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ACOUSTIC TRANSMISSION AND REFLECTION BY A SHEAR DISCONTINUITY SEPARATING HOT AND COLD REGIONS

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Acoustic transmission and reflection is analyzed for plane waves propagating from a hot moving medium, impinging on a plane shear discontinuity into a cold stationary region. It is shown that incident waves originating in the hot region and propagating in the flow direction are transmitted into the cold region at almost right angles to the interface. The result is employed to examine the strong side radiation of internal noise transmitted through the exhaust duct of a turbojet engine.

1. INTRODUCTION

The reflection of sound by an interface separating two regions in relative motion, originally examined briefly by Rayleigh [1] was subsequently treated by Rudnick [2], Keller [3], and Franken and Ingard [4]. Miles [5], in an extensive examination of the problem, corrected an erroneous interfacial kinematic condition used in the earlier work. Utilizing the properly formulated problem, Miles [5] and Ribner [6] discussed the effects of the velocity discontinuity on the transmission, reflection, and the possibility of resonance, amplification and instability of the interface.

The effect of a simultaneous temperature discontinuity was not studied in detail, however, and it is our purpose to show that a temperature discontinuity has a major effect on the direction of propagation on the transmitted waves.

A large temperature difference exists between the warm primary exhaust gases of a turbojet and the surrounding atmosphere. The results permit simple interpretation of certain observations of the internal noise radiation.

2. ANALYSIS OF THE PROBLEM

In conformity with references [5] and [6], consider a plane interface at $y = 0$, separating two regions with distinct sound speeds and flow velocities. In region 1, $y > 0$, the sound speed is c_1 and the flow Mach number is M. Assume for simplicity that region 2 ($y < 0$) is stationary and has a sound speed $c_2 < c_1$. The pressure fluctuations are $p_i \exp(-i\omega t)$, $Rp_r \exp(-i\omega t)$, $Tp_t \exp(-i\omega t)$ for the incident, reflected and transmitted waves, respectively, and $v(x)\exp(-i\omega t)$ is the y-displacement of the vortex sheet. The incident wave has a normal direction Θ_1; the reflected wave propagates in the direction $-\Theta_1$, and the transmitted wave in the direction Θ_2 (see Figure 1):

$$p_i = \exp\left(-i\frac{k_1 x\cos\Theta_1}{1 - M\cos\Theta_1} - i\frac{k_1 y\sin\Theta_1}{1 - M\cos\Theta_1}\right), \tag{1}$$

$$p_r = \exp\left(-i\frac{k_1 x\cos\Theta_1}{1 - M\cos\Theta_1} + i\frac{k_1 y\sin\Theta_1}{1 - M\cos\Theta_1}\right), \tag{2}$$

$$p_t = \exp(-ik_2 x\cos\Theta_2 - ik_2 y \sin\Theta_2), \tag{3}$$

with

$$k_1 = \omega/c_1, \tag{4}$$

$$k_2 = \omega/c_2, \tag{5}$$

$$0 < \Theta_1 < \pi.$$

All the waves have the same wavenumber in the x-direction; thus,

$$\cos\Theta_2 = \frac{(c_2/c_1)\cos\Theta_1}{1 - M\cos\Theta_1}. \tag{6}$$

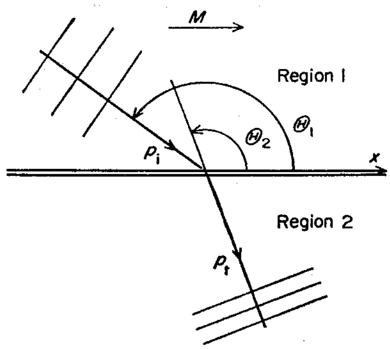

Figure 1. Geometry of the problem.

To obtain the transmission coefficient T and reflection coefficient R, the conditions of continuity of pressure and displacement at the interface must be applied:

$$(p_i + Rp_r)_{y=0_+} = (Tp_t)_{y=0_-}, \tag{7}$$

$$\left(-i\omega + U\frac{\partial}{\partial x}\right)^2 \nu(x) = -\frac{1}{\rho_1}\frac{\partial}{\partial y}(p_i + Rp_r)_{y=0_+}, \tag{8}$$

$$-(-i\omega)^2 \nu(x) = -\frac{1}{\rho_2}\frac{\partial}{\partial y}(Tp_t)_{y=0_-}. \tag{9}$$

Equations (7), (8) and (9) lead to

$$1 + R = T, \tag{10}$$

$$1 - R = (\rho_1 c_1^2/\rho_2 c_2^2)(\sin 2\Theta_2/\sin 2\Theta_1)\,T, \tag{11}$$

and yield the transmission coefficient:

$$T = 2/(1 + \rho_1 c_1^2 \sin 2\Theta_2/\rho_2 c_2^2 \sin 2\Theta_1). \tag{12}$$

The static pressures on both sides of the interface are equal and consequently the ratio $\rho_1 c_1^2/\rho_2 c_2^2$ reduces, for perfect gases, to γ_1/γ_2. It is assumed, for simplicity, that $\gamma_1 = \gamma_2$.

3. RESULTS

When the temperature is uniform, $c_2/c_1 = 1$, and the transmission angle Θ_2 is as shown on Figure 2. For an incidence Θ_1 below the direction of total reflection, $\Theta_{1R} = \arccos[1/(1 + M)]$, the transmission angle is complex, the transmitted waves are attenuated exponentially, and

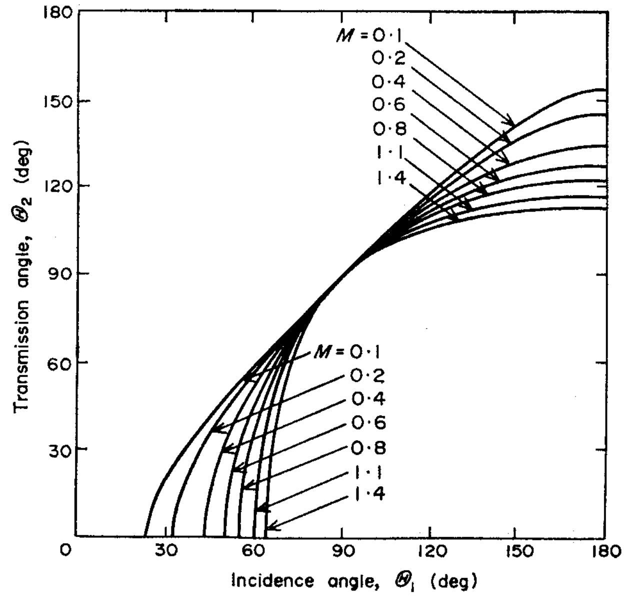

Figure 2. Transmission angle Θ_2 for a uniform temperature across the interface. Sound speed ratio $c_2/c_1 = 1$.

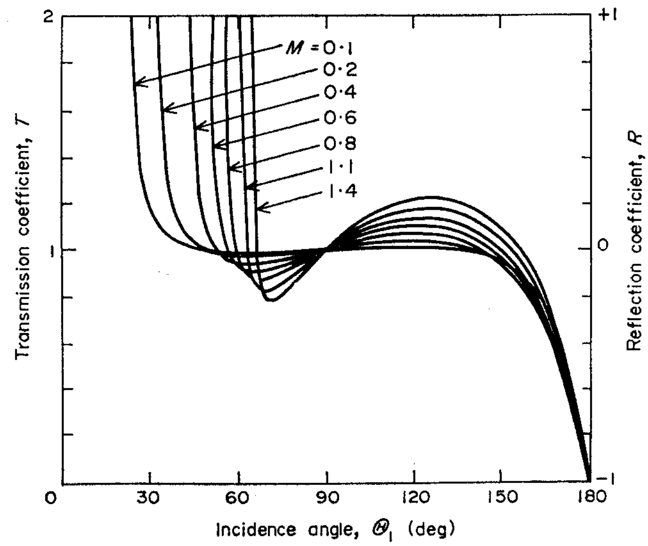

Figure 3. Transmission and reflection coefficients for a uniform temperature across the interface. Sound speed ratio $c_2/c_1 = 1$.

$|R| = 1$. Figure 3 shows the reflection and transmission coefficients, R and T. For normal incidence,

$$T(\Theta_1 = 90°) = 1,$$

and T remains around this value for a large range of incidence angles.

Next consider a strong temperature difference to exist between the streams, $c_2/c_1 = 0{\cdot}3$. Figure 4 shows that in this case the transmission angles, Θ_2, are strikingly different from those shown in Figure 2 for uniform temperature. When the value of Θ_1 is between 90° and 180°

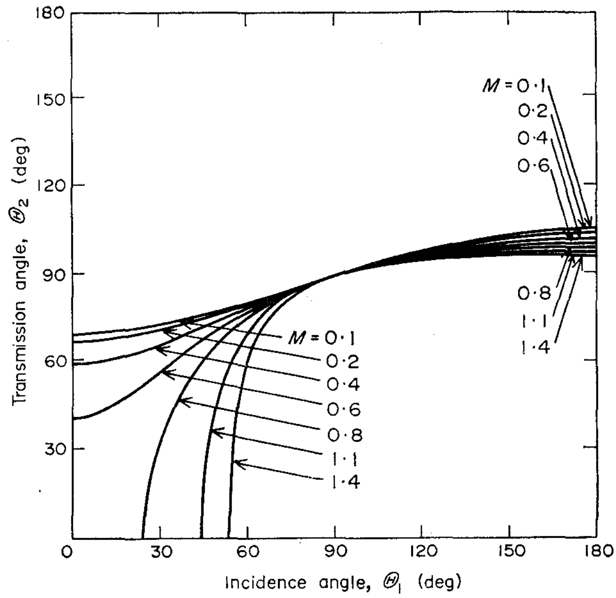

Figure 4. Transmission angle Θ_2 for a temperature discontinuity across the interface. Sound speed ratio $c_2/c_1 = 0{\cdot}3$.

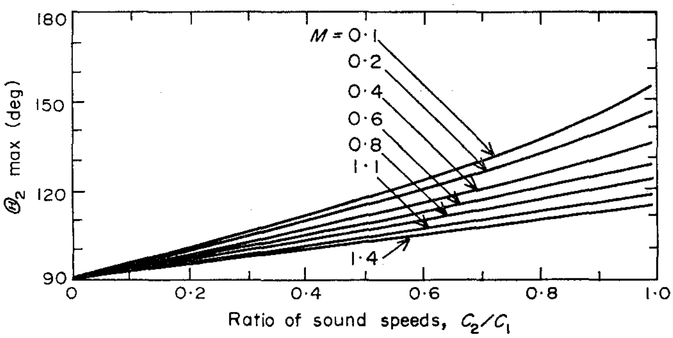

Figure 5. Maximum transmission angle $\Theta_{2\,\mathrm{max}}$. (The corresponding incidence angle is $\Theta_1 = 180°$.)

the transmission angles crowd in a small interval near $\Theta_2 = 90°$. For instance, when $M = 0{\cdot}8$, the transmission angle remains between 90° and 99°. The transmission angle is maximum when $\Theta_1 = 180°$; then

$$\Theta_{2\,\mathrm{max}} = \arccos\left(\frac{c_2/c_1}{1+M}\right). \tag{13}$$

This upper bound for Θ_2 has been represented on Figure 5 and remains below 110° for $c_2/c_1 \leqslant 0{\cdot}6$ and $M \geqslant 0{\cdot}8$. Figure 6 shows the transmission and reflection coefficients corresponding to $c_2/c_1 = 0{\cdot}3$. For normal incidence, the transmission coefficient is now

$$T(\Theta_1 = 90°) = 2/(1 + c_2/c_1) \tag{14}$$

and T remains close to this value for a large range of angles, between 70° and 150°.

The above results were derived for a plane configuration and an infinitely thin layer of discontinuity. However, even if the transition between regions 1 and 2 occurs continuously over a finite region, relation (6) between incidence and transmission angles remains valid.

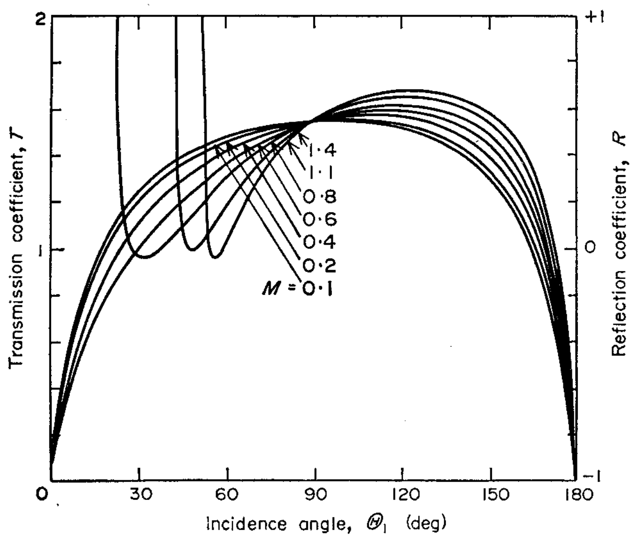

Figure 6. Transmission and reflection coefficients for a temperature discontinuity across the interface. Sound speed ratio $c_2/c_1 = 0{\cdot}3$.

The consequences for a turbojet engine may be inferred from the above ideal problem. The internal noise transmitted through the primary exhaust duct of a turbojet will be radiated in directions nearly at right angles to the jet axis. Such strong side radiation of internal noise has been observed by Grande [7] in a study of the JT8D engine and by Hawkins and Hoch [8] for the Olympus 593. The Olympus 593 has a particularly warm core when the afterburner is operating and also a slightly supersonic jet; the side radiation of internal noise is thus more pronounced.

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