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SOLID ROCKET MOTOR INSTABILITIES WITH  
AP COMPOSITE PROPELLANTS**

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# Numerical Analysis of Solid Rocket Motor Instabilities With AP Composite Propellants

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## ABSTRACT

A non-steady model for the combustion of ammonium perchlorate composite propellants has been developed in order to be incorporated into a comprehensive gasdynamics model of solid rocket motor flow fields. The model including the heterogeneous combustion and turbulence mechanisms is applied to nonlinear combustion instability analyses. This paper describes the essential mechanisms and features of the model and discusses the methodology of non-steady calculations of the combustion instabilities of solid rocket motors.

## INTRODUCTION

This work is part of the Multi-University Research Initiative (MURI) on Combustion Instability. The objective of this MURI is to develop capabilities and understandings that will assure the future stability of solid rocket motors employing advanced energetic propellants. A starting point is to work with ammonium perchlorate (AP) composite propellants because of their long history and current and near-term interests.

Several efforts are under way to develop comprehensive non-steady gasdynamics models of the rocket motor. These include approximate nonlinear analysis to obtain an overview in the form of mathematical solutions<sup>1</sup> and an exact numerical/computational fluid dynamics analysis of the motor flow field.<sup>2</sup> In recognition of the

importance of vortex generation in the coupling of the acoustics with the fluid dynamics in many applications, an analysis focused upon vorticity mechanisms is also in progress.<sup>3</sup> To further the development of these analyses, constitutive models for turbulence and the propellant combustion process are required. The purpose of this work is to incorporate meaningful models of the combustion and turbulence into the numerical/computational scheme. An ancillary goal is to provide mechanistic reasons as to why the ad-hoc model of Levine and Baum<sup>4</sup> was so successful in predicting features of nonlinear instability.

This paper will provide a general description of the gasdynamics model for the rocket motor and the approach to model the non-steady combustion response process. The work of Beddini<sup>5</sup> will be used to model the turbulence interactions. Current work is proceeding with the non-steady analysis. The non-steady results will be the subject of future papers.

## APPROACH

The objective of transient combustion modeling is to compute the instantaneous burning rate of a given composite propellant in response to imposed pressure and velocity waves. Since propellant combustion is coupled with the gasdynamics of a rocket motor, the modeling of combustion response functions has to embrace linear sinusoidal perturbations and nonlinear finite-amplitude oscillations under various motor conditions.

Approximate analysis of mean flow and turbulence effects on response thresholds in solid propellant rockets is also necessary. The acoustic transition/threshold analysis includes a numerical approach involving a turbulence model including mean flow effects on threshold (both injection and axial components of mean flow). If the acoustic erosion threshold can be avoided, a significant source of nonlinear gain could be eliminated from

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stability considerations.

For steady-state erosive burning, several researchers agree that turbulence formed by higher speed axial flow toward the aft end of the grain penetrates into propellant flame zone to enhance apparent diffusivity. One only needs small percent diffusivity enhancement to cause significant increase in the steady-state propellant burning rate. Laminar mechanisms and composite flame bending provide very small levels of augmentation in comparison with turbulence interaction when realistic flow/propellant parameters are considered. For the acoustic velocity response, the essential mechanism is similar except that no mean flow is required to produce turbulence. This is confirmed by a number of experiments for duct acoustics. Scaling behavior of unsteady flows is much more difficult to establish from the experimental literature, in comparison with steady state flow, especially when large surface mass is present.

The Cohen and Strand model for AP composite propellants<sup>6</sup> provides the basis for the combustion modeling in this work. It has been successful in explaining various features of the steady-state burning of such propellants. Later papers made use of this model to describe effects of AP particle size on combustion response in terms of classical response function theory.<sup>7,8</sup> For non-steady cases, the model considers that the AP and binder respond differently to imposed disturbances, yielding cyclic changes in their proportionings in the gas phase which affect flame temperature and other parameters. However, formal transient calculations have not been performed. Therefore, a coupled combustion model with the motor gasdynamics is being developed with the proper treatment of AP particle size effects in order to analyze motor instabilities.

The modeling has started with steady-state solutions coupling the Cohen and Strand model with the gasdynamics analysis of the rocket motor flow field. In order to obtain perturbed AP and binder solutions for any arbitrary wave form, the transient heat conduction equation in the solid phases must be solved with the incorporation of Beddini's transient effects into the model of the gas phase. An iteration method is to be used to converge surface temperature gradients and mass flows at each time step. Separate energy balances are used for AP and binder, in accordance with the steady-state approach. Phase shifts between AP and binder can be expected to result, which is considered to be a credible mechanism for multi-peaked response functions which have been observed.<sup>9</sup> The diffusion flame temperature will shift with AP/binder proportioning changes, affecting response magnitudes. AP particle size effects on non-steady behavior will be an important area of study. In the limit of small perturbations, calculation results will be compared with measured response functions for available cases. For large disturbances, features of non-

linear response will be studied with the view of explaining the results of Levine and Baum and of Culick (e.g., the ad-hoc "velocity-coupling" including the importance of threshold effects). Emphasis will be placed on describing mechanisms and ways to reduce combustion response.

For motor stability analysis, arbitrary wave form inputs will be replaced by a coupled solution for the motor gasdynamics. The main analysis will include mechanistic studies of various motor features known to affect motor instabilities from experience: propellant burn rate/AP particle size, pressure/nozzle throat size, grain geometry/web fraction/mean flow.

## FLOW FIELD MODEL

The combustion chamber consists of the gas phase motor cavity and the condensed phase propellant grain. The analysis of the gas-phase processes is based on the complete conservation equations of mass, momentum, and energy. Turbulent transport is considered to address the effect of turbulent flow disturbances. The work of Beddini<sup>5</sup> is used to model the turbulence interactions. In vector notation, the set of conservation equations for a 2-D axisymmetric combustion chamber is shown in Refs. 10 and 11. The set of governing equations converted into two-dimensional Cartesian forms<sup>11,12</sup> is solved numerically by means of a finite-volume approach. A dual time-stepping integration method<sup>13</sup> proven to be quite efficient and robust at all speeds has been adopted to study the unsteady behavior in the combustion chamber of the solid rocket motor. A fully-coupled implicit formulation is used to enhance numerical stability and efficiency.

The condensed phase including flame zones is treated as boundary conditions for the gas phase. The boundary conditions near the propellant surface of the gas phase are specified by the mass flow rates and flame temperatures obtained from the combustion model of the condensed phase explained in the previous work.<sup>14</sup> Mass conservation and iteration methods are used to specify the other variables of the boundary conditions. Fluid dynamic analysis, therefore, does not include the complicated flame zone near the propellant surface in its computational domain.

The specification of velocities among the boundary conditions near the propellant surface becomes important and crucial in pressure-oscillatory environments of rocket motors since the interfacial boundary conditions between gas and condensed phases specify the fluid variables slightly above the propellant surface. The powerful and robust conditions of no-slip and constant velocity are no longer valid on the interfacial boundary conditions. Velocities on the interfacial boundary fluctuate according to the acoustic oscillations. These velocity

fluctuations can be obtained after they are assumed as harmonic oscillations, which can be calculated based on the cold flow simulation of Flandro.<sup>15</sup> The simulation considers only thermodynamic and transport properties of the core region because the considered control volume for current calculations excludes the flame zone. Vorticity effect on the mass flow rate on the propellant surface can be also considered as another important factor to the velocity fluctuations. Efforts are placed on the specification of complicated velocity fluctuations.

After obtaining a steady-state solution of the flow field for the rocket motor, acoustic oscillations can be generated by the imposition of disturbances. There are two cases. Where details of physical nozzle are not necessary, the nozzle is replaced by boundary conditions at the aft end of the chamber and the disturbances are imposed at the chamber exit. Where the nozzle is included disturbances can be imposed at any location to simulate the evolution of longitudinal standing or traveling wave motions.

## COMBUSTION MODEL

### Basic Features

The combustion model is based upon the composite propellant model of Cohen and Strand<sup>6</sup>. Its application to unsteady burning is intended to describe the important effects of ammonium perchlorate (AP) particle size. It consists of a model describing the monopropellant combustion of AP incorporated into a multiple flame model, as derived from the BDP model<sup>16</sup>, describing the interaction of AP and binder. An important feature for the unsteady burning analysis, as for steady-state, is that separate energy balance are written for the AP and binder. Heat feedback to the AP and binder surfaces are in accordance with the computed flame geometry which is a function of the propellant formulation and pressure. For steady-state, the AP and binder mass flows are constrained to obey the continuity of the propellant formulation. This is relaxed in the unsteady analysis, whereby continuity is maintained only as an average over cycles of oscillatory burning. Solution are obtained for AP and binder surface temperatures, yielding component mass flows and overall propellant mass flow.

The original Cohen and Strand model treated erosive burning by a semi-empirical addition to the energy balances<sup>8</sup>. Functional forms representing effects of turbulence on the augmented heat transfer were in terms of crossflow speed, pressure, AP particle size (surface roughness) and burn rate (surface blowing). The expressions were calibrated from a data base, yielding

good agreement with the data. This is now being modified, for steady-state (erosive burning) and unsteady analysis (acoustic erosivity or "velocity-coupling"), by Beddini's turbulence model<sup>5</sup> because of its mechanistic bases whereby turbulence modifies the transport properties. Use of the Cohen and Strand model to explain effects of AP particle size and pressure on steady-state burning rate characteristics and on linearized response functions were presented in the original papers<sup>6-8</sup>.

### Oscillatory Combustion

The unsteady combustion analysis is intended to be a general non-linear analysis, with linearizations that may be of interest computed numerically in the very small perturbations. It is necessary to solve the transient heat conduction equation in the solid for each the AP and binder, the two being coupled through the flame structure that provides the heat feedback to each. The quasi-steady gas assumption is applied to the gas phase (flame) processes, valid over the range of axial mode frequencies of interest.<sup>17</sup>

An important feature of Beddini's model is the dependence of erosive threshold speed on oscillatory frequency or, in the non-linear context, on rates of change of the flow disturbances. This is due to enhanced turbulence penetration of the combustion zone under oscillatory flows. Thus acoustic erosivity can be greater than steady-state erosive burning. It has important consequences for non-linear response and triggering properties because the response to crossflow involving a threshold speed has been found to be crucial in non-linear analyses<sup>4</sup>. Understanding effects of propellant formulation (including AP particle size) and motor design (pressure and flow) variables on these properties is an important part of this work.

The unsteady solution begins with the steady-state solutions for AP and binder. The corresponding steady-state thermal profiles are computed for each as the initial condition. The perturbed pressure and velocity at the first time step are then imposed. Using the steady-state values for the binder parameters, the energy balance at the surface of the AP is solved for the new surface temperature and iterated to satisfy the thermal gradient boundary condition at its surface. Using these updated values for the AP, the binder is then similarly solved. This sequential procedure is repeated in an outer loop until the updated AP and binder values no longer change. Computations then proceed to the next time step.

At this stage of the work, only one AP particle size is treated in the propellant formulation (i.e., monomodal AP). Although the steady-state model can treat up to three AP sizes (trimodal), it is considered that success should first be achieved with one size to assure that

the procedure is well-behaved and to limit the scope of transient heat conduction computations required. Also, there is much to learn from examining each AP size individually. The manner of treating multimodal AP is controversial and is being assessed for future work.

An important feature of this unsteady model approach is the phase shifts between AP and binder mass flows that occur under oscillatory conditions because of their own differing response properties. These produce compositional fluctuations which lead to fluctuations in the thermochemical flame temperature. It can be a larger effect than the classical transient effect upon the flame temperature, and inherently non-linear. This feature not only affects the magnitudes of the response, but also the frequency-dependence such that a secondary peak or peaks can occur in linearized response function curves.

The unsteady combustion analysis can be performed independently, using arbitrary wave forms as inputs, or be coupled with the motor gasdynamics. The independent computations are convenient to study effects of propellant formulation variables. Similarly, detailed refinements of the motor gasdynamics analysis can proceed independently without a comprehensive combustion model. The two are being coupled as the results of the independent operations show that each is performing satisfactorily. In the coupled analysis, the combustion model computations have to be performed at every finite-different element of the propellant grain in the motor.

## DISCUSSIONS and FUTURE PLANS

Steady-state results have been obtained from turbulent flow calculations of the combustion chamber incorporating the combustion of ammonium perchlorate composite propellants into a comprehensive gasdynamics model of solid rocket motor flow fields.<sup>14</sup> For unsteady calculations, the combustion model has been developed after modification of energy balance equations, effects of turbulence and transient heat conduction. In order to obtain the convergent solution, iteration loops have been added into the calculation process of the combustion model. Additionally, acoustic erosivity under oscillatory flows has been emphasized in the combustion model. Currently efforts are placed on development of the interfacial boundary conditions for unsteady calculations to simulate combustion instabilities of the rocket motors. The problem is how to specify the coupled variations of flux components (propellant mass flux, axial and radial velocities) responding to the acoustic changes inside the combustion chamber and distribute propellant mass flux fluctuations into axial and radial flow components. Propellant mass flux is specified by the combustion model and, for harmonic oscillations, the ve-

locity fluctuations can be specified from Flandro.<sup>15</sup> If the interfacial boundary conditions are well established even though the specification of velocity fluctuations is difficult and complicated, the combustion model of any kind of propellant grain can be combined with gasdynamics in numerical analyses of combustion instabilities of solid rocket motors. Details of the fluid dynamics and combustion aspect will be covered in future papers.

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