# Comments on a Hanging Soap Film

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IN a recent note,[1] an apparent paradox has been encountered in the study of a soap film hanging on a horizontal circular frame. If one assumes that the "tensile force per unit length" within the film is constant at every point, the equations for static equilibrium of an element of area are not consistent. It is known, however (see, e.g., Ref. 2), that films, foams, etc., are stable in a gravitational field only if the surface energy (i.e., surface tension) is variable over the surface. The surface energy for a pure substance depends, essentially, on the temperature only, whereas for a liquid mixture it depends strongly on the relative concentrations of the constituents as well.

Films such as that under consideration are observed to be stable only if a liquid mixture, e.g., a soap solution, is used. Hence, when the equilibrium conditions are investigated, variations in the surface tension must be accounted for; the simplest example is a flat vertical film. In the present case, one may take $T$ (in the notation of Ref. 1) to be a function of $r$ only. The equations of horizontal and vertical equilibrium subsequently are found to be

$$\frac{d\phi}{dr} + \frac{\tan\phi}{r}\left(1 - \frac{d\ln T}{d\ln r}\right) = 0$$

$$\frac{d\phi}{dr} + \frac{\tan\phi}{r}\left(1 + \frac{d\ln T}{d\ln r}\right) - \frac{w}{2T\cos^2\phi}$$

from which follows the necessary condition

$$dT/dr = w/2\sin 2\phi$$

The last relation determines the manner in which $T$ varies if static equilibrium exists. Thus the paradox appears to be resolved; the more difficult problem of determining the shape of the film remains.

## References

[1] Gellatly, R. A., "A note on a soap-film paradox," J. Aerospace Sci. **29**, 1487 (1962).

[2] Adam, N. K., *The Physics and Chemistry of Surfaces* (Oxford University Press, London, 1941), 3rd ed., Chap. III.

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