

tations, which strengthens the conviction, or rather the belief, that the figure perceived is just the calculated image.

Then one disregards the fact that the perceived figure is a psychological entity, reconstructed and localized by the observer on the basis of the information supplied (mainly) by the eye and filtered through a number of processes, of which the one most widely investigated at present is the process concerned with the eye and in particular with the retina. That is, assuming that the calculated and the perceived images are the same thing, one disregards such facts as the aberrations and the irregularities of the optical system of the eye, the cellular structure of the retina, the diffusion of the energy in the optical and in the retinal media, the inhibition mechanism and, above all, the fact that the retinal process of vision is by no means static, but is considerably facilitated by the accommodation movements of the crystalline lens and, furthermore, by the micromovements of the physiological nystagmus, through which the retinal cells explore the energy distribution in the ethereal image. As is shown by the Mach approach, all this leads to a quite different representation of the energy distribution of the ethereal image, especially when luminance gradients are present.

That is a field widely investigated nowadays, and its importance is already manifest. But it is not to be forgotten that the visual process does not end at the retina and that nerve transmission to the cortical area concerned with visual sensations, as well as the psychological interpretation of the process, induce further distortions whose nature, up to now, is unfortunately quite unknown. The foregoing is known to everyone, but I have purposely summarized these few arguments just to stress the importance of discriminating among the perceived psychological image, the ethereal image, and the calculated image. Considerations of this nature ensure that, at least for the time being, there is no hope of reaching a complete knowledge of the ethereal image and that the calculated image is only a very rough representation of the actual and complete natural phenomenon.

With regard to energy, it is worthwhile to remark that the retinal process reiterates automatically a number of times per second, so that in prolonged observations the response is determined by the power, e.g., the luminous flux, and not by the total luminous energy.

Yet the time factor has an influence on the resolving power of the detected images, while obviously it does not affect the RP of the calculated images. It is odd that, with regard to calculated images, most, if not all the importance has been assigned to the diffraction effects, which are quite negligible in the case of most detected images. When the eye forms the image of a point source on the retina, the aberrations and irregularities of the eye prevent the formation of a regular centric.

Optics researchers of the old school did not object to applying an experimental control performed under conditions having nothing to do with centrics to an argument dealing with centrics.

The experiment with the two crossed slits<sup>1</sup> is puzzling. It shows that the resolution does not follow at all the rules of diffraction even when the pupil of the eye is so small that the ethereal image on the retina has an irradiance distribution approaching that computed with the known integrations.

Even under these conditions, the determining factor is still the energy. There seems no need to report here the successful series of experimental controls obtained both with artificial sources and with celestial double stars at a time when nobody dared to question the validity of the conventional RP rule.

So far I have dealt only with images detected by the eye as receiver. Analogous though quite different reasoning may be applied to images detected by photosensitive emulsions. Of course, in such a case one must take into account the granular structure of the emulsion, the diffusion of radiation in the absorbing layer, the influence of the development process, and consequently the temperature. The exposure time is of extreme importance; the possibility of accumulating the energy absorbed at successive instants makes the active energy factor the total energy received by the emulsion, and not the flux. A series of

experiments, also performed some time ago, has proved that the RP of photographic images is a function of the exposure, all other factors being constant, and that this function has a well-determined maximum. In order to obtain the maximum RP from a given combination of objective and film, the highest accuracy is demanded in choosing the right exposure. Of course, also all the other circumstances capable of influencing the result must not be neglected. They are not few and not so easy to identify.

The following episode is characteristic in this regard. About 25 years ago, some of my collaborators had been charged to carry out precision measurements on the RP of a given film. The experimental conditions had been so exactly determined that successive tests gave values constant within 1%. It was really a rare success, crowning almost a year of research. Then suddenly the result showed a variation of about 10%. An accurate re-examination of the data was required to find out the reason of the variation: the earlier measurements had been taken in August and the later measurements in September. Room temperature had varied, changing the development effect. From that time on the development was carried out with temperature control, and this cause of variation was eliminated.

I will not give here the many and intricate causes influencing the RP of images. It may be of interest, though, to remark that the diffraction of radiant energy is the least of these causes. A similar investigation may be extended to photoelectric images, so much used today in television and in electronic receivers. Of course, the RP of such images depends essentially on the energy involved and the role of diffraction is quite negligible. However, not being well acquainted with this subject, I leave it to the specialists on photoelectricity.

I realize myself that the facts I have reported here are not new, but many of them are not sufficiently appreciated. That was why I thought it justified to recall them in this note and to insist once again on the conclusion which follows from them. The RP of an instrument is not to be regarded as a characteristic of the instrument itself. One might speak of the *conventional resolving power* derived on Lord Rayleigh's assumption that the centrics must be at the separation limit when the maximum of one coincides with the minimum of the other. Let us do it, for the sake of simplicity. However, it must be borne clearly in mind that it has no definitive value in practice. To avoid misconception and misunderstanding, it would be perhaps more appropriate to state, instead of the RP, the diameter of the objective lens for a telescope, the numerical aperture for a microscope, the base length of the prism for a spectroscope, the total number of spacings for a grating. These data are unequivocal, and the researcher may use them as he sees fit. In fact, the conventional RP concerns only calculated images and it has a very little meaning compared with the RP of detected images which is instead the only practical information of interest for those who work with instruments.

The definition of RP for detected images is far more difficult, and I have already recalled that it involves the knowledge of the sensitivity characteristics of the receiver, of the energy conditions under which the detection takes place, and also possibly of other secondary factors. Let us hope it will not be too long before these arguments will become the heritage of everyday practice. It will eliminate a lot of misunderstanding and misconception and will be to the benefit of better instrumental projects.

<sup>1</sup> M. Bendetti, *Ricerca Sci. e Ricostruz.* 5, 6 (1945).

### Approximate Infrared Emissivity Calculations for HCl at Elevated Temperatures\*

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IN a recent paper,<sup>1</sup> Plass has presented the results of infrared emissivity calculations for HCl at elevated temperatures. The data were obtained through the use of machine computations by

employing the best available spectroscopic data. We wish to point out that numerical values in good accord with Plass' spectral absorption coefficients may be obtained, when the weak-line approximation holds, by utilizing the results of a relatively simple analytical treatment to the harmonic oscillator approximation. Our method is similar to an analysis first used by Kivel, Mayer, and Bethe<sup>2</sup> in emissivity calculations on the ultraviolet bands of NO. This method of treatment for electronic band systems has been refined recently by Keck, Camm, Kivel, and Wentink<sup>3</sup> in order to account for the influence of vibrational structure on emissivities.

Consider<sup>4</sup> transitions between the ground state with vibrational quantum number  $n''$  and rotational quantum number  $j''$  and the upper state  $n'$ ,  $j'$ . Then

$$hc\omega \approx (\epsilon_{n'} - \epsilon_{n''}) \pm 2j'hcBe,$$

where  $\omega$  is the observed wave number for the transition  $(n'', j'') \rightarrow (n', j')$  if  $\epsilon_{n''}$  and  $\epsilon_{n'}$  are the corresponding vibrational energies. If the dominant contributions are made by the higher rotational energy levels, then

$$\epsilon_{j'} \approx (j')^2 hcBe = [\omega/2Be - (\epsilon_{n'} - \epsilon_{n''})/2hcBe]^2 hcBe$$

and<sup>5</sup>

$$\frac{df_{n', n''}}{d\omega} = \frac{1}{2} f_{n', n''} \frac{|hc\omega - (\epsilon_{n'} - \epsilon_{n''})|}{2BekT} \times \exp\left\{-\frac{[hc\omega - (\epsilon_{n'} - \epsilon_{n''})]^2}{4hcBekT}\right\}. \quad (1)$$

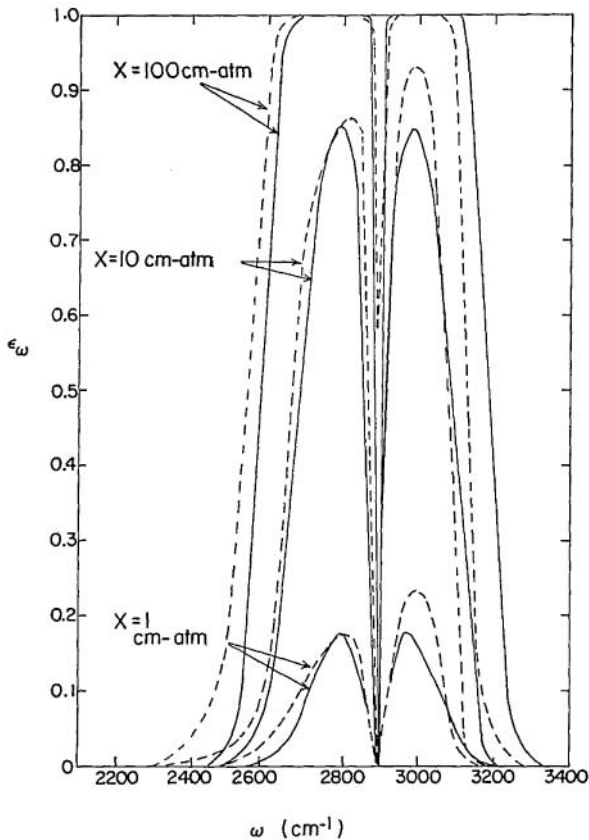


FIG. 1. Comparison of spectral emissivities in the weak-line approximation obtained by numerical calculations<sup>1</sup> (dotted curves) and spectral emissivities obtained by using Eq. (3a) (solid curves) for various optical depths at 600°K.

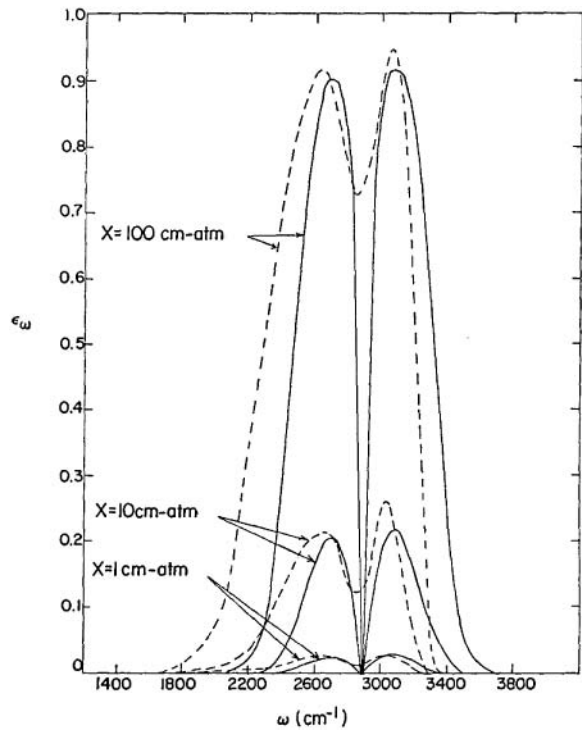


FIG. 2. Comparison of spectral emissivities in the weak-line approximation obtained by numerical calculations<sup>1</sup> (dotted curves) and spectral emissivities obtained by using Eq. (3a) (solid curves) for various optical depths at 2400°K.

In Eq. (1),  $f_{n', n''}$  is the total  $f$  number for the  $n' \rightarrow n''$  transition,  $k$  is the Boltzmann constant, and  $T$  denotes the temperature.

It is convenient to write

$$f_{n', n''} = f_{n'-n'', 0} \chi^2_{n', n''}, \quad (2)$$

where  $f_{n'-n'', 0}$  is the  $f$  number for the transition from the ground state that falls in the same spectral region as the transition  $n' \rightarrow n''$ . Since<sup>6</sup>

$$P_{\omega n', n''} \approx \frac{\pi e^2 N_{n''}}{mc^2 p} \frac{df_{n', n''}}{d\omega} (1 - e^{-hc\omega/kT}),$$

where  $P_{\omega n', n''}$  is the spectral absorption coefficient at the wave number  $\omega$  produced by the vibrational transition  $n' \rightarrow n''$  ( $e$  and  $m$  denote, respectively, the electronic charge and mass;  $N_{n''}/p$  represents the total number of molecules in the ground vibrational state per unit volume per unit pressure), we find for the spectral absorption coefficient  $P_{\omega}$  the following explicit relation:

$$P_{\omega} = \sum_{n''} P_{\omega n', n''} \approx \frac{1}{2} \frac{\pi e^2}{mc^2} f_{n'-n'', 0} \frac{N_T}{pQ_v} \frac{hc}{2BekT} [1 - \exp(-hc\omega/kT)] \times \sum_{n''} \chi^2_{n', n''} [\exp(-\epsilon_{n''}/kT)] |\omega - \omega_{n', n''}| \times \exp\left[-\frac{hc(\omega - \omega_{n', n''})^2}{4BekT}\right], \quad (3)$$

where  $N_T/p$  is the total number of molecules of the emitting species per unit pressure per unit volume. Equation (3) takes a particularly simple form to the harmonic oscillator approximation since only the transitions  $n' \rightarrow n' - 1$  are allowed in emission,  $\omega_{n', n''} \approx \omega_0 = \text{constant}$ ,  $\epsilon_{n''} = (n'' + \frac{1}{2})hc\omega_0$ ,  $\chi^2_{n', n''} = n' = n'' + 1$ . Thus

$$P_{\omega, h.o.} = \frac{1}{2} \frac{\pi e^2}{mc^2} f_{1, 0} \frac{N_T}{pQ_v} [1 - \exp(-hc\omega/kT)] [\exp(-hc\omega_0/2kT)] \times [1 - \exp(-hc\omega_0/kT)]^{-2} \frac{hc|\omega - \omega_0|}{2BekT} \left[ \exp - \frac{hc(\omega - \omega_0)^2}{4BekT} \right], \quad (3a)$$

where the subscript h.o. identifies results that are applicable to the harmonic oscillator approximation and  $\omega \approx \omega_0$  for harmonic transitions.<sup>7</sup>

Spectral emissivities

$$\epsilon_{\omega} \approx 1 - \exp(-P_{\omega, \text{h.o.}} l),$$

computed by the use of Eq. (3a), are compared with Plass' data in Figs. 1 and 2.<sup>8</sup> Reference to Figs. 1 and 2 shows that Eq. (3a) represents an acceptable approximation for the spectral absorption coefficient.

\* Supported by the Office of Naval Research under contract.

<sup>1</sup> V. R. Stull and G. N. Plass, "Spectral emissivity of hydrogen chloride from 1000-3400  $\text{cm}^{-1}$ ," Publ. No. U-461. (Aeronutronic, Newport Beach, California, May 15, 1959).

<sup>2</sup> B. Kivel, H. Mayer, and H. A. Bethe, *Ann. phys. (N. Y.)* 2, 57 (1957).

<sup>3</sup> J. C. Keck, J. C. Camm, B. Kivel, and T. Wentink, *Ann. phys. (N. Y.)* 7, 1 (1959).

<sup>4</sup> A more complete account of the theoretical treatment will be published elsewhere by S. S. Penner, K. G. P. Sulzmann, and C. B. Ludwig.

<sup>5</sup> S. S. Penner, *Quantitative Molecular Spectroscopy and Gas Emissivities* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts, 1959), p. 398.

<sup>6</sup> Reference 4, Eqs. (14-7) and (2-19).

<sup>7</sup> Equation (3a) is identical with Eq. (11-141) of reference 4.

<sup>8</sup> In our calculations we used a numerical estimate for  $f_{1,0}$  equivalent to that employed by Plass for the sum of the isotopic species  $\text{HCl}^{35}$  and  $\text{HCl}^{37}$ . This value is probably somewhat low (cf. reference 4).

## Photoelectric Measurement of Polarized Light by Means of an ADP Polarization Modulator.

### I. Photoelectric Polarimeter

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A SENARMONT compensator has been made that uses a crystal of ammonium dihydrogen phosphate<sup>1</sup> (ADP) as a retardation plate. The arrangement is shown in Fig. 1.

An alternating voltage is applied across the ADP crystal, which voltage causes the azimuth of the transmitted linearly polarized light to oscillate in synchronism with the applied voltage. The optical system of Fig. 1, not including the polarizer, is named the polarization azimuth vibrator (PAV) of Senarmont type.

Another type of PAV can be made with a different arrangement, shown in Fig. 2. This type is named the PAV of circular polarizer type. The two kinds of PAV affect the principal rays in the same way, but they differ in the way that they affect oblique rays; the conoscopic patterns are different.

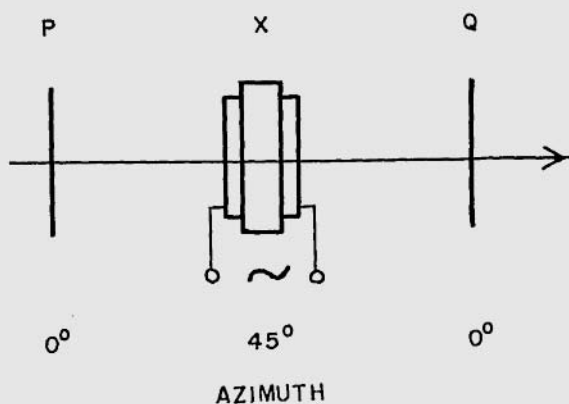


FIG. 1. The polarization azimuth vibrator of Senarmont type. The element P is a linear polarizer, X is an ADP crystal with electrodes on both surfaces, and Q is a quarter-wave plate.

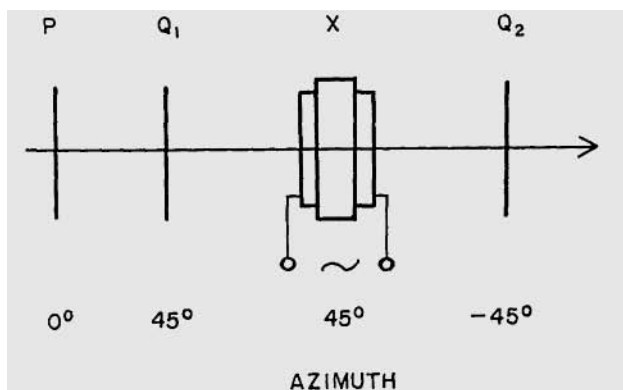


FIG. 2. The polarization azimuth vibrator of circular polarizer type. As in Fig. 1, P is a polarizer, X is an ADP crystal, and Q<sub>1</sub> and Q<sub>2</sub> are quarter-wave plates.

A photoelectric polarimeter<sup>2-4</sup> has been made that uses a PAV, as shown in Fig. 3.<sup>5</sup> The transmitted light flux is calculated<sup>2</sup> to be

$$F' = \frac{1}{4} F_0 (1 + \cos 2\theta \cos \epsilon - \sin 2\theta \sin \epsilon), \quad (1)$$

where  $\theta$  is the azimuth angle of the polarizer and  $\epsilon$  is the angular retardation of the ADP.

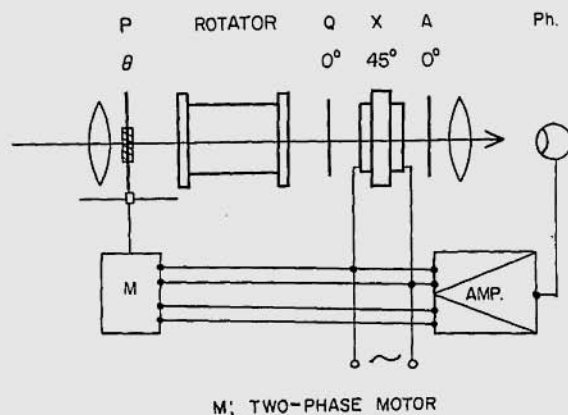


FIG. 3. A photoelectric polarimeter. The servo system rotates the polarizer P so that its rotation compensates the rotation produced by the material under study.

It is now supposed that the angle  $\epsilon$  varies sinusoidally in time

$$\epsilon = KE \sin \omega t \quad (2)$$

and that  $\epsilon$  is so small that  $\sin \epsilon$  is equal to  $\epsilon$ , and  $\cos \epsilon$  is represented by  $1 - \frac{1}{2} \epsilon^2$ . Then

$$F' = \frac{1}{4} F_0 \left\{ (1 + \cos 2\theta) - \frac{1}{4} K^2 E^2 \cos 2\theta + \frac{1}{4} K^2 E^2 \cos 2\theta \cos 2\omega t - KE \sin 2\theta \sin \omega t \right\}. \quad (3)$$

In this expression the first and second terms on the right represent dc outputs, the third term is an ac signal with twice the driving frequency, and the last term has the fundamental frequency and is proportional to  $\sin 2\theta$ . This signal is amplified by a frequency selective circuit and is fed to a balancing motor that rotates the polarizer so as to make  $\theta$  zero.

A PAV is a kind of half-shade device. The optimum half-shade angle  $\epsilon$  is a function of the extinction constant of the optical system, and is usually in the range from  $0.5^\circ$  to  $3.0^\circ$ . Accuracy of the polarimeter has been found to be about 1 min of angle.

This polarimeter has the following advantages:

1. The half-shade angle is easily adjusted to the desired value.