Effect of flow on the acoustic reflection coefficient at a duct inlet

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The effect of duct Mach number upon the acoustic reflection coefficient at the inlet of a duct with mean flow is investigated. An analysis, which models the duct inlet as a very short, one-dimensional nozzle over which the mean flow is accelerated from rest, gives good agreement with some recent experimental results. Discrepancies between the analysis and the experimental results are discussed in terms of radiation losses at the inlet and real fluid-flow effects within the duct.

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Recent experiments by Ingard and Singhal[1] have shown that R_2, the acoustic reflection coefficient at the inlet of a duct, depends strongly upon the Mach number of the mean flow in that duct. The magnitude of R_2 was plotted as a function of duct Mach number M for measurements at frequencies such that $\omega a/c \approx 0.20$ (ω = radial frequency, a = duct radius, c = sound speed). The general trend was a monotonic decrease of $|R_2|$ with increasing Mach number. The data were represented quite well by

$$|R_2| = 0.95\left(\frac{1-M}{1+M}\right)^{1.33}. \tag{1}$$

This function has been plotted in Fig. 1 (as the solid line) for the range of M in the experiments reported by Ingard and Singhal. In those experiments, the duct was square in cross section, $\frac{3}{4}\times\frac{3}{4}$ in., and about 100 in. long, and the data [upon which Eq. (1) was based] were for frequencies of 1000, 1200, and 1400 Hz.

This behavior is quite striking, especially when compared to the weak dependence upon M of the reflection coefficient at the duct exit (see Ref. 1). No attempt was made in Ref. 1 to explain the strong dependence of R_2 upon the Mach number, but it seems that quite a simple explanation is possible.

Ignoring real fluid effects first, we idealize the duct inlet region as a one-dimensional nozzle of zero axial length in which the flow is accelerated from rest to Mach number M. Any acoustic disturbance upon this region may be viewed as quasisteady if the disturbance has finite wavelength. The problem of acoustic response of quasisteady nozzles has been investigated by Marble.[2] The solution is quite simple and may be found by matching stagnation temperature, mass flow, and entropy across the nozzle. For a nozzle with inlet Mach number M_1 and exit Mach number M we find[2]

1498

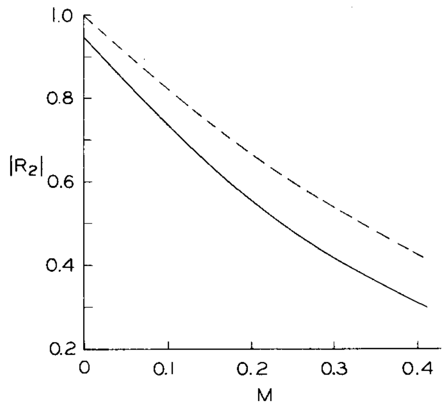

FIG. 1. Magnitude of acoustic reflection coefficient at inlet of duct with flow versus duct Mach number. Solid curve is $|R_2| = [(1-M)/(1+M)]^{1.33}$ [Eq. (1)], as reported in Ref. 1. The dashed line is $|R_2| = (1-M)/(1+M)$, which results from the quasi-steady analysis [Eq. (3)].

$$R_2 = \left(\frac{M-M_1}{1+M}\right)\left(\frac{M-1}{M_1+M}\right)\left[\frac{1-\frac{1}{2}(\gamma-1)M_1M}{1+\frac{1}{2}(\gamma-1)M_1M}\right] . \tag{2}$$

We idealize the duct inlet used by Ingard and Singhal as such a nozzle with $M_1 = 0$. Equation (2) then simplifies to

$$R_2 = -\frac{1-M}{1+M} . \tag{3}$$

The magnitude of this expression, $(1-M)/(1+M)$, is plotted in Fig. 1 as the dashed line. Clearly, the quasi-steady nozzle effect is responsible for the major part of the dependence of R_2 on the Mach number. However, Ingard's[1] experimentally measured data lie consistently below the curve given by Eq. (3). It is interesting to speculate on the reasons for this systematic difference. There are several possibilities which have to do with radiation losses and viscous flow effects.

The model we have used in deriving Eq. (3) cannot take into account any three-dimensional effects. It is clear that the radiation from the inlet will be three-dimensional, and that our model can not consider these losses. However, as the wavelength of the disturbance becomes very large (compared to the duct radius), we would expect these three-dimensional effects to decrease. It will be interesting to see what happens to low-frequency data because Ingard and Singhal[1] report that for $\omega a/c \ll 1$ the radiation resistance of the duct inlet is proportional to the square of $\omega a/c$. Thus, at low frequencies, if the radiation losses are important, the reflection coefficient should approach the compact solution limit.

Three viscous flow effects are ignored in the discussion of Ingard and Singhal. The first is the vena contracta-separated region which develops at the uncontoured inlet of a duct and which will hasten the development of turbulent pipe flow in the duct inlet region. Second, in quoting a single Mach number to characterize the duct flow, Ingard and Singhal are ignoring both the radial and axial variation of Mach number produced by wall friction.[3] The duct studied here is about 150 diameters long ($l = 150\ d$) and apparently has an unshaped inlet contour. Because of the frictional effects, one would expect that for an inlet Mach number of about 0.40, the exit Mach number would be very close to 1.0.

Because the authors of Ref. 1 make no statement concerning where measurements were made in the duct and what Mach number was used to describe the flow in the duct, it is not possible to assess the magnitude of these effects on their reported values of $|R_2|$. However, some idea of the magnitude of the effect produced by the longitudinal Mach number gradient can be gained by examining results obtained from a linearized one-dimensional analysis of the acoustic equations which is valid for $\omega a/c > 0.01$.[4] Table I shows the effects on the upstream and downstream transmission coefficients (T_u and T_d) of the change in mean Mach number produced in a pipe with fixed friction factor, $f = 0.017$, and inlet Mach number M_{inlet}. The outlet Mach number is calculated from M_{inlet} and $fl/d = 2.6$; here, T_u is the ratio of the amplitudes A of a wave propagating up the duct such that $T_u = A(\text{upstream end})/A(\text{downstream end})$. Similarly, $T_d = A(\text{downstream end})/A(\text{upstream end})$. Similar ratios are given for a region corresponding to the first quarter of the duct.

These calculations show that, for the postulated Mach number gradient in the duct and for inlet Mach numbers greater than 0.3, the attenuation of the upstream propagating wave cannot be neglected. Further, since the amplification of a downstream propagating wave is relatively small, i.e., $|T_u| \cdot |T_d|$ is always less than one, measurements of $|R_2|$ made at any point in the duct will lead to a value less than the real value, and this error will increase with the distance of the observation station from the duct entrance. For example, compare $|T_u| \cdot |T_d|$ values given in Table I for $l/d = 38$ and 150.

The conclusion, then, is that the inlet of a duct with flow can be modeled as a very short region over which the mean flow is accelerated from rest. The reflection coefficient based on this model gives good agreement with some recent measurements. The discrepancies seem to be related to radiation losses which the model does not consider and to some real fluid effects which may not have been considered when the measurements of Ref. 1 were made.

TABLE I. Effect upon magnitude of upstream and downstream propagating waves of friction in a duct with mean flow.

M_{inlet}	M_{outlet}	$\lvert T_u\rvert$	$\lvert T_d\rvert$	$\lvert T_u\rvert \cdot \lvert T_d\rvert$
		$l/d = 150$		
0.30	0.38	0.78	1.07	0.83
0.35	0.52	0.60	1.10	0.66
0.38	0.80	0.22	1.17	0.26
		$l/d = 38$		
0.30	0.31	0.97	1.00	0.97
0.35	0.38	0.92	1.01	0.93
0.40	0.44	0.86	1.03	0.89

[1]U. Ingard and V. K. Singhal, "Effect of flow on the acoustic resonances of an open-ended duct," J. Acoust. Soc. Am. 58, 788–793 (1975).
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[3]A. H. Shapiro, *The Dynamics and Thermodynamics of Compressible Fluid Flow* (Ronald, New York, 1953).
[4]M. S. Bohn, "Interaction of Acoustic Waves and Entropy Waves with High-Speed Nozzle Flows," Ph. D. thesis (California Institute of Technology, 1976) (unpublished).