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Propagation of an Acoustic Pulse of Finite Amplitude in a Granular Medium

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Abstract—A study of propagation of a wide-band acoustic signal in a granular medium is reported. Experimental data on the propagation of pulses with an amplitude up to 3 MPa and characteristic length about 1 μs through a sample of cobalt-manganese nodules are compared with a computer model of the process. An anomalous signal absorption in the high-frequency range observed with relatively weak sounding pulses is explained under the assumption of a fractal sample structure on a certain scale. When the signal amplitude increases, the absorption assumes a normal power form which is evidence of substance structural changes.

Remote sounding of porous marine sediments give rise to problems of sound propagation in a wide frequency range and in sediment structure. In this case, the level of the sounding signal may be high enough to take into account nonlinear distortions arising in signal propagation [1–3].

A special feature of the general theory of sound propagation in fluid-saturated porous media [4–7] is that it takes into account viscous losses incurred in liquid motion relative to a hard skeleton. These losses increase as f^2 at low frequencies and as $f^{1/2}$ at high frequencies. The losses due to friction of hard particles turn out to be proportional to the frequency f. A combination of these losses is used to describe the majority of experimental data on acoustic absorption in porous media. At the same time, in the high-frequency range, such sound attenuation in inhomogeneous structured media may depend on frequency to a much higher degree [8].

This situation poses a question about the role of the porous substance structure in sound propagation in a wide frequency range. This paper gives the results of an experimental study and a computational modeling of the propagation of finite-amplitude acoustic pulses through a sample of cobalt-manganese nodules (CMN). Such nodule formations cover vast areas of the oceanic floor in regions with hydrothermal sources. The porosity of such nodules is 50–60%, while their floor arrangement represents densely packed spherical granules of the size of several tenths of a millimeter.

In our experiment (Fig. 1), a powerful acoustic pulse with an amplitude up to 30 MPa was excited in water in the process of absorption of an optical pulse from a CO_2 laser. Laser radiation was injected into the liquid through a 1-cm-thick plate of zinc selenide transparent for CO_2 lasing wavelengths. Thus, the plate played the role of a rigid boundary where optical generation of an almost unipolar acoustic pulse occurred. In Fig. 1, this plate is shown as a shaded area. The length of an excited pulse was about 1 μs. The area of acoustic signal excitation measured 20 mm. A 40-mm-wide and 15-mm-thick CMN sample was positioned at a distance of 50 mm from the area of acoustic signal excitation. Acoustic sensors utilizing a polarized HDPE film were positioned both behind and ahead of the sample.

Figures 2 and 3 show characteristic oscilloscopic traces of both incident and propagated acoustic pulses. Due to absorption in the CMN sample, the leading edge of the pulse is slightly spread. As the amplitude increases, the pulse crest flattens, the negative phase of the propagated signal decreases, and completely vanish at large amplitudes. Thus, pulse distortions reveal a pronounced nonlinear character, which depends on the sounding signal amplitude. We note that a 30 MPa acoustic pulse produced in the experiment does not undergo noticeable distortions in the process of propagation to a distance of 1.5 cm.

This variation of pulse shape is illustrated in Fig. 4. It shows the dependence of the normalized momenta M for both incident and transmitted signals on the incident signal amplitude. Here $M = \frac{1}{p_m}\int p(t)\,dt$, where $p(t)$ is the acoustic pulse variation in time, and p_m is the maximum pressure in the pulse. In this notation, a normalized momentum essentially determines the characteristic length of a pulse and has a dimension of time. While the normalized momentum of the incident signal almost does not change in a wide range of excitation laser energy (this fact reflects the linear character of signal excitation in the experimental vessel), the momentum of the signal passed through CMNs clear increases with the amplitude. Thus, the ratio of the momenta of the transmitted signal and the sounding signal increases with the sounding signal amplitude.

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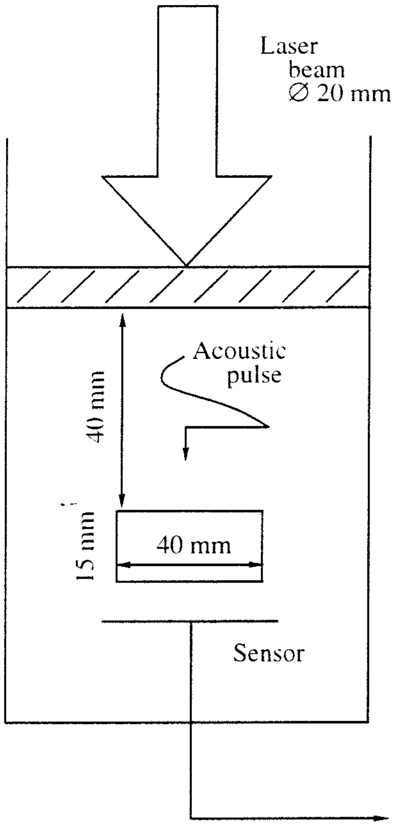

Fig. 1. Geometry of the experiment.

Taking into account the momentum conservation law, we come to the conclusion that this behavior may be caused by a change of CMN arrangement associated with a motion of nodules induced by a high amplitude pulse. In particular, we assume that granules, which do not interact in the range of small sounding amplitudes, approach each other with the growth of mechanical stress applied to the sample and, thereby, create new bonds.

At the same time, attention must be drawn to the fact that, despite a clearly pronounced nonlinear behavior of the pulse shape, its amplitude changes almost linearly with the incident signal amplitude (see Fig. 5). Thus, the nonlinear pulse shape transformation mainly lies in changes occurring at the pulse trailing edge.

The pointed pulse shape variations are reflected in the corresponding spectral parameters. Figure 6 shows typical spectra of incident and sample transmitted pulses of different amplitudes. While the spectra of incident pulses remain similar, the spectra of transmitted signals change drastically in both high- and low-frequency ranges. We note that the sounding of a CMN sample by a small amplitude signal in the high-frequency range (above 700 kHz) entails an anomalously high absorption of the signal, which increases with frequency faster than the characteristic profiles of $f^{-(1-2)}$ [1, 2]. It must be noted that the impedance characteristics of water-saturated CMN distributions were close to that of water; therefore, a considerable part of incident energy was transmitted through the CMN sample. Traditional methods of wave theory do not always adequately describe the acoustic processes in disordered media whose structure is far from chaotic. As examples of such media, we can mention polymer melts, amorphous bodies, and inhomogeneous geological structures including granular compounds like cobalt-manganese skin nodules. The reason for this is the presence of a middle-range order in the position of microscopic components. Such partially ordered media frequently have the property of scale invariance (scaling) in the statistical sense, and are described by fractal models [8]. It is difficult to describe fractal models analytically. They admit only qualitative relationships. On the other hand, fractals may be modeled with the help of computers, thus enabling numerical studies of processes in media with fractal structures.

Let us consider the wave properties of a granular material with fractal fragments. It differs from common media by the existence of localized phonon vibrational

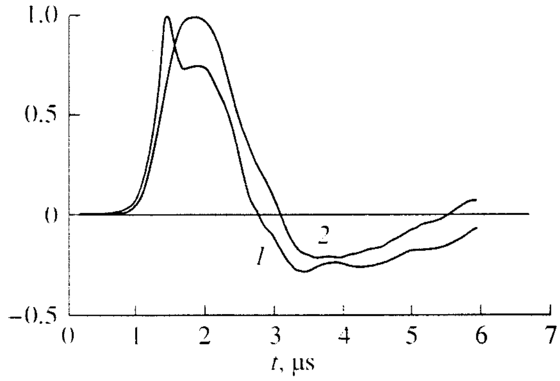

Fig. 2. Normalized oscilloscope patterns of acoustic pulses: (*1*) incident and (*2*) passed through the sample. Small amplitude. Optical radiation energy density $E = 0.08$ J/cm^2.

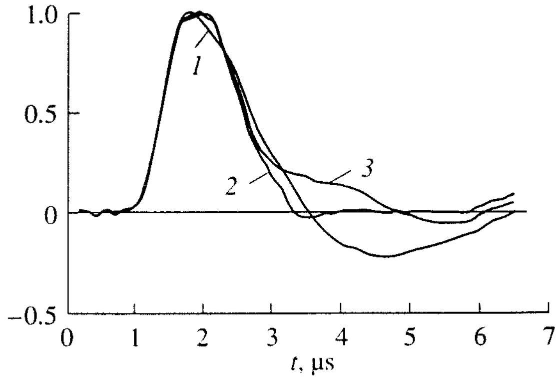

Fig. 3. Normalized oscilloscope patterns of acoustic pulses of different amplitudes passed through the sample: (*1*) $E =$ 0.08 J/cm^2, (*2*) $E = 0.8$ J/cm^2, and (*3*) $E = 1.2$ J/cm^2.

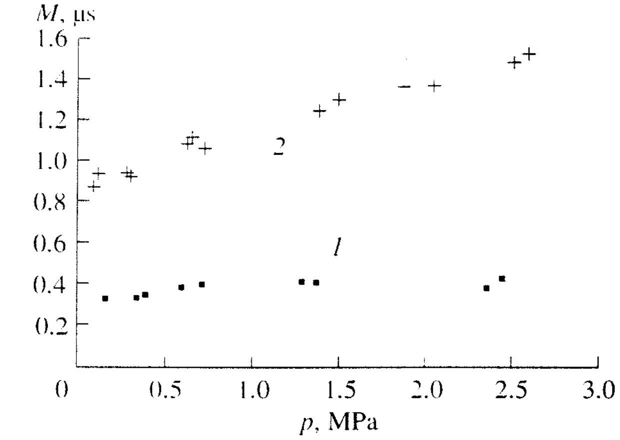

Fig. 4. Dependence of the normalized momenta of acoustic pulses on amplitude: (*1*) incident pulse; (*2*) pulse passed through the sample.

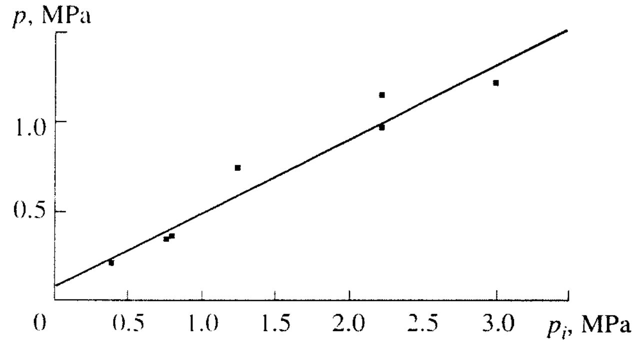

Fig. 5. Dependence of the amplitude p of the acoustic pulse propagated through the sample on the incident pulse amplitude p_i.

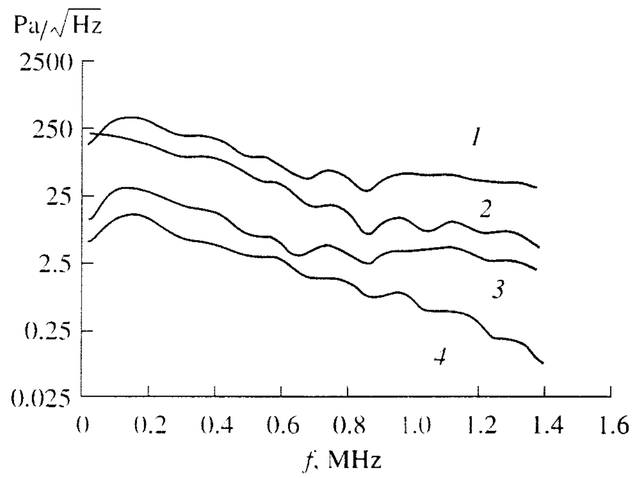

Fig. 6. Frequency spectra of signals: (*1*) incident pulse, E = 1.2 J/cm^2; (*2*) propagated pulse, E = 1.2 J/cm^2; (*3*) incident pulse, E = 0.08 J/cm^2; and (*4*) propagated pulse, E = 0.08 J/cm^2.

states called fractons, which replace common phonon states at frequencies exceeding a certain transition frequency.

Real fractal materials have the maximum scale l limiting the domain of fractal behavior. At scales exceeding l and, hence, at low frequencies, which do not exceed a certain transition frequency $f_c(l) \sim c/l$, a common phonon spectrum exists. A transition to a fracton spectrum takes place at higher frequencies. This spectrum characterizes localization of oscillation energy of sound waves in clusters. Thus, for acoustic pulses propagating in materials with a fractal structure in a certain scale range, an anomalous absorption of high frequencies must be observed. The low frequencies of an incident pulse pass through the medium "without noticing" fractal areas, while the high-frequency components get trapped by this structure. Sensors behind the sample should detect a narrower pulse more flat than the one passing through an ordinary medium. The absorption coefficient of an ordinary medium is determined by one power function of frequency for the whole frequency range $\alpha(f) \sim f^g$. The exponent g is 2 in media where energy losses are connected with viscosity and heat conduction [9]. At high frequencies, the exponent even decreases down to 0.5 in the Biot model [10], which describes sound attenuation in some types of marine bottom sediments. In the inhomogeneous friction model treating propagation of acoustic waves in soils and rocks, $g = 1$. This means that, according to known models of sound propagation in inhomogeneous media, the exponent in the function $\alpha(f) \sim f^g$ decreases as frequency increases. At the same time, the function of the absorption coefficient in fractal media will have a bend at the transition frequency, and, at high frequencies, g will exceed the respective exponent for the absorption function at low frequencies.

In order to analyze the propagation of finite amplitude sound in a granular medium, we conducted computational experiments assuming that this medium has a fractal structure. Fractal objects were formed on a two-dimensional square grid. A random number p from the interval (0; 1) was assigned to each node of the grid. If $p < p_0$, where p_0 is a constant, then a lumped mass (a granule) was placed into this node. We determine a cluster as a group of occupied grid nodes connected with the closest neighbor in the vertical or horizontal direction. Two nodes occupied by granules belong to a single cluster if they are connected by a path connecting grid nodes occupied by granules. A cluster, which extends from one side of a grid to the other, is called a connecting (percolation) cluster. A definite threshold probability exists in the limit of an infinite grid such that, for $p_0 > p_c$, there is a single connecting cluster, for $p_0 < p_c$, no single cluster exists, and all clusters are finite. In the particular case of a square two-dimensional grid, p_c = 0.59275. We conducted our calculations at p_0 = 0.8. The structure of such a fractal cell is shown in Fig. 7.

An incident acoustic pulse was modeled by a perturbation at the left boundary caused by a force acting on extreme left granules of a percolation cluster. The propagating pulse was calculated as the overall effect of extreme right granules on the right rigid boundary. The grid was infinitely extended upwards and downwards in order to avoid boundary distortions in the vertical direction. If a granule is present (absent) in a position (x, y), then it is present (absent) in all positions $(x, y + ka)$ for any integer k, where a is the spatial period along the y-axis. Thus, the grid may be presented as either quasi-infinite in the vertical direction, or rolled into a cylinder. In order to avoid distortions in the horizontal direction, which are connected with double reflection of the leading edge, the pulse length and the horizontal size of the grid were selected such that the trailing edge reached the right boundary earlier than the leading edge after two reflections (from the right and left boundaries). The pulse propagates along the cluster owing to the presence of an interaction potential between the granules. The left boundary of the medium is deemed to be free, while the right boundary is fixed.

Only the closest neighbors can affect granules. The interaction potential between neighboring granules was taken in the form:

$$U = \frac{1}{2}(dr)^2 + \frac{1}{3}(dr)^3 + \frac{10}{4}(dr)^4, \quad (1)$$

where dr is the variation of the initial distance between granules. The potential accounts for the nonlinearity of the interaction up to the third order terms. If the distance between the granules, which initially have not been closest neighbors, falls below a certain value, then the potential (1) gives rise to a repulsive force between them. In this case, dr is the difference of the distance between the granules and the grid period.

For a more adequate description of the physical processes, we introduced a weak attenuation as a force acting on the granule. This force is proportional to the oscillation velocity of the granule and opposite to it. It may be compared with a viscous drag affecting a finite granule in a flow. Thus, these percolation clusters model a porous medium consisting of solid particles submerged in liquid.

We calculated the pulse propagation in a grid of m granules (atoms) using the model equation

$$\frac{d^2 h_i}{dt^2} = f(i, h_1(t), \ldots, h_m(t)), \quad i = 1, \ldots, m, \quad (2)$$

where h_i is the two-dimensional displacement vector of atom i, and $f(i, h_1, \ldots, h_m)$ is the two-dimensional vector-function describing the granule interaction forces and attenuation. The interaction force was calculated as the derivative of expression (1). The viscous drag force was taken proportional to the granule velocity and opposite to it. The differential equation (2) was replaced by a difference equation according to the Euler scheme [11]:

$$\frac{h_i((n+1)\tau) - 2h_i(n\tau) + h_i((n-1)\tau)}{\tau^2} = f(i, h_1(n\tau), \ldots, h_m(n\tau)), \quad n = 1, \ldots, N. \quad (3)$$

The incident pulse was specified in the form:

$$p(t) = b_1 t\exp(-b_2^2 t^2), \quad t = n\tau. \quad (4)$$

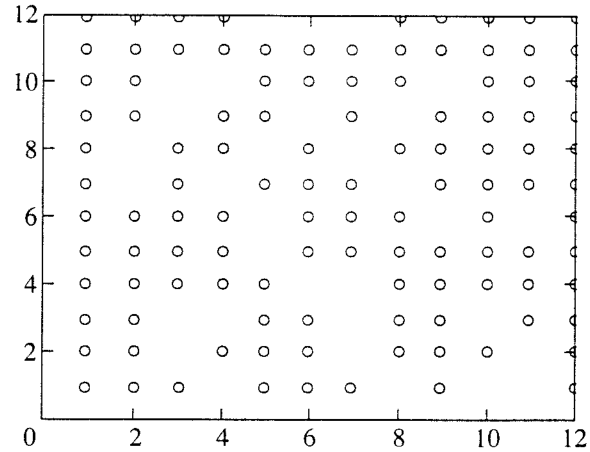

Fig. 7. Structure of a fractal cell.

Waves were studied in a medium represented by a square mesh grid rolled in a cylinder, some nodes of which were occupied by granules.

Computations were performed with a grid containing 12 × 12 nodes with 112 atoms. A sample cell is shown in Fig. 7. Pulses of small (linear case, $b_1 = 10^{-5}$) and large (nonlinear case, $b_1 = 0.3$) amplitude were considered. Since the grid period, mass of a granule, and bond elasticity are assumed to be unities [see expressions (1) and (2)], then the parameter b_1 characterizes the structural deformation beyond a certain threshold value at which the bonds between medium elements get disturbed. The parameter b_2 is selected from the condition that the pulse length must be significantly larger than the grid period and much smaller than the doubled grid size, in order to avoid the effect of multiple reflection from the boundaries. This condition can be written as $1 \leq 1/b_2 \leq 24$. Calculations were performed with $b_2^2 = 0.044$. The form of traveled pulses is shown in Fig. 8, and their transfer characteristics S/S_0 plotted to a semilogarithmic scale are given in Fig. 9. Here, S_0 and S are the spectra of the incident and transmitted pulses, respectively.

One can see that as the amplitude of deformations increases, the distortions of the pulse profile first occur at the trailing edge. The phenomenon of signal localization in the range of high frequencies at a small incident

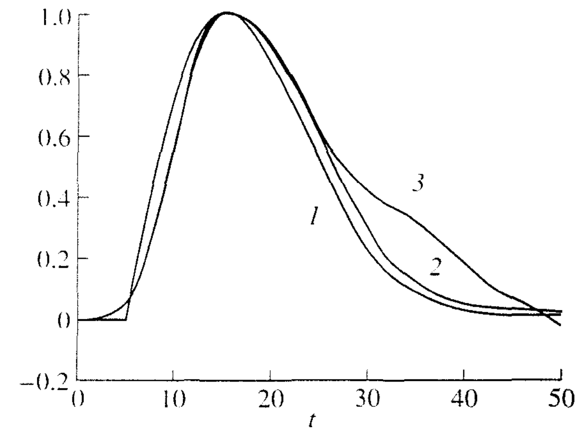

Fig. 8. Normalized pulse profile: (*1*) incident pulse, expression (4); (*2*) pulse passed through a fractal structure ($b_1 = 10^{-5}$); (*3*) pulse traveled through a fractal structure ($b_1 = 0.3$).

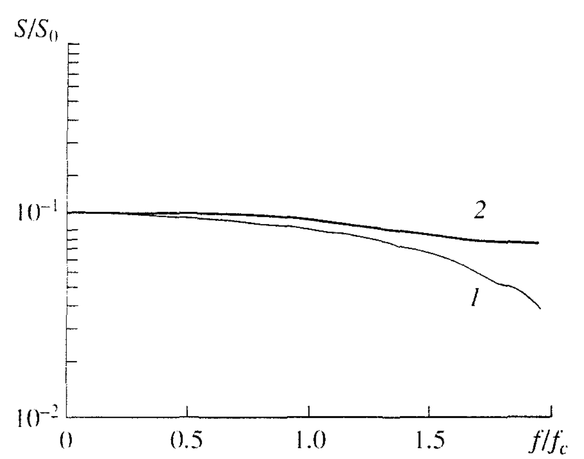

Fig. 9. Transfer characteristics of a fractal structure: (*1*) for small amplitude perturbations ($b_1 = 10^{-5}$); (*2*) for large amplitude perturbations ($b_1 = 0.3$).

pulse amplitude is reflected in a rapid decrease of the transfer characteristic at high frequencies. A sufficiently large amplitude changes the fractal structure, and those granules which were not the closest neighbors initially, come closer to a distance at which they begin to interact. The fractal medium loses its structure and becomes a randomly inhomogeneous medium. In this case, the absorption in the entire frequency range tends to the values described by a common power function of frequency. No anomalous absorption occurs at high frequencies, and the conditions for a kind of bleaching of the fractal medium in the range of high frequencies arise.

A comparison of the results of these computational studies of pulse propagation in media with a fractal structure and the experimental data on propagation of acoustic pulses through a natural fluid-saturated granular structure (see Figs. 5 and 8), reveals a certain qualitative similarity of the character of signal distortion and the corresponding transfer characteristics of these processes. First of all, this is a spreading of the pulse trailing edge as its amplitude grows (this effect increases the effective length) and a decrease of anomalous signal absorption in the high-frequency range. These findings show that porous marine bottom sediments of the type of cobalt-manganese nodule distributions have acoustic properties similar to that of the considered model of a fractal structure.

Measurements of mechanical properties of cobalt-manganese skin buildups show that their compression strength is within 1–7 MPa, and their tensile strength is one order of magnitude lower. These values are of the same order of magnitude as the compression and rarefaction parameters produced by a sounding pulse in the experiment. Therefore, the limiting values of medium deformation at $b_1 = 0.3$ used for computational modeling correspond in the order of magnitude to the experimental conditions.

Thus, a consideration of the fractal properties of a substance structure allows a qualitative modeling of the propagation of acoustic pulses of a finite amplitude through a granular medium. Modeling reveals anomalous absorption in the high frequency range at small signal amplitudes, distortions of the trailing edge of a pulse at high amplitudes, and a bleaching of the medium in the high-frequency range of the spectrum. A more detailed modeling, which reflects qualitative characteristics of processes accompanying the propagation of finite amplitude acoustic pulses in a granular medium, is possible if an adequate form of the granule interaction potential could be devised.

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