(*Reprinted from* JET PROPULSION, *October, 1957*)

Apparent Emission Intensities From a Turbulent Flame Composed of Wrinkled Laminar Flames[1]

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THERE has recently been some discussion on the use of the model of a wrinkled laminar flame for an approximate description of turbulent flame structure. In particular, Summerfield has claimed that some of his observations contradict theoretical predictions based on the use of this model.[4] It is the purpose of the following discussion to clarify some of the consequences of the wrinkled laminar flame model.

Apparent Product and Radical Profiles for Wrinkled Laminar Flames

Consider a turbulent flame. We assume that the structure of the one-dimensional steady-state laminar flame for the given combustible mixture is known, either from theory or experiment. Then the turbulent flame is assumed to consist of an ordinary highly wrinkled laminar flame which moves back and forth very rapidly through the spatial region occupied by the turbulent flame zone. With an instrument, the time resolution of which is long compared with the oscillation time of the wrinkled laminar flame, one sees only the time average of the motion of the laminar flame front, and therefore the turbulent flame pattern appears to be constant in time. Furthermore, regardless of the instrumental time resolution, the measured intensities will always be averaged over space (see Figs. 1 and 2).

The picture implied by the sketches shown in Figs. 1 and 2 is admittedly quite approximate. If a laminar flame front were really oscillating rapidly about a mean position, it would probably not have exactly the same structure as a steady-state laminar flame. Our model does not predict the length of time which the laminar flame spends at each point in the turbulent flame zone.

We shall use approximate representations for the radical distributions in the laminar flame in order to predict the corresponding turbulent flame structure. The roughest approximation to the distribution of radicals and products through a steady-state laminar flame involves radical concentrations which are zero outside an effective width a for the laminar flame and constant within a (see Fig. 3). We also assume that the product concentration is zero to the left of the center of the laminar flame and constant to the right (see Fig. 3).[5]

For the thickness corresponding to ordinary (wrinkled) laminar flames, it is reasonable to assume that a space average is equivalent to a time average. We may assign a probability function $p(x)$ to the turbulent flame in such a way that $p(x)dx$ is the probability that the center of the oscillating laminar flame will be found at a position x in the range dx in the turbulent flame zone. The laminar flame structure

Received June 28, 1957.

[1] Supported by the Office of Ordnance Research, U. S. Army, under Contract DA-04-495-Ord-446. The authors are indebted to S. S. Penner for suggesting this investigation and for helpful discussions.

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[4] Summerfield, M., Reiter, S. H., Kebely, V., and Mascolo, R. W., "The Structure and Propagation Mechanism of Turbulent Flames in High Speed Flow," JET PROPULSION, vol. 25, Aug. 1955, pp. 377–384.

[5] The representation used in Fig. 3 describes roughly the *emission intensity* profiles for CH and C_2 ("radicals") and for H_2O ("products") in laminar flames.

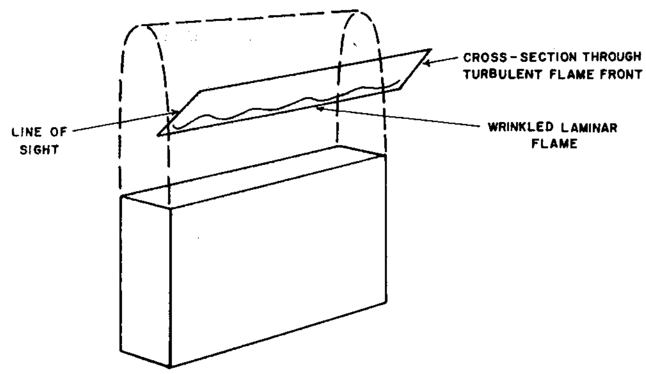

Fig. 1 Schematic illustration of space average obtained by observing an open two-dimensional turbulent flame

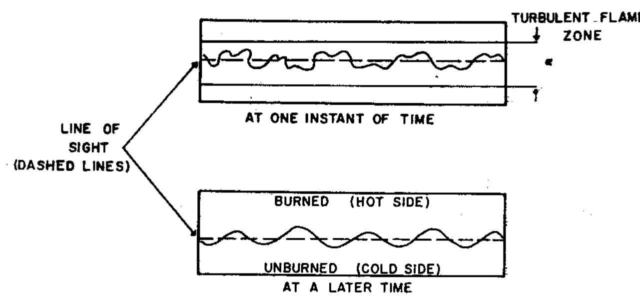

Fig. 2 Plane cutting the wrinkled laminar flame which separates the unburned from the burned gases

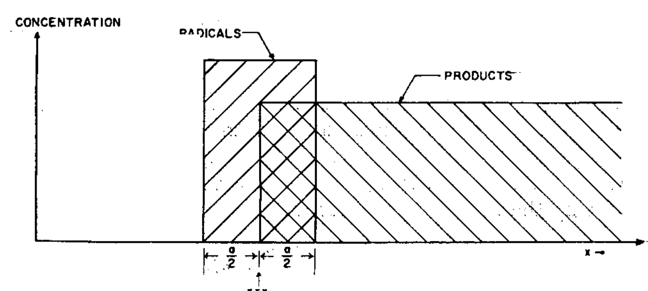

Fig. 3 Model of steady-state laminar flame

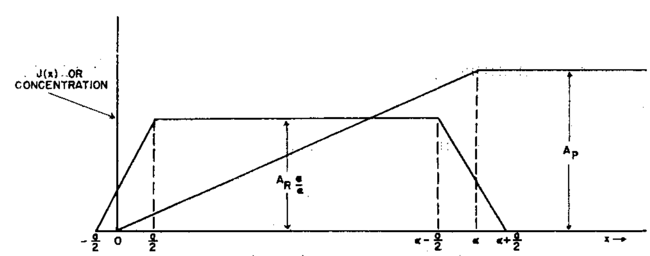

Fig. 4 The quantity $J(x)$ as a function of x for the assumed function $p(x)$

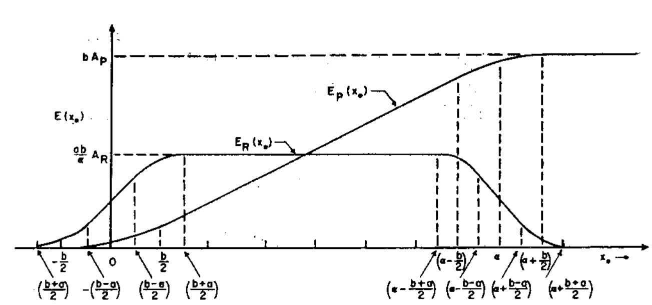

Fig. 5 Instrument response when observations are made with a slit width $b > a$ and $J(x)$ has the x-dependence shown in Fig 4

and the probability function $p(x)$ determine completely the results of emission intensity measurements on turbulent flames.

Consider the results of intensity measurements for a specified spectral line with an instrument of slit width b for a constant optical path-length d in the flame front perpendicular to the direction of propagation. Neglecting self-absorption, we may then calculate the energy per second received by the instrument, which is equivalent to the intensity reading of the instrument.

The total intensity $I_R(x, x_1)$ emitted from radicals at a position x by a laminar flame with center at x_1 is

$$I_R(x, x_1) = \begin{cases} 0 & \text{for } x < x_1 - a/2 \\ A_R & \text{for } x_1 - a/2 \leq x \leq x_1 + a/2 \\ 0 & \text{for } x > x_1 + a/2 \end{cases} \qquad [1]$$

similarly, the total intensity emitted from reaction products is

$$I_P(x, x_1) = \begin{cases} 0 & \text{for } x < x_1 \\ A_P & \text{for } x \geq x_1 \end{cases} \qquad [2]$$

where the subscripts R and P refer to radicals and products, respectively, and A_R and A_P are constants. Actually the intensity corresponding to the average over-all positions x_1 of the center of the laminar flame is observed for a real turbulent flame. If we call $J(x)$ the intensity observed for the turbulent flame at the spatial position x, then

$$J(x) = \int_{-\infty}^{\infty} p(x_1)I(x, x_1)dx_1 \qquad [3]$$

The probability is, of course, normalized in such a way that $\int_{-\infty}^{\infty} p(x)dx = 1$; $I(x, x_1)$, which depends only on $(x - x_1)$, may be called the intensity kernel. The quantity $J(x)$ is evidently proportional to the average concentration of emitters in the turbulent flame.

Actually, $J(x)$ is proportional to the reading of an instrument with an infinitesimal slit. With an instrument of finite slit width b and uniform sensitivity over b, which is centered at the position x_0, the energy per second $E(x_0)$ received by the instrument would be

$$E(x_0) = \int_{x_0 - (b/2)}^{x_0 + (b/2)} J(x)dx \qquad [4]$$

i.e., the actual reading of the instrument is proportional to

$$E(x_0) = \int_{x_0 - (b/2)}^{x_0 + (b/2)} dx \int_{-\infty}^{\infty} p(x_1)I(x, x_1)dx_1 \qquad [5]$$

Once $p(x)$ is known, $E(x_0)$ can be evaluated from Equation [5].

A reasonable approximate choice for $p(x)$ is

$$p(x) = \begin{cases} 0 & \text{for } x < 0 \\ \dfrac{1}{\alpha} & \text{for } 0 \leq x \leq \alpha \\ 0 & \text{for } x > \alpha \end{cases} \qquad [6]$$

where α is a constant approximately equal to the turbulent flame width. Equation [6] gives the probability distribution if it is equally likely to find the wrinkled laminar flame anywhere in the turbulent flame zone. Using Equation [6], we find

$$J_P(x) = \begin{cases} 0 & \text{for } x < 0 \\ A_P \left(\dfrac{x}{\alpha}\right) & \text{for } 0 \leq x \leq \alpha \\ A_P & \text{for } x > \alpha \end{cases} \qquad [7]$$

and

$$J_R(x) = \begin{cases} 0 & \text{for } x < -a/2 \\ A_R \dfrac{x + (a/2)}{\alpha} & \text{for } -a/2 \leq x < a/2 \\ A_R \dfrac{a}{\alpha} & \text{for } a/2 \leq x \leq \alpha - a/2 \\ A_R \dfrac{\alpha + (a/2) - x}{\alpha} & \text{for } \alpha - a/2 < x \leq \alpha + a/2 \\ 0 & \text{for } x > \alpha + a/2 \end{cases} \qquad [8]$$

These functions $J_P(x)$ and $J_R(x)$ are plotted in Fig. 4.

If one makes a measurement with an instrument of slit width $b > a$, which is often the case, then another integration yields

$$E_P(x_0) = \begin{cases} 0 & \text{for } x_0 < -b/2 \\ \dfrac{A_P}{2\alpha}[x_0 + (b/2)]^2 & \text{for } -b/2 \leq x_0 < b/2 \\ A_P \left(\dfrac{b}{\alpha}\right)_{x_0} & \text{for } b/2 \leq x_0 \leq \alpha - b/2 \\ \dfrac{A_P}{2\alpha}\{2\alpha b - [x_0 - \alpha - (b/2)]^2\} & \text{for } \alpha - b/2 < x_0 \leq \alpha + b/2 \\ A_P b & \text{for } x_0 > \alpha + b/2 \end{cases} \qquad [9]$$

and

$$E_R(x_0) = \begin{cases} 0 & \text{for } x_0 < -\left(\dfrac{b+a}{2}\right) \\ \dfrac{A_R}{2\alpha}\left[x_0 + \dfrac{(a+b)}{2}\right]^2 & \text{for } -\left(\dfrac{b+a}{2}\right) \leq x_0 < -\left(\dfrac{b-a}{2}\right) \\ A_R\left(\dfrac{a}{\alpha}\right)[x_0 + (b/2)] & \text{for } -\left(\dfrac{b-a}{2}\right) \leq x_0 < \left(\dfrac{b-a}{2}\right) \\ \dfrac{A_R}{\alpha}\left\{ab - \dfrac{1}{2}\left[x_0 - \dfrac{(b+a)}{2}\right]^2\right\} & \text{for } \dfrac{b-a}{2} \leq x_0 < \dfrac{b+a}{2} \\ A_R\left(\dfrac{ab}{\alpha}\right) & \text{for } \dfrac{b+a}{2} \leq x_0 \leq \alpha - \left(\dfrac{b+a}{2}\right). \end{cases} \qquad [10]$$

Evidently $E_R(x_0)$ is symmetric about $x_0 = \alpha/2$. The functions $E_P(x_0)$ and $E_R(x_0)$ are plotted in Fig. 5. For b somewhat less than a there is only a slight modification of the curves of $E(x_0)$; as $b \rightarrow 0$ the corners become sharper and the readings $E(x_0)$ approach $J(x_0)$, which is shown in Fig. 4.

The temperature distribution through a laminar flame follows a curve which is similar to that for the product concentration shown in Fig. 3. Therefore, the resulting average

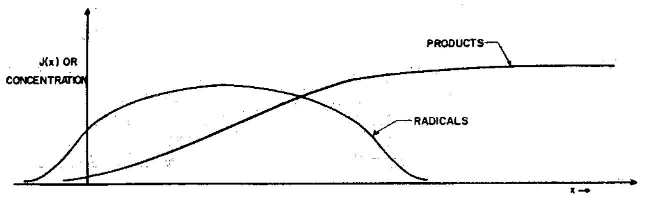

Fig. 6 Approximate concentration in the turbulent flame for a continuous function $p(x)$

temperature distribution through a turbulent flame would be approximately that shown for the products in Fig. 4, although this quantity is probably unobservable by ordinary spectrographic techniques.

Discussion

From the preceding simplified analysis a number of conclusions may be drawn regarding apparent intensity profiles for a turbulent flame composed of many wrinkled laminar flames. It is apparent that the wrinkled laminar flame model does not generally predict a rapid rise in the product concentration to its maximum value in a distance of the order of one laminar flame width. On the contrary, with the probability function $p(x)$ used by us, the product concentration rises gradually and does not reach its maximum value until near the end of the turbulent flame zone. Furthermore, it is obvious that a Gaussian distribution for $p(x)$ would lead to an even more gradual rise in the product concentration as is illustrated in Fig. 6. In order to obtain a sharp rise in product concentration one would have to use a $p(x)$ strongly skewed toward the cold end of the turbulent flame zone, which seems physically less attractive.

The wrinkled laminar flame model is also seen to predict that the radical concentration rises to its maximum value over a distance which is of the same order of magnitude as the laminar flame width or the slit width, whichever is larger, and then remains roughly constant throughout the remaining portion of the turbulent flame zone; it decreases to zero over approximately the same distance in which it rose to its maximum value. The distance over which the radical concentration appears to rise or fall is approximately proportional to the slit width for large slit widths ($b \gg a$) and is independent of the slit width for $b \ll a$.

Our particular model for $p(x)$ leads to the conclusion that the product concentration (and also the average temperature) reaches half of its maximum value in the center of the turbulent flame zone, i.e., at $x = \alpha/2$ (compare Fig. 4). It should be possible to check this conclusion experimentally.

The preceding conclusions are valid only for constant $p(x)$; they should remain nearly true for any $p(x)$ which is symmetric about $x = \alpha/2$. However, for skewed distributions the predictions do not apply.

The experiments of Summerfield seem to indicate that the onset of product emission lags the onset of radical emission by more than a laminar flame width. In the experimental studies the time of exposure was adjusted in such a way that the apparent maximum intensities of radical and product emission were the same. For our model the product intensity should increase much more slowly than the radical intensity. Adjusting the exposure time in such a way that radical and product peak intensities are equal will eliminate intensities below a well-defined minimum value. If this minimum is $(1/m)$ times the maximum intensity, then we predict a separation of about $(1/m)$ times the turbulent flame width in the apparent onset of emission of radicals and products. For most instruments m is a number between 2 and 10. Thus the separation should be $\frac{1}{2}$ to $\frac{1}{10}$ of a turbulent flame width, as has been observed actually by Summerfield and his collaborators.[4]