A REVIEW OF THE ONR/NAVAIR RESEARCH OPTION COMBUSTION INSTABILITIES IN COMPACT RAMJETS, 1983–1988

F. E. C. Culick
California Institute of Technology
Pasadena, CA, U.S.A.

K. C. Schadow
Naval Weapons Center

ABSTRACT

This paper consists of two parts summarizing two portions of the ONR/NAVAIR Research Option. The option began in 1983 and continued for five years, involving 11 organizations. Simultaneously, similar or related programs supported by other agencies or institutions were being carried out in several other places. Results of those programs have been briefly summarized in five papers collected in a document to be published by C.P.I.A. This paper contains two of the five papers in that document.

Here we cover the subjects of approximate analyses and stability; and large-scale structures and passive control. The first is concerned chiefly with an analytical framework constructed on the basis of observations; it is intended to provide a means of correlating and interpreting data, and predicting the stability of motions in a combustion chamber. The second is a summary of recent experimental work directed to understanding the flows in dump combustors of the sort used in modern ramjet engines. Much relevant material is not included here, but may be found in the remaining papers of the document cited above. For completeness, we note briefly the substance of those repors.

In their summary "Spray Combustion Processes in Ramjet Combustion Instability," Bowman (Stanford), Law (University of California, Davis) and Sirignano (University of California, Irvine) review several aspects of spray combustion relevant to combustion instabilities. The objectives of the works were: (1) to determine the effect of spray characteristics on the energy release pattern in a dump combustor and the subsequent effects on combustion instability; (2) to gain a fundamental understanding of the coupling of the spray vaporization process with an unsteady flow field; and (3) to investigate methods for controlling and enhancing spray vaporization rates in liquid-fueled ramjets.

During the past five years considerable progress has been made in applying methods of computational fluid dynamics to the flow in a dump combustor including consequences of energy release due to combustion processes. Jou has summarized work done at Flow Research, Inc. and at the Naval Research Laboratory in his paper "A Summary Report on Large-Eddy Simulations of Pressure Oscillations in a Ramjet Combustor."

The serious effects of combustion instabilities on the inlets of ramjet engines were discovered in the late 1970's in experimental work at the Aeropropulsion Laboratory, Wright Field, the Naval Weapons Center and the Marquardt Company. The most thorough laboratory work on the unsteady behavior of inlets has been accomplished at the McDonnell-Douglas Research Laboratory by Sajben who has reviewed the subject in his paper "The Role of Inlet in Ramjet Pressure Oscillations."

I. APPROXIMATE ANALYSIS AND STABILITY OF PRESSURE OSCILLATIONS IN RAMJETS

F. E. C. Culick

Abstract

This paper summarizes work accomplished during the past five years on analysis of stability related to recent experimental results on combustion instabilities in dump combustors. The primary purpose is to provide the information in a form useful to those concerned with design and development of operational systems. Thus most substantial details are omitted; the material is presented in a qualitative fashion.

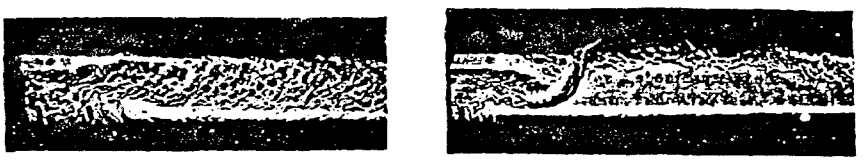
A major part of the work at Caltech, Georgia Tech and the University of California, Berkeley, has been supported under the ONR Research Initiative on Pressure Oscillations in Ramjets, the remainder being supported by AFOSR (Caltech) and by the Air Force Aero Propulsion Laboratory (California State University, Sacramento). It is convenient to begin this summary with a brief résumé of recent experimental work.

1. Introduction

Recent experimental programs are summarized more thoroughly in two other papers in this report. Al-

though the emphasis here is on analytical methods and results, it is useful to begin with a brief résumé of the observations that have motivated the analyses. The latest laboratory work has been accomplished chiefly at the Aero Propulsion Laboratory, Georgia Tech, Naval Weapons Center and University of California, Berkeley. Results have been obtained over broad ranges of fuel/air ratio, flow rate and frequency. Geometrical changes have been investigated, usually with the basic shape of a dump combustor: the primary reaction zone is stabilized by the recirculating flow on the leeward side of a rearward facing step. Extensive tests have also been carried out at Georgia Tech with premixed flames stabilized on wires supported transversely to the flow direction.

The most striking qualitative conclusion of those observations is that unsteady and periodic combustion in large vortices shed from the step is a significant contribution to excitation and sustenance of unsteady pressure oscillations. As one example, Figure 1 is a reproduction of shadowgraphs taken in the Caltech work with a two-dimensional duct. The flow downstream of the step is shown under stable conditions and when significant pressure oscillations are present. In the latter case, large vortices are shed periodically at the same frequency as the pressure oscillations. One cycle of the motion is illustrated in Figure 2 with a trace of the pressure. Smith and Zukoski (1985) have proposed the following mechanism.



Stable Flow **Unstable Flow**

Figure 1

A vortex is initiated at the edge of the step at a time determined partly by the local acoustic velocity. The vortex propagates downstream, releasing energy at a rate that seems to reach maximum when the vortex impinges on the wall. In order for the time of impingement to be at a favorable time during the acoustic oscillation, the propagation rate and hence strength of the vortex must increase with frequency. Because the vortex strength depends on the magnitude of velocity fluctuation initiating the motion of the lip, it is necessary that the steady amplitude of the acoustic field increase with frequency. That behavior is observed. Moreover, numerical calculations by Hendricks (1986) have shown quite similar behavior for the unsteady flow induced by an abrupt change of velocity past a rearward facing step. Figure 3 is a sketch taken from Hendricks' work showing the development of a vortex calculated for those conditions.

Similar observations have been made recently by Schadow et al. (1987a, 1987b) at the Naval Weapons Center and by Yu et al. (1987a, 1987b, 1988) at Berkeley in axisymmetric combustors. See also Schadow's paper in this collection. Keller et al. (1982) had previously observed vortex shedding in the turbulent reacting shear layer shed from a step, although not within the context of investigating combustion instabilities.

Although these flows share certain obvious similarities, the connections cannot presently be made quantitative: each device has to be treated as a special case. What is clear from these observations is that vortex shedding from bluff bodies is likely a dominant mechanism for combustion instabilities in ramjet engines and probably in afterburners as well. The earliest examples of this phenomenon were reported by Kaskan and Noreen (1955) and by Rogers and Marble (1955). Taken all together, those works illustrate why much of the analytical and computational work has taken the course it has in recent years. We must emphasize, however, that the more general forms of approximate analysis accommodate other mechanisms as well.

Results of computational fluid mechanics applied to this problem are summarized by Jon elsewhere in this report. We confine our remarks here to analytical work. In this context, 'computational' means numerical solution to the partial differential equations of conservation, intended to provide detailed results for all characteristics of the flow field. 'Analytical' implies a broad spectrum of methods, ranging from heuristic formulations giving rough estimates of behavior, to more-or-less rigorously founded approximate analysis involving solution of ordinary (in contrast to partial) differential equations. Ultimately for applications in design of actual systems, a combination of computational and analytical methods will be required — each approach has its advantages and shortcomings.

In the ideal (and unattainable) limit, computational results should provide precise and complete information. Apart from numerical inaccuracies, which can in principle be reduced to acceptable levels, uncertainties

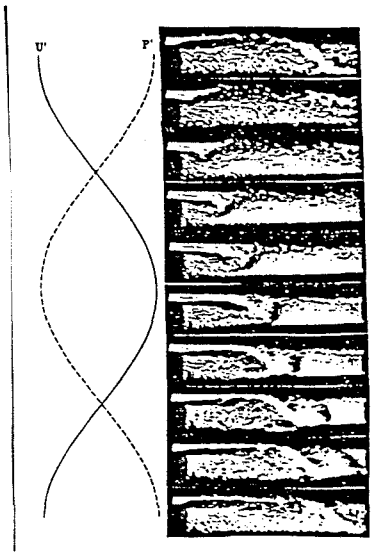


Figure 2

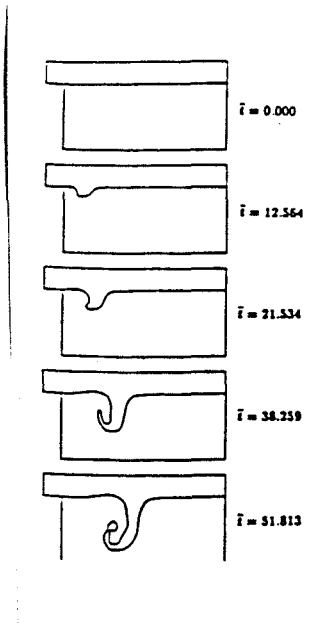


Figure 3

will likely always exist in necessary input data associated with chemical processes and with turbulent flows. In any event, a numerical calculation provides results for one case only. It is inefficient and tedious to use computational methods as a basis for perceiving trends of behavior and understanding the qualitative consequences of design changes. Nevertheless, once a design is fixed, computational fluid mechanics promises to be a valuable aid to confirming desired performance. That should reduce the amount of expensive testing.

On the other hand, because computational methods do treat only individual cases, they are inconvenient for preliminary design work, investigating parametric changes and for planning experiments. For those purposes, analytical methods are greatly superior. Successful development of approximate results should also provide simple 'rules-of-thumb' to guide development work. Because assumptions and approximations are required, the accuracy of any approximate analysis is difficult to assess. Only two means are available: comparison with experimental data or comparison with exact (numerical) solutions. Experimental data carry their own uncertainties, due both to measurement inaccuracies (not always well known) and to uncertainties in ancillary data that may be required for the comparisons. Additionally, at the present time detailed results are scare: often only global information is available, insufficient for persuasive confirmation of predictions.

Comparison with computational results is the most satisfactory way to check the validity of approximate analysis. A major difficulty in doing so is ensuring that the same problem really is being solved by the two methods. That concern arises because some of the physical processes — notably boundary conditions — may not be represented in the same fashion in the two approaches. The strategy has been followed with some success in applications to combustion instabilities in solid rockets, but not yet for ramjets for which it is more difficult to carry out this procedure.

2. General Remarks on Approximate Analysis

All forms of approximate analysis of combustion instabilities amount to extensions of ideas developed in classical acoustics. Indeed, the view generally taken is that the organized unsteady motions in combustion chambers (in contrast to turbulence) may be regarded as classical acoustic motions with perturbations due to combustion processes, mean flow and influences of the boundaries, such as the inlet diffuser and the exhaust nozzle. In its most elementary form, this approach then furnishes estimates of the frequencies and mode shapes of observed steady oscillations. Quite satisfactory results can usually be obtained with remarkably little effort. The values of the frequencies are usually dominated by the geometry of the chamber, although significant shifts may be due to the inlet and exhaust apparatus. Thus in first approximation, frequencies and modes may be calculated for the same geometry as the actual chamber but with all influences of flow and combustion ignored. This leads, for example, to the conclusion that the frequencies are proportional to the speed of sound divided by a characteristic length ℓ: $f \sim \bar{a}/\ell$. That is a basic scaling law: frequencies of combustion instabilities decrease

with increased size and increase with higher temperature. Moreover, the mode shapes of instabilities may also be crudely assessed on this basis. For example, one may gain some idea of the regions where velocity and pressure fluctuations are large. Velocities tend usually to be larger and pressures smaller near the boundary.

No such simple method supplies estimates of stability. The reason is that stability depends on the relative strength of processes whose effects are small compared with those governing the classical motions. In the latter case conservation of mass and momentum are the governing principles. Stability on the other hand is controlled by rather more detailed matters affecting the rates at which energy is supplied to or extracted from the motions. The dominant destabilizing processes in combustion instabilities are associated with the energy released by burning reactants. Direct conversion of heat released to mechanical energy of the unsteady motions may occur; or the transfer may be associated with coupling between the mean flow (itself due to combustion) and the oscillations. Important losses may be due to viscous effects, at the boundaries or interfaces between the gases and condensed material, and acoustic energy may be radiated through the open portions of the boundary. It is a fundamental characteristic that however significant and large those processes may appear to the stability of motions, their influences on the frequencies and mode shapes are secondary in most practical situations. They may therefore in that sense be treated as perturbations. That is not to say that the accompanying shifts of frequencies or distortions of mode shapes are necessarily unmeasurable. The main point is that this view is a fruitful starting point for approximate analysis.

Similar remarks apply to nonlinear behavior. Viewed in the general context of the theory of classical mechanical systems, combustion instabilities are "self-excited" motions. That is, to an external observer the motions appear to grow "out of the noise" with no external influence. Thus the system is truly unstable linearly; such a motion grows exponential in time and will reach a limiting form only if some sort of nonlinear process is active. As for the processes responsible for the linear instability, experience has established that nonlinear effects may be treated as pertubations.

Another feature of combustion instabilities allows one to analyze their behavior quite independently of the intended steady performance of the combustion system. The fundamental reason is that even the most severe unsteady motions require an amount of energy utterly negligible compared with that furnished by the continuous combustion of the reactants. Totally unacceptable — i.e. destructive — amplitudes can be generated with essentially no effect on, for example, the thrust of a propulsion system. Thus the unsteady motions have essentially no influence on the steady flow which may therefore be assumed known for the purposes of approximate analysis.

Even with the preceding observations, one should still wonder why the organized or collective motions we call 'acoustic' seem to be so well defined in combustion chambers containing obviously large amounts of fluctuations such as noise and turbulence. A fundamental reason for this result flows from early work by Chu and Kovasznay (1957) carried out in a different connection. Following earlier work by Kovasznay (1953), they showed that in general, a small unsteady disturbance in a compressible fluid may be synthesized of three contributions: an acoustic motion carrying pressure and velocity but no entropy fluctuations; a viscous wave having velocity and entropy but no pressure fluctuations; and an entropy or temperature wave also having no velocity and pressure fluctuations. The last is an example of convective waves. There are of course nonlinear interactions between the three sorts of waves, but this result provides a theoretical basis for anticipating that acoustic waves might be usefully considered without regard for many of the other apparent complications in a general flow.

3. Mechanisms of Instabilities

An approximate analysis based on the ideas discussed in the introduction will provide a framework in which the stability of unsteady motions can be studied. The analysis is grounded in classical acoustics, so the gasdynamics is accounted for in some approximation. Other contributing processes require additional considerations. These are associated broadly with the five pieces of the complete problem:

i.) the exhaust nozzle;
ii.) the inlet diffuser;
iii.) the mean flow field;
iv.) liquid/gas interactions;
v.) mechanisms for instabilities.

We concentrate here on the last, which is not entirely independent of the first four but it is convenient for the present discussion to introduce this arbitrary classification. Because an extended survey of all these matters has recently been prepared for liquid-fueled propulsion systems [Culick (1988)] the following remarks are much abbreviated.

The behavior of the exhaust nozzle has long been studied for small amplitude motions [Tsien (1952); Crocco and Cheng (1956); Crocco and Sirignano (1967)]. For the case of longitudinal motions, in which the velocity

fluctuations are parallel to the axis, the nozzle tends to attenuate the waves. It is essentially a complicated radiative process, associated with interactions between the acoustical motions and the strong gradients of mean flow properties in the convergent section. As a result, the nozzle does not act simply as an open hole, but rather causes energy losses at a rate proportional to the Mach number and area at the entrance.

Within the inlet, a system of shock waves exists to provide the mass flow and stagnation conditions demanded by the conditions set in the combustion chamber and exhaust nozzle. Under normal operating conditions the shocks are located downstream of the geometric throat in the expanding supersonic flow. The position of the shock depends chiefly on the stagnation pressure in the combustion chamber; increasing the stagnation pressure causes the shocks to move upstream where the Mach number and therefore loss of stagnation pressure are less. It is this sensitivity of the flow in the inlet to pressure changes downstream that has caused longitudinal oscillations to be such a serious concern in ramjet engines. In the late 1970's [Hall (1978, 1980); Rogers (1980a, 1980b)] first qualitative and later limited quantitative relations were established between the amplitudes of pressure oscillations and the loss of dynamic pressure margin.

Since these early works, extensive tests by Sajben and co-workers [Chen, Sajben and Kroutil (1979); Sajben, Bogar and Kroutil (1984); Bogar, Sajben and Kroutil (1983a, 1983b)] have shown that the unsteady behavior is greatly more complicated due to flow separation and instability of shear layers. High speed schlieren pictures [see also Schadow et al. (1981)] have shown large shock oscillations as well as the formation of vortex structures. Although computations based in the one-dimensional approximation to flow in the diffuser [Culick and Rogers (1983); Yang (1984); Yang and Culick (1984, 1985, 1986)] are useful and seem to capture some of the dominant features of the behavior, it is quite clear that the true motions can be simulated well only by numerical analysis based on the Navier-Stokes equations for two-or three-dimensional flows [Hsieh, Wardlaw and Coakley (1984); Hsieh and Coakley (1987); and references cited there].

There is evidence that under some conditions inlets exhibit self-excited or 'natural' oscillations. Energy is transferred from the mean flow to the fluctuations associated at least partly with separated flow. Although a one-dimensional calculation [Culick and Rogers (1983)] and an approximation to some of Sajben's data by Waugh et al. (1983, Appendix D) suggest the possibility that the inlet may drive combustion instabilities, there is no firm evidence from tests with combustors that those conclusions hold. Most experimental results strongly suggest that the major source of driving unstable motions is likely associated with processes in the combustion chamber. Nevertheless, because the flow from the inlet is the initial state for flow in the chamber, it is fundamentally important that processes in the inlet be well-understood.

So far as the computation of stability of motions in the combustion chamber are concerned, it is often convenient to represent the effects of both the exhaust nozzle and inlet diffuser as boundary conditions. However, because the volume of the inlet duct is large, the influence on the characteristics of the modes observed (both frequency and mode shape) is usually significant and must be treated with care, as some examples given in Section 5 will illustrate.

The mean flow field in a ramjet is not simple because flow separation at the dump plane is an essential feature. Full numerical calculations should provide the necessary information (see the results discussed by Jou in this report). For approximate analysis of stability, simpler representations seem to work fairly well, although the subject still demands attention. One approach to computing the field in two-dimensional configurations has been pursued by Yang (1984) and Yang and Culick (1983, 1984a, 1984b). The major assumptions are that the vorticity is uniform within the recirculation zone and that combustion occurs in an infinitesimally thin flame front. Although some success has been achieved, the analysis has not been extended to axisymmetric flows.

Liquid/gas interactions includes the entire spectrum of processes connecting the injection of liquid fuel to the combustion of gaseous reactants. A possible major cause of combustion instabilities is the unsteady vaporization and burning of fuel drops or, more generally, fuel sprays in which interactions among the drops themselves are not necessary negligible. Despite the considerable body of work on this subject, primarily for application to liquid rockets [see Harrje and Reardon (1972) and Culick (1988)] relatively little has been done to account for situations peculiar to ramjet combustors. There is no general theory useful for incorporation in an approximate analysis. Bowman, Law and Sirignano furnish a brief description of recent experimental and theoretical work in another paper in this volume. An important matter yet to be addressed is the distribution of fuel/oxider ratio over the inlet plane to a dump combustor, information which is essential to a complete theory of combustion instabilities.

The primary mechanisms for instabilities in liquid-fueled systems have been reviewed in the recent AGARD paper cited above [Culick (1988)]. For ramjets, it appears that there are three possible dominant causes: atomization, vaporization and combustion of liquid fuel; periodic shedding and combustion in large vortices; and convective waves. In the past, widespread use has been made of an idea suggested first by Karman, but developed chiefly by Crocco and his co-workers to interpret the stability of oscillations in terms of a time lag (τ) and a "pressure interaction index" (n), the "$n - \tau$ model". While that approach provides a convenient framework for representing the coupling between unsteady motions and destabilizing processes, it does not

constitute a basis for identifying the true physical mechanisms for instabilities. It has been particularly useful for correlating data collected for the development of liquid rockets [Harrje and Reardon (1972)] and, as discussed below, has more recently been used by Reardon to interpret one extensive series of laboratory tests with ramjet combustors. We confine our brief comments here to the two remaining mechanisms: convective waves and vortex shedding. The following material has been taken from Culick (1988) with only minor revision.

3.1 Convective Waves

The analysis by Chu and Kovasznay cited earlier demonstrated the independence of acoustic, viscous, and entropy waves for small disturbances within a volume. However, both viscous effects and nonuniform entropy may affect the acoustic field indirectly through processes at the boundaries. We examine here the possible influences of entropy fluctuations. These fall within the general class of convective waves, that is, disturbances that are carried with the mean flow: their propagation speed is the average flow speed. Entropy fluctuations are associated with the portion of temperature fluctuations not related isentropically to the pressure fluctuation, such as non-uniformities of temperature due, for example, to combustion of a mixture having non-uniformities in the fuel/oxidizer ratio. In general, an entropy wave may be regarded as a nonuniformity of temperature carried with the mean flow.

As shown by Chu (1953) pressure waves incident upon a plane flame will cause generation of entropy waves carried downstream in the flow of combustion products. Thus one should expect that when combustion instabilities occur, there must be ample opportunity for the production of entropy fluctuations. That process has negligible effect directly on stability (the coupling between acoustic and entropy waves is second order within the volume) but there has long been interest in the possible consequences of entropy waves for the following reason.

When an entropy wave is incident upon the exhaust nozzle, it must pass through a region containing large gradients of mean flow properties. A fluid element must retain its value of entropy and for this condition to be satisfied, the pressure and density fluctuations cannot be related by the familiar isentropic relation, $\delta p \sim \gamma \delta \rho$. As a result, within the nozzle pressure changes are produced that will generate an acoustic wave that will propagate upstream. Thus, an entropy wave incident upon an exhaust nozzle can produce an acoustic wave in the chamber, augmenting the acoustic field due to other sources.

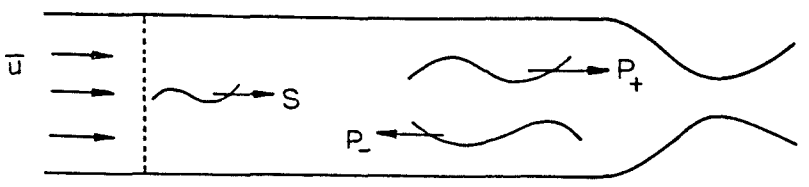


Figure 4

An artifical elementary example will illustrate the proposition. Consider a chamber admitting uniform constant mean flow at the head end, say through a choked porous plate; the flow exhausts through a choked nozzle (Figure 4). Suppose that at the head end a heater is placed, arranged so that its temperature can be varied periodically, with frequency ω. This action produces a continuous temperature or entropy wave convected with the flow. An experimental realization of this situation has been described by Zukoski and Auerbach (1976). We assume no losses within the flow, so a fluid element retains its entropy; small perturbations s' of the entropy satisfy the equation

$$\frac{\partial s'}{\partial t} + \bar{u}\frac{\partial s'}{\partial z} = 0 \tag{1}$$

If S is the amplitude of the fluctuation at the heater ($z = 0$), the solution for s' is

$$s' = Se^{-\omega(t-\frac{z}{\bar{u}})} \tag{2}$$

To simplify the calculations, assume that the flow speed is vanishingly small so that we may ignore its effect on acoustic waves. Then the acoustic pressure and velocity fields can be expressed as sums of rightward

and leftward traveling plane waves:

$$p' = \left[P_+ e^{ikz} + P_- e^{-ikz}\right] e^{-i\omega t}$$
$$u' = \left[U_+ e^{ikz} + U_- e^{-ihz}\right] e^{-i\omega t} \qquad (3)a,b$$

As usual, the complex wavenumber is $k = (\omega - i\alpha)/\bar{a}$ where $\bar{a}$ is the average speed of sound. The acoustic pressure and velocity must in the problem satisfy the classical acoustic momentum equation,

$$\bar{\rho}\frac{\partial u'}{\partial t} + \frac{\partial p'}{\partial z} = 0 \qquad (4)$$

Separate substitution of the forms for the rightward and leftward traveling waves shows that $U_\pm$, $P_\pm$ are related by

$$\bar{\rho}\,\bar{a}U_+ = P_+ \quad ; \quad \bar{\rho}\,\bar{a}U_- = -P_- \qquad (5)$$

Assume that the head end acts as a perfect reflector for the acoustic waves, so

$$u' = 0 \quad ; \quad \frac{\partial p'}{\partial z} = 0 \qquad (z = 0) \qquad (6)$$

In a real case (e.g. if the heater were actually a flame) the pressure fluctuations would cause fluctuations of entropy at the head end. To represent this effect, set s' proportional to p' at $z = 0$:

$$s' = A_0 p' \qquad (z = 0) \qquad (7)$$

Tsien (1952), Crocco (1953) and Crocco and Cheng (1956) have shown that the boundary condition at the nozzle entrance may be written in the form

$$p' + \bar{\rho}\,\bar{a}A_1 u' + A_2 s' = 0 \qquad (z = L) \qquad (8)$$

We may now show that the problem formulated here admits solutions representing steady acoustic oscillations in the chamber, whose stability depends on the values of the coefficients A_0, A_1, A_2. We eliminate the unknown amplitudes S, P_+, P_- and obtain a characteristic equation for the complex wavenumber k, by satisfying the boundary conditions (5)–(8). Substitute equations (3) and (5) into (6) to find

$$P_+ - P_- = 0 \qquad (9)$$

With (2) and (3)a, the condition (7) is satisfied if

$$S = A_0(P_+ + P_-) \qquad (10)$$

Finally, substitution of (2), (3)a,b and (10) in (8) gives

$$\left[\left(1 + A_1\right)e^{ikL} + A_0 A_2 e^{i\frac{\omega}{\bar{u}}L}\right]P_+ + \left[\left(1 - A_1\right)e^{-ikL} + A_0 A_2 e^{i\frac{\omega}{\bar{u}}L}\right]P_- = 0 \qquad (11)$$

With $P_- = P_+$ from (9) we have the characteristic equation

$$e^{i2kL} = \frac{-1}{(1 + A_1)}\left[1 - A_1 + 2A_0 A_2 e^{i(k + \frac{\omega}{\bar{u}})L}\right] \qquad (12)$$

Generally A_0, A_1, A_2 are complex numbers. The real and imaginary parts of (12) provide transcendental equations for the real and imaginary parts $(\omega/\bar{a}, \bar{\alpha}/\bar{a})$ of k. The solutions are unstable if $\alpha > 0$, corresponding to self-excited waves. Note that in the limiting case of no entropy fluctuations ($A_0 = 0$) and a rigid wall ($A_1 \to \infty$) at $z = L$, (12) reduces to $e^{i2kL} = +1$ or $\cos 2kL = 1$ and $\sin 2kL = 0$. Then $k = n\pi/L$ and the allowable wavelengths are $\lambda = 2\pi/k = 2L/n$, the correct values for a tube closed at both ends.

This example suggests the possibility for producing instabilities *if* entropy waves are generated and *if* those waves interact with the boundary in such a way as to produce acoustic disturbances. It is in fact a genuine possibility that has been considered both in laboratory tests and as an explanation of instabilities observed

in actual engines. The difficulties in applying this idea are largely associated with treating the processes responsible for causing the entropy waves.

In a combustion chamber, possible sources of entropy fluctuations may be distributed throughout the chamber. Burning of non-uniform regions of fuel/oxidizer ratio and interactions of pressure distributions with combustion zones are important causes, both producing non-isentropic temperature fluctuations. Thus in general the property that in inviscid flow free of sources an element of fluid has constant entropy, is inadequate. A proper description of entropy waves should be placed in the broader context accounting also for convective waves of vorticity as worked out first by Chu and Kovasznay (1957).

The sort of approximate calculation described above can be carried out equally well for vorticity: just replace the entropy fluctuation by the vorticity fluctuation. That is the gist of a simplified model discussed by Jon elsewhere in this report.

Those computations have produced two main results: they confirm the view that convective waves constitute a possible mechanism for instabilities; and they show that the frequencies of coupled acoustic/convective wave modes can be significantly different from those of perturbed classical acoustic modes. Some of the numerical results and some experimental tests as well, have shown peaks in power spectral densities that apparently are not related to excitation of classical modes. Those observations strongly suggest that convective waves participate in some combustion instabilities, although incontrovertible proof has not been given.

Waugh treated two models of instabilities associated with entropy waves [Waugh et al. (1983); Waugh and Brown (1984)]. In one model, the source of entropy fluctuations was concentrated at a single axial location, and in the second, several concentrated sources were used. The calculations required are modest extensions of the example given in Section 3.3. According to those results, distributed combustion tends to be more stable than concentrated combustion when the chief mechanism for instability is the convected entropy wave.

In a work intended to investigate the stability of unsteady motions with combustion in a dump combustor, Humphrey and Culick (1986, 1987a, 1987b) used the results worked out by Chu (1953) for the unsteady behavior of a plane flame. The upstream boundary condition at the inlet was set with the one-dimensional analysis of the shock response [Culick and Rogers (1983)]. Those works once again established the existence of coupled acoustic/entropy modes that do not reduce to classical modes when the entropy fluctuations vanish: they arise in addition to the classical modes which themselves are of course slightly modified when entropy fluctuations are present.

Prompted by high speed films of the unsteady flow in a dump combustor [Davis (1981)], Abouseif, Keklak and Toong (1984) postulated that the instabilities were due to coupling between entropy waves and acoustic waves. The basic model was essentially that described above. Periodic shedding of hot spots from the recirculation zone near the dump plane was interpreted as a consequence of periodic heat release causing oscillations of temperature. Predictions of the frequency were about 10 per cent below the observed values. The authors speculated that the difference may be due to their assumption that the combustion zone — and hence the source of entropy waves — was concentrated at the dump plane. Apparently no effort was made to model a distributed combustion zone and no comparison was made between those coupled modes and classical acoustic modes that could be excited directly by interaction with shed vortices. The stability of the modes was calculated (i.e. values of the growth constant) but data was not available for comparison.

Waugh and Brown (1984) also applied their analysis of acoustics with convective waves to Davis's data. They noted that Abousseif et al. had used an incorrect boundary condition at the nozzle. The corrected calculations produced frequencies quite close to those observed, and the mode shapes as well showed better agreement with test results.

3.2 Vortex Shedding and Combustion Instabilities

The presence of swirling, spinning or vortex motions in propulsion systems has long been recognized as a serious problem. They fall broadly into two classes: those with angular momentum directed along the axis, usually (if the rocket itself isn't spinning) related to standing or spinning transverse acoustic modes of the chamber; and those having angular momentum mainly perpendicular to the axis, associated with vortex shedding from bluff bodies or rearward facing steps.

Motions identified as forms of transverse or tangential modes do not normally qualify as mechanisms: they are themselves the combustion instability. Here we are concerned with vortex motions growing in unstable shear layers. Those vortices, now commonly called "large coherent structures" [Brown and Roshko (1974)] are convected downstream at approximately the average speed of the two streams forming the shear layer. In propulsion systems, the shear layers in question are generally formed in flow past bluff body flame holders (in thrust augmentors) or past rearward facing steps (in ramjet engines).

Observations of vortex shedding from flameholders, and recognition of the importance of this process as a possible mechanism for combustion instabilities were first independently reported by Kaskan and Noreen (1955)

and by Rogers (1954) and Rogers and Marble (1956). Both experiments used premixed gaseous fuel and air flowing past a flameholder in a rectangular channel. However, the particular mechanisms proposed were very different. Figures 5 and 6 taken respectively from Kaskan and Noreen (1955) and Rogers and Marble (1956) clearly show the vortex shedding.

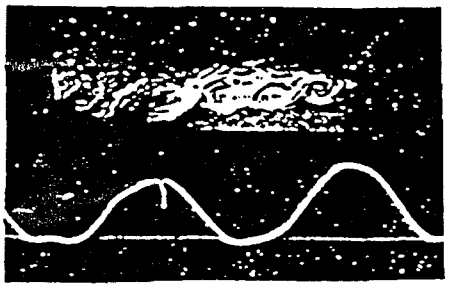


Figure 5

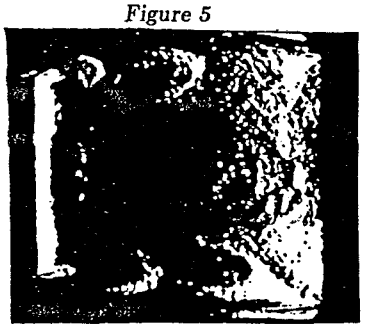


Figure 6

Motivated partly by earlier observations of Blackshear (1953) and Putnam and Dennis (1953), Kaskan and Noreen speculated that stretching of the flame front accompanying roll-up in the vortex causes a pressure disturbance. Periodic disturbances generated by periodic vortex shedding may then sustain either transverse or longitudinal acoustic fields. (The observed both in their tests.) As a quantitative basis for interpreting their results they modified a theoretical relation derived by Chu (1953) for plane flames. Although they had modest success comparing their reasoning with their data, Kaskan and Noreen did not provide a complete explanation of the closed-loop process required to generate self-excited oscillations. This mechanism has not subsequently received much notice as a cause for combustion instabilities, although the idea has recently been revived in connection with work on ramjet combustion. [Hegde et al. (1986, 1987, 1988); Reuter et al. (1988)].

Rogers and Marble gave detailed reasoning to support their idea that delayed periodic combustion in shed vortices generates periodic pressure pulses that serve as sources of the acoustic field (transverse in their tests). The fluctuating velocity of the acoustic field itself initiates the vortex shedding, thereby closing the loop. Rogers and Marble drew on earlier data for the ignition delay in flow past bluff bodies [Zukoski and Marble (1955)] to demonstrate that vortex combustion could in fact occur in proper phase to support the acoustic vibrations.

During the past six years, the idea that vortex shedding is a dominant factor in mechanisms for many combustion instabilities has gained growing support. Practically all of the work has been motivated by problems of longitudinal oscillations in ramjet engines. Even though the frequencies are substantially lower than those of the oscillations treated by Rogers and Marble, the essentials of the idea seem to hold true.

The problem of longitudinal oscillations in small ramjet engines was apparently first recognized by Hall (1978). Rogers (1980a, 1980b) gave thorough summaries of the available experimental work. Those reports served as the basis for an early analysis of the problem by Culick and Rogers (1983); that work did not include a satisfactory mechanism. Concurrently, Byrne (1981, 1983) proposed that vortex shedding in a dump combustor appeared to be a likely cause of the observed instabilities. Apparently unaware of the earlier work by Rogers and Marble on transverse oscillations, he based his argument on known results for cold jet flows.

Since the early 1980's a great deal of attention has been given to the role of vortex shedding in dump combustors, both in cold flow and in laboratory combustion tests [e.g. Keller et al. (1982); Smith and Zukoski (1985); Biron et al. (1986a, 1986b); Schadow et al. (1987); Sterling and Zukoski (1987); Poinsot et al. (1987); Yu et al. (1987)]. There is little doubt now that indeed the coupling between periodic energy released by combustion in shed vortices and the acoustic field is the dominant mechanism in dump combustors.

Extensive experimental work on vertex shedding in shear layers and jets at room temperature has provided a fairly complete picture of the formation of vortices; vortex pairing; and the general features of the flow without heat addition. Tests in various configurations, including those appropriate to ramjets [e.g. Flandro et al. (1972); Culick and Magiawala (1979); Dunlap and Brown (1981); Brown et al. (1981, 1983, 1985); Schadow et al. (1984)] established the ability of shed vortices to drive acoustic resonances over a broad range of flow conditions. The works cited above have extended that conclusion to flow with large heat addition accompanying combustion under circumstances simulating those found in actual ramjet engines.

The obvious qualitative importance of combustion in large vortices has prompted several recent analytical investigations of the process. Broadly the idea is that the shear layer is formed at the edge of a bluff body, the high speed stream consisting of an unburnt mixture of reactants; the low speed stream is composed largely of hot combustion products forming the recirculation zone behind the body. As Smith and Zukoski (1985) and Sterling and Zukoski (1987) have shown, the shear layer exhibits widely varying degrees of stability depending on the operating conditions. We are concerned here with cases when the layer is highly unstable, a situation encouraged by the action of the acoustic velocity forcing oscillations of the layer at the lip. Large vortices may then rapidly form, entraining unburnt mixture on one side of an interface, with the combustion products on the other side. A flame is initiated at the interface and the question to be answered is: how does the rate of combustion, and therefore heat release, vary as the vortex rolls up and propagates downstream?

Marble (1985) treated an idealized case of a diffusion flame initiated along a horizontal plane when simultaneously the velocity field of a line vortex is imposed along an axis in the interface. Elements of flame initially in the interface are caused to execute circular motions and are stretched by the vortex field, causing an increase in the rate at which reactants are consumed. The expanding core contains combustion products but as the vortex roll-up continues, the rate of consumption always remains greater than that for flame in the flat interface having the same length as that in the rolled-up vortex. Karagozian and Marble (1986) carried out a similar analysis accounting for the influence of stretching along the axis of the vortex. They found that, following a transient period during which the core grows to its asymptotic form, the augmented consumption rate is unaffected by axial stretching. In those cases the rate of heat release reached a constant value monotonically: there is no distinguished period of pulsed combustion as required for the mechanism for instability described above.

More recently, Laverdant and Candel (1987, 1988) have treated both diffusion and premixed flames in the presence of vortex motion with finite chemical kinetics. Their analysis is entirely numerical giving good agreement with those of Karagozian and Marble and Karagozian and Manda (1986) for a vortex pair.

Norton (1983) also analyzed the influence of finite chemical kinetics in the problem posed and solved by Marble (1985) who had assumed infinite reaction rates. Under some conditions, the heat release rate shows a modest peak in time. However, neither his results, nor those of Laverdant and Candel, suggest the sort of time delay to pulsed combustion one might like to see to complete the picture.

No work has been accomplished to determine whether or not the augmented reaction rates found in the analysis are sufficient to explain the mechanism of instabilities driven by vortex combustion. On the other hand, the experimental results reported by Smith and Zukoski (1985), Sterling and Zukoski (1987), and Yu et al. (1987) show vividly and beyond doubt that unsteady combustion associated with vortex motions is a vigorous source indeed.

Although most experimental work related to vortex shedding in ramjets has been done with two-dimensional or coaxial configurations, the phenomenon has also been found in side-dump combustors. Stull et al. (1983) have reported early work with the geometry and Nosseir and Behar (1986) have examined similar cases in a small scale. More extensive results with full-scale hardware were discussed by Zetterstõm and Sjõblum (1986) who investigate configuration having two or four inlets. Visualization in a water tunnel revealed the presence of vortex shedding. Instabilities in the operating engines were avoided by modifying the fuel injection systems in such a fashion as to minimize combustion within the vortices. That's an important practical result clearly supporting the general picture of vortex shedding as a dominant mechanism.

The essential ideas of vortex combustion as a mechanism for driving instabilities can be easily incorporated in the approximate analysis described in Section 5. There is ample evidence that large vortices in cold flow can sustain resonances in a duct; Flandro (1986) has shown one means of handling the process analytically based on direct fluid mechanical coupling between vortical and acoustic motions. See also Aaron and Culick (1985) for an elementary model of coupling associated in the impingement of a vortex on an obstacle. Tests with combustors have shown, however, that the amplitudes of oscillation are substantially greater when burning occurs. That result is most likely due to the unsteady energy release. We therefore assume that this is the main source of driving. This appears as a forcing function in the equation for the time dependent amplitude η_n, of the nth

mode. If only this energy source Q' is retained, the equation is that for a forced harmonic oscillator,

$$\frac{d^2\eta_n}{dt^2} + \omega_n^2\eta_n = \frac{\gamma - 1}{\bar{p}E_n^2}\int \psi_n \frac{\partial Q'}{\partial t} dV \tag{13}$$

where ψ_n is the spacial mode shape, $E_n^2 = \int \psi_n^2 dV$ and the integral is the volume of the chamber. Thus the problem comes down to constructing a model for Q'. No analysis of this process has been published, but the following simple example illustrates the idea.

Consider excitation of longitudinal modes and assume that the vortex travels parallel to the axis. The problem may be treated within the one-dimensional approximation, in this case meaning that the influences of the vortex are averaged over planes transverse to the axis. Figure 7 is a sketch of the situation. The origin $z = 0$ is at the dump plane which is not the location of a pressure anti-node. In fact, we must allow the acoustic velocity to be non-zero at the beginning of the shear layer.

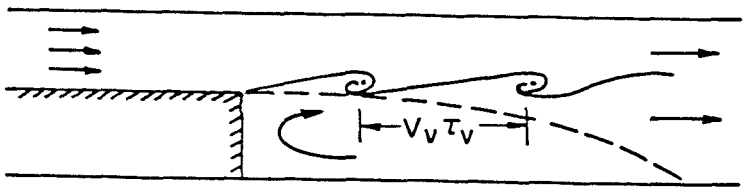


Figure 7

The vortices can be approximated as point sources of energy propagating with constant speed and launched periodically from the step. Pulses of energy may be assumed to be generated at each vortex, at some time following its initiation. Moreover, as suggested in the introduction, the strength of each vortex and the accompanying energy release can be related to the velocity fluctuation at its birth. No purpose is served by developing the formal description here; further details are given by Culick (1988) but the calculation has not been carried to completion. The main point is that, as for the case of convective waves discussed above, this mechanism is fairly easily accommodated within the approximate analysis.

4. The Time Lag Model Applied to Combustion Instabilities in Ramjet Engines

During the past seven years, Reardon (1981, 1983, 1984, 1985) has used the time lag model to correlate and interpret the extensive data taken by Davis (1981). Using his experience with liquid rockets, he has applied the time-lag method to analyze extensive data taken at the Air Force Wright Aeronautical Laboratories. He has restricted his attention to cases in which the oscillations in the combustion chamber were evidently bulk modes for which the pressure is closely uniform in space but pulsates in time. The work has been recently summarized by Reardon (1988).

The time lag model is unwieldy (at best) to use if combustion is allowed to be distributed and the time lag is variable. Hence as in many previous applications to liquid rockets, Reardon assumes that the energy release is concentrated in a transverse plane, that the parameters (n, τ) are constant, and that the flow field is one-dimensional. Then the combustion response is given by the part of equation (3.14) depending on frequency; to represent concentrated combustion, the average distribution is replaced by δ-function. A modest change in the argument allows one to use this form for the unsteady conversion of liquid to vapor, or for unsteady energy release.

Reardon assumes that the oscillations observed by Davis are bulk modes in the combustor: the pressure is essentially uniform in space and pulsates in time. Hence the mode shape $\psi(\vec{r})$ is approximately constant and one may assume that the total unsteady energy release due to combustion processes in the chamber, $\dot{E}_c$, is given by

$$\dot{E}_c = \dot{E}_0 n(1 - e^{-i\omega\tau})\frac{p'}{\bar{p}}$$

The rate of change of energy in the chamber is the net result of energy released by combustion and the rates at which energy is convected in and out of the combustor:

$$\frac{dE}{dt} = \dot{E}_c + \dot{E}_{in} - \dot{E}_{out}$$

This relation is the basis for Reardon's treatment of the experimental results.

In applications of the time lag model to instabilities in liquid rockets, both parameters (n, τ) have been determined by matching a theoretical result to experimental results for the stability boundary. The idea then is that those values of (n, τ) can be used to predict the stability characteristics for new (but in some sense similar) designs. Here, Reardon has chosen to use values of n calculated by Crocco and Cheng (1956) and to compute the time lag independently, using previous results obtained by others. In short, Reardon essentially assumes that the combustion model is known (defined by the two parameters (n, τ) with concentrated combustion) and then uses the relation for the balance of energy in the chamber to correlate data.

Stability of oscillations may be determined by application of the Nyquist criterion after the unsteady energy balance is re-written by using the Laplace transform. This possibility arises because the problem of self-excited combustion instabilities can be interpreted as a linear system with a negative feedback loop. The stability criterion, expressed with the growth constant α, depends on other processes included in the energy balance. The formal result may therefore be used to test the importance of those processes by comparison with data.

In addition to examining possible effects of vortex shedding, Reardon has considered fluctuations of the energy release due to variations in the flow rates of reactants; influences of pressure and temperature variations in the flame zone; and convective waves, generating pressure waves as a result of regions of non-uniform entropy (temperature) carried into the exhaust nozzle by the average flow. He attempted to correlate data for the frequencies, amplitudes and incidence of oscillations in terms of geometric and operating characteristics. His chief conclusion is that oscillations in the feed rates of fuel and air seem to be dominant causes of oscillations in the cases he examined. The results are limited — it is difficult to see how they can be generalized – and are contingent upon several assumptions of the values for crucial parameters. Nevertheless, he has succeeded in bringing some order to a large set of data.

5. Approximate Analysis Based on Galerkin's Method

With the recent developments in high speed computers serious consideration must be given to extensive numerical analysis of internal flows and combustion instabilities based on the complete equations of motion. That is a formidable task, particularly when the formation and combustion of liquid sprays is treated. It appears that the most extensive current program of that sort is being carried out in France [Garnier et al. (1988)]. All such work must rely to some extent on correlations of data, particularly in representations of some of the processes involved in sources of energy and mass. Similar considerations are required as part of an approximate analysis, although the form of the results described here is quite forgiving because spacial and time averaging tend to smear inaccuracies.

The analytical work described here has been concerned primarily with answering the question: what causes unsteady motions in combustion chambers to grow and reach the limiting amplitudes observed in practice? Thus the formulation encompasses both the causes or mechanisms of instabilities, and the linear and nonlinear processes influencing the time evolution of the motions. This analysis offers several important advantages.

First, because it begins by replacing the partial differential conservation equations by total differential equations, the expense of obtaining specific results is greatly reduced. Second, the formulation is generally applicable to combustion instabilities in any combustion chamber. One purpose of the recent review [Culick (1988)] of instabilities in liquid-fueled systems was to suggest how the various phenomena can be accommodated within the framework of the approximate analysis used here. In particular, one may treat any known mechanism; the problem of analyzing a special case comes down to modeling the important processes. Thus it becomes possible to assess quantitatively the relative influences of the energy gains and losses associated with the physical processes accounted for.

Third, for theoretical purposes, the formulation is convenient because any combustion instability is represented as a system of coupled nonlinear oscillators. This form permits easy interpretation of the behavior and is accessible to analysis by contemporary methods of nonlinear dynamical systems [Paparizos and Culick (1988a, 1988b)]. A corollary is that for practical purposes, numerical computations can be routinely performed with assurance that the truth of the results can be established, within, of course, the physical approximations used to establish the initial system of equations. Insufficient work has been done to determine broadly the accuracy of the method. Early work some years ago, and a limited amount of recent work [Culick and Yang (1989)] have shown that the accuracy is very good indeed for both linear and nonlinear behavior, at least when the amplitudes of pressure oscillation are less than 10%.

Finally, the structure of the formulation, in which an arbitrary motion is expressed as a synthesis of normal modes, lends itself directly to the general problem of active control. The recent developments of high-capacity, light-weight and inexpensive high-speed computers makes conceivable the active control of combustion instabilities in full-scale propulsion systems. A few laboratory tests at Cambridge University and Ecole Centrale have shown some success controlling oscillations in small devices. Application to large systems is far off and will no doubt require many sensors and control inputs. Development of practical systems will require considerable further research, both theoretical and experimental.

We furnish here only the briefest description of the analysis. In general, the presence of a liquid phase must be accounted for. The conservation equations for the gas and liquid phases can be combined in the following convenient form [Culick (1988); Culick and Yang (1989)]:

$$\frac{\partial \rho}{\partial t} + \vec{u}_g \cdot \nabla \rho = \mathcal{W} \tag{14}$$

$$\rho \frac{\partial \vec{u}_g}{\partial t} + \rho \vec{u}_g \cdot \nabla \vec{u}_g = -\nabla p + \vec{\mathcal{F}} \tag{15}$$

$$\frac{\partial p}{\partial t} + \bar{\gamma} p \nabla \cdot \vec{u}_g = -\vec{u}_g \cdot \nabla p + \rho \tag{16}$$

where $\vec{u}_g$ is the gas velocity, but ρ is the mass-averaged density of the two-phase mixture. The formulas for the source terms are given in the references cited and need not be repeated here.

We shall drop the subscript on the velocity, $\vec{u}_g \rightarrow \vec{u}$, and all of the following discussion will be phrased as if we are dealing with a single gas. That simplifies matters without excluding any essentials of the formalism. In our work, we have accounted explicitly for the liquid phase in only one specific problem reported by Yang and Culick (1984a).

All dependent variables are now written as sums of mean and fluctuating values and terms retained to the desired order. Eventually a nonlinear wave equation for the pressure fluctuation can be formed, with its boundary condition:

$$\nabla^2 p' - \frac{1}{\bar{a}^2} \frac{\partial^2 p'}{\partial t^2} = h \tag{17}$$

$$\hat{n} \cdot \nabla p' = -f \tag{18}$$

We now use a form of the method of least residuals, essentially Galerkin's method. This approach was first applied to combustion instabilities in liquid rockets by Zinn and Powell (1968, 1970). Independently, essentially the same idea was worked out for solid propellant rockets by Culick (1971, 1975, 1976), the basis for the discussion here.

For most problems of practical interest, the motions may be approximated quite well as combinations of a small number of classical acoustic modes, denoted here $\psi_n(r)$. Thus it is reasonable to expand the field in the normal modes with time varying amplitudes:

$$p' = \bar{p} \sum \eta_n(t) \psi_n(\vec{r}) \tag{19}$$

$$\vec{u}' = \sum \frac{1}{\gamma k_n^2} \dot{\eta}_n(t) \nabla \psi_n(\vec{r}) \tag{20}$$

The classical modes satisfy the equations for problems without sources:

$$\nabla^2 \psi_n + k_n^2 \psi_n = 0 \tag{21}$$

$$\hat{n} \cdot \nabla \psi_n = 0 \tag{22}$$

In some situations, it may be useful to use an inhomogeneous boundary condition for ψ_n. It is a practical necessity to ensure that the ψ_n are orthogonal,

$$\int \psi_n \psi_m dV = E_n^2 \delta_{nm} \tag{23}$$

Substitution and some manipulations with equations (17) and (18) lead to the equation for η_n:

$$\frac{d^2 \eta_n}{dt^2} + \omega_n^2 \eta_n = F_n \tag{24}$$

where

$$F_n = -\frac{\bar{a}^2}{\bar{p}E_n^2}\left\{\int h v_n dV + \oiint f \psi_n ds\right\} \tag{25}$$

These two equations are the basis for the approximate nonlinear analysis.

A major part of the effort in analyzing specific problems divides into two parts: construction of appropriate contributions to the functions h and f; and solution to the linear problems associated with the physical situation being studied. Then nonlinear problems can be treated. The coupled set of nonlinear equations (24) can be solved numerically after F_n has been given explicit form. However, there is considerable advantage in recognizing that the observed behavior of interest here generally consist of oscillations having slowly varying amplitudes and phases. Thus it is sensible to write the amplitudes $\eta_n(t)$ in the form

$$\eta_n(t) = \gamma_n(t)\sin(\omega_n t + \phi_n(t) = A_n(t)\sin\omega_n t + B_n(t)\cos\omega_n t \tag{26}$$

Application of the method of averaging leads eventually to the set of coupled first order equations for $A_n(t)$ and $B_n(t)$:

$$\begin{aligned} \frac{dA_n}{dt} &= \alpha_n A_n + \theta_n B_n + f_n \\ \frac{dB_n}{dt} &= \alpha_n B_n - \theta_n A_n + g_n \end{aligned} \tag{27}a,b$$

where f_n, g_n are nonlinear functions of the A_n, B_n. The constants α_n, θ_n are the growth rate and frequency shift due to linear processes. If f_n and g_n are neglected, equations (27)a,b readily solved to give

$$\begin{aligned} A_n &= A_{n0}e^{\alpha_n t}\cos\theta_n t \\ B_n &= -B_{n0}e^{\alpha_n t}\sin\theta_n t \end{aligned} \tag{28}a,b$$

The assumed form (25) for the amplitude is then

$$\eta_n(t) = A_{n0}e^{\alpha_n t}\sin(\omega_n + \theta_n)t \tag{29}$$

Thus the matter of solving linear problems comes down to computing α_n and θ_n: if α_n is positive the nth mode is unstable. Generally the frequency shift θ_n is not of interest in practice. Calculation of α_n and θ_n requires carrying out the integrals defining F_n, equation (25). If all quantities are assumed to vary harmonically in time, $\eta_n = \hat{\eta}_n e^{i\bar{a}kt}$, etc., where $k = (\omega - i\alpha)/\bar{a}$, one finds [Culick (1988)] the formulas

$$\begin{aligned} \theta_n &= -\frac{1}{2\omega_n}F_n^{(r)} \\ \alpha_n &= -\frac{1}{2\omega_n}F_n^{(i)} \end{aligned} \tag{30}a,b$$

Two pieces of information are needed before F_n can be computed: the contributing processes represented in h and f must be modeled; and the unperturbed mode shapes ψ_n must be found. The most important processes are those associated with the mechanisms of instabilities. Two have been discussed in Section 2. More extended discussions may be found in the references.

In a recent series of works, Yang, Paparizos, Culick, Humphrey and Kim (1983–1988) have studied the application of an approximate analysis based on Galerkin's method and time-averaging to explain combustion instabilities. The analysis provides a general framework for interpreting and predicting the behavior of unsteady motions generally in combustion chambers. It accommodates longitudinal and transverse modes and, because of its broad nature, any mechanism can be studied. The results are quantitative, giving formulas for linear stability as well as limited results for the existence and stability of periodic limit cycles. Moreover, explicit representations of Rayleigh's criterion have been derived [Culick (1987, 1988)].

The mode shapes must be computed as the solutions to the eigenvalue problem posed as equations (21) and (22). In some situations a different boundary condition may be required to produce acceptable results. For example, in a ramjet combustor, the influence of motions in the inlet diffuser are substantial. Comparison with experimental results has confirmed the correctness of the approach followed here.

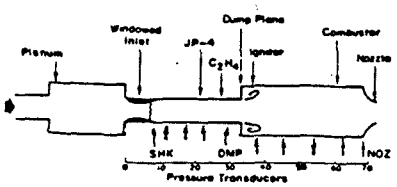
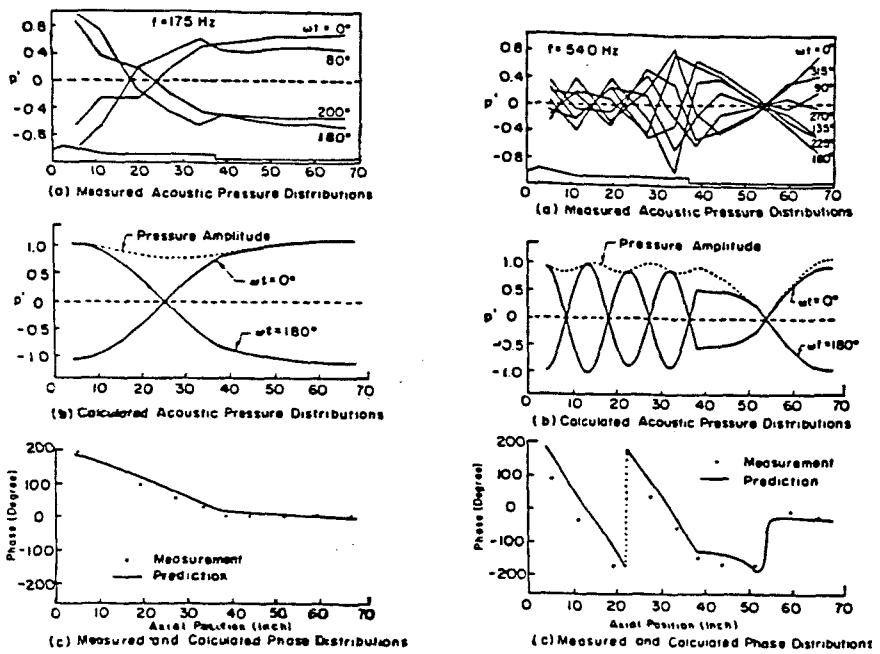


Figure 8



(a) (*b*)

Figure 9

The most extensive measurements of mode shapes in dump combustors were made at the Naval Weapons Center by Schadow and co-workers. A summary of the results, with references to the previous work, was published by Crump et al. (1986). Figure 8 shows the geometry of the sub-scale laboratory device; some results of measurements and analysis are reproduced in Figure 9. A case in which a bulk mode is excited in the combustion chamber (175 Hz) is shown in Figure 9(a); the fundamental wave mode was excited in the chamber excited for the case shown in Figure 9(b) (540 Hz). The calculated results were based on a one-dimensional analysis [Yang (1984)] in which combustion was ignored and the mean flow was accounted for only in the inlet. The good agreement is further evidence of the point emphasized in Sections 1 and 2, that the mode shapes and frequencies for combustion instabilities are often well-approximated by results based on classical acoustics. Here we also find that the one-dimensional approximation works well. For those calculations, the inlet shock was represented with the admittance function computed by Culick and Rogers (1983). It is apparently a good approximation that for these cases, the shock system is highly absorbing: the reflected wave has much smaller amplitude than the upstream-traveling incident wave. That fact, and the presence of the high speed average flow, explains why the relative phase varies linearly in the inlet.

Clark and Humphrey (1986) have also reported fairly good results obtained with a one-dimensional analysis

applied to a side-dump configuration. The engine was supplied from a large plenum through inlets that were not always choked. Although the frequencies of oscillation, phase distributions throughout the device, and amplitude distributions within the combustor were predicted well, the amplitude distributions within the inlets diffuser considerably from the measured results. The reasons for the differences are not known. Yang and Culick (1985) later carried out a numerical analysis including vaporization of the liquid fuel and were able to predict quite well both the distribution and level of the pressure field.

6. Concluding Remarks

The general approach described here for approximate analysis of combustion instabilities has proven very effective. Grounded firmly in classical acoustics, the method allows one to accommodate any processes occurring in combustion chambers. Limitations are set chiefly by one's ability to construct faithful models of the processes. The accuracy of the formal results has been demonstrated for some simple cases by comparison with "exact" numerical solutions to the conservation equations. In that respect more work is required.

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