

Relation Between Gas Absorptivities and Emissivities*

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IN previous theoretical studies,¹⁻⁴ expressions have been obtained between infrared gas absorptivities and emissivities for several descriptions of vibration-rotation bands. Although these expressions provide a good correlation of the (limited) available experimental data, it is of interest to supplement the earlier results by the relation corresponding to absorption of radiation by molecular vibration-rotation bands composed of randomly distributed dispersion lines with arbitrary intensity probability distribution function.

The absorptivity (α_{ab}) of a gas at temperature T_g and of optical depth X for blackbody radiation at temperature T_s is given by the relation

$$\alpha_{ab} \approx \frac{1}{\sigma T_s^4} \sum_i [\bar{K}^0 \omega_i(T_s)] \int_{\Delta\omega_i(T_g)} \{1 - \exp\{-[P_{\omega}(T_g)X]\}\} d\omega, \quad (1)$$

where σ represents the Stefan-Boltzmann constant, $\bar{K}^0 \omega_i$ is the blackbody radiancy at the wave number corresponding to the center of the i th band and P_{ω} is the spectral absorption coefficient of the gas. For the statistical model corresponding to a wave number interval of effective width $\Delta\omega_i(T_g)$,

$$\alpha_{ab} \approx \frac{1}{\sigma T_s^4} \sum_i [\bar{K}^0 \omega_i(T_s)] \left\{ 1 - \exp\left[-\frac{A_i(T_g)}{d_i^*(T_g)}\right] \right\} \Delta\omega_i(T_g), \quad (2)$$

where

$$\bar{A}_i = \int_0^\infty P(\bar{S}_i/S) A_{Li} d(\bar{S}_i/S)$$

is the weighted mean value of the line absorption A_{Li} computed with respect to the normalized intensity probability distribution function $P(\bar{S}_i/S)$, and d_i^* is the mean line spacing for lines in the i th band. For dispersion lines of half-width b , the line absorption is $A_{Li} \approx SX$ for $SX/2\pi b < 2/\pi$ and $A_{Li} \approx 2(SbX)^{1/2}$ for $(SX/2\pi b) > 2/\pi$. For the linear region^{1,3,4}

$$\alpha_{ab} = \left(\frac{T_g}{T_s}\right) \epsilon(T_s, X). \quad (3)$$

On the other hand, for the square root region,

$$\bar{A}_i = 2(b_i X)^{1/2} \int_0^\infty P(\bar{S}_i/S) S^{1/2} d(\bar{S}_i/S) = \text{constant} (\bar{S}_i b_i X)^{1/2}. \quad (4)$$

Hence, assuming that $\bar{S}_i = \alpha_i / (\Delta\omega_i / d_i^*)$ where α_i is the integrated absorption for the i th band, $\Delta\omega_i(T)$ proportional to $T^{1/2}$, α_i proportional to T^{-1} , b_i proportional to $T^{-1/2}$, and

$$d_i^*(T_g) / d_i^*(T_s) = \left(\frac{T_s}{T_g}\right)^\eta,$$

we obtain

$$\frac{\bar{A}_i(T_g)}{d_i^*(T_g)} = \text{constant} \frac{[\bar{S}_i(T_g) b_i(T_g) X (T_g/T_s)^{2-\eta}]^{1/2}}{d_i^*(T_s)},$$

whence Eq. (2) becomes

$$\alpha_{ab} \approx (T_g/T_s)^\eta \epsilon[T_s, X(T_g/T_s)^{2-\eta}]. \quad (5)$$

For water vapor η is about unity⁴⁻⁶ and Eq. (5) reduces to

$$\alpha_{ab} \approx (T_g/T_s)^\eta \epsilon[T_s, X(T_g/T_s)] \quad (6)$$

which is very nearly the same as Hottel's empirical formula for water vapor, *viz.*,

$$\alpha_H = (T_g/T_s)^{-0.46} \epsilon[T_s, X(T_g/T_s)].$$

For well overlapped and "just overlapping" lines it has been shown previously¹⁻⁴ that

$$\alpha_{ab} = (T_g/T_s)^\eta \epsilon[T_s, X(T_g/T_s)^{1/2}] \quad (7)$$

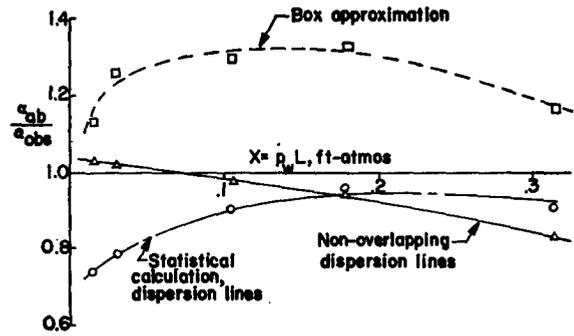


FIG. 1.

whereas, for nonoverlapping dispersion lines with $\eta = 1$,^{1,3,4}

$$\alpha_{ab} = (T_g/T_s) \epsilon[T_s, X(T_g/T_s)^2]. \quad (8)$$

A representative comparison of observed and calculated results is shown in Fig. 1 for water vapor. Reference to Fig. 1, and to data for larger values of X which are not plotted, shows that Eq. (6) gives a fair representation for X greater than about 0.1 or 0.2 ft-atmos; Eq. (7) holds for very small values of X and also for X greater than about 0.5 ft-atmos; Eq. (8) applies for X less than about 0.2 ft-atmos. These results are consistent with the approximations involved in the derivation of Eqs. (6) to (8). Equation (6) should apply when the absorption is dominated by dispersion lines in the square root region irrespective of the extent of overlapping between lines.

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¹ A. Thomson, Ph.D. thesis, California Institute of Technology, Pasadena, June, 1958.

² S. S. Penner and A. Thomson, *J. Appl. Phys.* **28**, 615 (1957).

³ Thomson, Hooker, and Penner, *J. Appl. Phys.* **30**, 124 (1959).

⁴ S. S. Penner, *Quantitative Molecular Spectroscopy and Gas Emissivities* (Addison-Wesley Publishing Company, Inc., Reading, Massachusetts) (to be published).

⁵ A. Thomson, "An approximate analytic expression for the engineering emissivity of water vapor," Technical Note No. 4, Gruen Applied Science Laboratory, Pasadena, California, April, 1957.

⁶ S. S. Penner and A. Thomson, *Transport Properties in Gases*, (Northwestern University Press, Evanston, Illinois, 1957), pp. 151-173.

Errata: Note on Stress Rupture in Glass

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THE following reference should be added to the note:

¹ E. F. Poncelet, *Glass Ind.* **38**, 551 to 558, 617 to 623 (1957).

Books Reviewed

Prompt, noncritical reviews appear in this column. Critical reviews of many of the books described here will appear in *Physics Today*, *The Review of Scientific Instruments*, or *American Journal of Physics*.

Vacuum Tube and Semiconductor Electronics. JACOB MILLMAN. Pp. 644 lxxvii. McGraw-Hill Book Company, Inc., New York, 1958. Price \$10.00.

This volume is another in the McGraw-Hill series of texts on electrical engineering. It is essentially an extension of the book entitled *Electronics* by Millman and Seely to include more material on semiconductors. The increasing applications