Applications of Various Methods of Analysis to Combustion Instabilities in Solid Propellant Rockets

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Abstract

Instabilities of motions in a combustion chamber are consequences of the coupled dynamics of combustion processes and of the flow in the chamber. The extreme complexities of the problem always require approximations of various sorts to make progress in understanding the mechanisms and behavior of combustion instabilities. This paper covers recent progress in the subject, mainly summarizing efforts in two areas: approximate analysis based on a form of Galerkin's method, particularly useful for understanding the global linear and nonlinear dynamics of combustion instabilities and numerical simulations intended to accommodate as fully as possible fundamental chemical processes in both the condensed and gaseous phases. One purpose of current work is to bring closer together these approaches to produce more comprehensive and detailed realistic results applicable to the interpretation of observations and for design of new rockets for both space and military applications. Particularly important are the goals of determining the connections between chemical composition and instabilities; and the influences of geometry on nonlinear behavior.

I. Introduction

The idea of spatial averaging, a tactic replacing partial differential equations by total differential equations in time entered the subject of combustion instabilities in solid rockets with publication of works by Zinn and his students (Lores and Zinn, 1973). Those analyses, extensions of the method of least residuals, vs. Galerkin's method, were another application of the procedure used in calculations for instabilities in liquid rocket engines (Zinn and Powell, 1970). Probably the most important aspect of those works was the introduction of a formulation allowing convenient investigation of nonlinear behavior. From a quite different starting point, Culick (1971, 1976) independently applied a form of Galerkin's method, first to interpret T-burner data and later as the basis for analyzing general nonlinear behavior in solid propellant rockets. The method has been refined and extended in the past two decades to give results established within the context of contemporary nonlinear dynamical systems theory. Some results obtained in the past year for pulsed instabilities and the influences of noise are covered in Section 2.

Only quite recently have the first results been reported for CFD applied to essentially the complete problem of unsteady two-dimensional flows in a solid propellant rocket (I.-S. Tseng, 1992; C.-F. Tseng, 1993; T.-S. Roh, 1995; C.-F. Tseng et al, 1994; and Roh et al, 1995). The importance of those works arises partly from the fact that, at some level of approximation, all combustion processes (including detailed chemical kinetics) and the influences of turbulence are accounted for, thereby giving numerical representations of realistic flows; and the results should serve eventually as part of the basis for assessing the validity of approximate methods that are more suitable for routine use in practical applications and design. However even the most elaborate and expensive calculations potentially have practical value, offering the prospects for determining the consequences of modifying the chemical composition of the propellant. Because compositional changes will likely remain the successful means of passively controlling combustion instabilities in solid propellant rockets, the importance of the approach is obvious. In Section 3 recent progress in extending the earlier work in two-dimensional simulations to axisymmetric flows is described.

2. An Approximate Framework for Analyzing and Predicting Combustion Instabilities

In principle, numerical analysis of the complete problem, as described in the following section, should eventually be capable of capturing all possible events in a combustor. It's a possibility to be realized in the future, and always imperfectly. As a practical matter a literal or formal theoretical and analytical framework provides a more convenient setting for interpreting and understanding both computations and observations.

In the limit of small amplitudes, any disturbance in a compressible medium may be synthesized of three types or 'modes' of wave motion: acoustic waves, vorticity waves and entropy (or temperature waves), a result most thoroughly treated by Chu and Kovazsnay (1958). Moreover, in a uniform flow* the three kinds of waves propagate independently and may be coupled at boundaries. Therefore, because the observed instabilities have frequencies, and mode shapes as well, predicted quite accurately by classical acoustics, we conclude that the synthesis of wave types is satisfied quite closely. That means that a reasonable approach theoretically is to construct a wave equation representing classical acoustic propagation for the pressure fluctuations with all other phenomena appearing as perturbations.

Such a wave equation, with the associated boundary condition, can be constructed by a two-parameter expansion procedure [Culick 1963, 1966, 1975, 1976, 1996] giving results having the form

$$\begin{aligned} \nabla^2 p' - \frac{1}{a_0^2}\frac{\partial^2 p'}{\partial t} &= h \\ \hat{n}\cdot\nabla p' &= -f \end{aligned} \qquad (1)\text{ a,b}$$

where a_o is the average speed of sound. The functions h and f contain all perturbations of the classical problem including nonlinear acoustics, interactions between the fluctuations and the mean flow, spatial nonuniformities of temperature, combustion processes, interactions with condensed material, ... etc.

Recently, the splitting procedure of Chu and Kovazsnay has been explicitly used in this formulation to include the possible influences of vorticity and entropy waves (Culick et al 1991; Burnley, 1996; Burnley and Culick, 1996). The formulation includes vorticity waves of the sort discussed in Section 3, but the primary motivation so far has been to begin investigating the consequences of nonlinear acoustics/noise interactions.

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* The caveat that the flow should be uniform is a bit unclear in Chu and Kovazsnay's paper (it is implied by a formal step early in their analysis) and is often overlooked or forgotten (as the present author has done ocasionally in the past).

The idea of Galerkin's method is to expand the pressure field in a series of basis functions $\psi n(\bar{r})$ (essentially a spatial Fourier decomposition of the field) having time-dependent amplitudes $\eta_n(t)$:

$$p'(\bar{r},t)=\bar{p}\sum_{n=1}^{\infty}\eta_n(t)\psi_n(\bar{r}) \tag{2}$$

Usually the $\psi_n(\bar{r})$ have been taken as the classical acoustic mode shapes for a volume having the same shape as the combustion chamber in question and enclosed by rigid walls. However there is no rigid rule for selecting the ψ_n. Requiring the classical boundary condition $\hat{n}\cdot\nabla\psi_n=0$ for rigid impermeable walls works well if the exhaust nozzle is choked; other cases may be better served by imposing different boundary conditions on the ψ_n. In any case, the use of (7) does not intrinsically constitute a restrictive approximation.

Spatial averaging and manipulations covered thoroughly in several of the references, and other works cited there, eventually lead to the set of equations for the time-dependent amplitudes,

$$\frac{d^2\eta_n}{dt^2}+\omega_n^2\eta_n=F_n \tag{3}$$

where the 'forces' are

$$F_n=-\frac{a_0^2}{\bar{p}E_n^2}\left\{\int\psi_n h dV+\oint\psi_n f dS\right\} \tag{4}$$

The set (10) governs the behavior of a collection of coupled nonlinear oscillators, one associated with each of the modes $\psi_n(\bar{r})$ of the acoustic field. This formulation therefore produces an appealing representation of the general problem and one which has been found to be very useful for both practical and theoretical purposes.

2.1 Application of the Approximate Method

Equation (3), with the definitions (4) for F_n and the functions h, f not reproduced here form the basis of what is conveniently referred to as the approximate method. It should be apparent even from this brief exposition that the method is not one that flows from first principles but rather provides a framework for investigating particular problems. Problems are distinguished by the processes contained in F_n which in general has the form

$$F_n=2\alpha_n\dot{\eta}_n+2\omega_n\theta_n\eta_n-\sum_{i=1}^{\infty}\sum_{g=1}^{\infty}\left[A_{nij}\dot{\eta}_i\dot{\eta}_j+B_{nij}\eta_i\eta_j\right]+\left(F_n\right)_{nonlin}^{other} \tag{5}$$

where the double sum responds the effects of nonlinear gasdynamics to second order in the fluctuations. The first two terms contain <u>all</u> linear contributions computed to first order in the characteristic Mach number if the mean flow, or a comparable small parameter. All other nonlinear processes are represented by the last term*.

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* Note that linear coupling between modes is absent from (12). The reason, explained by Culick (1996), is grounded in the formalism showing that linear coupling affects the

Culick (1994) has summarized the results obtained over several years for two-dimensional and axisymmetric problems. Solutions have been obtained both for the complete oscillator equations (5) as well as for the set of first order equations obtained by applying the method of averaging. The idea has been to determine how the behavior of the solutions depends on the linear parameters varied over ranges roughly appropriate to practical situations. Rather than use numerical simulations, direct solutions to the equations, effective use has been made of a continuation method. That method produces bifurcation diagrams showing the locus of stable, or unstable, limit cycles as one or two linear parameters are varied Usually α_1, the growth constant of the first mode has been used. Then a bifurcation curve shows the amplitude of oscillation (the amplitudes of all modes are computed) versus α_1. A typical example is shown in Figure 1. For α_1 negative, all modes are

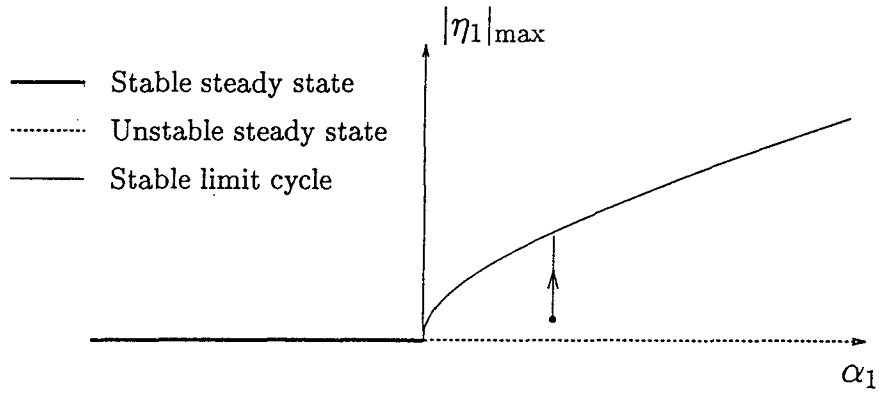

Figure 1

stable and no limit cycles exist. When α_1 passes through zero, a Hopf bifurcation exists and for α_1 positive, stable limit cycles exist. In this way, the answer to P 1 can, in principle, be found.

Only in very simple cases has it been possible to give literal results for the conditions of existence and uniqueness of stable limit cycles. Based on the results to date, the following general conclusions have been established, without formal proof:

1) For given values of the α_n and θ_n, stable limit cycles exist for all $\alpha_1 > 0$ and $\alpha_n < 0$ for $n > 1$. This conclusion may depend on the values of the frequency shifts θ_n , but the dependence is not known. If the first mode is stable and one of the higher modes is unstable, the situation is more complicated because further bifurcations occur as the growth constant of the unstable mode is increased.

2. If nonlinear gasdynamics is the only nonlinear process and the system is linearly stable, no stable limit cycles exist, a partial answer to P 2. Apparently nonlinear coupling between modes serves only to re-distribute the energy provided by the initial pulse and oscillations cannot be sustained. This result is, of course, directly a consequence of the structure of the nonlinear terms. Notably, nonlinear interactions between the fluctuations and the mean flow are not included.

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eigenvalues only to second order and therefore must be dropped because the equations are valid only to first order.

3. Nonlinear combustion, with no nonlinear gasdynamics or other nonlinear processes, will not sustain stable limit cycles (Burnley, 1996).

4. The combination of nonlinear combustion and second order nonlinear gasdynamics does contain pulsed instabilities, providing the combustion response is proportional to the magnitude of the velocity with a threshold (Burnley, 1996). It is necessary that the threshold velocity be non-zero. This conclusion is a partial answer to the problem P 2 and much remains to be accomplished.

5. When time-averaging is used, the results are valid in some region near the Hopf bifurcation mentioned in item 1. The region extends to larger α_1 as the number of modes considered is increased.

For reference we show the result cited in item 4. Figure 2 is an example of the velocity coupled combustion response with non-zero threshold velocity, $u_t = 0.2$. A bifurcation diagram is reproduced in Figure 3

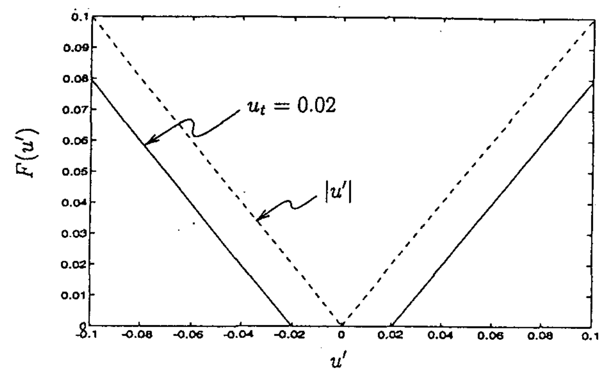


Figure 2

having the characteristics noted. If $u_t = 0$, the Hopf bifurcation is shifted to -33 sec^{-1} but there are no possibilities for pulsed instabilities: the nonlinear dependence on $|\bar{u}'|$ shifts the value of the growth constant to negative values for linearly unstable waves. 'Triggering' exists in the range $-16 < \alpha_1 < 0$ where there are three possible stats of the system, one of which is the quiescent state, one is unstable and one represents a periodic limit cycle. In a small range of positive $\alpha_1, 0 < \alpha_1 \tilde{<} 3$, four states exist; two limit cycles are possible and two states (ne of which $|\eta_1| = 0$) are unstable. Which limit cycle is reached in practice depends, of course, on the size of the initial disturbance.

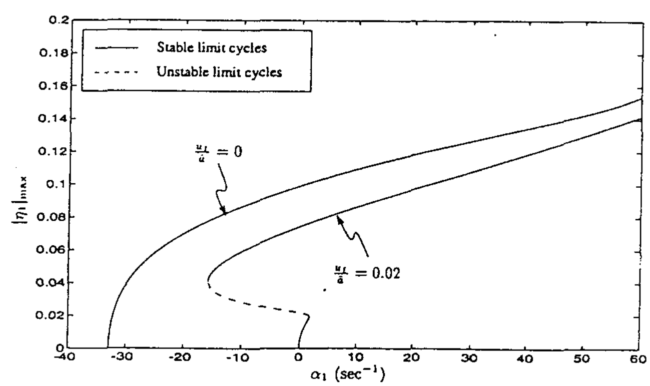


Figure 3

2.2 Including Vorticity and Entropy Waves in the Approximate Method

It is an obvious observation that all combustors are noisy. Normally it is an accepted fact, perhaps environmentally irritating, but any attempts at noise reduction have been exercised outside the chamber, as, for example, in the case of heating units or in protecting payloads on spacecraft. Occasionally the question has been raised: can disturbances identified as "noise" drive combustion instabilities? If the system is unstable, then the answer is that noisy disturbances will evolve into limit cycles but those are not driven oscillations. The question raised has to do with the consequences of noise in a stable system, a much more complicated matter.

The idea is to use Chu and Kovazsnay's principle of splitting arbitrary disturbances within the approximate analysis. It's initially a straightforward matter substituting the additive expression for the total velocity in the integrals defining F_n: $\vec{u}'=\vec{u}_a'+\vec{u}_\Omega'+\vec{u}_s'$, the sum of the velocity fluctuations associated with the three types of waves. At this point no attention is paid to the sources of the entropy and vorticity waves. Eventually the nonlinear oscillator equations have the form

$$\ddot{\eta}_n+\omega_n^2\eta_n=2\alpha_n\dot{\eta}_n+2\omega_n\theta_n\eta_n+\left(F_n\right)_{nonlin}^{GD}+\sum_{i=1}^{\infty}\left[\xi_{ni}^{v}\dot{\eta}_i+\xi_{ni}\eta_i\right]+\Xi_n+\left(F_n\right)_{nonlin}^{other} \tag{6}$$

where the ξ_{ni}^v, ξ_i and Ξ_n are random functions representing the stochastic or noise processes. Those functions have explicit definitions involving integrals in which the velocity fluctuations $\vec{u}_\Omega'+\vec{u}_s'$ appear, as well as the mean velocity $\vec{\bar{u}}$. The acoustic velocity has been replaced, as usually, by its approximate series representation (8). The noise sources can of course be expanded further by separating them according to the velocity fluctuations associated with the entropy and vorticity waves. The terms in (13) have the form of 'multiplicative' or parametric noise and additive noise.

To gain some experience in solving these equations, and to learn whether or not the formulation will give reasonable results, the calculations done so far simply assume that the functions ξ_{ni}^v, ξ_{ni} and Ξ_n are specified as white noise. Moreover, only two or four modes have been included with noise sources in the lowest or lowest two equations.

As an example we consider here only the case of excitation of the acoustic modes when the system is stable and only gasdynamical nonlinearities are accounted for. A result is shown in Figure 4 for the probability density function computed when two modes are accounted for and of the random functions, only ξ_1^v is non-zero. This amounts to assuming a 'noisy growth rate' for the first mode. Since the system is linearly stable, the average value of the amplitude r, is zero. If the mode is linearly unstable, then the average value of the amplitude is

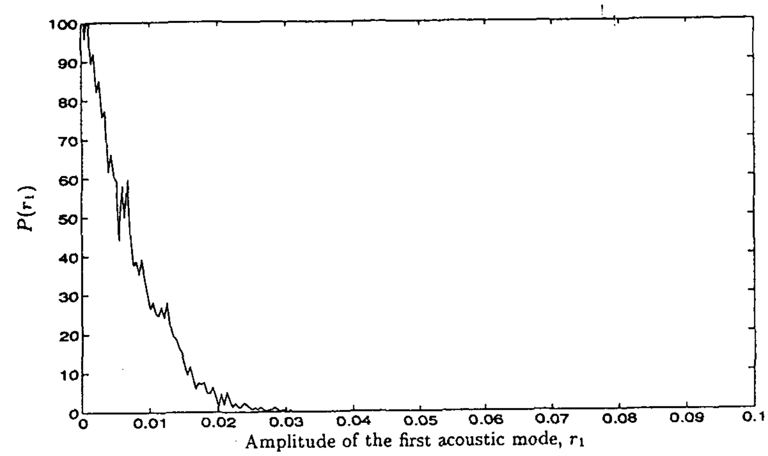


Figure 4

nonzero, as in Figure 5. Without noise, the "probability density function is a delta function at $r_1 = 0.07$.

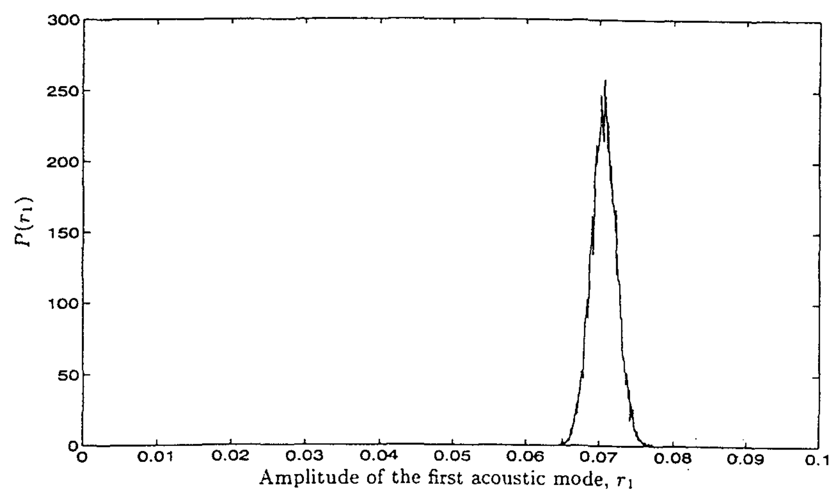


Figure 5

Figure 6 is the time history of pressure for such a case. The result of this calculation is indistinguishable from the typical experimental record if the data are suitably filtered.

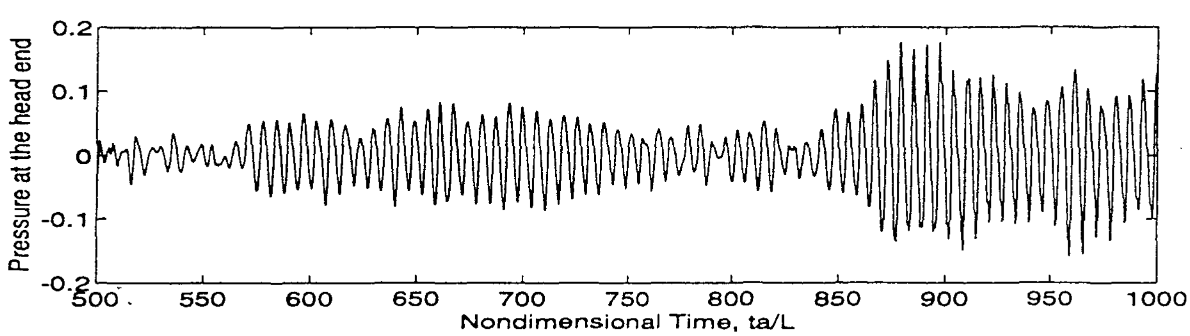


Figure 6

3. Numerical Analysis Of Combustion Instabilities

The major obstacle in approaching combustion instabilities of solid propellants arises from difficulties in treating the various complicated physico-chemical processes. Specifically, interactions between chamber gas dynamics and transient combustion responses of the propellants need to be resolved in order to understand the key mechanisms responsible for driving instabilities in rocket motors. Several modes of interactions between acoustic oscillations and transient combustion have been observed, the most important of which are pressure and velocity coupled responses, denoting the sensitivity of combustion processes to local pressure and velocity fluctuations, respectively. While pressure coupling has been extensively studied and the origins of the physical behavior are reasonably well understood, the general problem of velocity coupling remains largely unresolved. Velocity coupling implies that transient combustion response is strongly influenced by the velocity oscillations parallel to the burning surface, rather than pressure oscillations (Price, 1979). Recent studies of turbulent transition behavior of the acoustic boundary layer (Ma et al., 1990) and the turbulent reactive acoustic boundary layer (Roberts and Beddini, 1989 and 1990) have been conducted, revealing that the turbulent acoustic boundary layer on a solid propellant surface could be one of the primary mechanisms in velocity-coupled erosive burning. The modification of transport properties caused by turbulent fluctuations in the region between the gas phase flame and the propellant surface (Bulgakov, 1993 and King, 1993) is regarded as the underlying physical process in this type of velocity coupling.

In a series of studies on the interactions between acoustic motions and propellant combustion (Tseng and Yang, 1994 and Roh and Yang, 1995), an analysis capable of treating complicated physico-chemical processes involved in unsteady combustion of homogeneous propellants has been developed for two-dimensional combustion chambers. Turbulence closure has been achieved by means of a well-calibrated two-layer model taking into account the effect of propellant surface transport properties. Turbulence penetrates into the primary flame zone and consequently increases the propellant burning rate, a phenomenon commonly referred to as erosive burning. Momentum and energy transport enhanced by turbulence significantly alter the oscillatory flow characteristics in the gas phase. It has been shown that the turbulent reactive acoustic boundary layer on a solid propellant surface, could be one of the primary mechanisms in velocity-coupled erosive burning.

Investigation of unsteady behavior in the axisymmetric combustion chamber of solid rocket motors has been conducted to overcome a lack of numerical simulation (Roh and Yang, 1996), even though a physical insight into the gaseous dynamics and unsteady propellant response can be obtained from unsteady motions in two-dimensional combustion chambers. Since the mass flow rate and its influence on turbulence intensity increase in the axisymmetric combustion chamber, the reduced chamber in axial direction has been used in axisymmetric calculations instead of comparatively long two-dimensional chambers. The short computational domain enhances the resolution of grid aspect ratio under limitation of computational resources. Axisymmetric calculations have improved the description of flame structures in terms of the heat-release mechanism of the gas phase and heat transfers between the gas and condensed phases. Various distinct features of turbulent flows in an oscillatory motor environment and their ensuing influences on propellant combustion response have been studied.

3.1 Theoretical Formulation

The formulation with an improved reaction mechanism follows the model established in the previous work (Roh and Yang, 1996). The analysis of the gas-phase processes is based on the complete conservation equations of mass, momentum, energy, and species concentration, and takes into account finite-rate chemical kinetics and variations of thermophysical properties with temperature. Turbulent transport is considered to discuss the effect of turbulent flow disturbances on unsteady heat-release mechanisms. Turbulence closure has been achieved based on the two-layer model of Rodi (Rodi, 1991) because of its superior performance over conventional low-Reynolds-number k-ε schemes in terms of numerical accuracy and convergence. For unsteady calculations, the empirical constants in the turbulent equations have been corrected based on the former study (Fan and Lakshminarayana, 1993).

Owing to the difficulties in establishing a complete chemical kinetics scheme and limitations of computational resources, a thorough consideration of all physical and chemical processes does not appear feasible. A reduced reaction mechanism is therefore used to describe the combustion wave structure in both the gas and condensed phases. The chemical kinetics used here follow the model established in the previous study (Roh and Yang, 1995). This model is a viable alternative which provides well-resolved and reasonably accurate information about major chemical kinetic pathways.

The condensed phase consists of a preheated zone and a superficial degradation layer in which both thermal decomposition of the propellant and reaction of the decomposed species take place simultaneously. If we ignore the bulk motion, mass diffusion, and axial thermal diffusion, and assume constant thermo-physical properties, the formulation governing condensed-phase processes reduces to a set of one-dimensional equations. The detailed discussion of the formulation is given in the previous work (Roh and Yang, 1996).

In order to match the gas phase with the condensed phase at the burning surface, matching conditions such as mass energy balances are required. Conservation laws are applied to mass and species balances at the gas-solid interface. The energy balance at the interface is established for unsteady calculations (Roh and Yang, 1996).

3.2 Numerical Method

In order to avoid a singularity problem inevitably encountered in certain cases of two-dimensional axisymmetric calculations, governing equations can be modified and solved as two-dimensional Cartesian equations. Each variable in vector notation of gorverning equations has y term which represents the radial coordinate. New variable vectors can be obtained by removing y term and generating a new source vector, which includes convection and diffusion vectors in radial direction (Roh and Yang, 1996). The new source vector allows the modified governing equations to be solved in two-dimensional Cartesian manner even though they are two-dimensional axisymmetric equations.

The method of a finite-volume approach has been used to numerically solve the new set of governing equations. An implicit dual time-stepping integration method proven to be quite efficient and robust for reacting flows at all speeds (Tseng and Yang, 1994) has been adopted to study the unsteady behavior in the combustion chamber of the solid rocket motor. A fully-coupled implicit formulation is used to enhance numerical stability and efficiency.

3.3 Results And Discussions

Figure 7 shows the situation examined here, an axisymmetric rocket motor loaded with a double-base homogeneous propellant grain. The computational domain for treating various distinct features of turbulent flows in an oscillatory motor environment are described in the previous work (Roh and Yang, 1996). The steady-state flowfield must be obtained first to provide the initial conditions for analysis of unsteady motions. The effect of turbulence on the temperature field is clearly seen in Fig. 8, which shows the radial profiles of temperature at various axial locations. The motor internal flow exerts almost no influence on the combustion wave structure in the upstream region. The most significant observation, however, is that the dark zone gradually disappears in the downstream region. The luminous flame zone expands and eventually merges with the primary flame zone because of the turbulence-enhanced heat transfer in the dark zone, where the elevated temperature facilitates reduction of NO into final products (Tseng and Yang, 1994). Figure 9 shows the axial distributions of surface temperature and burning rate. The turbulence in the downstream region penetrates into the

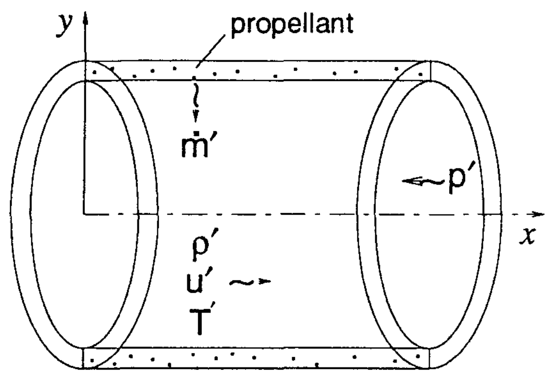
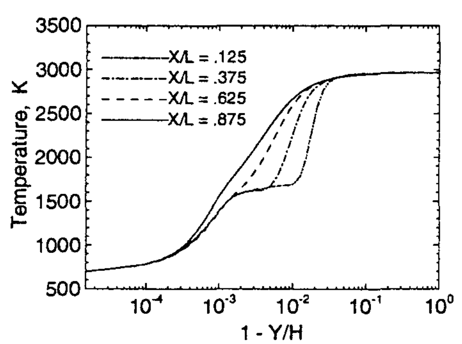


Figure 7

primary flame zone, enhancing mixing processes and modifying the local heat transfer rate at the propellant surface. The consequent increase in burning rate results from the increased surface temperature due to enhanced heat transfer from the gas phase. This shows that the primary flame behavior is most strongly related to the combustion characteristics of the condensed phase, rather than the secondary flame zone (Roh and Yang, 1995).



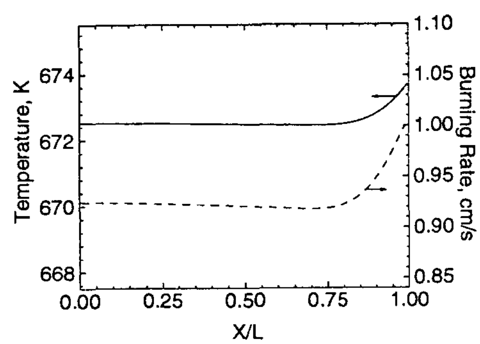


Figure 8

Figure 9

After obtaining a steady state solution, periodic pressure oscillations are imposed at the exit to simulate acoustic oscillations of a longitudinal standing wave in the combustion chamber. The amplitude of the pressure oscillation is 2.0% of the mean pressure, and the frequency is 880 Hz corresponding to the first longitudinal acoustic mode. The transition from laminar to turbulent flow can be examined in Figs. 10 and 11, which show the radial distributions of the axial velocity fluctuation u'. Both amplitudes and phases are presented to achieve a better understanding of this complicated structure. Also included is the flame thickness, defined as the location at which the temperature reaches 97% of the final flame temperature. The axial velocity fluctuation u' near the head end region has almost the same shear wave pattern as the previous study (Roh and Yang, 1995). Beyond x/L = 0.25, the shear wave is already smoothed and shows only traces of the oscillation. While the velocity fluctuation appears flat at x/L = 0.75, it increases more and penetrates closer to the surface near the exit than in the other regions. In the core-flow region, the behavior of u' seems to follow the isentropic relation with the acoustic pressure. The phase of the velocity fluctuation in the head end only shows variations along the radial direction, while most of the downstream region shows a 90 degree phase lag with respect to p'.

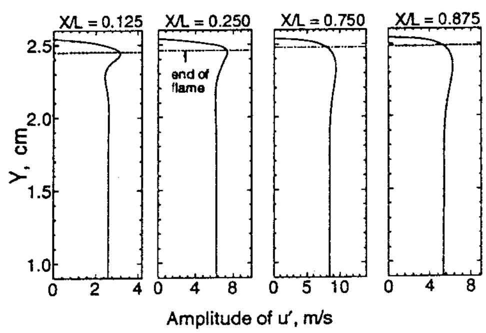


Figure 10

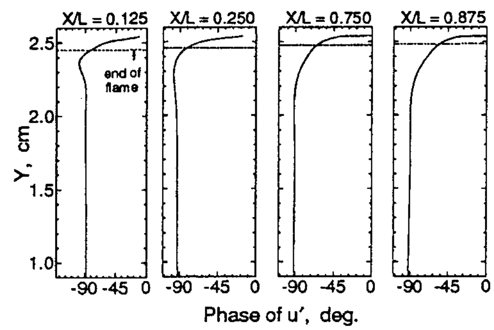


Figure 11

Figures 12 and 13 show the radial distributions of fluctuating temperature at various times. Temperature fluctuations near the head end show similar patterns as observed in the laminar case considered in the previous study (Roh and Yang, 1995). The most significant difference from the laminar case is the occurrence of a large temperature oscillation within the primary flame zone in the downstream region.

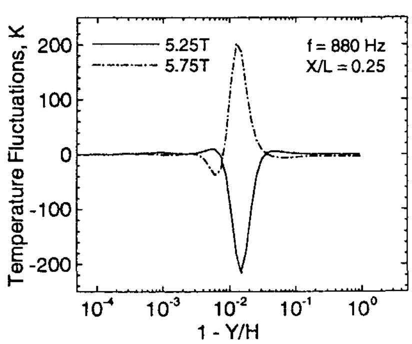


Figure 12

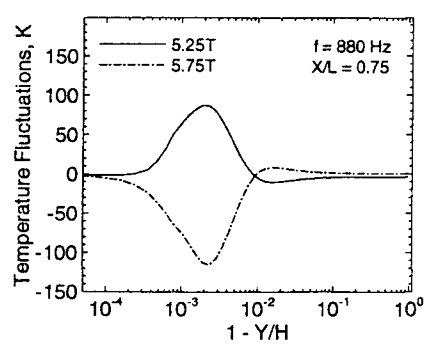


Figure 13

Time evolution of the heat-release fluctuations are presented in Figs. 14 and 15. Large fluctuations occur in the secondary flame zone near the head end region, while the downstream region shows a small amplitude of heat release fluctuations, similar to the behavior observed for the temperature fluctuations. Since enhanced mixing processes due to turbulence distribute the chemical reaction over the entire reacting zone (rather than having a strongly exothermic reaction concentrated in the thin secondary flame zone), the amplitudes of temperature and heat-release fluctuation decrease in the downstream region.

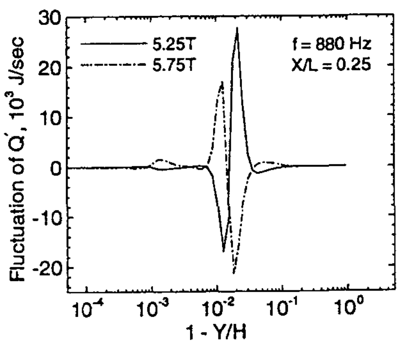


Figure 14

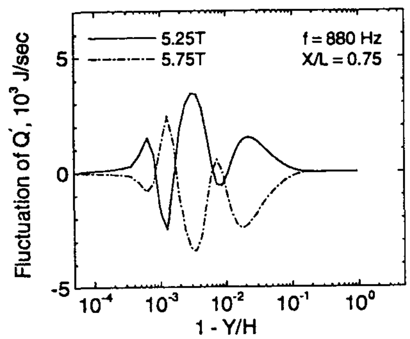


Figure 15

The effect of heat release on motor stability characteristics can be investigated using Rayleigh's criterion (Rayleigh, 1954 and Culick, 1987), which determines the conditions for driving or suppressing flow oscillations when thermal energy is added (or subtracted) periodically to (or from) the acoustic field. The Rayleigh's criterion results for the first mode of oscillation are presented in Fig. 16. Owing to the reduced heat-release fluctuations, the Rayleigh parameter $p'q'$, contains small values in the downstream region. The primary flame zone plays a decisive role in motor stability characteristics in the turbulent downstream region. Overall, the nonsteady heat release arising from interactions between the acoustic wave and the propellant combustion in the turbulent-flow region tends to drive the flow oscillations in a rocket motor.

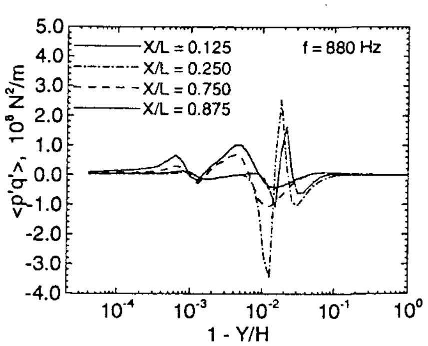


Figure 16

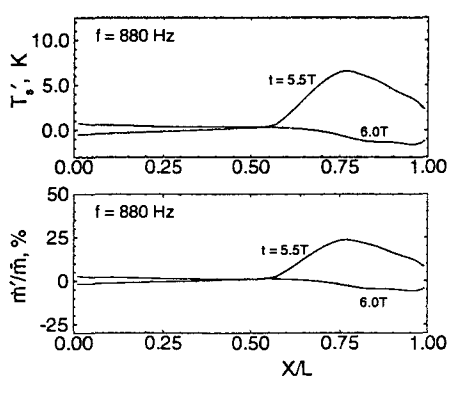


Figure 17

The axial variations of surface temperature and burning-rate fluctuations are presented in Fig. 17 It is shown that the burning rate at the propellant surface oscillates as a result of the temperature fluctuation. The fluctuation of surface temperature shows a pressure-coupled behavior in the head-end region. Large increases of both temperature and burning rate are observed in the downstream region but the fluctuation is asymmetric.

4. Concluding Remarks

The term 'combustion instabilities' refers to one sort of unsteady motion in a combustor, distinguished by several characteristics: one or more well-defined frequencies commonly close to classical acoustic frequencies; possibly large amplitudes due to the large reservoir of energy available from combustion processes; behavior determined by the dynamics of the coupled system comprising the chamber, the compressible medium within the chamber, and the combustion processes themselves. There are other unsteady motions in the chamber, some related to the combustion of reactants and some arising with flow separation and instabilities of a purely fluid-mechanical nature. Hence for both theoretical and practical reasons, combustion instabilities must be investigated within that broader context.

It may never be possible to construct from first principles a complete description of the flow in a combustor, including all combustion and flow processes. Hence successful results require a combination of efforts covering theory, analysis, and experiment. Although this paper has been concerned only with theory and analysis, the primary motivation has been behavior observed in actual rockets, and appeal to experimental results is at least implicit in all the work reported here.

One of the chief purposes of this paper has been to explain the complementary nature of the two types of analysis and to clarify the role of each. The first author's past and continuing choice has been to work with the approximate analysis described in Section 2. That approach is adaptable to handle virtually any problem arising in practice providing the necessary modeling can be accomplished. In any case the calculations are cheaper and quicker than those performed with a full numerical analysis and provide broad understanding of the dynamical behavior of combustors. The price for that advantage is reduced accuracy and limitations on the range of applicability in some respects (e.g. amplitude of oscillations). Moreover, results cannot be obtained with the approximate method without providing input information obtained from ancillary modeling, experiments, numerical results produced by the numerical analysis discussed in Section 3 or literal results obtained with analytical methods applied to specific problems. Thus the approximate method in a certain sense is at the mercy of the other two kinds of analysis. However, the approach is a much more convenient context in which to understand the global linear and nonlinear behavior of a combustor.

Numerical analysis has two important functions in this field: to provide comprehensive, and as accurate as possible, calculations of the complete nonlinear problem including turbulence, realistic kinetics mechanisms, and accurate representations of combustion processes. Certain basic parts of the results, such as combustion response functions and the vorticity field, are required for the approximate method. The numerical analysis itself requires input from the basic chemistry and kinetics of propellant combustion, including experimental results for combustion processes. Thus in addition to its value for providing basic understanding of unsteady flows in combustors, CFD serves as a bridge between fundamental processes of combustion and the approximate analysis of the global dynamics.

Not covered here is a third class comprising analytical methods such as those developed by Flandro (1995a, 1995b) provide understanding and interpretation of phenomena that cannot be obtained by numerical analysis alone. Vorticity generation and the history of the vorticity in a chamber is an important example. The investigations on that subject, even for one-dimensional flows provide a basis for understanding that would be hard to come by with numerical analysis alone.

Continued progress in understanding unsteady motions in combustors, including especially combustion instabilities, will rest on appropriate melding of the three classes of analysis, with theoretical and experimental results for the chemistry and combustion of propellants.

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References

Bulgakov, V. K., Karpov, A. I., and Lipanov, A. M. (1993) ``Numerical Studies of Solid Propellant Erosive Burning," *Journal of Propulsion and Power*, Vol. 9, No. 6.

Burnley, V. S. (1996) "Nonlinear Combustion Instabilities and Stochastic Sources," Ph.D. Dissertation, Dept. of Aero., California Institute of Technology.

Burnley, V. S. and Culick, F. E. C. (1996), "Some Dynamics of Acoustic Oscillations with Nonlinear Combustion and Noise." Proceedings, Fourth International Conference on Special Topics in Combustion. *Challenges in Propellants and Combustion 100 Years after Nobel*, Stockholm, Sweden, 27-31 May.

Chu, B.-T. and Kovazsnay, L. S. G. (1958) "Nonlinear Interactions in a Viscous Heat-Conducting Compressible Gas" *J. Fluid Mech.*, Vol. 3, No. 5, pp 494-514.

Culick, F. E. C. (1963) ``High Frequency Oscillations in Liquid Rockets," *AIAA J.* Vol. 1, No. 5 , pp.1097–1104.

Culick, F. E. C. (1966) ``Acoustic Oscillations in Solid Propellant Rockets," *Astronautica Acta*, Vol. 12, No. 2 , pp. 113–126.

Culick, F. E. C. (1971) ``Non-linear Growth and Limiting Amplitude of Acoustic Oscillations in Combustion Chambers," *Combustion Science and Technology*, Vol. 3, No. 1, pp. 1–16.

Culick, F. E. C. (1972) `` Interactions Between the Flow Field, Combustion, and Wave Motions in Rocket Motors," Naval Weapons Center, NWC TP-5349 .

Culick, F. E. C. (1975)``Stability of Three-Dimensional Motions in a Combustion Chamber," *Combustion Science and Technology*, Vol. 10 , pp. 109–124.

Culick, F. E. C. (1976)``Nonlinear Behavior of Acoustic Waves in Combustion Chambers," Parts I and II, *Acta Astronautica*, Vol. 3 , pp. 714–757.

Culick, F. E. C. (1987) ``A Note on Rayleigh's Criterion" *Combustion Science and Technology*, Vol. 56, pp. 159-166.

Culick, F. E. C. (1988)``Combustion Instabilities in Liquid-Fueled Propulsion Systems — An Overview," AGARD, 72B Specialists' Meeting of the Propulsion and Energetics Panel, AGARD CP 450.

Culick, F. E. C., (1991), ``Combustion Noise and Combustion Instabilities in Propulsion Systems," (with L. Paparizos, J. Sterling, and V. Burnley), AGARD Conference on Combat Aircraft Noise, Conference Proceedings 512.

Culick, F. E. C. (1994) "Some Recent Results for Nonlinear Acoustics in Combustion Chambers," *AIAA Journal*, Vol. 32, No. 1, pp 146-169.

Culick, F. E. C. (1996) "On the Ordering Procedure and the Insignificance of Linear Coupling in Combustion Instabilities." Submitted to *Combustion Science and Technology.*

Fan, S. and Lakshminarayana, B. (1993) ``Low-Reynolds-Number k-e Model for Unsteady Turbulent Boundary-Layer Flows." *AIAA Journal,* Vol. 31, No. 10, pp. 1777-1784.

Lores, E. M. and Zinn, B. T. (1973) "Nonlinear Longitudinal Instability in Rocket Motors," *Combustion Science and Technology,* Vol. 7, No. 6, pp. 245-256.

Ma, T., Van Moorhem, W. K., and Shorthill, R. W. (1990) ``An Innovative Method of Investigating the role of Turbulence in the Velocity Coupling Phenomenon," *Journal of Vi bration and Acoustiics,* Vol. 112.

Ma, T., Van Moorhem, W. K., and Shorthill, R. W. (1990) ``Experimental Investigation of Velocity Coupling in Combustion Instability," *Journal of Propulsion* Vol. 7, No. 5.

Rayleigh, J. W. S. (1954) *Theory of Sound,* Vol. II., Dover, New York.

Roberts, T. A., and Beddini, R. A. (1988) "Response of Solid Propellant Combustion to the Presence of a Turbulent Acoustic Boundary Layer," *AIAA* 88-2942.

Roberts, T. A. and Beddini, R. A. (1989) "A Comparison of Acoustic and Steady-State Erosive Burning in Solid Rocket Motors," AIAA 89-2664.

Rodi, W. (1991) ``Experience with Two-Layer Models Combining the k-e Model with a One-Equation Model near the Wall," AIAA 91-0216.

Roh, T.-S. (1995) "Numerical Simulation of Combustion Instabilities of Double-Base Homogeneous Propellants in Rocket Motors" Thesis in M.E., The Pennsylvania State University.

Roh, T.-S., Tseng, I.-S., and Yang, V. (1995) "Effects of Acoustic Oscillations in Flame Dynamics of Homogeneous Propellants in Rocket Motors" *J. Propulsion and Power,* Vol. 11, No. 4, pp 640-650.

Roh, T. S. and Yang, V. (1995) ``A Comprehensive Analysis of Combustion Instabilities of Homogeneous Propellants in A Rocket Motor," *AIAA* 95-2863.

Roh, T. S. and Yang, V. (1996) ``Numerical Analysis of Combustion Instabilities of Homogeneous Propellants in Axisymmetric Rocket Motors," *AIAA* No. 96-2623.

Tseng, C.-F. (1993) "Interactions Between Acoustic Waves and Premixed Flames in Porous Chambers" Thesis in Fuel Science, The Pennsylvania State University.

Tseng, I.-S. (1992) "Numerical Simulation of Velocity-Coupled Combustion Response of Solid Rocket Propellants" Thesis in Mechanical Engineering, The Pennsylvania State University.

Tseng, I. S. and Yang, V. (1994) ``Combustion of a Double-Base Homogeneous Propellant in a Rocket Motor," *Combustion and Flame,* Vol. 96, pp. 325-342.

Zinn, B. T. and Powell, E. A. (1970) "Application of the Galerkin Method in the Solution of Combustion Instability Problems," *Proceedings of the Nineteenth International Astronautical Congress on Propulsion, Re-Entry Physics,* Vol. 3, pp. 59-73.