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ONE DIMENSIONAL MODELLING OF FAILURE IN LAMINATED PLATES BY DELAMINATION BUCKLING

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Abstract—When low speed objects impact composite laminated plates delamination may result. Under in-plane compression such delaminations may buckle and tend to enlarge the delaminated area which can lead to loss of global plate stability.

This process is modelled here in a first attempt by a delaminating beam-column wherein the local delamination growth, stability and arrest are governed by a fracture mechanics-based energy release rate criterion.

NOTATION

A_3 midspane transverse deflection in Section 3
D_i $\frac{Et_i^3}{12(1-\nu^2)}$—bending rigidity in "$i$th" section
E Young's modulus
G strain energy release rate (s.e.r.r.) per unit area
$\bar{G}$ $G/\{Et^5L^{-4}(1-\nu^2)^{-1}\}$
$G_a \ldots G_e$—s.e.r.r. associated with models $a \ldots e$
h delamination thickness
$\bar{h}$ h/t
L total length of plate
l delamination length
$\bar{l}$ l/L
l_1 $(L-l)/2, l_2 = l_3 = l$
l^* $l/\{h\Gamma_0^{*-1/4}\}$
l_0 initial film length
l_A, l_B significant film length
l_0^* $l_0/\{h\Gamma_0^{*-1/4}\}, l_A^* = 3.376, l_B^* = 2.221$
l_{a^*}, l_{e^*} delamination length which maximize G_a, G_e respect
$\bar{l}_{a^*}$ $l_{a^*}/L, \bar{l}_{e^*} = l_{e^*}/L$
P_i total load in "ith" section
t total plate thickness
t_1 $t, t_2 = t-h, t_3 = h$
U strain energy
U_i, U_{ii}, U_{iii} strain energy for three different states
$\bar{U}$ $U/\{Et^5L^{-3}(1-\nu^2)^{-1}\}$
u_i $\frac{l_i}{2}\sqrt{(P_i/D_i)}, i = 1,2,3$ normalized total load in "ith" section
Γ_0 energy required to produce a unit of new surface
Γ_0^* $\Gamma_0 \Big/ \left\{\frac{Eh}{2(1-\nu^2)}\right\}$
δ end deflection of delamination
ϵ_0 loading strain
ϵ_i $\frac{P_i}{Et_i}, i = 1,2,3$ midplane (membrane) strain in "ith" section
ϵ_L $\frac{\pi^2}{3(1-\nu^2)}\left(\frac{t}{L}\right)^2$ buckling strain of plate
ϵ_{cr} $\frac{\pi^2}{3(1-\nu^2)}\left(\frac{h}{l}\right)^2$ buckling strain of delamination
$\bar{\epsilon}_0$ $\epsilon_0/\epsilon_L, \bar{\epsilon}_i = \epsilon_i/\epsilon_L, i = 1,2,3, \bar{\epsilon}_{cr} = \epsilon_{cr}/\epsilon_L = (\bar{h}/\bar{l})^2$
ϵ_0^* $(1-\nu^2)\epsilon_0\Gamma_0^{*-1/2}$
ϵ_{0A}^* $0.866, \epsilon_{0B}^* = 1.000$
η $u_3 - \pi$ expansion parameter
θ end rotation of delamination
$\bar{\theta}$ $\theta L/t$
κ $1 - \bar{h} + \bar{h}\bar{l}$
ν Poisson's ratio

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1. INTRODUCTION

Fiber reinforced plastics and, in particular, graphite fiber composite materials enjoy a definite strength to weight advantage over many standard engineering materials used in weight critical applications. This assessment must, however, be made with respect to applications where the primary stresses are aligned with the fiber direction such as the extension (tension or compression) or bending of a thin plate where stresses normal to the plane of the plate are small. If such a plate is subjected to impact, considerable damage can be caused since the cohesive strength of the plate through its thickness is quite low. This in turn can lead to degradation of the extensional or bending strength of the plate[1–5].

The mechanism of this strength degradation has been the subject of a recent investigation[6]. Although the details of the initial degradation process are poorly understood, it is believed that the strength degradation under compressive in-plane loading is the result of coupled delamination and delamination buckling.

An experimental investigation into the failure mechanism using high-speed photography[7] has shown that the failure process can be divided roughly into two phases. In the first phase the plate is impacted and the resulting response causes interlaminar separation. The size of this damage area is a function of the impactor parameters and the plate material, lay-up, etc,[6]. For the present discussion it will be assumed that the dimension of the damage area is large compared to the laminate thickness but small compared with the plate size.

In the second phase the damage area spreads to the undamaged area of the plate through a combination of laminate buckling and further delamination. It is this failure phase with which we are concerned in the following development. In order to elucidate the dominant physical phenomena in a readily tractable analytical manner it appears prudent to deal first with a geometrically simpler situation than the full plate problem illustrated in Fig. 1: the treatment of that problem depends heavily on numerical computations. Instead we shall deal here only with the one dimensional plate analogue represented by the cross section in Fig. 1 which geometry and loading are considered to be invariant along the coordinate normal to the plane of the figure. In the subsequent analysis which is condensed from references[8–11] we shall start from the assumption that a delamination exists in the plate. The latter may be initially unloaded or under an in-plane compressive load when the delamination appears. In either case the analysis will study the growth (under load) of the damage area. Quasistatic conditions will be assumed and the analysis will draw on the theory of ordinary beams as well as a rate independent fracture criterion based on the energy release rate.

Growth of the delamination is assumed to occur in its own plane in keeping with the laminate character of layered composites. Yet, for simplicity reasons the properties of the plate are assumed homogeneous, isotropic, and linearly elastic. We note, however, that impact damage in a fiber composite of, say, quasi-isotropic and symmetric lay-up generates in general two or more delaminations none of which possess the same properties themselves. Such material behavior can be readily dealt with at the expense of introducing additional parameters into the problem; but, because neither the physical principles involved in the analysis nor the character of the results will change, we omit attention to that detail.

Depending on the thickness and number of delaminations relative to the total plate thickness several further approximations may be considered as illustrated in Fig. 2. In Fig. 2(a), the unbuckled portion of the plate has been made infinitely thick; this is called the "thin film" model. A finite thickness (assumed large compared to the delamination) is introduced in the delamination "thick column" model Fig. 2(b). The case of several delaminations can be analyzed (Fig. 2c) as well as a symmetrical split (Fig. 2d). The most general case analyzed in this report is represented in Fig. 2(e).

The analyses for all these models are delineated in this report. The "thin film" model is analyzed first since the results are quite simple and illustrative of the results for the more complete models†.

†We wish to point out that after the typing of the manuscript we became aware of publications dealing with the thin film problem[12] as well as delamination of a ring under external pressure[13].