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Effect of Scale Size on a Rocket Engine With Suddenly Frozen Nozzle Flow[1]

RONALD WATSON[2]

California Institute of Technology, Pasadena, Calif.

Recent analytical work by Bray indicates that single element, chemically reacting systems, e.g., $H_2 \leftrightarrows 2H$, may suddenly freeze their composition at some point in a high speed nozzle and then remain at fixed composition throughout the remainder of the expansion. This sudden freezing or "quenching" phenomenon was also apparent in some theoretical calculations reported by Hall et al. and has been verified experimentally by Wegener. It is the purpose of this note to show qualitatively how Bray's sudden-freezing criterion is related to engine size by the scale factor for geometrically similar engines having nonequilibrium nozzle flows and in which a propellant system is used for which Bray's analysis is valid.

Theory

USING Lighthill's model for an "ideal" dissociating diatomic gas (1),[3] Bray (2) calculated properties of initially dissociated oxygen and nitrogen systems during expansion through a hypersonic nozzle. His results showed that the flow was near equilibrium in the upstream region of the nozzle but that a relatively sudden[4] freezing of composition occurred downstream of the throat. He attributed the sudden freezing to the effective vanishing of atom recombination because of the low densities; i.e., he assumed, the recombination rate depends on density cubed (since it is a three-body collision process) and thus is very low at low gas densities.

Since in near-equilibrium flow, the rates of both recombination and dissociation are of comparable magnitudes, and since in near-frozen flow the dissociation rate is negligibly small, Bray reasoned that the quench point must be in the neighborhood where the dissociation rate is comparable to the time rate of change of the mass fraction of the atomic species during an equilibrium expansion. That is

$$\left|\frac{d\alpha}{dt}\right| \ll r_D \text{ (for near-equilibrium)}$$

$$\left|\frac{d\alpha}{dt}\right| \gg r_D \text{ (for near-frozen)}$$

Therefore

$$\left|\frac{d\alpha}{dt}\right|_Q \cong r_D \text{ (for the quench point)} \qquad [1]$$

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[2] NSF Fellow, Daniel and Florence Guggenheim Jet Propulsion Center.

[3] Numbers in parentheses indicate References at end of paper.

[4] The "suddenness" of the freezing depends on both the reaction rate constants and geometry. In a personal communication Bray pointed out that for nozzles with very small expansion angles, the freezing stretches out over quite some length and is far from sudden.

where

α = mass fraction of the atomic species
r_D = dissociation reaction rate
Q = subscript signifying sudden freezing point

Subsequent calculations by Bray showed that Equation [1] was indeed a satisfactory criterion for the quench point. Calculations by Hall et al. (3) and experiments by Wegener (4) confirm the essentials of Bray's conclusions.

Following Penner [see (5), chaps. 17 and 18], the quench point criterion may be rewritten in a form which is more easily related to the size of the engine, namely

$$\left|\frac{d\alpha}{dx}\right|_Q = \frac{k_R P K_P (1-\alpha)}{2u\, R^2\, T^2} \qquad [2]$$

where

x = distance along the nozzle
k_R = specific recombination rate constant
P = static pressure
K_P = pressure equilibrium constant
u = gas velocity
R = universal gas constant
T = static temperature

Application to Geometrically Similar Engines

The shape of the downstream section of the nozzles of a set of geometrically similar rocket engines may be characterized by the formula

$$A_i = A_0^*(C_i^2 + bx_i^2) \qquad [3]$$

where

A_i = nozzle area of the ith engine at any distance from the throat
A_0^* = throat area of the reference engine which is constant for a set of similar engines
C_i = scale size factor for the ith engine (see Eq. [4])
b = shape factor, which is constant for a set of similar engines
x_i = distance from the throat in the ith engine

At large distances from the throat, the nozzle geometry of Equation [3] corresponds to a cone of half-angle $\sqrt{bA_0^*/\pi}$. The distance from the throat is then

$$x_i = C_i\sqrt{(1/b)(A_i/A_i^* - 1)} \qquad [4]$$

where A_i^* = throat area of the ith engine.

Thus, for a common nozzle area ratio, a characteristic length in any two of these similar engines is related by the ratio of their scale factors, C_i/C_j.

For a given set of chamber conditions, the performance of a quenched flow engine would depend only upon the area ratio at which the flow is frozen, since this fixes the exhaust conditions. Since the distances from the throat to a given quench point area ratio for geometrically similar engines are related through the scale factors C_i, C_j, etc., and since the gas conditions at these quench points are all the same (same area ratio and equilibrium flow up to the quench point), then the slopes of the equilibrium mass fractions at these quench points are all related through Equation [4] by

$$C_1\frac{d\alpha}{dx_1} = C_2\frac{d\alpha}{dx_2} = \ldots$$

or

$$C_i \frac{d\alpha}{dx_i} = \text{constant (for same area ratio quench point)} \quad [5]$$

Then from Equation [2], for these engines to have the same exhaust conditions would require that

$$C_1 \, k_{R_1} = C_2 \, k_{R_2} = \ldots$$

or

$$k_{R_i} = (C_j/C_i)k_{R_j} \quad [6]$$

Equation [6] shows that a smaller specific recombination rate constant is required to quench the flow in a large engine at the same area ratio as in a small engine. Conversely, for the same rate constant, Equations [2 and 4] indicate that the quench point would be delayed to a lower $|d\alpha/dx|_Q$ (with resulting higher specific impulse values) in a large engine by very roughly the scale factor C_i, although no "clean" analytical expression shows this.

Bray's quenching criterion thus reinforces the intuitive feeling that recombination should proceed further in a large engine because of the longer gas residence time in the nozzle (which coincidentally increases linearly with the scale factor C_i for engines with the flow completely in equilibrium or completely frozen throughout).

Thus from the point of view of chemical kinetics only, it appears that for a given total thrust, there would be an improvement in specific impulse obtained by using one large engine rather than by "stacking" several small ones, if sudden freezing occurred somewhere in the nozzle.

References

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3 Hall, J. G., Eschenroeder, A. Q. and Klein, J. J., "Chemical Nonequilibrium Effects on Hydrogen Rocket Impulse at Low Pressures," ARS Journal, Feb. 1960, vol. 30, no. 2, pp. 188–190.

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