

Relation Between Gas Absorptivities and Emissivities*

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IN previous theoretical studies,¹⁻⁴ expressions have been obtained between infrared gas absorptivities and emissivities for several descriptions of vibration-rotation bands. Although these expressions provide a good correlation of the (limited) available experimental data, it is of interest to supplement the earlier results by the relation corresponding to absorption of radiation by molecular vibration-rotation bands composed of randomly distributed dispersion lines with arbitrary intensity probability distribution function.

The absorptivity (α_{ab}) of a gas at temperature T_g and of optical depth X for blackbody radiation at temperature T_s is given by the relation

$$\alpha_{ab} \approx \frac{1}{\sigma T_s^4} \sum_i [\bar{R}^0_{\omega_i}(T_s)] \int_{\Delta\omega_i(T_g)} \{1 - \exp\{-[P_{\omega_i}(T_g)X]\}\} d\omega, \quad (1)$$

where σ represents the Stefan-Boltzmann constant, $\bar{R}^0_{\omega_i}$ is the blackbody radiance at the wave number corresponding to the center of the i th band and P_{ω_i} is the spectral absorption coefficient of the gas. For the statistical model corresponding to a wave number interval of effective width $\Delta\omega_i(T_g)$,

$$\alpha_{ab} \approx \frac{1}{\sigma T_s^4} \sum_i [\bar{R}^0_{\omega_i}(T_s)] \left\{ 1 - \exp\left[-\frac{A_i(T_g)}{d_i^*(T_g)}\right] \right\} \Delta\omega_i(T_g), \quad (2)$$

where

$$\bar{A}_i = \int_0^\infty P(\bar{S}_i/S) A_{Li} d(\bar{S}_i/S)$$

is the weighted mean value of the line absorption A_{Li} computed with respect to the normalized intensity probability distribution function $P(\bar{S}_i/S)$, and d_i^* is the mean line spacing for lines in the i th band. For dispersion lines of half-width b , the line absorption is $A_L \approx SX$ for $SX/2\pi b < 2/\pi$ and $A_L \approx 2(SbX)^{1/2}$ for $(SX/2\pi b) > 2/\pi$. For the linear region^{1,3,4}

$$\alpha_{ab} = \left(\frac{T_s}{T_g}\right) \epsilon(T_s, X). \quad (3)$$

On the other hand, for the square root region,

$$\bar{A}_i = 2(b_i X)^{1/2} \int_0^\infty P(\bar{S}_i/S) S^{1/2} d(\bar{S}_i/S) = \text{constant} (\bar{S}_i b_i X)^{1/2}. \quad (4)$$

Hence, assuming that $\bar{S}_i = \alpha_i / (\Delta\omega_i / d_i^*)$ where α_i is the integrated absorption for the i th band, $\Delta\omega_i(T)$ proportional to $T^{1/2}$, α_i proportional to T^{-1} , b_i proportional to $T^{-1/2}$, and

$$d_i^*(T_g) / d_i^*(T_s) = \left(\frac{T_s}{T_g}\right)^\eta,$$

we obtain

$$\frac{\bar{A}_i(T_g)}{d_i^*(T_g)} = \text{constant} \frac{[\bar{S}_i(T_s) b_i(T_s) X (T_s/T_g)^{2-\eta}]^{1/2}}{d_i^*(T_s)},$$

whence Eq. (2) becomes

$$\alpha_{ab} \approx (T_g/T_s)^{1/2} \epsilon[T_s, X (T_s/T_g)^{2-\eta}]. \quad (5)$$

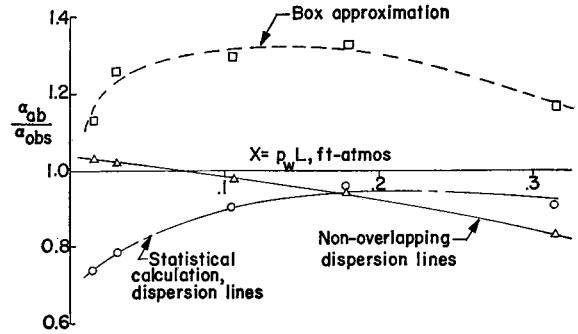


FIG. 1.

For water vapor η is about unity⁴⁻⁶ and Eq. (5) reduces to

$$\alpha_{ab} \approx (T_g/T_s)^{1/2} \epsilon[T_s, X (T_s/T_g)] \quad (6)$$

which is very nearly the same as Hottel's empirical formula for water vapor, viz.,

$$\alpha_H = (T_g/T_s)^{-0.45} \epsilon[T_s, X (T_s/T_g)].$$

For well overlapped and "just overlapping" lines it has been shown previously¹⁻⁴ that

$$\alpha_{ab} = (T_g/T_s) \epsilon[T_s, X (T_s/T_g)^{1/2}] \quad (7)$$

whereas, for nonoverlapping dispersion lines with $\eta = 1$,^{1,3,4}

$$\alpha_{ab} = (T_g/T_s) \epsilon[T_s, X (T_s/T_g)^2]. \quad (8)$$

A representative comparison of observed and calculated results is shown in Fig. 1 for water vapor. Reference to Fig. 1, and to data for larger values of X which are not plotted, shows that Eq. (6) gives a fair representation for X greater than about 0.1 or 0.2 ft-atmos; Eq. (7) holds for very small values of X and also for X greater than about 0.5 ft-atmos; Eq. (8) applies for X less than about 0.2 ft-atmos. These results are consistent with the approximations involved in the derivation of Eqs. (6) to (8). Equation (6) should apply when the absorption is dominated by dispersion lines in the square root region irrespective of the extent of overlapping between lines.

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⁶ S. S. Penner and A. Thomson, Transport Properties in Gases, (Northwestern University Press, Evanston, Illinois, 1957), pp. 151-173.