation that matters. The most significant qualitative result of the theory is the following:

> *Any unsteady motion in a combustion chamber can be interpreted in terms of the time-evolution of a collection of coupled nonlinear oscillators, each oscillator corresponding to an acoustic mode of the chamber.*

This conclusion is enormously helpful in understanding even the simplest observations of actual behavior as well as theoretical developments. A large part of the foundation of the theory consists in establishing the connections between the unsteady gas dynamics—the acoustic modes—and the corresponding collection of oscillators. Application of the theory rests chiefly in working out quantitatively the connections between the parameters characterizing the system of oscillators and the processes in a combustion chamber. Then known mathematical methods can be used to compute the dynamical behavior of a combustor.

3. Combustor Dynamics

It is fundamentally important to understand that the phenomenon called a "combustion instability" in a rocket motor is *not* a matter of lost stability of propellant combustion. Rather, as observations of the sort described in Section 2 clearly show, it is the propulsion *system* that is unstable. Figure 7 is the simplest way of illustrating the general point. The motor is the system (the 'plant' in the language of controls technology) which is observed to be unstable. But for the purposes here, the motor must be viewed as a system comprising two main parts: the gaseous medium within the chamber, perhaps a two-phase mixture of gas and condensed material, consisting largely of combustion products and labeled 'combustor dynamics' in Figure 7; and the burning propellant, simply labeled 'combustion dynamics' in Figure 7. It is the combination of the *combustor* and *combustion* that is observed to be unstable, exhibiting combustion instabilities. As the external and internal 'inputs' we have indicated fluctuations of mass, m_e' and m_i', a point we return to in Section 4.3.

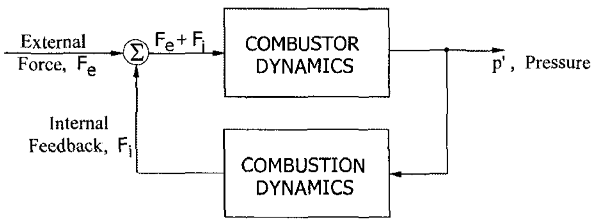


Figure 7

3.1 Frequencies and Mode Shapes

A rocket chamber with no combustion and flow is of course stable: viscous losses ensure that any disturbance will decay. The following is a simple method often used to determine the frequencies and pressure distributions of the normal modes of a rocket motor, those motions most likely to appear as combustion instabilities. Construct a cavity—of plastic, wood, or metal—having the same shape as the actual motor and closed with a rigid boundary at the entrance[2] to the nozzle. Direct the output of a speaker through a hole at the boundary so acoustic waves can be supplied continuously to the cavity. The frequency of the waves can be changed by driving the speaker with a variable frequency oscillator; suitable means can be provided to keep the amplitude of the injected waves constant. When the frequency is fixed at some value, a steady wave system (a 'standing wave') is quickly established in the chamber. Its amplitude can be measured with a microphone mounted in the boundary, the head end being a favored location.

When the frequency of the injected waves is raised from zero to some large value (say up to 1KHz or more) the response of the chamber measured with the microphone will show a sequence of peaks, as sketched in Figure 8a. If the chamber is slender, the frequencies at which the peaks occur, called *natural*, *normal*, or *resonant* frequencies, have values found in classical acoustics for an organ pipe of length L, $f_l = la/2L$ where a is the speed of sound, and $l = 1, 2$. Quite generally a closed cavity has resonant frequencies determined by the shape of the boundary and the speed of sound; the values of the frequencies are generally not related in simple fashion to one another. The widths of the peaks are proportional to the damping in the system at the resonant frequencies.

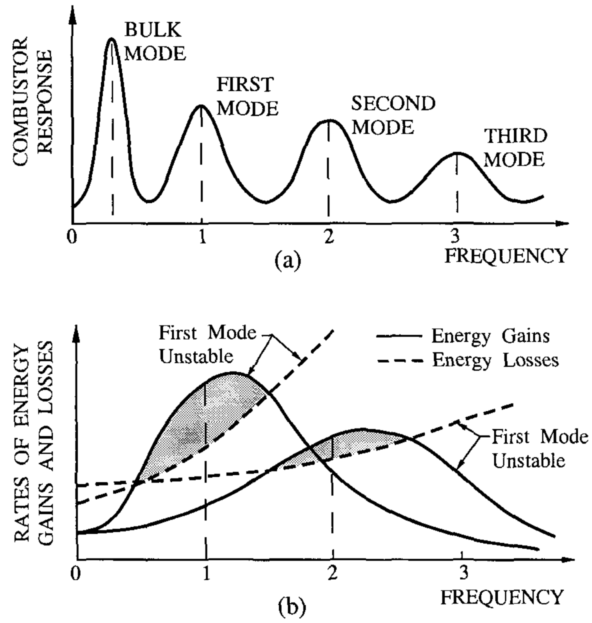


Figure 8

It is fundamental, and fortunate, that even when combustion and flow are present in a cavity, the frequencies and spatial features of the modes are not much changed from the classical values referred to in the preceding paragraph. For example, the distribution of the pressure amplitude in the fundamental longitudinal mode of a slender rocket motor has closely the form of half of a cosine wave with maximum and minimum at the ends[3]. That general result, established both theoretically and experimentally has far-reaching consequences for investigating and understanding combustion instabilities. Stability of disturbances is another matter, a central issue in problems of combustion instabilities. The shapes of peaks in the cold flow tests are necessarily different from those that would be found for a motor. *Hence one cannot infer stability of a motor from this sort of testing.*

In the remainder of this paper we are concerned chiefly with the stability, nonlinear behavior, and mechanisms of combustion instabilitites. We assume that the frequencies and forms of all the normal acoustic modes for a combustor are known, either by tests of the sort described above, or by calculations. As explained further below, we may then regard any unsteady motion—any combustion instability—as a formal synthesis of normal modes. In practice, one mode—commonly the fundamental, is dominant. That is, one

[2] This choice of downstream acoustic boundary is correct if the actual rocket is fitted with a choked nozzle having Mach number less then, roughly, 0.5 at the entrance. Adjustments can be made on theoretical grounds for other cases.

[3] This result has the important a consequence of producing oscillations of thrust approximately equal to $\pm 2S_c\Delta p$ where S_c is the port area and Δp is the peak value of the pressure oscillation. Hence a weak pressure oscillation can produce noticeable fluctuations of thrust.

mode is the most unstable. Other modes may appear either due to coupling to the unstable mode, or they may themselves be unstable.

3.2 Gains and Losses of energy in Unsteady Motions

An unsteady motion in a chamber is unstable if it gains energy at a rate faster than it loses energy. If the motion is oscillatory, we can speak simply of the energy gained or lost per cycle. This idea was given a more explicit form, for the phenomenon of acoustic waves excited in a tube by heat addition, by Lord Rayleigh (1878, 1945). Culick (1987, 1992) has discussed Rayleigh's criterion in the context of the analytical framework summarized in Section 3.4.3.

The idea can be illustrated quite simply as sketched in Figure 8(b). In solid propellant rockets the rate of energy gain is almost entirely due to the fluctuations of mass in flow associated with the dynamics of surface combustion, represented by a 'combustion response' (Section 4.1). The energy gains from the combustion processes depend significantly on frequency, commonly showing a broad peak in the range of the natural acoustic frequencies of rocket combustors.

On the other hand, the energy losses commonly increase with frequency. Despite the presence of several dissipative processes, the energy losses for a solid propellant rocket typically increase monotonically with frequency. As a result, it is not unusual that in some range of frequency the energy gains will exceed the losses. Hence, acoustic modes in that range will be driven, i.e. they are linearly unstable. Two possibilities are suggested in Figure 8(b). The shaded regions indicate the ranges where the gains exceed the losses. In case (1), the first mode for the chamber whose dynamics are sketched in Part (a) of the figure will be driven; and in case (2) the second mode is unstable.

Figure 8 is the simplest illustration of the way in which the combustor dynamics and combustion dynamics interact to produce combustion instabilities.

3.3 A Mechanical Analog for Combustion Instabilities

The interpretation based on representing motions in terms of modes is best addressed by considering a simple analog, two masses suspended with springs as shown in Figure 9. Gravity is ignored, and the masses are assumed to move horizontally only, say in a frictionless tube.

Figure 9

Because there are two masses moving in one dimension, this system has two degrees of freedom. Consequently there are two normal modes: in one both masses move together back and forth, a fixed distance apart; and in the other, they oscillate in opposition. Figure 10 illustrates the motions in the normal modes. One complete cycle of each oscillation is shown, but the periods differ; the asymmetric mode on the left has a lower frequency.

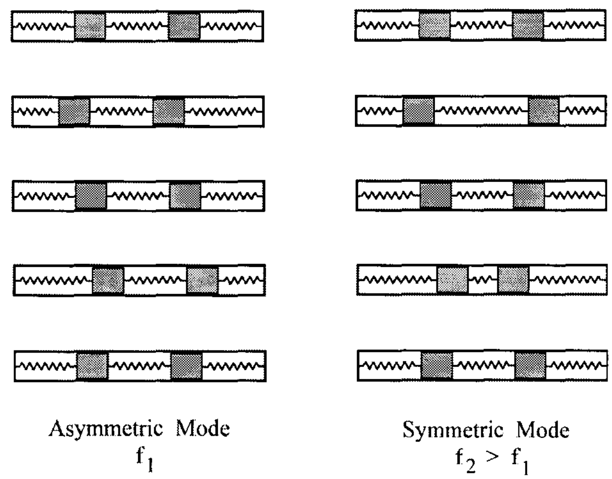


Figure 10

It is a result of the theory of linear systems that if the initial positions and velocities of the masses are specified at $t = 0$, then the unique motions of both masses can be computed for $t > 0$. Moreover, the motions can be expressed as a sum of the two normal modes, perhaps with time-varying coefficients, a consequence of the fact that the relative contributions of the modes to the motion may vary with time. However, the normal modes are not coupled. If the initial conditions are such that only one of the two normal modes is excited, then the subsequent motion will consist of that mode only. Note the important distinction that although the motions of the two masses in Figure 10 are coupled by the spring, the normal modes are mutually independent.

Now imagine that external forces can be applied to one or both masses: the motion of the system can be forced, allowing measurement of the frequencies and the relative motions of the two masses in the two normal modes. The procedure is entirely analogous to that described in connection with Figure 8 for an acoustic cavity.

If the systems in Figures 9 and 10 were free of energy losses, then when the forcing frequency equals one of the resonant frequencies, the displacements of the masses become infinitely large. However, if there is some dissipation, e.g. due to friction, then a result like that in Figure 8 will be found, but with only two finite peaks.

Although the system in Figure 9 is very simple, it exhibits many of the basic properties of the motions in a combustor. The reason for that is that we can rigorously, with a formal analysis not covered here (see Culick 1999), pass from the system of two masses to the continuous system of gases in a combustor. The idea is to add more masses and springs in such a way that in the limit of infinitely many masses and springs, the mass per unit length is proportional to the density of the gaseous medium, and the spring constant per unit length is proportional to the compressibility.

With that relation in mind, we can examine the problem of instability with the two-mass system. In fact, even more simply we consider the single mass in Figure 11.

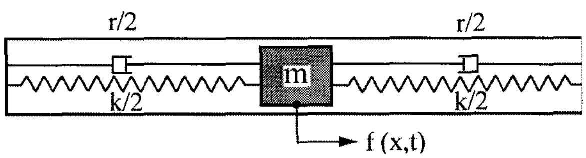


Figure 11

We now add linear energy dissipation represented by the two dashpots, and suppose that a force $mf(x,t)$ is exerted on the mass. The equation of motion is

$$m\ddot{x} + r\dot{x} + kx = mf(x,t)$$

and after division by *m*,

$$\ddot{x} + 2\alpha\dot{x} + \omega_0^2 x = f(x,t) \tag{3.1}$$

The system shown in Figure 11 is the simplest example serving to illustrate several fundamental ideas which in elaborated form are central to the scheme of combustion instabilities illustrated in Figure 1. Only simple computations are required to make the points.

3.3.1 Forced Motions

If the force $f(x,t)$ is independent of *x*, then the motion is, quite generally, a forced motion. However, the case usually considered is $f(t)$ sinusoidal, represented in complex form as

$$f(t) = \hat{f}e^{i\omega t} = \hat{f}(\cos\omega t + i\sin\omega t)$$

For a linear system, as (3.1) is, the response $x(t)$ is also sinusoidal and can be assumed to have the form

$$x(t) = \hat{x}e^{i\omega t}$$

Substitution in (3.1) gives the transfer function relating the amplitude of the forcing function to the complex amplitude $\hat{x}$ of the response

$$\frac{\hat{x}}{\hat{f}} = \frac{1}{(\omega_0^2 - \omega^2) + i(2\alpha\omega)} = |G|e^{i\arg G} \tag{3.2}$$

where

$$|G| = \sqrt{(\omega^2 - \omega_0^2)^2 + (2\alpha\omega)^2} \tag{3.3 a, b}$$

$$\arg G = \tan^{-1}\frac{(-2\alpha\omega)}{(\omega_0^2 - \omega^2)}$$

To state that $\hat{x}$ is complex is a simple way of stating that the response does not occur in phase with the applied force, but rather lags ($\omega < \omega_0$) or leads ($\omega > \omega_0$).

If the system is initially at rest and the sinusoidal force is suddenly switched on to a constant amplitude, the response is also sinusoidal and grows to a constant value set by the balance of energy: the work done per cycle by the applied force equals the energy dissipated per cycle by the losses represented by the dashpots in Figure 11. The maximum excursion of the displacement of *m* grows as shown in Figure 12, the envelope of the oscillations being concave downward during the transient period.

Suppose now that the forcing function is a complicated function of time, perhaps containing 'noise' or some sort of 'random process'. That means that the system is exposed to forces over a broad band of frequencies; its response will correspondingly be non-zero over a broad band, with a peak in the vicinity of the resonant frequency. The width of the peak will be related to the damping constant α with some 'error' λ, (Seywert and

Culick 1999). Naturally, the response of a more complicated system will show peaks at all of the modal frequencies within the range of the forcing function, although the heights of the peaks may be very small. Thus it is certainly possible to gain information about the basic characteristics of a system when it is exposed to a broadband force.

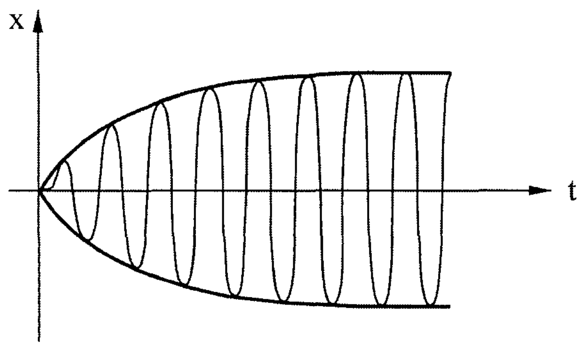


Figure 12

3.3.2 Unstable or Self-Excited Motions (Internal Feedback)

If the system in Figure 11 is to have internal feedback, then part of the force must be dependent on the motion itself. To keep the system linear, we assume that f is a sum of parts proportional to the displacement and the velocity:

$$f = \Delta\omega_0 x + 2\Delta a\dot{x} \tag{3.4}$$

In this case we assume that a solution exists having the form

$$x(t) = \hat{x}e^{i\lambda t}$$

Substitution of this expression and (3.4) into (3.1) leads eventually to the solution

$$x(t) = e^{-(\alpha-\Delta\alpha)t}e^{i\Omega t} \tag{3.5}$$

with

$$\Omega = (\omega + \Delta\omega_0)\sqrt{1 - \left(\frac{\alpha - \Delta\alpha}{\omega + \omega_0}\right)^2} \tag{3.6}$$

Much can be said about this result, but the main points to be emphasized here are:

If $\Delta\alpha < 0$ and $|\Delta\alpha| > \alpha$ then $-(\alpha-\Delta\alpha)$ is positive and the motion grows without limit. That can happen because the force f, equation (3.4) depends on the displacement itself, and when $\Delta\alpha$ is negative, represents an input of energy to the oscillator. For this problem that process constitutes positive feedback which then can be identified as the cause of the instability.

(ii) If (as commonly happens) $\Delta\alpha$ and $\Delta\omega_0$ are small, then two observational consequences follow: (i) many periods (cycles) of oscillation are required for the amplitude to grow significantly; and (ii) the frequency of the unstable oscillation is not much different from the unperturbed ($\Delta\alpha=\Delta\omega_o=0$) resonant frequency ω_0.

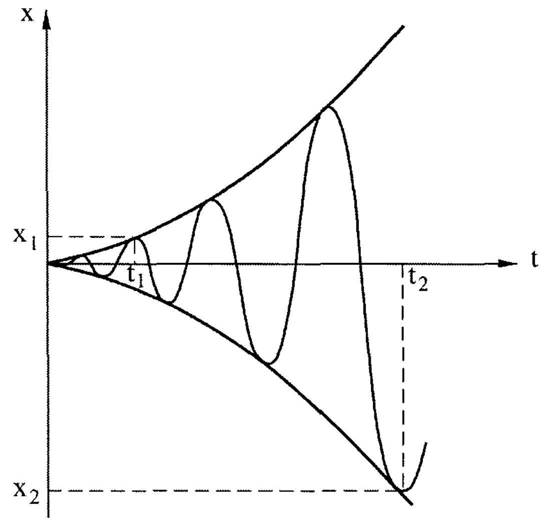


Figure 13

A useful rule of thumb for the growth of oscillation follows directly from (3.5). Figure 13 is a sketch of an unstable disturbance. The envelope of the oscillations is the magnitude of (3.5); the ratio of the amplitudes at two times t_1, t_2 is

$$\left|\frac{x_2(t_2)}{x_1(t_1)}\right| = e^{-(\alpha+\Delta\alpha)(t_2-t_1)} = e$$

Now choose the interval $t_2 - t_1$, such that x_2 is e times x_1, so the left-hand side equals e as shown. Then the equality is satisfied only if $-(\alpha + \Delta\alpha)(t_2 - t_1) = 1$. Let τ_o be the period of the oscillation, $\tau_o = 1/f_0$ and the number n of periods in the interval is $(t_2 - t_1)/\tau$. Hence we find

$$n = \frac{1}{-(\alpha + \Delta\alpha)\tau_0} = \frac{f_0}{-(\alpha + \Delta\alpha)} \tag{3.7}$$

Recall that here $\alpha+\Delta\alpha$ must be *negative* for an unstable motion. Thus the number of cycles for an e-fold increase of amplitude is the frequency divided by the growth constant. This parameter, frequency divided by the growth constant, is therefore a useful measure of the intensity of an instability.

It is particularly important to realize that the envelope of unstable oscillations is concaved upward, an obvious distinction from the case for forced oscillations, shown in Figure 12 (e.g. Figure 3). It is often the case in practice that data taken during the transient growth of an oscillation in a motor will reveal whether the motion is driven or is a linear instability. That is why we are justified in citing the cases shown in Figures 3-5 as examples of combustion instabilities. However, in practice, the data may not be sufficiently clean to provide conclusive evidence.

Another cause of oscillations in combustors is shedding of coherent vortices from edges, or as a natural phenomenon originating in the region close to the lateral surface. There is still a question whether the oscillations are self-excited or forced. In any case, their amplitudes are always small; there are no examples of growths to amplitudes greater than a percent or so of the mean pressure; see, for example, LeBreton et al, 1999.

3.4 Analysis of the Global Dynamics of a Combustion Chamber

The combustion processes and flow in a combustion chamber are astonishingly complicated. Yet the pressure recorded is a simple, accurate observation and could be interpreted, for example, the recording of the displacement of a mass within a simple mechanical system such as that discussed in Section 3.2. It must therefore be possible, in some degree of approximation, to find a simple quantitative representation of the behavior behind such a pressure record. To be most useful and informative, such an approximation should not itself involve direct solutions of partial differential equations describing all chemical and physical behaviors defining the general problem.

For many years the approach summarized in this section has proven to be productive of basic understanding and of results having direct practical applications. The principal qualitative consequence has been stated at the end of Section 2, that the dynamics of a combustor can be expressed in terms of the simple oscillators described in the preceding paragraphs.

The following remarks constitute a superficial summary of the basis for this conclusion.

In the first instance the basic physical principles are formulated mathematically as partial differential equations. Those equations contain terms representing all processes and govern the total unsteady motions. If the necessary information is provided to allow quantitative representation of the processes, it is possible to carry out very useful numerical simulations described in a companion paper (Yang, 2000). However here we are concerned with application of some methods of analysis and for that purpose the equations must be approximated. The basis for doing so is an expansion of the dependent variables. In problems of combustor dynamics there are two small parameters characterizing the flow and suitable for formulating such an expansion: the Mach members of $\overline{M}_r$ of the mean flow and M_r' of the unsteady field. A two parameter expansion can then be worked out and used to define several classes of problems including linear stabilities and several sorts of nonlinear behavior (e.g. see Culick 1999). The details are irrelevant here: what is important to realize is that a systematic procedure exists to construct equations which experience has shown accurately describe observed behavior.

Because such a description based on partial differential equations is local in space, applications to macroscopic systems require that a considerable gap be spanned. One procedure for doing so is spatial averaging. Several methods exist for spatial averaging, a general approach that has been widely used for more than a century in many fields. The earliest forms of the method were developed for applications to structures; e.g. Rayleigh's method, Galerkin's method and others that can be viewed as forms of the *method of least residuals*. In general, spatial averaging has two great advantages: the partial differential equations are reduced to total differential equations; and the formulation is applicable to both linear and nonlinear problems.

It was the group at Georgia Tech who first applied a modified form of the method of least residuals to linear and nonlinear combustion instabilities in liquid and solid rockets (Powell 1970, Zinn and Powell 1970, Jones and Zinn 1972 and other publications). Most of those works have technical restrictions intrinsic to particular details of the methods used, because the average velocity is taken to be uniform and the unsteady velocity is assumed to be derivable from a potential. The second is a serious matter partly, but not completely, overcome in the method used here[4]. This method originated in the use of Green's functions to treat linear problems of combustion instabilities (Culick, 1961). Modification for nonlinear problems beginning in 1971 (Culick, 1971, 1975) carries with it a restriction, not apparent until recently, that places limits

[4] Seywert and Culick (1998) have given an incomplete discussion and comparison of some formal aspects of the two approaches to application of spatial averaging in this field. Further considerations of details are in progress.

on certain applications. That restriction arises because the pressure is used as the sole independent variable. Although using the pressure rather than the velocity potential already permits broader applications, serious limits remain. Specifically, it does not seem possible to account vigorously for unsteady vorticity within that scheme; and it is unclear how to accommodate certain kinds of higher order terms, even in the absence of unsteady vorticity. One promising approach to a more generalized formulation is based in the use of both a velocity potential and vector potential (Malhotra, 2000, work in progress).

Whatever procedure of spatial averaging is used, the starting point is the set of partial differential equations of conservation for a reacting mixture. Further modeling is required to represent combustion and other processes, but that task may be accomplished later in the analysis. Because observations are made of pressure waves, it is reasonable to seek a wave equation for pressure with perturbations—source terms in the volume and at the boundary that represent the distinctions between problems of combustion dynamics and classical acoustics. The derivation follows the procedure used to obtain the classical acoustics equation, difficult only because of technical complications arising with the many terms in the equations.

3.4.1 Acoustic, Vorticity and Entropy (Temperature) Waves

The flow is viewed in the conventional way as the sum of an average or mean field and the unsteady fluctuations. All unsteady disturbances are accounted for, including those associated with the acoustic field, vorticity and entropy. It is fundamentally important to make the distinction between these three types of disturbances, referred to as waves because in combustion chambers, they all propagate, although with different speeds. The physical properties of those waves, and their contributions to observed behavior, are especially noticeable in combustor flows for the following reasons.

Combustion instabilities have always been observed as anomalous behavior of the pressure field, appearing usually as reasonably well-defined oscillations. It seems now obvious that, given the frequencies observed, the oscillations in pressure records must betray the presence of acoustic waves. But that conclusion becomes less clear when one considers the medium in which the waves must propagate: there are variations of the average velocity and temperature; regions of separated flow are often present: and in full-scale combustors, the flow is inevitably turbulent, i.e. it is characterized by a broad spectrum of vorticity fluctuations. It should seem then surprising that well-defined acoustic waves are present: the reasons that acoustic waves are indeed identifiable under these conditions have been explained by Chu and Kovasznay (1956).

Taking advantage of previous special results, Chu and Kovasznay showed that, quite generally in the limit of small amplitudes, any disturbance in a compressible medium can be constituted of the three types or modes of waves: *acoustic, vorticity*, and *entropy* waves. Moreover, in a uniform average flow, those waves propagate independently of one another. Acoustic waves propagate at the speed of sound while vorticity and entropy waves 'propagate', or better, are convected, with the speed of the mean flow[5]. Entropy waves can be regarded as 'spots' of temperature different from the average value.

Thus, the first step to understanding the physical basis of combustion instabilities is to realize that there is scientific basis for the obvious presence of organized acoustic waves even within a complicated turbulent flow field. That they seem so often to dominate the unsteadiness observed in the pressure field is due chiefly to three reasons:

the two other types of waves (vorticity and entropy) carry no pressure fluctuations in the limits of small amplitudes;
even in non-uniform flow the acoustic waves are only weakly affected by interactions with the vorticity and entropy waves; and
small local changes of pressure or in some circumstances, velocity, in a reacting flow can cause quite substantial changes in the rate of the energy released in chemical reactions; only a small fraction of that energy converted to mechanical energy may produce further significant local fluctuations of pressure.

It is the general process (iii) that is responsible for the internal feedback loop shown in Figure 7. Hence once again we conclude that the key to understanding, and controlling, combustion instabilities is associated with the coupling between combustion and combustor dynamics.

[5] It is this difference in propagation speeds that causes difficulties in the current form of the averaging and expansion procedure described here.

3.4.2 The Wave Equation for Unsteady Flow in a Combustor

The remarks in the preceding section describe the physical basis for modeling a broad class of unsteady notions, in any combustion chamber, with the equation for waves of pressure, but containing some additional forms not arising in classical acoustics. Those additional terms represent sources of mass, momentum and energy associated with combustion processes; interactions between the acoustic waves and the mean flow; and interactions among the three modes of motion, acoustic, vorticity and temperature waves. The details are unimportant here, so we simply quote the formal result, the perturbed wave equation for the pressure with its boundary condition on the pressure gradient:

$$\nabla^2 p' - \frac{1}{\bar{a}^2}\frac{\partial^2 p'}{\partial t^2} = h \tag{3.8}$$

$$\nabla p' = -f \tag{3.9}$$

Several points must be emphasized but without complete explanation:

(i) Equation (3.8) and its boundary condition (3.9) are valid only in an approximation, which is well defined and understood. The approximation has been shown by long experience to be valid for a wide variety of practical conditions.
(ii) The functions h and f contain formal representations of all relevant physical and chemical processes, constructed to the approximation cited in (i). However, to obtain explicit and quantitative results, those processes must be modeled, tasks outside the formal procedure.
(iii) Equations (3.8) and (3.9) are nonlinear, but in the limit of small amplitudes of motion, describe linear behavior, including stability.

3.4.3 Spatial Averaging; Combustor Dynamics as a Collection of Oscillators

In classical acoustics, with no combustion and flow, the functions h and f are missing unless there are other sources present, such as moving portions of the boundary. The idea now is to compare the two general problems, with and without the source functions. The second problem is the unperturbed classical problem defining the natural modes and frequencies described by the homogeneous wave equation and boundary condition:

$$\nabla^2 \psi_n + k_n^2 \psi_n = 0 \tag{3.10}$$

$$\hat{n} \cdot \nabla \psi_n = 0 \tag{3.11}$$

For some cases, the equation governing ψ_n is more complicated because the ψ_n must be calculated for the same geometry *and* distribution of average temperature as exists in the actual problem. Nonuniform temperature leads to additional terms in (3.10). In practical applications, determining the ψ_n may be an expensive procedure, but it can always be accomplished. Work in progress will provide a convenient automated means of computing the Ψ_n and k_n for arbitrary configurations (French 2000).

The italicized statement at the end of Section 2 is the grand interpretation of the point of view founded in the basic ideas discussed in Sections 3.1-3.3 and especially in 3.3.1, the idea that any *small* disturbance can be regarded as a synthesis of three types of waves. For the matters in question here, experience has shown that the idea can be made even more precise: the dynamical motions called combustion instabilities consist chiefly of pressure perturbed by the influences of vorticity and entropy waves. Therefore, we take as the primary representation of the field a synthesis of acoustic waves, the ψ_n defined by the above two equations, having time-dependent amplitudes η_n:

$$p'(\vec{r};t) = \bar{p}\sum_{o}^{\infty} \eta_n(t)\psi_n(\vec{r}) \tag{3.12}$$

After the two problems defined above (the actual and the unperturbed problems) are compared (subtracted) and averaged with a weighting function (Ψ_n) over the volume of the chamber, we find the basic equations for the theory of global chamber dynamics:

$$\frac{d^2\eta_n}{dt^2} + \omega_n^2 \eta_n = F_n \tag{3.13}$$

The forcing functions F_n are computed with the formula

$$F_n = -\frac{\bar{a}^2}{\bar{p}E_n^{\,2}}\left\{\iiint h\psi_n dV + \oiint f\psi_n dS\right\} \tag{3.14}$$

and

$$E_n^2 = \int \psi_n^{\,2} dV \tag{3.15}$$

Equation (3.13) is the basis for the statement emphasized at the beginning of this section. The left hand side represents the motion of a simple undamped oscillator, its 'displacement' being the amplitude of the corresponding natural mode of the chamber. Although not shown by the symbolic form (3.14), the force F_n contains linear parts—attenuation, frequency shifts and mode coupling—as well as nonlinear behavior, also including nonlinear coupling.

Thus we can split F_n into several meaningful parts and write the oscillator equations in the form

$$\begin{aligned}&\frac{d^2\eta_n}{dt^2}+D_{nn}\frac{d\eta_n}{dt}+\left(\omega_n^2+E_{nn}\right)\eta_n\\&=-\sum^{i\neq n}\left[D_{ni}\dot{\eta}_i+E_{ni}\eta_i\right]+F_n^{NL}\end{aligned}\qquad(3.16)$$

where F_n^{NL} stands for all nonlinear contributions to the force. The left-hand side of (3.16) represents the behavior of a damped oscillator and is identical with (3.1); the summation on the right side contains all the linear coupling terms, and F_n^{NL} is the nonlinear part. Thus equation (3.15) is the theoretical basis for the italicized assertion at the end of Section 2.

Remarks:

(i) It is especially important to understand that the function F_n defined by (3.14) contains *formal* representations of all relevant physical processes, but is by itself inadequate to give quantitative results. In passing from (3.12) to (3.16) we have written F_n

$$F_n=-D_{nn}\dot{\eta}_n-F_{nn}\eta_n-\sum^{i\neq n}\left[D_{ni}\dot{\eta}_i+E_{ni}\eta_i\right]+F_n^{NL}$$

thereby identifying the various coefficients D_{nn}, D_{ni}...etc. Comparison of (3.14) and (3.17) suggests what is true, that those coefficients are defined by integrals over the volume and on the surface of the chamber. The integrals are functions representing the various processes. Modeling, based either in theory or experimental results, is required to provide the necessary representations.

(ii) Each linear oscillator—i.e. each normal mode—is characterized by three parameters, of which two arise from perturbations to a classical undamped system:

(1) natural frequency, ω_n;
(2) growth or damping constant D_{nn}=$-2\alpha_n$, negative (i.e. α_n >0) for unstable motions;
(3) the nonlinear physical processes contained in F_n^{NL} are those responsible for limiting linearly unstable motions, i.e. for forming limit cycles, and for producing subcritical bifurcation (triggered or nonlinear instabilities)
(4) parts of the coefficients shown and of F_n^{NL} follow from gas dynamics and hence are known; the remaining contributions must be found from models of the processes in question. In that regard, the combustion processes are the most important.

3.4.4 Linear Stability

The formalism described here has been widely used to investigate linear stability for more than thirty years. Linear problems are described by (3.16) with F_n^{NL} neglected:

$$\begin{aligned}&\frac{d^2\eta_n}{dt^2}-2\alpha_n\frac{d^2\eta_n}{dt}+\left(\omega_n^2-2\omega_n\delta\omega_n\right)\eta_n\\&=\sum^{i\neq n}\left[D_{ni}\dot{\eta}_i+E_{ni}\eta_i\right]\end{aligned}\qquad(3.17)$$

Almost all works dealing with linear stability concentrate on the stability of normal modes. If, as seems often to be the case, the coupling terms on the right hand side are small and can be ignored, (3.17) becomes

$$\frac{d^2\eta_n}{dt^2}-2\alpha_n\frac{d\eta_n}{dt}+\left(\omega_n^2-2\omega_2\delta\omega_n\right)\eta_n=0\qquad(3.18)$$

Use of (3.18) does not rest on ignoring the linear coupling. A well-known procedure in the theory of linear systems leads to the same form by a transformation of variables: that is the proper justification for using (3.18). In any case, the solution for η_n gives the time evolution of the amplitude of the n^{th} mode, following an initial disturbance $\hat{\eta}_n\cos\Phi_n$

$$\eta_n(t)=\hat{\eta}_n e^{\alpha_n t}\cos\left(\omega_n\sqrt{1-\zeta_n^2}\right)t\qquad(3.19)$$

and

$$\zeta_n=-\frac{\alpha_n}{\omega_n}\qquad(3.20)$$

In this field α_n is conventionally defined to be positive if the n^{th} mode is unstable. The stability boundary is therefore located by setting $\alpha_n = 0$. This is far from the empty result it may appear to be, because α_n is a sum of several contributions arising from the linear processes contained in F_n, equation (3.14). Those contributions represent the effects of the linear processes accounted for in the functions k and f defined in (3.8) and (3.9). As a consequence of the spatial averaging, α_n contains integrals over the volume of the chamber and over the boundary covering largely the burning surface, and closed by the area of the entrance to the nozzle. Schematically we can therefore express α_n as

$$\alpha_n = [\alpha_n]_{nozzle} + [\alpha_n]_{particle\,damping} + [\alpha_n]_{mean\,flow/acoustics} + [\alpha]_{vortex\,shedding}$$
$$+ [\alpha_n]_{distributed\,combustion} + [\alpha_n]_{surface\,combustion} + [\alpha_n]_{other} \quad (3.21)$$

Before discussing the pieces of α_n, we note a useful interpretation. The envelope of the pressure oscillations varies as $e^{\alpha_n t}$, i.e.

$$[p']_{amp} = ke^{\alpha_n t} = |\eta_n|_{max}$$

where k is independent of time. It is a result of classical acoustics that the time-averaged energy $\langle E_n \rangle$ of an oscillation is proportional to the square of the pressure amplitude, i.e. where k_1 is another constant in time. Differentiating this formula leads to

$$\alpha_n = \frac{1}{2}\left(\frac{1}{\langle E_n \rangle}\frac{\langle dE_n \rangle}{dt}\right) \quad (3.22)$$

The growth (or decay) constant is equal to one-half the fractional rate of change of time–averaged energy. Thus α_n given by (3.21) is most easily regarded as a sum of rates of energy losses and gains. We now discuss the various terms of (3.21), deferring the matter of surface combustion to Section 4.

Nozzle

The influence of the exhaust nozzle has long been known to be stabilizing. While some theoretical results suggest the possibility of small destabilizing effects for some cases involving transverse modes, no experimental evidence exists. Modes of oscillation involving velocity fluctuations parallel to the axis are always attenuated by the exhaust nozzle. It is a large contribution for purely longitudinal modes and is practically independent of frequency.

Particle Damping

Attenuation or damping of acoustic waves by small particles has also long been known. This is potentially a dominant effect in rockets using propellants that produce condensed combustion products. The value of α_n is proportional to the mass fraction of condensed material in the products (about 38% for a propellant containing 19% *Al*) but depends very significantly on the particle material density, the average particle size and frequency. For example, if the frequency of the oscillations is 500Hz, the optimum particle size of $Al_2 O_3$ maximum damping is about 10 microns.

Particle damping has long been understood to be a critical matter in the stability of motions in solid propellant rockets. It is a significant contribution to the decay constant for motors using metallized propellants, and its dependence on parameters characterizing the system are known. Hence the processes responsible for particle damping, especially how they can be changed, offer good possibilities for improving the stability of a motor. The idea is to charge the propellant composition, and hence ultimately the characteristics of the condensed products of combustion in such a way as to increase the particle damping. A good discussion of this strategy has been given by Derr et al (1979).

In addition to those mentioned in the previous paragraph, it is an important property that the maximum particle damping increases with frequency. Thus it is a practical consequence that particle damping is a controlling factor in stability of motions at thousands of Hertz but not at hundreds of Hertz. That is the chief reason, for example, that small amounts of inert particulate additives can be effective for suppressing high frequency instabilities often found when non-aluminized propellants are used. In contrast, low frequency instabilities are commonly found with propellants containing aluminum which, as noted above, produces substantial amounts of particulate material after combustion.

The theory used for computing particle damping is extremely accurate; simplified forms were derived long ago and have been shown beyond doubt to be accurate. Moreover, the results are applicable under the conditions existing in solid rocket motors only if certain requirements are met. The main requirements are (1) that the size distribution of the condensed particles must be known and (2) that the particles are neither burning or evaporating, nor growing due to further accumulation or agglomeration of material. Some results have been obtained in other connections for evaporating or condensing particles but they have not been confirmed experimentally. (Wooten 1966; Dupays and Vuillot 2000).

Production of the condensed material in a solid rocket takes place as part of the combustion processes. Thus, in the context of MURI, this is a matter for Task II. As discussion in the accompanying paper by Beckstead (2000) describes, considerable work has been done within the MURI programs, both experimentally and in modeling. However, no effort has been expended in the *dynamics* of aluminum combustion – anywhere.

Mean Flow/Acoustics Interactions

Interactions between fluctuations and the average flow is a significant matter in the field of combustor dynamics, certainly having importance second only to that of combustion dynamics. In all types of combustors, there are interactions between the unsteady and steady fields within the volume and at the boundary where the reactants enter and at the entrance to the exhaust nozzle where combustion products leave. The latter is in addition to the contribution to the attenuation due to the nozzle mentioned above, which arises from energy transfer between the waves and the non-uniform mean flow in the convergent section of the nozzle.

The first effect of linear interactions between the mean and unsteady fields involves both volumetric and surface processes. Formally, various integrals in the definition (3.14) of F_n can be combined to produce a single surface integral that can be interpreted as the net convection of acoustic kinetic energy through the boundary by the mean flow. Usually this is an important contribution to the growth/decay constant, both numerically and qualitatively because it varies with grain geometry in a manner similar to the contribution from surface combustion; see equation (4.13). Hence attention should be paid to this phenomenon during design of a motor.

Much more complicated—and still controversial—are consequences of the production of unsteady vorticity at the burning surface. The physical origin of the process is clear in general terms and not in question, namely the interactions between unsteady motions parallel to a surface through which flow is emerging. That this process contributes a loss of acoustic energy in a rocket motor, was discovered as part of an analysis based in the one-dimensional approximation (Culick 1973). The connection between that result (which became known as 'flow-turning') and the actual process of vorticity generation was not recognized until Flandro (1995) carried out a careful two-dimensional analysis. Qualitatively, what happens is the following.

Because this is a purely fluid-mechanical process, we need not account for combustion to understand the essential character of the problem. Consider a cylindrical chamber whose lateral boundary is permeable, admitting flow with velocity normal to the surface. The chamber is closed at one end and open at the other to allow the exhaust. A piston is fitted at the head end, so that acoustic waves can be generated. If the frequency and amplitude of the oscillating piston are held fixed, after a transient period of starting, a standing wave system will exist in the chamber, sketched in Figure 14. The acoustic velocity in the second longitudinal mode is shown by the double-headed arrows. For this mode the velocity is zero at roughly the mid-point of the chamber and is nearly zero at the ends. Associated with the oscillations between the modes is an oscillating pressure and pressure gradient which has maximum amplitude at the center of the chamber.

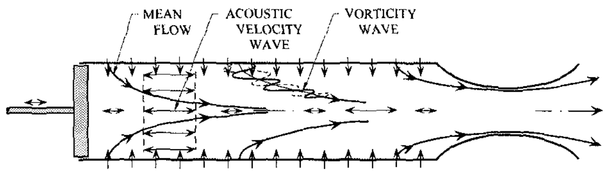


Figure 14

Ideal longitudinal modes possess planar phase fronts; that is, the velocity and pressure fluctuations have the same values at the lateral boundary as on the axis. In practice the behavior is only slightly different from the ideal. Hence the flow entering the chamber at the boundary is immediately exposed to an oscillating environment, the acoustic pressure acting to accelerate, periodically, the incoming flow in a direction normal to itself. Thus the incoming flow must not only turn on the average to the axial direction, but also suffers as an oscillatory 'turning'—thus the term 'flow turning effect'. It is during this process, essentially a shearing action on the incoming waves sketched in Figure 15, that waves of vorticity are generated. Because the acoustic pressure field does work in the incoming flow, which therefore gains kinetic energy, the process of generating the waves of vorticity at this level evidently appears as a loss of energy to the acoustic field. In other words, energy is transferred from the acoustic field to the oscillating vorticity connected with the mean flow. Both analysis (Flandro, 1995, Majdorani, 1998) and numerical simulation (Roh et al 1995) suggest that the vorticity waves are dissipated rapidly through the action of viscous stresses and interaction with turbulence.

The analysis by Flandro 1995, Malhotra and Flandro 1997 and Majdorani and Van Moorhem 1998 have shown that, in addition to the basic flow turning loss found originally in the one-dimensional analysis, there are two other contributions: 1) the gradient of oscillatory pressure parallel to the surface causes, by conservation of mass, a velocity fluctuation normal to the lateral boundary, thereby causing additional loss associated with momentum exchange between the incoming and bulk flows; and 2) within the chamber there are volumetric interactions between the vorticity waves and the acoustic waves, causing a gain of energy for the acoustic field. This gain may be partly responsible for small amplitude oscillations in large solid rocket boosters. (But see remark (iii) below.)

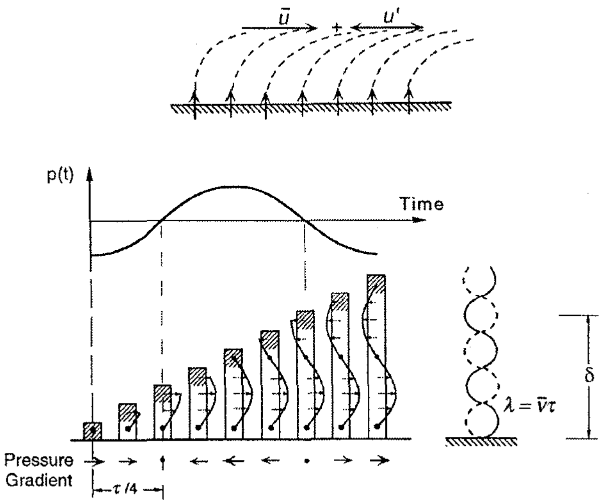


Figure 15

Figure 14 correctly suggests that even without combustion, the flow field in this situation is extremely complicated, involving mean flow, acoustic waves and vorticity waves. The simultaneous presence of the two types of waves existing on two quite different length scales is particularly troublesome. The frequencies of the waves, are identical, but the wavelengths are $\lambda_a = \bar{a}/f$ for the acoustic waves and $\lambda_v = \bar{u}_r/f$ for the vorticity waves, where $\bar{u}_r$ is a characteristic mean flow velocity. Hence their wavelengths are in the ratio of the mean characteristic Mach number $\bar{M}_r$ of the mean flow, one of the two small parameters cited in Section 3.3.1. We therefore expect that the presence of the vorticity waves should exercise in the acoustic field perturbations of the order of $\bar{M}_r$. That is true over much of the field, but because the two velocity fields (acoustic and vorticity) satisfy different boundary conditions, incorporating both wave phenomena in a single analysis poses difficult problems. It appears that the difficulties have been best resolved in recent analysis by Malhotra (2001; see a progress report by Malhotra and Culick (2000).

There is no longer disagreement that the losses (or gains) associated with the generation of acoustic waves must be accounted for in computations of linear stability. However, the precise form (and hence the *quantitative* aspects) of the contribution to the stability of actual motors remains controversial. It seems that this offers a good opportunity for special application of numerical simulations. Tentatively we offer the following conclusions as the best strategy for practical applications at this time:

(i) the original flow-turning loss, confirmed by Flandro (1995) in his analysis and subsequently by Majdorani and Van Moorhem (1998) and Malhotra (2001) should be included in computations of linear stability;

(ii) an additional contribution overlooked in the one-dimensional analysis and first identified by Flandro (1995) must be included: that is the part as discussed above, proportional to the local gradient of acoustic pressure parallel to the boundary; and according to the limited results of numerical simulations the vorticity waves seem to be rapidly destroyed following their generation, by viscous dissipation and interactions with turbulence in the flow; if that result holds, then it may be appropriate to ignore interactions of the vorticity waves with the acoustic field within the volume, and with the flow in the exhaust nozzle.

The general problem of the generation and consequences of unsteady vorticity remains a problem of current investigation. It seems now that the work by Malhotra (2001) may bring the matter to closure.

Vortex Shedding

A process in some respects related to the generation of unsteady vorticity discussed above, production or shedding of relatively large vortices is largely a distinct phenomenon contributing in its own way to stability of oscillations. Two types of vortex shedding, both apparently significant only in large solid rocket motors, have been subjects of research and practical concern in the past twenty-five years: shedding from edges or obstacles; and generation of vortices out of an instability of the mean flow created by mass injection at the lateral boundary.

The importance of vortex shedding from obstacles seems to have been first recognized by Flandro and Jacobs (1975). Subsequently, Flandro (197?) used the idea as the basis for successfully eliminating an instability in a full-scale test motor by rounding edges near the head end of the grain. The phenomenon is purely fluid mechanical and is conveniently studies in laboratory devices (Culick and Magiawala 1979; Dunlap and Brown 1981; Brown et al 1981; Nomoto and Culick 1982; Aaron and Culick (1985).

Flandro (1988) first worked out a linear analysis of vortex shedding from edges, based on instability of a shear layer. The shedding process is coupled to the acoustic field because the acoustic velocity acting at the origin of the shear layer (at an edge) encourages initiation of vortices. Then the vortices generate pressure waves in two ways: first as weak (approximately quadruple) moving sources; and second, more strongly as approximately dipole sources if they encounter an obstacle downstream. As part of an extensive research program concerned with instabilities found in the P230 solid rocket booster motors on the Ariane 5, Lupoglazoff and Vuillot (1992) carried out numerical simulations (2D) using both Euler and Nairer-Stokes equations. They concluded that "comparison with Flandro's method did not yield conclusive results."

That conclusion, and particularly the reasons behind it, more clarified with sub-scale tests with cold flow and with further numerical simulations (Vuillot et al 1993; Lupoglazoff and Vuillot 1996). Those works document the discovery of a new phenomenon, the second kind of vortex shedding referred to above, occurring without edges or obstacles, and called "parietal vortex shedding" (Lupoglazoff and Vuillot 1996 and 1998). The distinguishing characteristic of this kind of vortex shedding is that it arises from a hydrodynamic instability of the mean flow created by injection at the lateral boundary. Coupling to the acoustic field occurs, apparently, due to both the velocity and pressure fluctuations. The results suggest that background noise has some influence in the process, perhaps most importantly in initiating the instability (Lupoglazoff and Vuillot 2998). Moreover, unsteady surface combustion and residual combustion of aluminum near the lateral boundary exert significant influences. Le Breton et al (1999) have given a recent review of all aspects of parietal vortex shedding.

From a different point of view, Beddini and his students (Beddini 1988; Lee and Beddini 1999; Beddini and Roberts 1991) have investigated the flow field near a lateral boundary with mass injection. Their detailed numerical computations show the existence of an instability which seems likely to be identical with that leading parietal vortex shedding. However, establishing identity of the two sorts of phenomena remains to be accomplished. As part of the Caltech MURI Program, work is in progress to merge Beddini's results with the analysis of combustion dynamics being developed by Cohen et al (2000).

Although the expression (3.21) for the growth constant contains a contribution due to vortex shedding, no representation currently exists. In his early work, Flandro (1986) cast his results in that form, the corresponding result for parietal vortex shedding has not yet been reported. Obtaining the appropriate results remains a goal in Task III of the Caltech MURI.

Distributed Combustion

Distributed combustion really means, for practical purposes, residual combustion of aluminum. There have been occasional indications that the dynamics of residual combustion may have been a noticeable contribution to stability of full-scale motors. However, the evidence is not firm and no supportable conclusions have been drawn. There exist no analyses of the dynamics of aluminum combustion and no experimental work. Yet this is an important process to understand well because residual combustion produces condensed products and hence affects particle damping. The process clearly poses extremely difficult experimental problems even if one seeks only qualitative information.

4. Dynamics of Surface Combustion

It is fair to conclude that generally the theory, analysis and understanding of combustor dynamics are well in hand. Questions and problems remain but methods are available to treat them and in general we know how to proceed, both with formal analysis and with numerical simulations. In contrast, the processes comprising surface combustion and their dynamics unquestionably present the most important remaining problems relating to combustion instabilities. Serious deficiencies exist, both theoretically and experimentally. Perhaps the simplest summary of the matter is to state that here, in a thin region where solid material is converted to combustion products, we are up against the conflict of the three widely differing length scales cited in the abstract and it is in this region that the three sorts of dynamics come together, where the mating dance is held. Thus the three tasks of the MURI Program must also come together here, as emphasized in Figure 16.

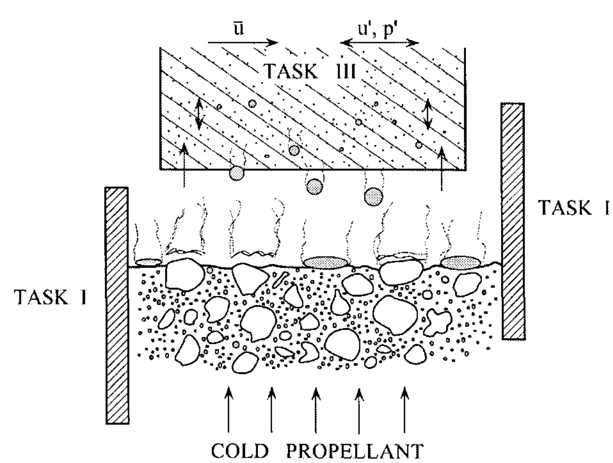


Figure 16

The shaded zones suggest the spatial extent of the processes to which the three tasks are devoted.

What is particularly important for solving the problem of chamber dynamics is the component of the velocity fluctuation at the edge of the combustion zone, perpendicular to the surface, $\hat{\boldsymbol{n}}\cdot\vec{u}'$ here denoted[6] $\vec{u}_b' = -\hat{\boldsymbol{n}}\cdot\vec{u}'$. The surface integral of $\vec{u}_b'$ arises directly in the definition (3.14) of F_n,

$$F_n^{(c)} = \frac{\gamma}{E_n^2}\iint \psi_n \frac{\partial \vec{u}_b'}{\partial t} dS \tag{4.1}$$

where the integration extends over the burning surface. For linear behavior, it is a long tradition in acoustics (e.g. see Morse 1935) to characterize nonreacting surfaces by admittance functions, simply as a convenience. That practice was taken over in the 1950's by the group under McClure at Johns Hopkins (Hart and McClure 1959). Partly due to historical practice, the admittance function A_p is defined as the dimensionless quantity[7]

$$A_p = \frac{u_b'/\bar{a}}{p'/\gamma\bar{p}} \tag{4.2}$$

which for sinusoidal motions $(u_b' = \hat{u}_b e^{i\omega t}, p' = \hat{p}e^{i\omega t})$ becomes

$$A_p = \frac{\hat{u}_b/\bar{a}}{\hat{p}/\gamma\bar{p}} = \overline{M}_b \frac{\hat{u}_b/\bar{u}_b}{\hat{p}/\gamma\bar{p}} \tag{4.3}$$

generally $\hat{n}A_b$ is a complex quantity and represents the fluctuation of normal velocity in the combustion products near a burning surface produced by imposition of a sinusoidal pressure fluctuation.

Analysis of the combustor dynamics quite naturally leads to the requirement that the velocity fluctuation must be specified as a boundary condition. On the other hand, analysis of the combustion dynamics leads most directly to results for fluctuations of the mass flux, m', perpendicular to the surface. From its definition, $m' = (\rho u)' = \rho'\bar{u} + \bar{\rho}u'$, and (4.2) can be written

$$A_p = \frac{1}{p'/\gamma\bar{p}}\frac{1}{\bar{\rho}\bar{a}}\left(m_p' - \rho_b'\bar{u}_b\right)$$

$$= \overline{M}_b\left(\frac{m_p'/\overline{m}_b}{p'/\gamma\bar{p}} - \frac{\rho'/\bar{\rho}_b}{p'/\gamma\bar{p}}\right) \tag{4.4}$$

The ratio involving m_p' has come to be known as the response function denoted here by R_p,

$$R_p = \frac{m_p'/\overline{m}_b}{p'/\gamma\bar{p}} \tag{4.5}$$

and (4.4) is

$$A_p = \overline{M}_b R_p - \overline{M}_b \frac{\rho'/\bar{\rho}_b}{p'/\gamma\bar{p}} \tag{4.6}$$

The boundary conditions and the definitions of A_p and R_p are somewhat more complicated if the presence of condensed material is accounted for in the combustion products. Here for clarity we assume a single phase gas flow.

Then the density fluctuations are related to pressure and temperature fluctuations by the perfect gas law,

$$\frac{\rho'}{\bar{\rho}_b} = \frac{p'}{\bar{p}} - \frac{T'}{\overline{T}_b} \tag{4.7}$$

The temperature fluctuations can be written as the sum of two parts, one related isentropically to the pressure fluctuation and the remainder due to non-isentropic changes, $\Delta T_f'$ in the flame zone,

$$\frac{T'}{\overline{T}} = \left(\frac{T'}{\overline{T}}\right)_{is} = \frac{\Delta T_f}{\overline{T}_b} = \frac{\gamma - 1}{\gamma}\frac{p'}{\bar{p}} + \frac{\Delta T_f'}{\overline{T}_b} \tag{4.8}$$

Substitution in the second term of (3.27) then leads to the results

$$A_p + \overline{M}_b = \overline{M}_b\left(R_p + \frac{1}{\gamma}\frac{\Delta T_f'}{\overline{T}_b}\right) \tag{4.9}$$

For pure sinusoidal oscillations, $\frac{\partial u_b'}{\partial t} = i\omega u_b'$ and with (4.9), the forcing function (4.1) can now be written $F_n^{(c)} = \hat{F}_n^{(c)} e^{i\omega t}$ with

[6] The minus sign is attached here because $\hat{\boldsymbol{n}}$ is the outward normal and for the surface combustion it is natural to define the inward velocity positive away from the surface.

[7] We use here the subscript $(\ )_p$ to indicate the response to a pressure fluctuation, instead of the more conventional subscript $(\ )_b$ used to denote the burning surface.

$$\hat{F}_n^{(c)} = \frac{i\omega}{E_n^2}\iint \psi_n \frac{\hat{p}}{\bar{p}_n}\left(R_p + \frac{1}{\gamma}\frac{\Delta\hat{T}_f}{\bar{T}_b}'\right)\bar{u}_b dS$$
$$-\frac{i\omega}{E_n^2}\iint \psi_n \frac{\hat{p}}{\bar{p}}\bar{u}_b dS$$

Within the approximations for the analysis of linear stability, $\frac{\hat{p}}{\bar{p}_n} \approx \psi_n$ and the last formula becomes

$$\hat{F}_n^{(c)} = \frac{i\omega}{E_n^2}\iint \psi_n^2\left(R_b + \frac{\Delta\hat{T}_f/\bar{T}_b}{p'/\gamma\bar{p}}\right)\bar{u}_b dS - \frac{i\omega}{E_n^2}\iint \psi_n^2 \bar{u}_b dS \tag{4.10}$$

It is a remarkable consequence of the calculations that the second integral is cancelled exactly by the contribution from the mean flow/acoustics *not* involving unsteady vorticity. Hence for the purposes here we may neglect the last term and take as the forcing function arising from the dynamics of surface combustion

$$\hat{F}_n^{(c)} = \frac{i\omega}{E_n^2}\iint \psi_n^2\left(R_b + \frac{\Delta\hat{T}_f/\bar{T}_b}{p'/\gamma\bar{p}}\right)\bar{u}_b dS \tag{4.11}$$

This is the *exact* result of the theory for linear behavior.

The non-isentropic temperature fluctuation $\Delta\hat{T}_f$ at the edge of the combustion zone is forced as part of the solution for the combustion dynamics. It has never been measured[8] and is routinely neglected. Therefore, we take as the theoretical result for the form of the driving force due to combustion for a pure sinusoidal oscillation of the n^{th} mode,

$$\hat{F}_n^{(c)} = \frac{i\omega}{E_n^2}\iint \psi_n^2 R_p \bar{u}_b dS \tag{4.12}$$

This result shows clearly why so much emphasis is placed on modeling and measuring the response function R_p. Note that if $\Delta\hat{T}_f/\bar{T}_b$ is neglected, the relation (4.9) becomes

$$A_p + \bar{M}_b = \bar{M}_b R_p \tag{4.13}$$

It is this combination which is inferred from the results of T-burner tests. That is a nice property of that kind of test, due essentially to the fact that a T-burner is really a special form of rocket motor for which the general analysis has been constructed.

Remarks

(i) The basic information required of the combustion dynamics is the fluctuation of velocity u_b' *normal* to the surface for use in (4.1). By construction of the oscillator equations, (3.13) u_b' is *defined* to be the value at the (perhaps ill-defined) edge of the combustion. Alternative definitions of u_b' can be accommodated, but proper account must be taken in the procedure of spatial averaging.

(ii) The steps leading from (4.1) to (4.12) apply only to the limiting case when the combustion processes are assumed to respond solely to oscillations of pressure. This case is conventionally referred to as *pressure coupling*. With several approximations, notably linear behavior, the required information has then been transformed from the normal *velocity* at the surface to the *mass flux* expressed in terms of the response function R_p. The implications of these definitions are particularly important for planning experimental determination of the combustion dynamics, the subject of Section 5.

(iii) While pressure coupling has received the greatest attention both theoretically and experimentally, the second limiting case of *velocity coupling* has long been regarded as a mechanism (McClure et al 1962) that is likely significant in practice. By velocity coupling we mean the dynamical response of surface combustion represented by u_b', to fluctuations of the unsteady velocity field in the chamber at the edge of the combustion zone. It is consistent with the relative spatial scales of the combustor and combustion dynamics to take that velocity to be parallel to the surface; υ_b' is of course always defined to be normal to the surface. In the sense defined here, there are no true theoretical results for velocity coupling based on analysis of the combustion dynamics. Approaches based on non-steady erosive burning – i.e. a kind of unsteady form of results valid in the first instance for steady combustion – can provide certain guidelines and seem to capture part of the behavior (Lengellé, 1975; Cohen and Strand, 1985; King 1981 and 1993) Renie and Osborne, 1981. The results are generally regarded as inadequate if only because the implied core flows assumed in the

[8] If $\Delta\hat{T}_f$ is non-zero, entropy waves are generated, which are then convected by the mean flow. Evidence for the existence of weak entropy waves was reported by Waesche (1965) and Krier (1969).

analyses do not reflect the unsteady processes at the boundary. Only limited experimental results are available (Beckstead and Butcher 1974; Brown et al 1986). They show several surprising characteristics still unexplained due, one may reasonably argue, to the absence of a theory. Recent work treating stability of the near field and including influences of turbulence in the combustion zone (Lee and Beddini 1999) offer considerable promise for correcting some of the deficiencies. In particular, that analysis may provide an explanation for the physical origin of the threshold velocity introduced in the heuristic model represented later by equation (5.1). The limited experimental results (Beckstead and Butcher, 1974; Brown et al, 1986)) have presented several surprising characteristics still unexplained, due, one may reasonably argue, to the absence of a theory. The phenomenon of velocity coupling is related to the problem of mean flow/acoustic interactions discussed in 3.4.4. This subject not closed and remains an object of current research.

4.1 Modeling Combustion Dynamics

According to the research described above, the greater part of results for modeling combustion dynamics have been obtained for linear pressure coupling, expressed as the response function R_p. The basic result still usually appealed to for explaining observed behavior of both combustion and combustor dynamics is the response function first reported in the explicit form (4.14) by Dennison and Baum (1961):

$$R_p = n\frac{AB}{\lambda + \frac{A}{\lambda} - (1+A) + AB} \tag{4.14}$$

Culick (1968) reviewed analyses of the pressure-coupled response function and showed that almost all reported results reduced to this form because they shared common assumptions in constructing the physical model:

(i) All processes in the gas phase and in the infinitesimally thin interfacial region are assumed to be quasi-steady;
(ii) The solid phase and flow in the gas phase are assumed to be spatially homogeneous having variations only in the direction normal to planes parallel to the surface;
(iii) Only unsteady heat transfer in the solid phase depends explicitly on time and hence produces true dynamical behavior.

The symbols in (4.14) have the following meaning:

λ: complex function of the dimensionless frequency $\Omega = \omega\bar{r}^2/\kappa$ where $\bar{r}$ is the mean burning rate and κ is the thermal diffusivity of the solid phase; λ satisfies the equation:

$$\lambda(\lambda -) = i\Omega \tag{4.15}$$

n: pressure index in $\bar{r} = a\bar{p}^n$; hence $n = \partial \log \bar{r} / \partial \log \bar{p}$, calculated for a fixed temperature of the cold propellant

A: a parameter depending on the surface temperature $\bar{T}_s$ at the interface and on the activation energy of the conversion of solid to gas assumed to obey an Arrhenius law, representing the decomposition process, $A = E_s(\bar{T}_s - T_c)/R\bar{T}_s^2$

B: a parameter depending chiefly on details of the model chosen for the processes in the gas phase, and characterizing the heat feedback to the condensed phase.

Hence R_p/n is a 2-parameter model, a complex function of the dimensionless frequency Ω; this class of response functions represented by (4.14) is commonly identified as the QSHOD model (Quasi-Static Homogeneous One-Dimensional) (Beckstead and Culick 1971).

Variations of (4.14) closely related to the QSHOD model arise when the following additional influences are taken into account:

(i) pressure-dependent interfacial processes;
(ii) unsteady variations of the burning rate—and hence m_b'—due to absorption of oscillatory radiation incident on the surface;
(iii) non-uniform properties of the solid phase in the direction normal to the surface;
(iv) exothermic reactions in the solid phase.

Other possibilities may exist but it seems that i)-iv.) cover the cases most closely related to the QSHOD model. It is characteristic of the sort of analysis required in these problems that it is relatively easy to investigate the consequences of different assumptions for the behavior because the only dynamics is due to unsteady heat transfer in the condensed phase, for which there is a single characteristic frequency, the real part of the response functions has only one pronounced peak. That is a nice result because if other peaks appear in experimental results, they must be due to different processes.

This simple form (4.14) for the response function has been extremely important for many years, despite its obvious deficiencies, precisely because it is simple. Other than the fact that it really does contain much of the real behavior, the result has been most useful because it suggests scaling laws, of which the most important is the following.

For given values of A and B (i.e. a given propellant), the quantity R_p/n is a function only of $\Omega = \omega\bar{r}^2/\kappa$. Thus the peak of the real part occurs at some value Ω_p of Ω. Because κ is relatively insensitive to mean pressure, then the real frequency ω_p of the peak varies with the pressure because of the dependence on linear burning rate, $\bar{r} = a_r\bar{p}^n$

$$\omega_p = \frac{\Omega_p}{\kappa}\bar{r}^2 = \left(\frac{\Omega_p a_r^2}{\kappa}\right)\bar{p}^{2n} \quad (4.16)$$

Thus the characteristic frequency of the combustion dynamics increases with pressure[9] Suppose for example that a motor of a given design, in particular the length is fixed, is stable at some pressure $\bar{p}$, for which the peak frequency is less than the frequency $\omega_1 = \pi\bar{a}/L$ of the fundamental longitudinal mode, $\omega_p < \pi\bar{a}/L$. But suppose that the nozzle is changed to produce a higher operating pressure. It is then conceivable that ω_p is increased so that $\omega_p \sim \pi\bar{a}/L$. If under these conditions the driving by the combustion processes is sufficiently large (i.e. $R_p^{(r)}$ is sufficiently large) then the new design could be unstable.

Corresponding reasoning can be established for the behavior as the values of A and B are changed due, for example, changes in the propellant, although it is helpful then to have some numerical results. One may then draw conclusions concerning the influences of certain properties (such as the surface activation energy) on the shape of the response function. The point here is not that the conclusions are necessarily always correct, but that already with the simplest QSHOD model of the combustion response, one gains some significant understanding of the dependence of the combustion dynamics on physical properties of the propellant[10]. Thus, in addition to providing a basis for concluding and interpreting experimental results, the simple literal results suggest several simple scaling laws. Further remarks on this matter are given in Section 4.3.

Several works (for example Tien 1972; Clavin and Lazmi, 1992; and Isella and Culick 2000) have investigated the consequences of relaxing the assumption of quasi-steady behavior of the gas phase. The results are not surprising: the response is increased in a range of frequencies greater than that for which the dynamics of the solid phase is dominant, and a second characteristic peak appears at higher frequencies.

All of the preceding discussion deals with generic analyses intended to expose general behavior. There has recently been, especially as part of the MURI programs, substantial effort to discover and understand more detailed aspects of the behavior. Analyses of this sort must begin with models of steady state burning for heterogeneous propellants, the business of Task II. Two recent papers serve presently as good introductions: Beckstead (2000) and Cohen et al (2000).

4.2 Measurements of Combustion Dynamics

Experimental data for the response function are essential for at least two reasons: presently it is impossible to compute the combustion dynamics sufficiently accurately for reliable *ab initio* predictions of stability in motors; and without reasonably good data as a guide, analysis alone tends to be sterile. A great advantage of the formula (4.14) is that only measurements of steady combustion are required to evaluate the parameters n, A, B and the reference frequency F^2x. For most results (see Culick 1968) based in detailed flame models, A and B are difficult combinations of material and chemical properties. It is a great advantage of the *Z-N* formulation (Zeld'ovich 1942; Novozhilov 1968 and many other works) that A and B can be expressed in terms of parameters relatively easily and routinely measured in the process of characterizing the combustion of a propellant:

$$r = \left(\frac{\partial\bar{T}_s}{\partial T_c}\right)_{\bar{p}} \quad (4.17)$$

$$n = \left(\frac{\partial\ln\bar{r}}{\partial T_c}\right)_{\bar{p}}\left(\bar{T}_s - T_c\right) = \sigma_p\left(\bar{T}_s - T_c\right) \quad (4.18)$$

[9] Similar reasoning may be worked out for simple models of the velocity-coupled response.

[10] This is strictly true if the parameters A and B are independent of pressure. In reality, while A is really constant, B varies significantly with pressure. Moreover, B depends weakly on the conditioning temperature. These details complicate the behavior, but do not invalidate the point.

where σ_p is the temperature sensitivity and $\overline{T}_s$ is the average surface temperature.

The relation connecting the two sets of parameters are:

$$A = \frac{k}{r} \tag{4.19}$$

$$B = \frac{1}{k} \tag{4.20}$$

While there are some distinct advantages of clarity and interpretation favoring the *Z-N* analysis of the response function, two points must be emphasized:

(i) although part of the formulation is very different, the physical content of the *Z-N* model is fundamentally identical to that of all other models leading to (4.14); and therefore
(ii) the form of the frequency dependence of the basic *Z-N* model is precisely (4.14) with *A* and *B* replaced by the formulas (4.18) and (4.19).

In principle, then, standard measurements taken during steady combustion of propellants will, with use of (4.14) or some other comparable analytical result, give the response function. Unfortunately, it has long been known that comparison of the available theories with experimental results show obvious deficiencies. That statement can be made even though current experimental methods are inadequate to give us the data we need under all conditions (frequency range and mean pressure) existing in operational motors. (Beckstead and Culick 1971, Beckstead 2000). Experimental results *do* confirm the general behavior described above—that the dynamics of unsteady heat transfer in the condensed phase dominates the dynamics in the lower frequency range where the longitudinal (axial) modes of solid propellant rocket motors are usually found.

Nevertheless, it is essential to have a method for measuring with confidence the response function[11] of propellants. It is a matter of both practical and research importance to be able to measure accurately and economically both the response function and its changes with small changes in composition. Much effort has been expended in the MURI programs to reach that goal, without notable success. The various methods have been critically reviewed (Culick 1998) and will be reported upon individually in the MURI documentation from the various research groups. Here we will simply list the methods without explanation, and only brief assessments.

MURI or related funding supported work on the following methods regarded as possible alternative to the standard T-Burner:

1) Laser-recoil devices: (University of Illinois at Urbana-Champagne; Pennsylvania State University); NAWC China Lake
2) Ultrasonic Doppler: (University of Alabama at Huntsville; University of Illinois at Urbana-Champagne)
3) Magnetohydrodynamic/Velocimetry (Pennsylvania State University)
4) Microwave Doppler (Institute of Chemistry and Chemical Kinetics, Novosibirsk)
5) Transient Device (University of Illinois at Urbana-Champagne)

Although some of the results obtained with these methods may be useful for clarifying certain aspects of unsteady combustion of the particular propellants tested, *none* are applicable to the problems encountered in motors. Each one suffers from one or more of the following deficiencies:

1) restricted low frequency range (< 600–1000Hz)
2) restricted to low pressure
3) inapplicable to metallized propellants
4) large uncertainties in the interpretation of data nonreproducibility of test results.

The last is particularly troublesome, especially because for the most part little attempt has been made to establish true error bars. Moreover, existing large differences between results obtained by different methods for the same propellant tested under the same conditions have not been explained. Those acquiring data have not been conscientious in this respect and therefore have failed to interpret the significance of their own results.

There seems also to be a lack of continued concerted effort to correlate data for the response function with (4.14). It is especially important to maintain recognition of the connection between (A, B) and (*k*, *r*), equations (4.16)–(4.19). Best fits of data with (4.14) and compilation, where possible, with measured values of (*k*, *r*) is a basic matter for this subject.

It is especially important, when making comparisons of results obtained with different devices, to account for the fact that different characteristics are measured. It's a point that seems to have been occasionally

[11] Here we are concerned only with the pressure-coupled response. See later remarks on the velocity-coupled response, Section 5.2.

overlooked. Any of the current methods produces data for one of the four quantities: fluctuation of velocity u'_b at the edge of the combustion zone; fluctuation of the mass flux m'_{bs} of material issuing from the condensed phase; the velocity fluctuation r' of the interface; or the combination represented in the dimensionless form $A_b + \overline{M}_b$ by (4.9). thus we can classify the available devices according to this scheme:

u'_b: magnetohydrodynamic/velocimetry

m'_{bs}: laser-recoil devices

r': ultrasonic Doppler; microwave Doppler

$A_b + \overline{M}_b$: T-burner

We will not pursue details here, but it is important to emphasize that when comparisons are made between a quantity associated with the interfacial region and condensed phase (M'_{bs}, r') and one characteristic of the chamber side of the gas phase $\left(U'_b, A_b + \overline{M}_b\right)$ two aspects of the behavior in the gas phase must be accounted for:

1) *Dynamics*: if quasi-steady behavior is assumed, then m'_b for the combustion products departing the combustion zone equals m'_{bs}. Otherwise, true unsteady behavior must be accounted for.
2) *Chemical Thermodynamics*: the value of $\Delta T'_f$; equations (4.8) and (4.9) must be specified. Normally it is neglected.

The standard practice, at least implicitly, has been to assume $\Delta T'_f=0$ and to neglect dynamics in the gas phase. Hence (4.13) is assumed to hold,

$$A_b + \overline{M}_b = \overline{M}_b R_b$$

with R_b equal to the normalized fluctuation of mass flux from the condensed phase (i.e. $m'_b = m'_{bs}$). Probably these assumptions should be examined whenever comparisons are made. For example, data has recently been reported for the same propellants tested in an ultrasonic Doppler device_____________ 2000) and with a magnetohydrodynamic/velocimeter/_______ 2000) of the matter just discussed.

Several suggestions have been offered for alternative approaches to measuring response functions (e.g. somehow making use of MEMS technology) but none have been shown to be feasible. The practical fact of the matter is that the experimental conditions are extremely difficult to face: high temperatures, high pressures and large bandwidths (preferably to several kilohertz.) and a medium which for metallized propellants is opaque to non-intrusive diagnostics based on lasers. Casual, gratuitous uninformed suggestions for other 'methods' have not been helpful.

Thus it remains true that the only method currently available for practical purposes is the T-burner. See Beckstead (2000) for a recent discussion of T-burner data for some un-metallized propellants, giving also a useful review of historical aspects. It's true that the T-burner has many drawbacks, but it is usable for practical ranges of pressure and temperature and, with experienced users, has in fact produced much data valuable for both applications and research. It is by no means obvious that the method should not receive investment for refining the method, for example in one place as a national.

In the past few months at Caltech we have been studying possible use of laser Doppler velocimetry (LDV). It's not a new idea, having been used briefly nearly twenty years ago (Caveny, Collins and Cheng 1981). However, for various reasons (valid or not is questionable) the matter has not been pursued further. We believe that LDV with the newer equipment and data processing available has genuine promise and we intend to carry out feasibility tests shortly. One great advantage of this method, if it works, is that it should provide *direct* measurement of the velocity fluctuation in the gas phase, exactly the quantity required for analysis (recall the formula 4.1).

4.3 'Rules of Thumb'

It follows from (4.15) that for a given propellant, the peak of the pressure-coupled response shifts to lower frequencies as the chamber pressure increases. This property has particularly serious practical implications now, due to the general trend to designs for higher chamber pressures. It's an important practical scaling law. There are many advantages associated with considering the behavior as a function of frequency rather than time. The connection between the two descriptions can be viewed in the following way.

In the language of control theory, R_p given by (4.14) is a transfer function relating the oscillating input (pressure) to the oscillating output (mass flux) for the burning propellant. It is R_p that appears in the feedback path shown in Figure 7 which was prepared implicitly time-dependent variables. It is often convenient to consider the behavior in frequency space as in the field of controls, an approach taken first by Sehgal and

Strand (1964). Here we imagine that the Laplace transform has been taken of all the time functions. With capital letters denoting the transforms and G_a standing for the combustor dynamics (acoustics), Figure 7 becomes Figure 17. We define the transforms M_e, M_i

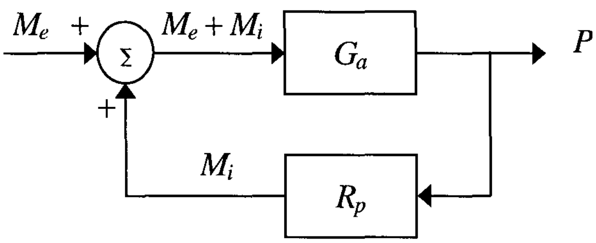


Figure 17

and P in dimensionless form so that (4.5) gives

$$R_p(\Omega) = \frac{M_i}{P} \tag{4.20}$$

and the transfer function for the combustor dynamics is

$$G_a = \frac{P}{M} = \frac{P}{M_e + M_i} \tag{4.21}$$

When R_p is developed as a transform from the equations governing the original problem, the transform variable is Ω, now complex

$$\Omega = \Omega_r + \Omega_i \tag{4.22}$$

It is part of the analysis that the function λ in (4.14) is also complex, satisfying the equation

$$\lambda(\lambda - 1) = i\Omega \tag{4.23}$$

The block diagram can be solved to give the ratio of output P to input M_e,

$$\frac{P}{M_e} = \frac{G_a}{1 - G_a R_p} \tag{4.24}$$

According to the principles of control theory, the behavior of the system is determined by the locations of the zeros and poles of the loop transfer function $G_a R_p$. While G_a does have simple poles, representing the resonances or damped natural acoustic modes, of the combustor, the transfer function R_p for the combustion dynamics does not have poles. However, there are conditions under which the denominator of R_p vanishes, representing intrinsic instabilitites. The boundary separating the regions of stable and unstable combustion in the sense represented by R_p is therefore defined by the equation

$$\lambda^2 + (AB - A - 1)\lambda + A = 0 \tag{4.25}$$

For a given pair of values of *(A, B),* equations (4.23) and (4.25) determine the real and imaginary parts of λ and Ω. Constraints are placed on the allowable values of A and B by two physical conditions:

(a) disturbances in the condensed phase, here waves of temperature generated at the interface must decay to vanishingly small values far from the interface: that condition requires $\lambda_r > 0_{\hat{2}}$.

(b) Transient disturbances in the condensed phase must be stable, requiring $\Omega_i > 0$ for the definitions implied here.

The constraint (*a*) has long been known: see Dennison and Baum, 1961, and Culick 1968 and references cited there; less attention has been paid to (*b*): see Levine and Culick (1974). Isella (2001) has recently re-examined the matter and has shown that the behavior is somewhat more complicated than previously understood, but the conclusion remains the same. Figure 18 shows the general result. The solid line, bounding the shaded region in which pairs (A, B) lead to unacceptable response functions, is the locus

$$A = \frac{B+1}{(B-1)^2}$$

or

$$B = \left((1 + \frac{1}{2A} \right) - \frac{1}{2A}\sqrt{1 + 8A} \tag{4.26}$$

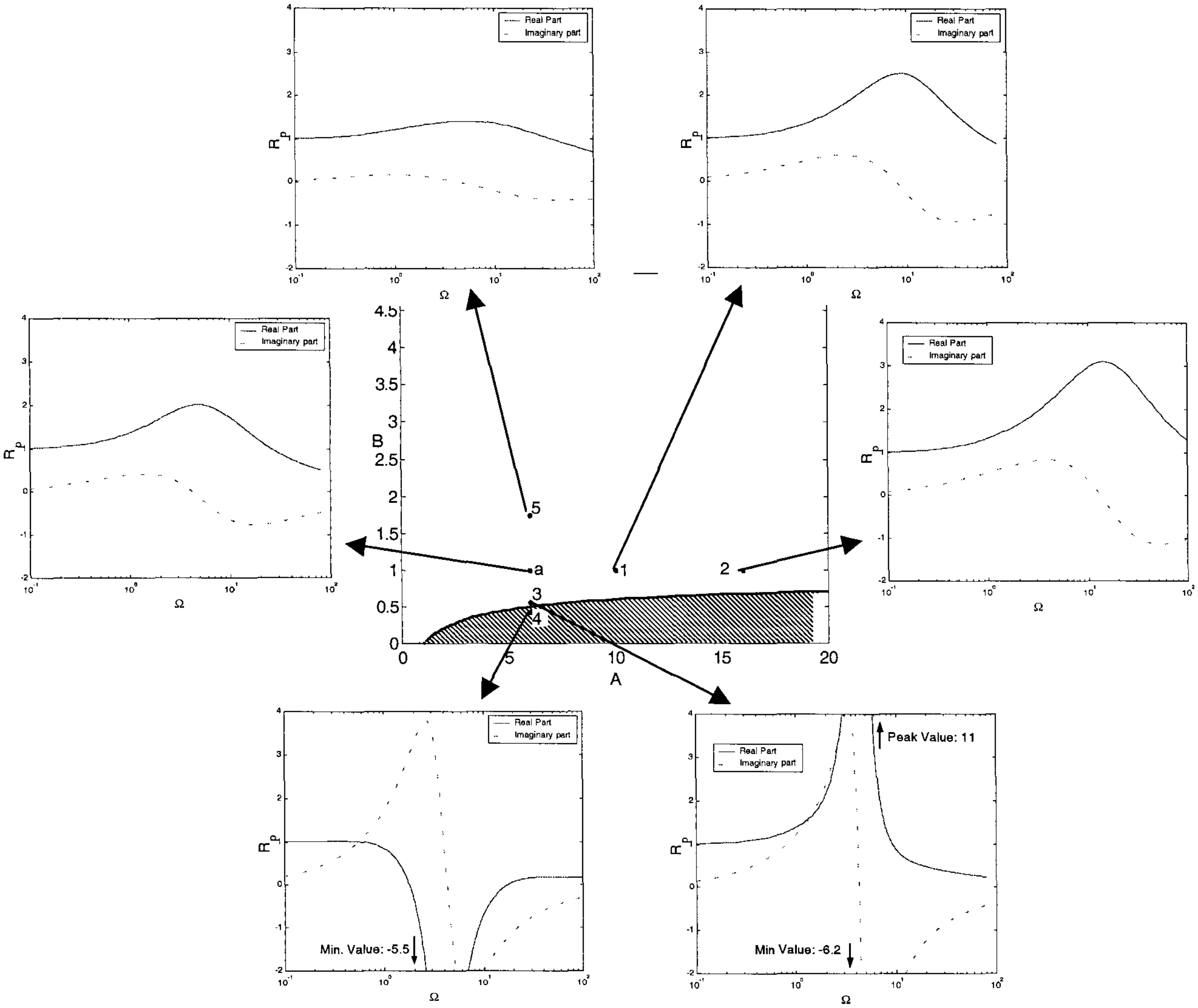


Figure 18

Only values of (A, B) within the cross hatched region and its boundary are excluded by the constraints (a) and (b). The allowed values give solutions representing temperature waves decaying with distance from the interface, in the condensed phase; and for which the transient response to changes of pressure decay in time.

Infinitely large values of the response function occur only on the boundary of the forbidden region in Figure 18. As one check of this theory, we should locate pairs of measured values of A, B for actual propellants. Associated with each pair is a unique response function given, by (4.14). Therefore this approach provides a means of assessing the theory by comparing, for a given propellant where values of A and B are known, measured values of the response function with those computed using (4.14).

There is a body of information, unfortunately scattered throughout the literature, including inaccessible reports within industrial and government literature, dealing with connections between the properties of propellants and their response functions. The conclusions reached are usually based on heuristic reasoning supported by experimental data. Three recent papers provide very good summaries of some of the more significant portions of the available information regarding the remarks here and greatly abbreviated (see Beckstead 2000; Beckstead and Meredith, 2000; Blomshield 2000; and Cohen et al 2000).

It is useful first to examine qualitatively the implications of the regions defined in Figure 18, and the QSHOD response functions computed for the six cases noted. We emphasize that according to the above remarks, only the cross-hatched region contains invalid pairs of the parameters.

From these results we conclude:

(i) For a fixed value of B well inside the stable region, increasing A from a relatively low value (6) first causes the peak value of the real part[12] of the response $R_b^{(r)}$ to decrease and then eventually increase, with little effect on the width of the response. The value of frequency at which the peak occurs increases monotonically with A.

(ii) As B is increased both the real and imaginary parts of the response function behave smoothly, exhibiting no infinitely large values.

(iii) In contrast, decreasing B with A constant at the reference value (6) causes the peak value of $R_b^{(r)}$ to increase, tending to infinitely positive values on the stable side of the lower boundary, and infinitely negative values on the unstable side. That discontinuity in magnitude is due to the behavior of the phase of R_b in the vicinity of the stability boundary.

Interpretation of this behavior shows why it is useful to be aware of the two different definitions of the parameters appearing in the QSHOD formula. We regard A as proportional to the surface activation energy (Denison and Baum 1961); and B as inversely proportional to T_p, the thermal sensitivity of the burning rate (see the relation 4.19).

Hence, according to (i), increasing the surface activation energy tends to favor driving instabilities at higher frequencies. This behavior has implications for possible consequences of energetic materials characterized by higher activation energies. The observations (ii) and (iii) imply that increasing *or* decreasing the thermal sensitivity of a propellant, from a state in the stable region, causes the combustion dynamics to approach a qualitatively different form; it is only at the boundary that infinite values occur. Increasing the thermal sensitivity is potentially more troublesome because such a change produces larger values of the response functions, indeed, infinitely large on the lower boundary. Note that even though a propellant might operate in the lower region of intrinsic instability, this does *not* mean that a motor with that propellant would be unstable. For the larger values of B, the values of $R_b^{(r)}$, and hence the corresponding contribution to the growth constant, remain relatively low[13].

The preceding remarks are concerned with fairly obvious connections between values A and B and the behavior of the QSHOD response function. That we may, in first approximation, regard A as proportional to the surface activation energy, and B as inversely proportional to the thermal sensitivity, is clearly a helpful start to understanding possible relations between propellant composition and the response function. But we know that the conclusions reached on that basis can only be approximately true and possibly can be misleading if the QSHOD model badly misses the mark. More useful to those concerned with practical applications are 'rules of thumb' more solidly grounded in experience. For that purpose it is convenient here to restrict attention to four kinds of changes in the propellant composition and operating condition: AP particle size; mean operating pressure; conditioning temperature; and aluminum in the propellant. The following include are based on views given the author privately by Messrs. Beckstead, Blomshield and Cohen.

a) AP Particle Size

If the burning rate $\bar{r}$ is held constant by adding rate catalysts, changing the distribution by removing fine AP tends to decrease the real part of the real part $R_p^{(r)}$ of the response function, thereby decreasing the driving[14]. Higher values of the particle loading also tend to reduce $R_p^{(r)}$. Double base propellants and nitramines commonly exhibit responses broader in frequency, encouraging instabilities in the higher frequency range for transverse modes. Ammonium perchlorate has a relatively low value of surface activation energy, leading to low values of A, and therefore lower values of the frequency for the peak response.

b) Mean Operating Pressure

The mean pressure has effects dependent on the size of AP particles, due to changes in the structures of the gas phase flames about individual particle sizes. As a result, increasing the pressure tends to have consequences similar to the effects of adding coarse AP at high burn rates, reducing the driving. This effect is simultaneous with the effect in the peak frequency represented by (4.15). Consequently, the response $R_p^{(r)}$ may increase or decrease with pressure, depending on the propellant in question. The conclusion cannot be translated directly to a statement about the stability of a motor because when the pressure is increased by reducing the area of the nozzle throat, the nozzle damping is reduced.

[12] Recall that the larger is $R_b^{(r)}$, the greater is the tendency for surface combustion to drive oscillations, i.e. the corresponding contribution to the growth constant is larger.

[13] In any event, R_p is proportional to the pressure index, so low values of n are favorable to reducing the combustion driving.

[14] Recall that in the formula for the contribution to α, equation (3.20), it is the real part of R_p that appears.

It therefore happens that higher pressures tend to be destabilizing.

c) **Conditioning Temperature, T_c**

For reasons similar to those cited in respect to the effects of operating pressure, increasing T_c seems often to favor lower response at lower frequencies and high burn rates, but higher response at high frequencies and low burn rates. Low burn rate sensitivity (σ_p) usually reduces the combustion response, in agreement with earlier remarks addressing the effect of higher $B \sim 1/\sigma_p$.

d) **Aluminum**

The greatest effect of aluminum on motor stability is the increased attenuation due to the production of condensed aluminum oxide in the combustion products. The size of the effect depends on frequency and on the oxide particle size, a quantity strongly dependent on the combustion processes. There seem to be only small effects at most on the actual dynamic response of the surface combustion processes. The properties of the combustion product Al_2O_3 depend on the properties of the aluminum initially particles and proportion of aluminum oxide remaining from the manufacturing process. Moreover, there are effects of other propellant variables such as *AP* particle size distribution leading to the "pocket model" for the formation of liquid aluminum drops on the burning propellant surface. The connections between the properties of the combustion products and the compositional variables of the unburnt propellant are poorly understood.

Work with propellants containing energetic ingredients is a subject of current research. Higher values of the surface activation energy give larger values of *A* with consequences noted above, but details of the combustion processes (effects on *B*) are important, particularly when the amount and size distribution of *AP* present are changed.

5. Calculations of Nonlinear Combustor Dynamics

The analysis described in Section 3 can be used to compute the linear stability of a motor, and also its nonlinear behavior, especially properties of limit cycles. Stability of course has always been—and should remain—the chief practical concern in this field. After all, if the motor is stable, nobody worries. However, the uncertainties in the required input information (above all, the response function) are such that one cannot guarantee stability (or instability) in any sense. Certain trends with changes of design are well-established, and there exist a considerable number of 'rules of thumb' of the sort discussed in Section 4.3

However, there remain many questions unanswered, and obvious gaps in our understanding. One of the most important is our inability to explain why the combustor dynamics should, in a number of documented cases, apparently be surprisingly sensitive to small changes of propellant composition.

'Sensitivity' in the context of combustion instabilities has two meanings: sensitivity of the linear stability of motions; and sensitivity of the properties of limit cycles. Examples of both sorts exist. In operational systems sensitivity of linear stability may be observed as the intermittent appearance of instabilities in a given design. Sensitivity of limit cycles is reflected in changes of the limiting amplitude without any intentional changes in propellant composition or motor design. It is the second sort that arose in the Minuteman vehicle, Figure 3. Midway in production the amplitude of the oscillation at $500Hz$ increased significantly with the change from one propellant lot to the next.

Possible sensitive dependence of linear stability on small changes can be understood from the fact that the net growth constant is commonly the small difference between two large numbers. That is, in equation (3.21), the total value of α_n for the contributions to damping, α_{nd}, has nearly the same value as α_{nc}, the part of α_n representing driving, α_{nc}, dominated by the combustion processes, and both are large compared with the difference. The net growth constant is

$$\alpha_n = \alpha_{nc} - \alpha_{nd} << \alpha_{nc}, \alpha_{nd} \tag{5.1}$$

This behavior, that the stability of a motor is a consequence of the small difference between two large numbers, is a also major source of experimental uncertainties in T-burner data.

The practical consequence of (5.1) for stability of motors is that a small change in the response function (for example due to a change of composition) may have an observably large effect on stability-even to the extent of causing a previously stable motor to become unstable if $|\delta\alpha_{nc}| > |\alpha_{nc} - \alpha_{nd}|$.

It is less easy to understand how a small change of composition producing a small change of the response function may have noticeable effects on the behavior of limit cycles. That was the situation that arose, for example, with the problem in the Minuteman II Stage 3 (Figure 5). During production, from one propellant lot

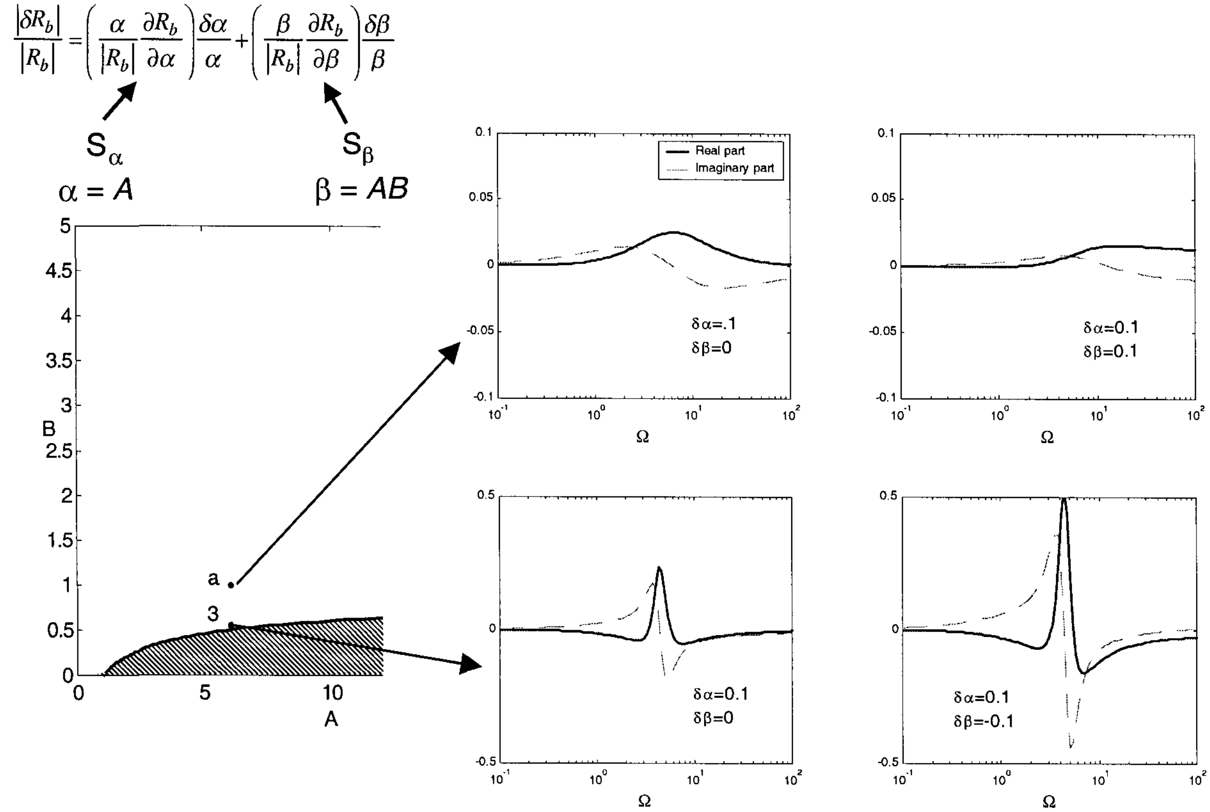


Figure 19

to the next, the amplitude of oscillations occasionally suffered unacceptably large increases of amplitude, causing flight failure. This and similar experiences with other motors were part of the motivation for investigating nonlinear behavior. The central question is: are there circumstances under which small – in fact undetectable with current experimental methods – changes in the propellant composition will cause substantial changes in limit cycles?

At this stage of the discussion, there are two obvious answers to the question:

(i) small changes of composition may in fact produce large changes in the response function which would be reflected both in the intensity of linear instability and in the properties of the limit cycles;

(ii) small changes of composition may produce only small changes of the response function which will not have significant effects on linear stability, but may, because of the nonlinear behavior, cause significant changes in the limit cycles.

To investigate the matter we assume the nonlinear acoustics of longitudinal modes in a straight cylinder, and to begin, the QSHOD model (4.14) for the pressure coupled response. The chief idea here is to work out a few examples to discover how small changes in the boundary conditions specified by the response function may be reflected in the limit cycles. In other words, we use the *combustor dynamics* as a kind of diagnostics to learn more about the quantitative influences of the *combustion dynamics* to learn more about the quantitative influences of the *combustion dynamics.*

In respect to item (i) above, we may assume that within the QSHOD model, changes of composition are interpreted as changes of the parameters *A and B.* See Section 4.3 for discussion of some possible connections. We should then be concerned with the sensitivities of the response function to variations of the parameters. It is a computational convenience to use the parameters $\alpha = A$ and $\beta = AB$; then we may define the sensitivity as the fractional change of R_p:

$$\frac{|\delta R_b|}{|R_b|} = \left(\frac{\alpha}{|R_b|}\frac{\partial R_b}{\partial \alpha}\right)\frac{\delta\alpha}{\alpha} + \left(\frac{\beta}{|R_b|}\frac{\partial R_b}{\partial \beta}\right)\frac{\delta\beta}{\beta} \qquad (5.2)$$

A few calculations show that the **sensitivity** $\left|\delta R_p / R_p\right|$ **is high if the reference values of *A* and *B* are selected close to the lower boundary for intrinsic instability identified in Figure 18.**

Figure 19 shows some results for the real and imaginary parts of the fractional charge of R_b, defined by (5.2), for small changes of $A=\alpha$ and $AB=\beta$. The fact that the sensitivity of the response function becomes high if the values of A and B lie close to the boundary has long been known by researchers in this field. A recent reminder by Professor B. N. Novozhilov during an informal workshop motivated the present calculations.

For two reasons, it seems that attempting to explain the extreme sensitivity in question by assuming values of A and B close to the intrinsic stability boundary is likely not correct:

(i) If its properties were to assume such values, it seems that (even before any small changes assessed here as $\delta\alpha$ and $\delta\beta$) combustion of the propellant either in test devices or in motors, would in the natural course of events exhibit serious problems of large excursions of pressure; and

(ii) We must not forget that evidences of sensitivity have been found in records of motor test firings and therefore reflect the dynamical behavior of the complete system of chamber and combustion by Figure dynamics as emphasized 7.

It is also an important practical matter—a hint to the sort of investigation one should pursue—that the basis for claiming unexplained sensitivities is not only linear instability, but properties of **observed** limit cycles (amplitudes and modal content). Therefore one should carry out calculations for the time evolution of motions in a motor which is linearly unstable, and determine the consequences of changing the combustion dynamics, i.e. the response function.

5.1 Limit Cycles with Pressure-Coupled Surface Combustion

We consider here only the special case of longitudinal oscillations. The oscillation equations simplify considerably (Culick 1975, 1994) and the behavior is less complicated because the unperturbed frequencies of the normal modes are integral multiples of the fundamental. For the values of α_n and θ_n we use the example of a small cylindrical motor given by Culick and Yang (1992) except that the contributions by surface combustion are computed for the particular response functions introduced here.

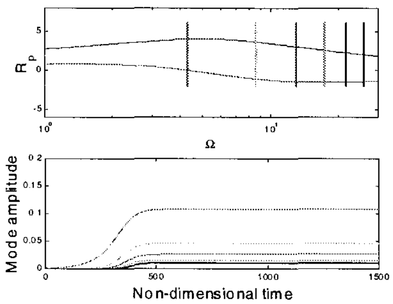
The strategy followed here is to compute the time evolution of amplitudes in the limit cycles[15] for a sequence of cases, beginning with the reference case using the QSHOD response function (4.14) with A =6.0 B = 1.0 and n = 0.5. All material properties are assigned the values chosen by Culick and Yang (1992). Several examples are then given of the corresponding results for slightly different values of A and B.

There have been a few analyses during the past decades to try to correct the obvious deficiencies of the simple QSHOD analysis. Mainly interest has been directed including the effects of dynamics of the gas phase (Tien 1972, Clavin and Lazmi 1992). As expected, the results show increased values of the response function in the frequency range above that where unsteady heat transfer in the condensed phase dominates. Recently we have reported a simplified analysis including both the dynamics of the gas phase and of a thin region at the interface of gas and condensed phases (Isella and Culick, 2000). The purpose of the latter is to try to capture some of the behavior associated with the transformation of solid to gas (formation of liquid layers in some constituents, agglomeration of molten aluminum drops, etc.) without becoming engaged in forbiddingly complicated calculations. Examples of response functions calculated in this way are then used in computations of the limit cycles.

In Figures 19-23, the top part shows the real and imaginary parts of the response function used in each case. The vertical lines identify the dimensionless frequencies of the six modes accounted for in the calculation of the combustor dynamics. The second part of each figure shows the evolution, in time, of the amplitudes of all six modes. A small table gives the values of the harmonic amplitudes in the corresponding limit cycle. A few cycles of the waveforms in the limit cycles are reproduced in the bottom parts of Figures 23-25. The minor bumps in the waveshapes arise because only a relatively small number of modes have been taken into account.

These examples lead to the following conclusion: **Unless the values of A and B lie close to the boundary of intrinsic stability, the properties of the limit cycles do not change greatly when the parameters are slightly changed.** Near the boundary, the response function is large and also is itself very sensitive to the values of A and B. Figures 19 and 20 illustrate the typical behavior for a 5% change of A when the values of A and B are not close to the boundary of intrinsic stability (Point a in Figures 18 and 19).

[15] For these calculations we use the time-averaged forms of the oscillator equations (Culick 1975, 1998; Jahnke and Culick 1999)

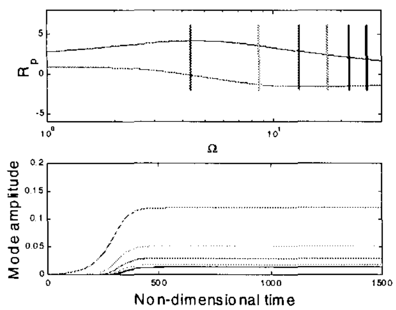


A=6; B=1

Limit amplitudes

Mode	Amplitude
1	0.11
2	0.046
3	0.026
4	0.016
5	0.011
6	0.010

Figure 19



$\delta A = 5\%$; $\delta B = 5\%$

Limit amplitudes

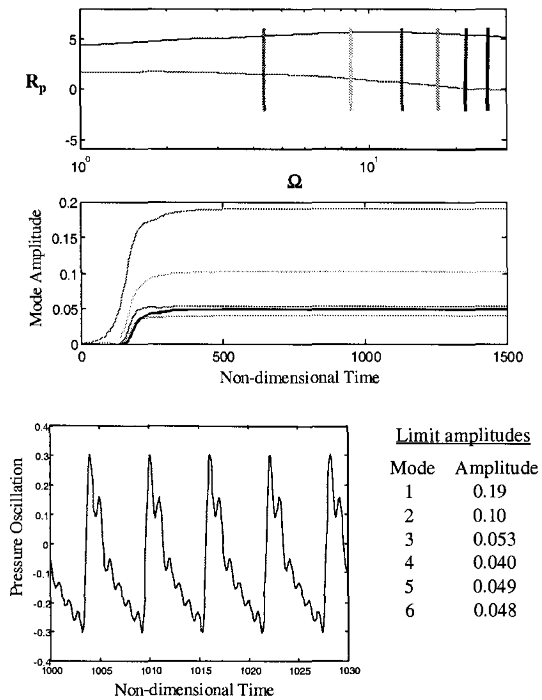
Mode	Amplitude
1	0.12
2	0.052
3	0.029
4	0.018
5	0.013
6	0.014

Figure 20

Because it is the only dynamical process (i.e. frequency dependent) contained in the model leading to (4.14), apparently unsteady heat transfer in the condensed phase apparently cannot alone account either for large sensitivities of the response function or for sensitivities of limit cycles. That conclusion led us to examine the possible consequences of the dynamics of the combustion processes in the interfacial region and in the gas phase. Figures 21-23 lead to the conclusion that **modifications and extensions of the model for the pressure-coupled response function do not lead to the large changes and sensitivity of the *combustor dynamics* that we seek.** Note that in these examples, the pressure-coupled response actually changes noticeably, but with only weak effects on the limit cycles of the combustor dynamics.

Therefore we have recently carried out similar analysis of limit cycles with a model for a velocity-coupled response function.

QS response function with thermal model of surface layer

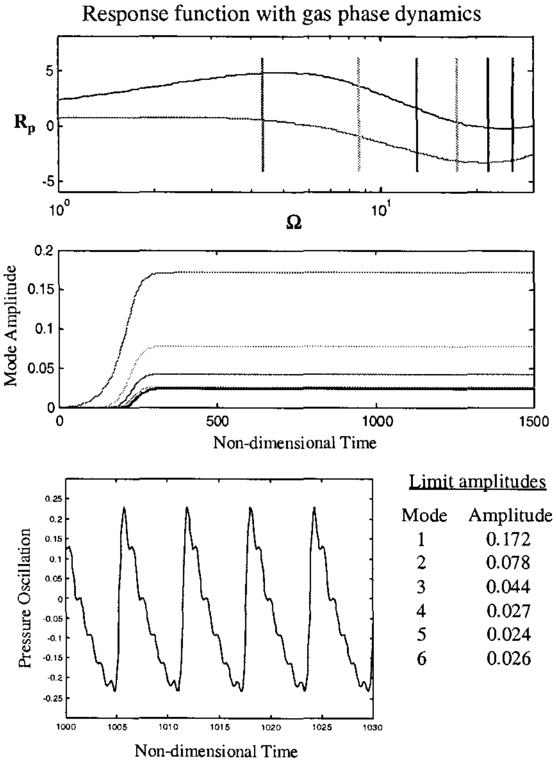


Limit amplitudes

Mode	Amplitude
1	0.19
2	0.10
3	0.053
4	0.040
5	0.049
6	0.048

Figure 21

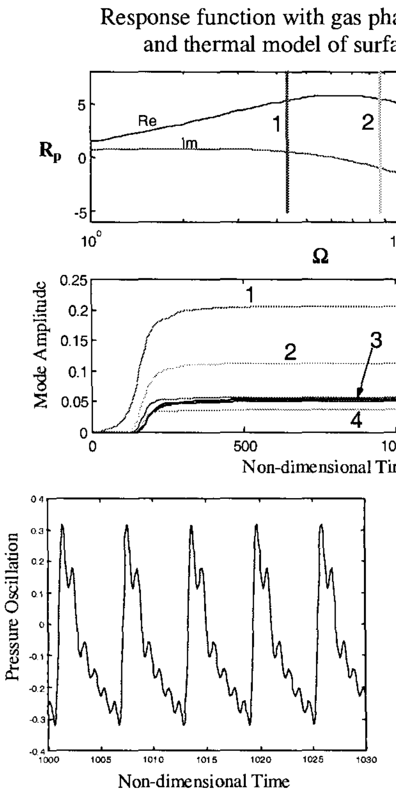
Response function with gas phase dynamics



Limit amplitudes

Mode	Amplitude
1	0.172
2	0.078
3	0.044
4	0.027
5	0.024
6	0.026

Figure 22



Limit amplitudes

Mode	Amplitude
1	0.20
2	0.11
3	0.056
4	0.037
5	0.050
6	0.053

Figure 23

5.2 Limit Cycles with Velocity-Coupled Surface Combustion

We use the *ad hoc* model for the velocity-coupled response function first proposed by Baum, Levine and Lovine (1988) in their successful numerical simulations of pulsed instabilities. Burnley (1996) later showed that the model also gives pulsed instabilities when used with the analysis described here in Section 3.3. The mass flow rate normal to the burning surface is

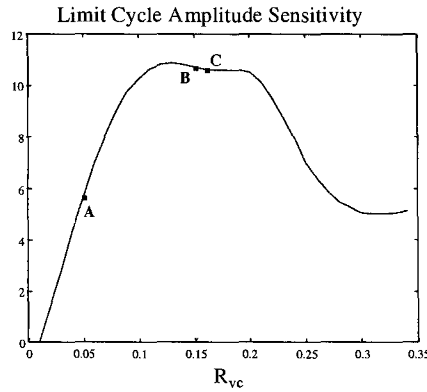
$$\dot{m}' = \dot{m}'_{pc}\left(1 + \tilde{R}_{vc}|\mathbf{u}' - \mathbf{u}'_T|\right) \qquad (5.1)$$

where $\dot{m}'_{PC}$ is the fluctuation due to pressure-coupling (here taken to be the QSHOD result (4.14), $\mathbf{u}'$ is the fluctuating velocity parallel to the surface and $\mathbf{u}'_T$ is a 'threshold' value. The parameter $\tilde{R}_{VC}$ is a measure of the assumed dependence on velocity.

Let A_{LC} denote the amplitude of the unstable first mode in the limit cycle, and define the sensitivity to be the fractional change of the amplitude when the velocity coupling parameter $\tilde{R}_{VC}$ is changed:

$$S_{VC} = \frac{1}{A_{LC}} \frac{\partial A_{LC}}{\partial \tilde{R}_{VC}} \qquad (5.2)$$

Figure 22 shows the calculated values of S_{VC} for limit cycles computed using (5.1) for the combustion dynamics, when the parameter $\tilde{R}_{vc}$ is



Fractional change of amplitude in the Limit Cycle

Figure 24

The points *B* and *C* represent cases differing by a 5% change of $\tilde{R}_{VC}$. Figure 25 shows time simulations for the three points *A*, *B*, *C*. in Figure 24. A 5% increase of $\tilde{R}_{VC}$ produces a 29% change in the amplitude of the first mode.

Of course this single example does not prove the point, but it is suggestive of the conclusion:

Unsteady surface combustion responsive to velocity fluctuations leads to sensitivity of the combustor dynamics, much greater than that found for pressure-coupled surface combustion.

The reason why this conclusion may be true is not known presently. The likely cause is probably related to the phase relations established between imposed fluctuations and the fluctuations of mass issuing from the burning surface.

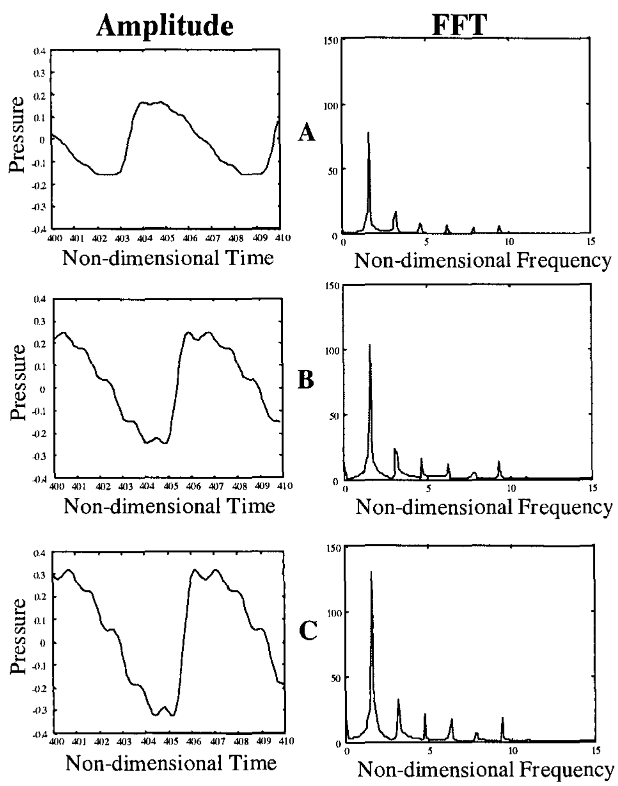


Figure 25

5.3 The Role of Numerical Simulations

As Yang has discussed in another paper (Yang 2000) the progress with numerical simulations applied to solid propellant rocket motors has been very substantial in the past decade or so. Nevertheless, there are serious limitations. Despite the great advances in computing resources, the demands placed by the applications discussed here are such that even approximations to the actual problems are expensive and time-consuming. Hence this approach taken alone is far from occupying a place in the routine work of design and development.

In a more fundamental respect, carrying out numerical simulations as an isolated activity is a very difficult way of gaining understanding. Each simulation is a particular case and to understand new phenomena requires many simulations. Thus an analysis of the sort used here will always occupy an important position in this field, for both theoretical purposes and for obtaining results quickly and cheaply.

Nevertheless, numerical simulations are extremely important in the total picture and must receive continuing support for at least two reasons. First, the long-term goal is to be able to simulate real problems 'exactly'. For a recent step in that direction, see Roh et al 1995; work of that sort is in progress and increasingly results are becoming available. When the ultimate goal of simulating most of the behavior present in real problems is reached, just as in other fields (notably external aircraft aerodynamics) sufficient confidence in the results will allow, the argument goes, less hot testing, thereby reducing costs. If experience in other fields is a reliable guide here, one should probably anticipate that effective capability for numerical simulations will not in fact reduce cost, but will provide the basis for better final products. For example, simulations allow application of principles of optimization that cannot, because of cost, be worked out empirically.

In the context of research, particularly in respect to the analytical framework discussed here (Section 3) numerical simulations can be used to assess the validity and accuracy of results obtained with approximations. That is an important part of the process that cannot be accomplished by comparison with experimental results, due to uncertainties. Examples of this application of numerical simulations have been reported by Culick and Yang (1992) and by Flandro, Majdorani and Malhotra (1998).

6.0 Noise and Forced Oscillations

Combustion processes and flow in all types of combustion chambers generate substantial noise. The precise origins are well-known and have received relatively little attention, partly because while the noise is an irritation, it contains very little energy, so changes in the noise level-even its complete elimination - would have no measurable effect on the designed performance of the system.

From time-to-time questions have been raised concerning possible connections between the noise field, regarded as distributed stochastic sources, and the dynamics of the combustor, in particular stability. The observed 'noise' is really a part of the pressure recorded by transducers. Hence, like the entire pressure record itself, the properties of the noise must in fact somehow reflect also the properties of the chamber. Is it possible to learn something about the stability of a motor by suitable processing of pressure records?

It is fairly easy to appreciate the basis for such an inquiry. Think of the combustor as a linear system, as it does behave so if the unsteady motions are sufficiently small. Then any unsteady pressure field within the chamber can be regarded as a collection of coupled oscillators whose amplitudes are governed by equations of the form (3.16) but with $F_n^{NL} = 0$ in this linear limit. Recent work, (Burnley, 1996; Burnley and Culick, 1999) has shown that if one accounts for stochastic sources following the ideas of Chu et al, Section 3.3.1, one can find the set of equations

$$\frac{d^2\eta_n}{dt^2} = 2\alpha_n \frac{d\eta_n}{dt} + \omega_n^2\eta_n = \sum^{i\neq n}\left[\xi_{ni}^{(v)}\dot{\eta}_i + \xi_{ni}\eta_i\right] + \Xi_n \tag{6.1}$$

By construction, the stochastic 'forces' Ξ_n do not depend on the amplitudes η_n. That is, they are independent of the acoustic field, and the pressure observed by a transducer is the sum of the responses of simple oscillators having resonant frequencies ω_n. Each oscillator has a response curve having the form of a broad peak with maximum close to ω_n and width proportional to α_n.

The source or forcing functions Ξ_n are 'broad-banded', that is they contain components over a broad frequency range. Hence the motion of each oscillator is driven over the entire frequency range covered by the response curve.

Consequently, the total field observed will consist of a sequence of broad peaks roughly centered at the natural frequencies of the acoustic modes. These are not perfectly defined smooth curves but rather have a stochastic or random character due to the random nature of the forces. Nevertheless, it is fairly clear that to some approximation, data for the pressure field excited by the noise sources in a chamber should yield values for the α_n, which may be regarded in turn as measures of the 'margins of stability' of the chamber.

Apparently a Russian group concerned with the stability of liquid rocket motors was the first to make use of the statistical properties of repeated test firings of identical motors (Denisov et al 1993) to assesss stability. In a series of papers Hessler (1982, 1998) has proposed the method just described characterizing it as a means for determining 'passive stability'. Seywert and Culick (1999; 2000) used the analytical framework described here, essentially equation (6.1) to clarify certain aspects of this method by carrying out numerical simulations in the following way.

We begin with a known system, all α_n and ω_n being specified. The Ξ_n are specified as white noise; at this time there is no basis for modeling the random forcing functions any differently. The equations 6.1 are solved for an example in which several modes are accounted for. The result is a pressure record such as that shown in Figure 24, which can be regarded as simulated experimental data. For this example, the first mode is unstable and all others are stable. Without noise, the amplitude of the limit cycles is 0.07. With noise, the amplitude is given by a probability distribution shown in Figure 25.

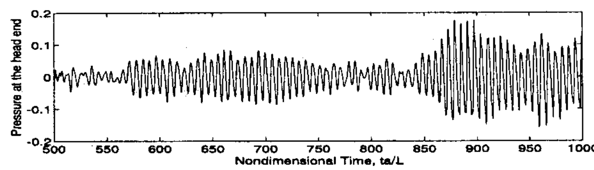


Figure 24

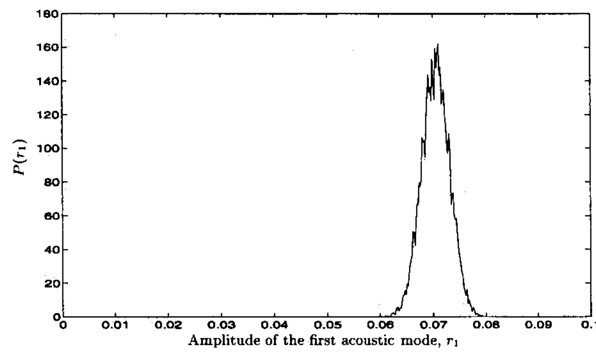


Figure 25

In the example just covered, we knew the (α_n, θ_n) and calculated the behavior of the unstable system with specified noise sources. Now suppose that the system is *stable* but contains, as it must in reality, noise sources. According to the discussion in Section 3.2 the noise must excite the natural dynamical motions of the system. A pressure record will reflect the excitation of those modes and hence should contain information about the damping in the system. The idea now is to process the data (here-simulated data) to discover how well we can recover values for (α_n, θ_n). This procedure has long been widely known in several fields as *system identification* (system ID)[16]

It is assumed that the behavior of the system for which the data is given can be represented by a simple model, here a collection of damped simple oscillators. Then come sort of 'curve-fitting' procedure is used to extract values for the parameters characterizing the model. In our work (Seywert and Culick 1999, 2000) we have used a 'maximum entropy method' suggested by Hessler and available as part of the MATLAB Toolbox.

[16] Hessler and colleagues introduced the term "passive linear stability margin". In my opinion this is an unnecessary and confusing term obscuring the true nature of the procedure. The usage should be discontinued in favor of the conventional and well-understood term 'system ID'.

Figure 27 shows one result, the reconstruction of a smoothed spectrum for the trace shown in Figure 26.

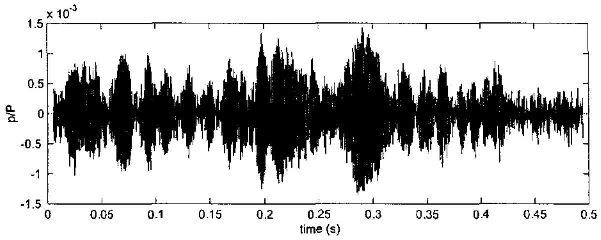


Figure 26

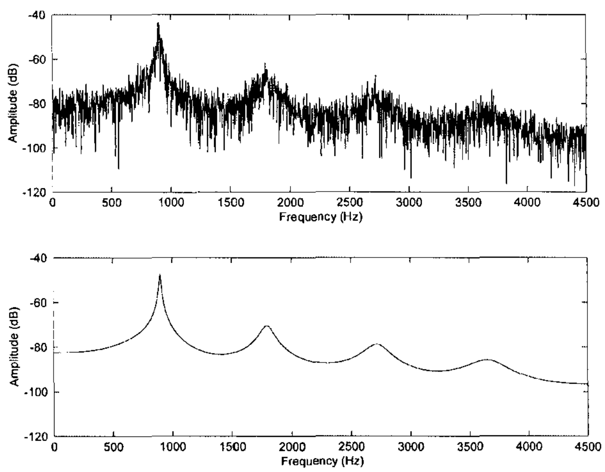


Figure 27

Our conclusion is that, due to the randomness of the forcing function, the values of the α_n carry uncertainties varying from ten to more than thirty percent. Whether or not the method is useful in practice therefore will depend very much on the particular problem in question. However, several general remarks follow from the calculations and examples we have done:

(i) Some amount of noise must be present to excite the response of the chamber. For realistic levels, as far as we can estimate, the consequent uncertainties in the inferred values of the decay constants α_n are too large for quantitative work;
(ii) For practical applications, full-scale motors must be fired, an expensive procedure.
(iii) It is especially important to realize that this method can be applied *only* to stable motors.
(iv) Moreover, the values of α_n are of course net values, differences between the damping and driving: there is no possibility with this method alone to learn anything about the damping and driving themselves. To do so requires separate determinations of the contribution due, say, to the damping processes.
(v) Finally, any results obtained with this method are strictly valid only for the motor in question. It is conceivable that with a sufficiently large body of empirical information collected in this fashion, something could be learned of general behavior. However, it seems an expensive and tedious approach.

It's an interesting and possibly useful idea to apply methods of system identification. However, just as in the field of controls where most of the subject of system ID has been developed, there is no substitute for understanding the system from first principles—particularly in the long term.

7. Concluding Remarks

With the accomplishments of the MURI programs during the past five years, we have a much firmer understanding of the mechanisms and nature of combustion in stabilities in solid propellant rocket motors. As part of that process, we also have better definitions of the problems to be solved and, for the most part, the best courses to follow for solutions. Because there really are three sorts of dynamics active on three different length scales, but necessarily collaborating for instabilities to occur, the overall problem *in total* is extremely complicated. Complete understanding and solution will require the largest available computing resources and some experimental methods that have yet to be devised.

To understand the current state of the field, and future needs, it is convenient, as we have done here, to approach the subject at the level of global combustor dynamics. The analysis serves as a kind of diagnostics to understand where the crucial problems lie. While numerical simulations will in principle give complete detailed descriptions of the behavior, practical restrictions prevent such results at the present time. In any case, it is difficult and tedious to extract general understanding with that approach. Hence we have emphasized the use of an approximate analysis which is capable of giving results that can be compared directly with observations of full-scale motors as well as laboratory devices. Calculations make use of known mathematical methods so one is entirely free to concentrate on the physical behavior and, especially the ancillary information necessary to find solutions.
From that point of view the most significant findings and conclusions have to do with the dynamics of surface combustion. The formal analysis cannot produce quantitative results alone; independent modeling and the relevant physical and chemical processes are required. It is in that activity that results of the MURI Tasks I and II are incorporated in Task III.

Therefore it is immediately apparent how the microscopic chemical dynamics and the combustion dynamics will be reflected in the global dynamics of a motor.

In that respect, perhaps the most informative part of this paper is Section 5. The examples there show explicitly that the dynamical response of surface combustion influences not only the stability of the system, but equally the nonlinear behavior detected as properties of the possible limit cycles in an unstable system. Here we have used those properties as a sort of diagnostic to suggest that the velocity-coupled response should be given far more attention than it has in the past.

It has long been known that velocity-sensitive surface combustion is an important, if not the dominant, mechanism for certain types of instabilities found commonly in high length/diameter motors. What we suggest here is that it is the sensitivity of surface combustion to velocity fluctuations parallel to the surface that may explain observations that small changes of propellant composition can occasionally have substantial consequences in the dynamics of a motor. Presently this should be taken as a preliminary but fairly well-founded conclusion. We cannot state at this time how general the conclusion is. The particulars, both physical and chemical, remain obscure.

This conclusion raises the importance of developing an effective means for measuring the dynamics of surface combustion with much greater accuracy then presently available. In fact, apart from variants of the T-burner (Beckstead and Butcher, 1974; Brown et al, 1986), there is presently no generally accepted method for measuring the velocity-coupled response. Those referenced have always been regarded with great skepticism at the least.

There is no question that the greatest need in this field is a new method for measuring the surface combustion response generally. It is presently impossible to take data having the accuracy necessary to settle the matters raised in the preceding paragraphs.

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