plete confirmation of the dimensionless parameters used in Figs. 6 and 7.

Since the small disturbances which have been observed on the surface of a liquid film flowing under the influence of a turbulent gas stream appear to be related to the gas-stream turbulence, a complete analytical investigation of their origin would be difficult. However, much useful information could perhaps be obtained from an analysis of the stability of Couette flow with two layers of fluid of different densities and viscosities. Such a flow might be stable to small oscillations for all Reynolds numbers, but it is possible that the wave lengths and velocities of the least-damped oscillations are related closely to the small disturbances observed on the surface of a liquid film. The fact that the wave length of the small surface disturbances has been observed to be approximately 10 film thicknesses makes the suggested analysis appear promising; a typical result of stability analyses is that the least-damped oscillations have a wave length of the order of ten times the characteristic length of the flow field.

The Mechanics of Film Cooling—Part 2

ELDON L. KNUTH[2]

Daniel and Florence Guggenheim Jet Propulsion Center, California Institute of Technology, Pasadena, Calif.

5 Evaporation From Stable Liquid Wall Films into Heated Turbulent Gas Streams

A Previous Studies

Of the several works published in recent years on evaporation from annular liquid wall films into heated turbulent gas streams, only the most comprehensive papers are reviewed here. These are the theoretical paper by L. Crocco (15) and the experimental paper by Kinney (13); both papers appeared in 1952.

Crocco extended Rannie's (16) approximate theory of porous-wall cooling for inert coolants to porous,[8] sweat, and film cooling for the case in which the coolant itself is reactive with the hot gas stream. The liquid film was assumed to be stable, and axial gradients were neglected in comparison with radial gradients. Crocco divided the gas stream into two regions: a central turbulent core where the gases are not affected by the addition of mass at the boundary, and a laminar sublayer adjacent to the boundary where all the effects of mass addition are confined. (The boundary referred to may be either a liquid-gas interface or a porous wall, the choice depending upon the type of cooling which is employed.) In the turbulent core, the Reynolds analogy was extended to read

$$\frac{\theta_\infty - \theta_\delta}{\dot{w}_\delta} = \frac{H_{t\infty} - H_{t\delta}}{\dot{q}_\delta} = \frac{u_\infty - u_\delta}{\tau_\delta} \qquad [1]$$

where θ is the oxidizer specific concentration (weight of oxidizer per unit total weight), $\dot{w}$ is oxidizer transfer per unit area and per unit time, H is enthalpy, $\dot{q}$ is heat transfer per unit area and per unit time, the subscript ∞ refers to bulk properties or average velocity, the subscript δ refers to the junction of the laminar sublayer and the turbulent core, and the subscript t refers to total (indicating that chemical energy, but not kinetic energy, of the fluid should be included). The thickness

[2] At present, Propulsion Engineer, Aerophysics Development Corporation, Pacific Palisades, Calif.

[8] Crocco defined porous cooling as "cooling through a porous wall with a gas or a liquid vaporized before entering the wall" and sweat cooling as "cooling through a porous wall where the coolant is liquid throughout the wall."

of the laminar sublayer in the gas stream δ was given by the relation

$$\frac{\rho_\infty \sqrt{\tau_0/\rho_\infty}\,\delta}{\bar{\mu}_M} = 5.6 \qquad [2]$$

where a bar over the symbol for a fluid property indicates that the appropriate averaged quantity should be used. (Equation [2] was extended, in the absence of better information, from Prandtl's assumption for isothermal pipe flows.) Crocco treated the Schmidt and Prandtl numbers (equal, respectively, to $\mu/\rho D$ and $c_p\mu/k$, where D is molecular mass diffusivity, c_p is specific heat at constant pressure, and k is thermal conductivity) as invariants with respect to distance in the laminar sublayer. He further considered the combustion gases to be diffused as a whole, the driving potential for mass transfer to be specific concentration, and the reaction times of the mixtures to be short in comparison with other times involved. Remarks are made in section 5-B concerning this treatment of the turbulent core and assumption concerning the driving potential for mass transfer. The assumption of negligible axial gradients in comparison with radial gradients is discussed in section 5-C.

Subsequently Crocco obtained a relation between the temperature of the boundary (either a liquid-gas interface or a porous wall) and the rate of mass addition at the boundary for given gas-stream conditions and coolant properties. This relation permits the wall temperature to be determined for porous cooling when the rate of mass addition at the boundary is given, or the evaporation rate to be determined for sweat or film cooling if the liquid-film temperature is known. (Crocco assumed the liquid-film temperature to be equal to the boiling temperature of the liquid under the prevailing pressure. A general method for calculating the liquid-film temperature, based on a proposed extension of the wet-bulb-thermometer equation, is included in section 5-A.) Results of numerical calculations for gasoline and for water, both of which were evaporating into products of combustion of gasoline and oxygen, were presented.

Kinney reported on investigations of liquid-film cooling in straight horizontal tubes. The results of the experiments were plotted on a single curve which relates the length of cooled surface with the coolant-flow rate when gas-stream parameters and fluid properties are specified.

In a comment (17) on Crocco's paper, Abramson compared the theoretical results of Crocco with the experimental results reported by Kinney. The theory predicts a greater liquid-cooled length for all coolant-flow rates than were observed experimentally; only the large deviations at high coolant-flow rates (the result of film instability) were explained by Abramson.

B Theoretical Analysis

A theoretical analysis of the mass-transfer process from a stable annular liquid wall film flowing under the influence of a fully developed turbulent heated gas stream in a duct is presented. The purpose is to show the relative importance of the several parameters which affect the evaporation rate and to determine the magnitude of the evaporation rate for given fluid properties and gas-stream parameters. It is believed that the analysis presented here differs from previous analyses in that the effects of mass addition on transport phenomena are given consideration in the turbulent core as well as in the laminar sublayer of the gas stream and the surface temperature of the liquid film is calculated instead of estimated. Although the case in which the coolant itself is reactive with the hot gas stream is not analyzed explicitly here, the results obtained may be extended in a manner analogous to that employed by Crocco to extend Rannie's work.

Assumptions. In order to facilitate computations (and still obtain useful results), a model will be considered which has the following characteristics: 1 Variations with respect to time may be neglected. 2 The effects of body forces may be neglected in comparison with the effects of viscous and inertia forces. 3 Work done by viscous and pressure forces may be neglected in comparison with heat transferred because of temperature gradients. 4 Mass transfer due to temperature gradient may be neglected in comparison with mass transfer due to partial-pressure gradient. 5 The liquid-film surface velocity may be neglected in comparison with the average gas velocity. 6 The gas stream may be divided into two regions: a center core in which the fluid flow is predominantly turbulent and a laminar sublayer (adjacent to the liquid film) in which the fluid flow is predominantly laminar. (Comparisons of heat-transfer rates obtained for turbulent pipe flows with heat-transfer rates predicted by the Prandtl-Taylor equation justify such a division into two regions when the Prandtl and Schmidt numbers do not vary from unity by more than a factor of 2. Most gases satisfy this restriction.) 7 Axial variations in the gas stream are small compared with radial variations, and the laminar sublayer thickness of the gas stream is small compared with the pipe diameter. These features permit the assumption that transfer processes in the laminar sublayer are one-dimensional. 8 The heat which is transferred to the liquid film from the hot gas by convection and condution is equal to that required for vaporization of the liquid. (This characteristic is attained if all liquid which leaves the film is in the vaporized form (i.e., the film is stable) and if the net heat which is transferred to the liquid film by radiation is equal to the heat which is transferred from the liquid film to the duct wall plus the heat which is required to warm the liquid from the injection temperature to the evaporation temperature. In many cases, these three heat quantities are negligible in comparion with the heat required for vaporization of the liquid.) 9 The eddy heat diffusivity, eddy mass diffusivity, and eddy viscosity are equal in magnitude. 10 Mass diffusion in the laminar sublayer may be treated as a binary process even when more than two molecular species are present. Such a treatment is nearly correct unless the gas stream contains large concentrations of species having widely different molecular weights, e.g., large quantities of hydrogen and carbon dioxide. (This simplifying assumption is not necessary for the turbulent core since the mass diffusion in the turbulent core is the result of macroscopic mixing rather than molecular processes.)

Solution of the Problem. Steps in the solution of the problem will include (a) the derivation of heat-, mass-, and momentum-transfer relations in the laminar sublayer, (b) the postulate of an extension of the Reynolds analogy to heat mass, and momentum transfer in the turbulent core of two-component turbulent pipe flow with unidirectional radial diffusion, and (c) the combination of results of *a* and *b* in order to obtain the desired relations between evaporation rate, fluid properties, and gas-stream parameters, applicable to stable annular liquid wall films flowing under the influence of fully developed heated turbulent gas streams when entrance effects are negligible.

First, consider the laminar sublayer. Since evaporated coolant is not being stored in the laminar sublayer, it follows immediately that $\dot{m}$ is not a function of y in the laminar sublayer, where y is distance into the gas stream from the liquid-gas interface measured perpendicularly to the film surface. Furthermore, assumption 8 implies that heat transferred by conduction across the liquid-gas interface is equal to $-\dot{m}_0\Delta H$, where ΔH is coolant latent heat of vaporization. Hence, one may write the heat, mass, and force balances for the laminar sublayer in the forms

$$-k_M \frac{dT}{dy} + \dot{m}_0\bar{c}_{pV}T = -\dot{m}_0\Delta H + \dot{m}_0\bar{c}_{pV}T_0 \qquad [3]$$

$$\frac{pD}{R_VT}\frac{d}{dy}\ln(p - p_V) = \dot{m}_0 \qquad [4]$$

$$-\mu_M \frac{du}{dy} + \dot{m}_0u = -\tau_0 \qquad [5]$$

where R is the gas constant. (Note that the driving potential for mass transfer is taken properly to be partial pressure instead of specific concentration; this treatment is essential when the molecular weight of the coolant differs greatly from that of the hot gas.) Rearrangement of Equations [3], [4], and [5] and integration between the limits 0 and δ yield the relations

$$\frac{\bar{c}_{pM}}{\bar{c}_{pV}} \ln \left(\frac{\Delta H + \bar{c}_{pV}(T_\delta - T_0)}{\Delta H} \right) = \bar{Pr}_M \frac{\dot{m}_0 \delta'}{\mu_{M_0}} \quad [6]$$

$$\frac{\bar{R}_M}{R_V} \ln \left(\frac{p - p_{V\delta}}{p - p_{V_0}} \right) = \bar{Sc}_M \frac{\dot{m}_0 \delta'}{\mu_{M_0}} \quad [7]$$

$$\ln \left(\frac{\tau_0 + \dot{m}_0 u_\delta}{\tau_0} \right) = \frac{\dot{m}_0 \delta'}{\mu_{M_0}} \quad [8]$$

where Pr is Prandtl number, Sc is Schmidt number, and

$$\delta' = \int_0^\delta \frac{\mu_{M_0}}{\mu_M} dy \quad [9]$$

Equations [6] through [9] describe completely the relationships between conditions at the liquid-gas interface and conditions at the junction of the laminar sublayer with the turbulent core.

Second, consider the turbulent core. A logical extension of the Reynolds analogy (see Ref. 18 for Reynolds' statement of the hypothesis) to heat, mass, and momentum transfer in the turbulent core of nonreacting, two-component pipe flows with unidirectional radial diffusion must specify that the rate of radial momentum transport at any point of the flow field under consideration bears the same relation to the gradients which produce momentum flow as the energy-transfer rate bears to the gradients which produce energy flow and as the mass-transfer rate bears to the gradient which produces mass flow. Whether heat and momentum carried in the radial direction by the diffusing vapor should or should not be included in an analogy of this type is not apparent immediately. However, since velocity, temperature, and partial-pressure profiles are joined most smoothly at the junction of the laminar sublayer and turbulent core when heat and momentum carried by the diffusing vapor are considered in the turbulent core as well as in the laminar sublayer, it seems reasonable that the effects of mass diffusion on heat and momentum transfer in the turbulent core should be included in the proposed Reynolds analogy extension. (Obviously, as was the case for Reynolds' original hypothesis, the merits of this suggestion can be established conclusively only by experimental means.) Following this suggestion, the extension of the Reynolds analogy to heat, mass, and momentum transfer in the turbulent core of two-component turbulent pipe flows with unidirectional radial diffusion is postulated to be

$$\frac{\dot{m}_\delta}{\frac{p}{R_V T} \frac{d}{dy} \ln (p - p_V)} = \frac{\dot{q}_\delta + c_{pV} T_\delta \dot{m}_\delta}{-\rho_M c_{pM} \frac{dT}{dy} + c_{pV} T \frac{p}{R_V T} \frac{d}{dy} \ln (p - p_V)} = \frac{-\tau_\delta + u_\delta \dot{m}_\delta}{-\rho_M \frac{du}{dy} + u \frac{p}{R_V T} \frac{d}{dy} \ln (p - p_V)} \quad [10]$$

(See Appendix B of Ref. 2 for the identification of $\dot{m}$, $\dot{q}$, and τ with time averages of turbulent fluctuations.) Equations [10] differ from Crocco's extension of the Reynolds analogy in that consideration has been given the effects of mass addition on transport phenomena in the turbulent core and that the driving potential for mass transfer is taken to be partial pressure. For given boundary conditions, Equations [10] prescribe completely the relations between heat, mass, and momentum transfer in the turbulent core of the model being considered.

The combination of the results of the previous two paragraphs will provide now the desired relations between evaporation rate, fluid properties, and gas-stream parameters. Noting that at $y = \delta$ Equations [3], [4], and [5] read

$$\dot{q}_\delta = -\dot{m}_0 \Delta H - \dot{m}_0 \bar{c}_{pV}(T_\delta - T_0) \quad [11]$$

$$\dot{m}_\delta = \dot{m}_0 \quad [12]$$

$$\tau_\delta = \tau_0 + \dot{m}_0 u_\delta \quad [13]$$

one may rearrange Equations [10] to read

$$\frac{R_M}{R_V} d \ln (p - p_V) = \frac{c_{pM}}{c_{pV}} d \ln [\Delta H + \bar{c}_{pV}(T - T_0)] = d \ln (\tau_0 + \dot{m}_0 u) \quad [14]$$

which, when integrated between $y = \delta$ and a point far into the turbulent core, yield

$$\frac{\bar{R}_M}{R_V} \ln \frac{p - p_{V_\infty}}{p - p_{V\delta}} = \frac{\bar{c}_{pM}}{\bar{c}_{pV}} \ln \frac{\Delta H + \bar{c}_{pV}(T_\infty - T_0)}{\Delta H + \bar{c}_{pV}(T_\delta - T_0)} = \ln \frac{\tau_0 + \dot{m}_0 u_\infty}{\tau_0 + \dot{m}_0 u_\delta} \quad [15]$$

Substituting from Equations [6], [7], and [8] in Equations [15],

$$\frac{\bar{c}_{pM}}{\bar{c}_{pV}} \ln \left(1 + \frac{\bar{c}_{pV}(T_\infty - T_0)}{\Delta H} \right) = \ln \left(1 + \frac{\dot{m}_0 u_\infty}{\tau_0} \right) + (\bar{Pr}_M - 1) \frac{\dot{m}_0 \delta'}{\mu_{M_0}} \quad [16]$$

$$\frac{\bar{R}_M}{R_V} \ln \left(1 + \frac{p_{V_0} - p_{V_\infty}}{p - p_{V_0}} \right) = \ln \left(1 + \frac{\dot{m}_0 u_\infty}{\tau_0} \right) + (\bar{Sc}_M - 1) \frac{\dot{m}_0 \delta'}{\mu_{M_0}} \quad [17]$$

(Note that when $\bar{Pr}_M = \bar{Sc}_M = 1$, Equations [16] and [17] are identical with the results which one obtains when the Reynolds analogy is applied to the entire gas stream including the laminar sublayer. This state of affairs is in accord with the premises upon which the Reynolds analogy was extended.) Equations [16] and [17] provide the desired relations between evaporation rate, fluid properties, and gas-stream parameters.

However, before Equations [16] and [17] can be used conveniently for the calculation of the evaporation rate for given fluid properties and gas-stream parameters, the shear stress at the liquid-gas interface τ_0, the gas-stream laminar sublayer thickness δ', and the vapor pressure at the liquid-gas interface p_{V_0} must be related to easily manipulated parameters. Consider the shear stress τ_0. Not enough experimental data concerning turbulent pipe flows with mass addition at the wall are available to permit one to make a precise prediction of the value of τ_0 for given gas-stream and mass-addition parameters. Hence, assume [with Rannie (12)] that the shear stress τ_δ at the junction of the laminar sublayer with the turbulent core is unaffected by mass addition at the wall and is the same as for ordinary turbulent pipe flows, i.e., that τ_δ can be related to the gas-stream parameters and the ordinary pipe-flow friction coefficient C_f by

$$\tau_\delta = \frac{C_f}{2} \rho_\infty u_\infty^2 \quad [18]$$

The consequence of this assumption is that τ_0 is now related to gas-stream and mass-addition parameters by

$$\tau_0 = \frac{\tau_0}{\tau_\delta} \tau_\delta = e^{-(\dot{m}_0 \delta'/\mu_{M_0})} \frac{C_f}{2} \rho_\infty u_\infty^2 \quad [19]$$

a relation which includes a simple correction for mass-addition effects and which reduces to the ordinary pipe-flow relation when the mass-addition rate vanishes. It is suggested that C_f be taken to be the friction coefficient corresponding to ordinary turbulent flows in smooth pipes when performing calculations for stable films. [Abramson's remark (17) implying that friction coefficients for stable liquid wall films are greater than those for smooth pipes is not well founded; the investigations referred to by Abramson, namely, those examined by Lockhart and Martinelli (19) and Bergelin (20), were conducted with liquid flows considerably out of the range found in stable liquid wall films.] This treatment of the shearing stress τ_0 will most certainly have to be modified (especially for large evaporation rates) when more complete information is available concerning the effects of mass addition on turbulent flows. In the meantime, Equation [19] indicates the trend of the effects of mass addition on shearing stress at the wall.

In analogy with ordinary turbulent pipe flows, identify δ' with the dimensionless laminar sublayer thickness δ^* by means of the defining relation

$$\delta^* = \frac{\rho_\infty \sqrt{\tau_\delta/\delta_\infty}\,\delta'}{\mu_{M_0}} \qquad \text{[20]}$$

so that the factor $\dot{m}_0\delta'/\mu_{M_0}$ which appears in Equations [16] and [17] may be written in the form

$$\frac{\dot{m}_0\delta'}{\mu_{M_0}} = \frac{\dot{m}_0}{\rho_\infty u_\infty}\sqrt{\frac{2}{C_f}}\,\delta^* \qquad \text{[21]}$$

Here δ^* is a parameter which cannot be evaluated except by experiment. But the laminar sublayer thickness for flow when mass is added at the wall has not been determined experimentally. Hence, a simple extension from results of ordinary pipe-flow experiments will be made. Prandtl (21) found that, for ordinary turbulent pipe flows, the assumption

$$\frac{u_\delta}{u_\infty} = \sqrt{\frac{C_f}{2}}\,\frac{\rho\sqrt{\tau_0/\rho}\,\delta}{\mu} = 1.1 \times Re^{-1/8} \qquad \text{[22]}$$

fits the experimental results; this assumption has led to the use of

$$\frac{\rho\sqrt{\tau_0/\rho}\,\delta}{\mu} = 5.6 \qquad \text{[23]}$$

for ordinary turbulent pipe flows. The simplest assumption for flows with mass addition at the walls which reduces to Equation [23] in the case of flow with no mass addition is

$$\frac{\rho_\infty\sqrt{\tau_\delta/\rho_\infty}\,\delta'}{\mu_{M_0}} = \delta \quad \cong 6 \qquad \text{[24]}$$

In the absence of better information, it is suggested that this relation be used. Note that the treatment of variable viscosity in the gas-stream laminar sublayer provides for laminar-sublayer-thickness corrections due to the effects of variable fluid properties in the direction suggested by Reichardt (22).

From the kinetic theory of gases (23) the relation connecting the vapor pressure p_{V_0}, the surface temperature T_0, and the evaporation rate $\dot{m}_0$ from the surface is

$$\dot{m}_0 = \frac{(p_{V_s} - p_{V_0})f}{\sqrt{2\pi R_V T_0}} \qquad \text{[25]}$$

where f is the evaporation coefficient (equal to or less than unity), and the subscript s refers to saturation conditions corresponding to the surface temperature T_0. In order to indicate the relative magnitudes of p_{V_s} and p_{V_0}, Equation [25] may be rearranged into the form

$$\frac{p_{V_s}}{p_{V_0}} = 1 + \left(\frac{v}{a}\right)_{V_0}\frac{\sqrt{2\pi\gamma_{V_0}}}{f} \qquad \text{[25a]}$$

where v_V is the diffusion velocity in the y-direction of the vapor relative to the evaporating surface, and a_V is the velocity of sound in vapor. Parameter values typical of those encountered in a film-cooling application are $f = 0.04$ (23), $\gamma_{V_0} = 1.3$, and $(v/a)_{V_0} = (1/1750)$. For these parameter values, Equation [25a] reads

$$\frac{p_{V_s}}{p_{V_0}} = 1 + \frac{1}{1750}\,\frac{\sqrt{2\pi \times 1.3}}{0.04} = 1.04 \qquad \text{[25b]}$$

so that one may write to good approximation

$$p_{V_0} = p_{V_s} \qquad \text{[26]}$$

where p_{V_s} is a known function of T_0 (see, e.g., Ref. 24 for tables of experimental data or use Clausius-Clapeyron equation). The relating of the unwieldy parameters τ_0, δ', and p_{V_0} to easily manipulated parameters is now completed, and Equations [16] and [17] may be written in the convenient form

$$\frac{\bar{c}_{pM}}{\bar{c}_{pV}}\ln\left(1 + \frac{\bar{c}_{PV}(T_\infty - T_0)}{\Delta H}\right) = \ln\left(1 + \frac{\dot{m}_0}{\rho_\infty u_\infty}\frac{2}{C_f}e^{\frac{\dot{m}_0}{\rho_\infty u_\infty}\sqrt{\frac{2}{C_f}}\delta^*}\right) + (\overline{Pr}_M - 1)\frac{\dot{m}_0}{\rho_\infty u_\infty}\sqrt{\frac{2}{C_f}}\,\delta^* \qquad \text{[27]}$$

$$\frac{\bar{R}_M}{R_V}\ln\left(1 + \frac{p_{V_s} - p_{V\infty}}{p - p_{V_s}}\right) = \ln\left(1 + \frac{\dot{m}_0}{\rho_\infty u_\infty}\frac{2}{c_f}e^{\frac{\dot{m}}{\rho_\infty u_\infty}\sqrt{\frac{2}{C_f}}\delta^*}\right) + (\overline{Sc}_M - 1)\frac{\dot{m}_0}{\rho_\infty u_\infty}\sqrt{\frac{2}{C_f}}\,\delta^* \qquad \text{[28]}$$

They provide the desired relationship (implicit, to be sure) between evaporation rate (or, alternatively, liquid-film surface temperature), fluid properties, and gas-stream parameters.

Discussion of Solution. Since Equations [27] and [28] reduce to

$$\frac{1}{C_h} = \frac{2}{C_f} + \delta^*\sqrt{\frac{2}{C_f}}(\overline{Pr}_G - 1) \qquad \text{[29]}$$

$$\frac{1}{C_M} = \frac{2}{C_f} + \delta^*\sqrt{\frac{2}{C_f}}(\overline{Sc}_G - 1) \qquad \text{[30]}$$

as $\dot{m}_0$ and p_V approach zero, where the gas-stream, heat-transfer coefficient C_h and the gas-stream, mass-transfer coefficient C_M are in this case defined by

$$C_h = \frac{\dot{q}_0}{\rho_\infty u_\infty \bar{c}_{pG}(T_\infty - T_0)} = \frac{\dot{m}_0 \Delta H}{\rho_\infty u_\infty \bar{c}_{pG}(T_\infty - T_0)} \qquad \text{[31]}$$

$$C_m = \frac{\dot{m}_0}{u_\infty(p_{V_s} - p_{V\infty})/R_V T_\infty} \qquad \text{[32]}$$

it is proposed that Equations [27] and [28] are extensions of the Prandtl-Taylor equation to heat transfer and mass transfer in the case of film cooling.

Note also that, for relatively small temperature and vapor-pressure differences, one may eliminate the evaporation rate from Equations [27] and [28] to obtain

$$\frac{p_{V_s}}{p - p_{V_s}} - \frac{p_{V\infty}}{p - p_{V_s}} \approx \frac{R_V}{\bar{R}_M}\frac{\bar{c}_{pM}(T_\infty - T_0)}{\Delta H} \times \left[\frac{1 + \sqrt{\frac{C_f}{2}}\,\delta^*(\overline{Sc}_M - 1)}{1 + \sqrt{\frac{C_f}{2}}\,\delta^*(\overline{Pr}_M - 1)}\right] \qquad \text{[33]}$$

Compare this approximate equation with the semi-empirical, wet-bulb-thermometer equation (valid for small temperature differences and small vapor pressures)

$$\frac{p_{V_s}}{p - p_{V_s}} - \frac{p_{V\infty}}{p - p_{V\infty}} = \frac{R_V}{R_G}\frac{\bar{c}_{pM}(T_\infty - T_0)}{\Delta H}\left(\frac{\bar{c}_M}{\overline{Pr}_M}\right)^{0.56} \qquad \text{[34]}$$

where

$$\frac{p_{V_s}}{p - p_{V_s}} - \frac{p_{V_\infty}}{p - p_{V_\infty}} = \frac{R_V}{R_G} \frac{\bar{c}_{pM}(T_\infty - T_0)}{\Delta H} \quad [35]$$

is the wet-bulb-thermometer equation presented by Lewis (25), and

$$\left(\frac{\bar{Sc}_M}{\bar{Pr}_M}\right)^{0.56} \quad [36]$$

is a modifying function based upon the correlation presented by Bedingfield and Drew (26) for data which were obtained with Schmidt numbers from 0.60 to 2.60 and Prandtl number equal to 0.70. (Although Klinkenberg and Mooy have given the name Lewis number to the ratio Sc/Pr in Ref. 27, this nomenclature is not used generally in current literature.) If the derivation leading up to Equation [33] is correct, then the theoretical factor

$$\left[\frac{1 + \sqrt{\frac{C_f}{2}}\delta^*(\bar{Sc}_M - 1)}{1 + \sqrt{\frac{C_f}{2}}\delta^*(\bar{Pr}_M - 1)}\right] \quad [37]$$

should be equal approximately to the empirical factor (Equation [36])

$$\left(\frac{\bar{Sc}_M}{\bar{Pr}_M}\right)^{0.56}$$

for the parameter values upon which the exponent 0.56 was based. A precise comparison of factors [36] and [37] cannot be made since C_f is not known precisely for the test conditions corresponding to the data examined by Bedingfield and Drew. However, one can make a reasonable approximate comparison by noting that, for adiabatic pipe flows at relatively low Reynolds numbers with no mass addition at the wall

$$\sqrt{\frac{C_f}{2}}\delta^* = \frac{u_\delta}{u_\infty} = \sqrt{\frac{0.0791}{2}} Re_G^{-1/8} 5.6 \approx \frac{1}{2} \quad [38]$$

and then by accepting this approximation as being also a reasonable approximation for general flows when low mass-transfer rates and small temperature gradients exist (as was the case in the tests corresponding to the data examined by Bedingfield and Drew). Subsequent calculations based on this approximation indicate that the value of factor [36] differs by only an insignificant amount from the value of factor [37] for Schmidt numbers from 0.60 to 2.60 and Prandtl number equal to 0.70. Hence, it is proposed that Equations [27] and [28], taken collectively, constitute an extension of the wet-bulb-thermometer equation to the case when relatively large temperature and partial-pressure gradients occur.

Confusion exists in current literature concerning the value of the surface temperature of a liquid film flowing under the influence of a heated gas stream. (It is important especially to know the value of this temperature when calculating values of the fluid properties at the gas-liquid interface.) Several authors have assumed the liquid-film surface temperature to be equal to the boiling temperature of the liquid under the prevailing static pressure in the duct; this assumption is perhaps an erroneous generalization of the observation that the surface temperature of a film evaporating into an atmosphere consisting of only its own vapor is very nearly equal to the boiling temperature of the liquid under the prevailing pressure. Such a generalization is not valid when the atmosphere into which the liquid film is evaporating contains gases other than the vapor corresponding to the liquid in the film (as is usually the case for film cooling); actually, the following statements hold: 1 The liquid-film surface temperature is very nearly equal to the boiling temperature of the liquid when under a pressure equal to the prevailing interfacial vapor pressure (see Equation [25a]). 2 The prevailing interfacial vapor pressure is less than the static pressure in the duct (and consequently the liquid-film surface temperature is lower than the boiling temperature of the liquid when under a pressure equal to the static pressure prevailing in the duct), provided that the ratio $\dot{m}_0 u_\infty/\tau_0$ is not infinite and that p_{V_∞} is not equal to p (see Equation [17]; do not forget assumption 8).

To indicate the effects of several parameter variations on the evaporation rate $\dot{m}_0$ and the liquid-film surface temperature T_0, the curves which are presented in Figs. 9 and 10 have been prepared. The figures indicate that, when efficient coolant usage is required for given gas-stream parameters, it is desirable that the coolant have a high specific heat and a large heat of vaporization.

Since it is not possible, in general, to obtain explicit relations for either the evaporation rate $\dot{m}_0$ or the film surface temperature T_0 from Equations [27] and [28], curves such as are presented in Figs. 9 and 10 are found to be useful aids in the determination of $\dot{m}_0$ and/or T_0 for given fluid properties

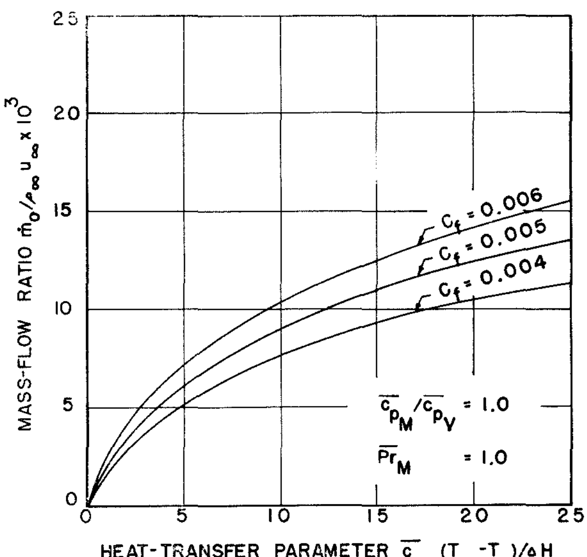

Fig. 9a Mass-flow ratio vs. heat-transfer parameter for several values of C_f

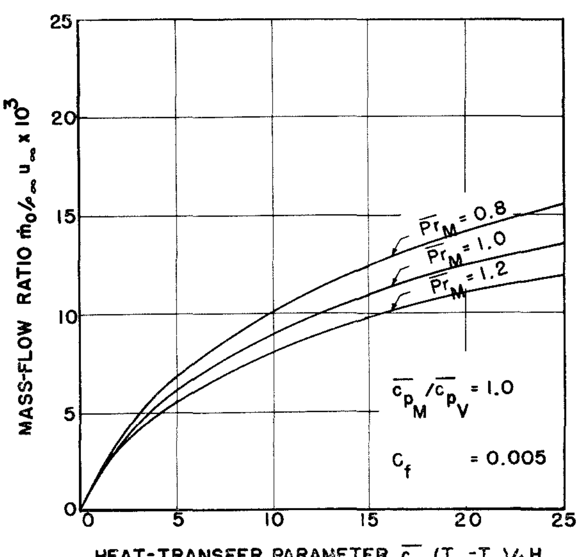

Fig. 9b Mass-flow ratio vs. heat-transfer parameter for several values of $\bar{Pr}_M$

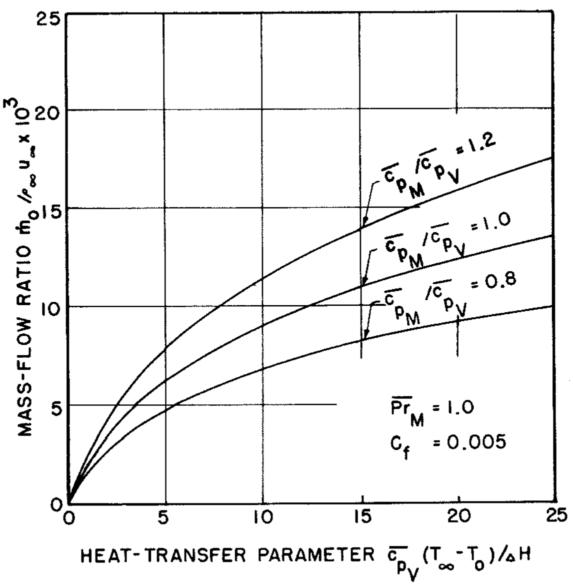

Fig. 9c Mass-flow ratio vs. heat-transfer parameter for several values of $\bar{c}_{p_M}/\bar{c}_{p_V}$

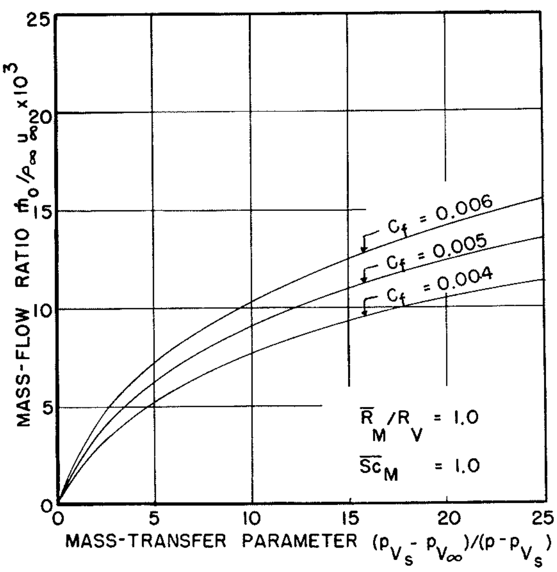

Fig. 10a Mass-flow ratio vs. mass-transfer parameter for several values of C_f

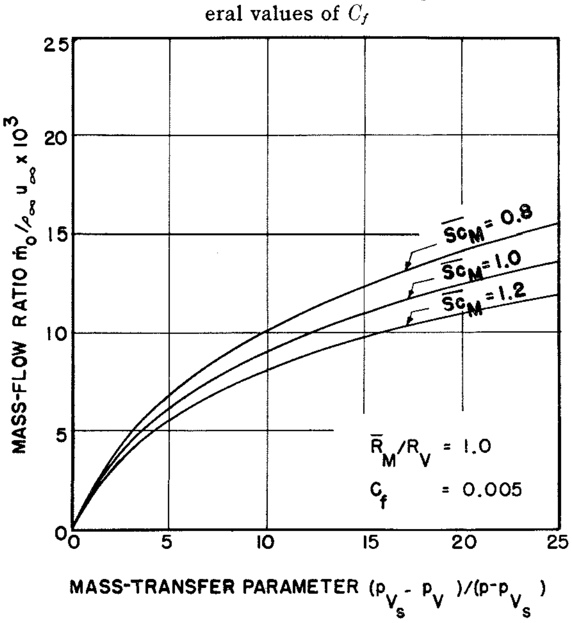

Fig. 10b Mass-flow ratio vs. mass-transfer parameter for several values of $\overline{Sc}_M$

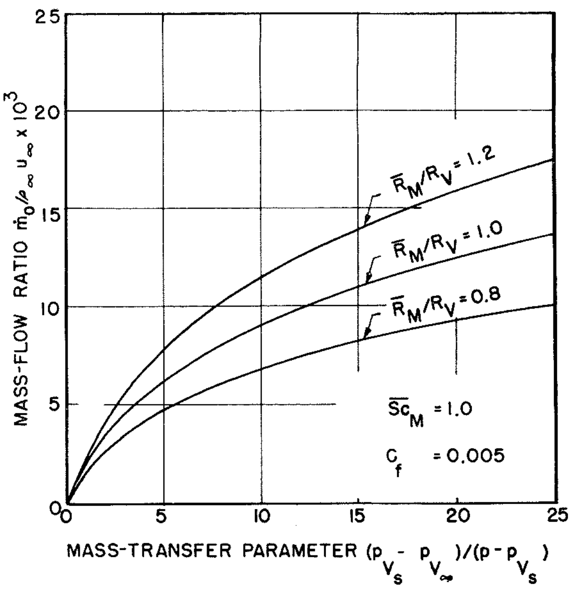

Fig. 10c Mass-flow ratio vs. mass-transfer parameter for several values of $\bar{R}_M/R_V$

and gas-stream parameters. The following procedure is suggested for obtaining the solution to a general evaporation problem: 1 For the given fluid properties and friction coefficient, prepare a curve of $\dot{m}_0/\rho_\infty u_\infty$ vs. $\bar{c}_{pV}(T_\infty - T_0)/\Delta H$ and a curve of $\dot{m}_0/\rho_\infty u_\infty$ vs. $(p_{V_s} - p_{V\infty})/(p - p_{V_s})$, calculating $c_{pV}(T_\infty - T_0)/\Delta H$ and $(p_{V_s} - p_{V\infty})/(p - p_{V_s})$ for selected values of $\dot{m}_0/\rho_\infty u_\infty$. 2 Estimate the value of T_0 and calculate $c_{pV}(T_\infty - T_0)/\Delta H$ based on this estimation. [Note, for relatively large values of T_∞, that $\bar{c}_{pV}(T_\infty - T_0)/\Delta H$ varies much more slowly with variations in T_0 than does $(p_{V_s} - p_{V\infty})/(p - p_{V_s})$.] 3 Read $\dot{m}_0/\rho_\infty u_\infty$ from the appropriate prepared curve, using the value of $\bar{c}_{pV}(T_\infty - T_0)/\Delta H$ calculated in step 2. 4 Read $(p_{V_s} - p_{V\infty})/(p - p_{V_s})$ from the appropriate prepared curve, using the value of $\dot{m}_0/\rho_\infty u_\infty$ obtained in step 3. 5 Calculate T_0 corresponding to the value of $(p_{V_s} - p_{V\infty})/(p - p_{V_s})$ obtained in step 4. 6 If the value of $\bar{c}_{pV}(T_\infty - T_0)/\Delta H$ based upon the value of T_0 as calculated in step 5 is appreciably different from the value of $\bar{c}_{pV}(T_\infty - T_0)/\Delta H$ based upon the value of T_0 as estimated in step 2, then repeat steps 3, 4, and 5 using the corrected value of $\bar{c}_{pV}(T_\infty - T_0)/\Delta H$. Such iteration is unnecessary usually when T_∞ is relatively large; then $\bar{c}_{pV}(T_\infty = T_0)/\Delta H$ is a very slowly varying function of T_0, and $(p_{V_s} - p_{V\infty})/(p - p_{V_s})$ is a very rapidly varying function of T_0.

Note that calculations are simplified greatly in the event that the Prandtl and Schmidt numbers are nearly equal. In this case the following procedure is suggested for obtaining the solution to a given evaporation problem: 1 Eliminate the evaporation rate from Equations [27] and [28] to obtain a relatively simple relation between T_∞ and functions of T_0 (which relation is independent of flow parameters)

$$\frac{\bar{c}_{pM}}{\bar{c}_{pV}} \ln\left(1 + \frac{\bar{c}_{pV}(T_\infty - T_0)}{\Delta H}\right) = \frac{\bar{R}_M}{R_V} \ln\left(1 + \frac{p_{V_s} - p_{V\infty}}{p - p_{V_s}}\right) \ldots [39]$$

Prepare curves of T_0 vs. T_∞ and a curve of $\dot{m}_0/\rho_\infty u_\infty$ vs. $\bar{c}_{pV}(T_\infty - T_0)/\Delta H$ for the given conditions, calculating T_∞ for selected values of T_0 and $\bar{c}_{pV}(T_\infty - T_0)/\Delta H$ for selected values of $\dot{m}_0/\rho_\infty u_\infty$. 2 Read T_0 from the appropriate prepared curve, using the given value of T_∞. 3 Calculate $\bar{c}_{pV}(T_\infty - T_0)/\Delta H$, using the value of T_0 obtained in step 2. 4 Read $\dot{m}_0/\rho_\infty u_\infty$ from the appropriate prepared curve, using the value of $\bar{c}_{pV}(T_\infty - T_0)/\Delta H$ calculated in step 3.

Evaporation rates predicted by Equation [27] of this paper

$$\frac{\bar{c}_{pM}}{\bar{c}_{pV}} \ln\left(1+\frac{\bar{c}_{pV}(T_\infty - T_0)}{\Delta H}\right) = \ln\left(1+\frac{\dot{m}_0}{\rho_\infty u_\infty}\frac{2}{C_f} e^{\frac{\dot{m}_0}{\rho_\infty u_\infty}\sqrt{\frac{2}{C_f}}\delta^*}\right) + (\overline{Pr}_M - 1)\frac{\dot{m}_0}{\rho_\infty u_\infty}\sqrt{\frac{2}{C_f}}\,\delta^* \ldots [27]$$

are compared in Fig. 11 with those predicted by Crocco's extension of Rannie's equation to the case of film cooling, which equation (in the present nomenclature) reads

$$\frac{\bar{c}_{pM}}{\bar{c}_{pV}} \ln\left(1+\frac{\bar{c}_{pV}(T_\infty - T_0)}{\Delta H}\right) = \frac{\bar{c}_{pM}}{\bar{c}_{rV}} \ln\left[e^{\frac{\dot{m}_0}{\rho_\infty u_\infty}\sqrt{\frac{2}{C_f}}\delta^*} + \frac{\bar{c}_{pV}}{\bar{c}_{pM}}\left(\frac{1}{\delta^*}\sqrt{\frac{2}{c_f}} - 1\right)\left(e^{\frac{\dot{m}_0}{\rho_\infty u_\infty}\sqrt{\frac{2}{C_f}}\delta^*} - 1\right)\right] + \left(\overline{Pr}_M - \frac{\bar{c}_{pM}}{\bar{c}_{pV}}\right)\frac{\dot{m}_0}{\rho_\infty u_\infty}\sqrt{\frac{2}{C_f}}\,\delta^* \ldots [40]$$

The divergence of the two curves is due to the fact that effects of mass addition on transport phenomena in the turbulent core have been treated differently in the two analyses.

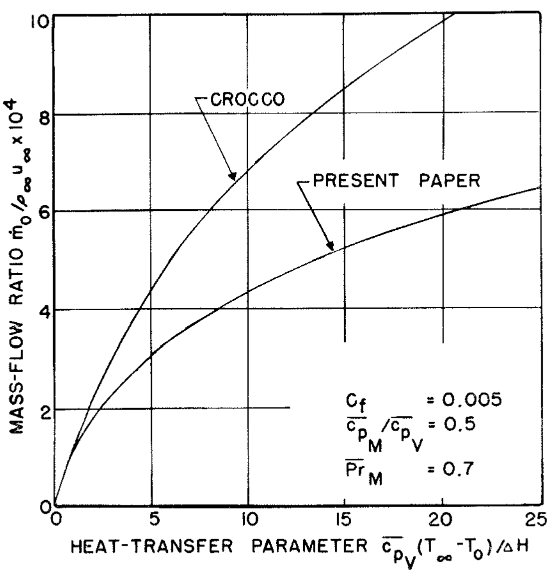

Fig. 11 Mass-flow ratio vs. heat-transfer parameter as predicted by Crocco and present paper

Summarizing, a method has been found for determining the evaporation rate and the surface temperature for a stable inert liquid wall film flowing under the influence of a high-velocity, fully developed turbulent gas stream in a duct. The method is based upon extensions of the Prandtl-Taylor equation to heat transfer and mass transfer in the case of film cooling which constitute collectively an extension of the wet-bulb-thermometer equation to the case when large temperature and partial-pressure gradients occur. An attempt has been made to take into account the effects of mass addition on transport phenomena in the turbulent core.

C *Experimental Study*

The data which were discussed in section 4 with respect to film stability are discussed in this section with respect to evaporation rate. The evaporation rates which are examined are those which correspond to the longest stable films obtainable for the several sets of gas-stream condition. It is useless to examine the evaporation rates for unstable films since the mass-transfer rate from unstable films has not been analyzed theoretically; shorter films will not be examined since they are more susceptible to large experimental errors than are long films.

Theoretical values of $\dot{m}_0/\rho_\infty u_\infty$ were obtained with the aid of Figs. 12 and 13, prepared in accordance with Equations

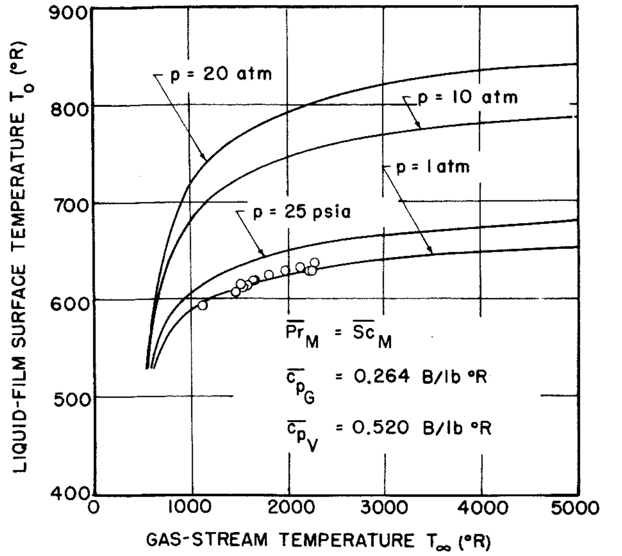

Fig. 12 Liquid-film surface temperature vs. gas-stream temperature for water film flowing under influence of fully developed turbulent hot-air stream in duct

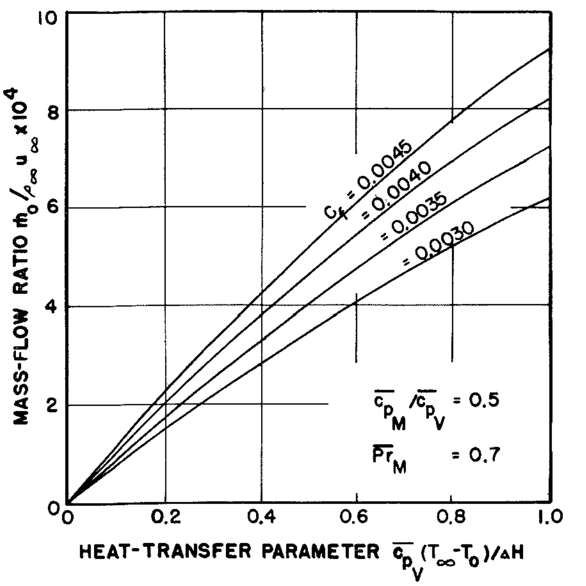

Fig. 13 Mass-flow ratio vs. heat-transfer parameter for water evaporating into air stream

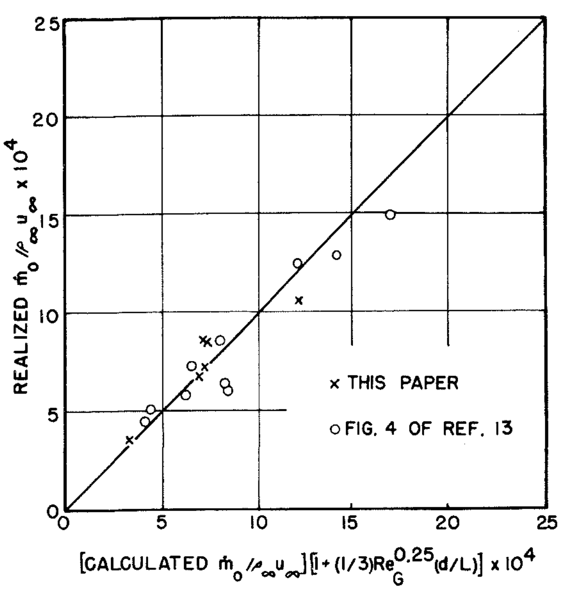

Fig. 14 Comparison of realized mass-flow ratio with calculated mass-flow ratio

[39] and [27], respectively. To obtain $\dot{m}_0/\rho_\infty u_\infty$ for given gas-stream conditions, read the film surface temperature T_0 from Fig. 12 for the given gas-stream static pressure and bulk temperature, then calculate the heat-transfer parameter $\bar{c}_{pV}(T_\infty - T_0)/\Delta H$ (a function of T_∞ and T_0), and read finally the mass-flow ratio from Fig. 13 for the appropriate value of $\bar{c}_{pV}(T_\infty - T_0)/\Delta H$ and the given value of C_f. (Fig. 12 emphasizes incidentally that, in general, although T_∞ may be much greater than the boiling temperature of the liquid at the prevailing pressure, the boiling point of the liquid is not reached necessarily at the liquid surface; see section 5-B.)

A preliminary examination of the available data [those reported in this paper and those summarized by Kinney (13)] discloses that the realized evaporation rates exceeded consistently the predicted values. The reason for this discrepancy becomes apparent when assumption 8 of section 5-B is compared with the test conditions which existed. Whereas it was assumed that axial variations in the gas stream were small compared with radial variations, test conditions were such that extremely large axial variations occurred at the point of liquid injection, i.e., at the test-section inlet; although the velocity profile at the test-section inlet was essentially that of fully developed turbulent pipe flow (since approach duct lengths of 10 and 20 pipe diameters were employed by Kinney, and approach duct lengths of 27 pipe diameters were employed in the current tests), the partial-pressure and temperature profiles at the test-section inlet were essentially square (since the air was virtually dry upstream from the test-section inlet and the approach duct was thermally insulated). Consequently, one would be surprised if the experimentally determined evaporation rates did not exceed the predicted values.

Calculations by Latzko (28) and experimental data by Boelter, Young, and Iverson (29) indicate that, when the temperature profile at the test-section inlet is that for fully developed turbulent pipe flow, then the ratio of the average heat-transfer coefficient for a finite-length heated test section (whose length is at least five times its diameter) to the local heat-transfer coefficient far downstream from the inlet of an infinitely long heated test section may be given by an expression of the form

$$1 + \text{const} \times Re_G^{0.25}(d/L)$$

where d is the duct diameter and L is the test-section length. (For air flows in polished tubes with no mass diffusion, Boelter, et al., determined the value of the constant to be approximately 0.1.) These results motivated Fig. 14, where experimentally determined values of $\dot{m}_0/\rho_\infty u_\infty$ are plotted as a function of calculated values of $\dot{m}_0/\rho_\infty u_\infty$ which have been multiplied by the parameter $1 + (1/3)Re_G^{0.25}(d/L)$. [Gas properties have been evaluated at the bulk temperature T_∞; in the event that the bulk temperature is extremely large compared with the wall temperature, it may be necessary to evaluate the gas properties at the average of the bulk and wall temperatures instead of at the bulk temperature, as suggested by Deissler (30).] Fig. 14 explains the excessively large evaporation rates found to occur for the very short films (see Figs. 6 and 7 of Ref. 1 and Fig. 4 of Ref. 13) and implies that even greater modifications of evaporation rates as predicted by Equations [27] and [28] will be necessary if the velocity profile at the test-section inlet is square as are also the partial-pressure and temperature profiles (see Ref. 29 for experimental data on the effects of various entrance conditions on heat-transfer coefficients for turbulent flows of gases in ducts).

The relative merits of Equation [27] and Crocco's equation connot be compared with existing experimental data since (a) the parameter $\bar{c}_{pV}(T_\infty - T_0)/\Delta H$ did not exceed unity in either the NACA or the present tests, and (b) the evaporation rates predicted by Equation [27] and Crocco's equation do not deviate appreciably until the parameter $\bar{c}_{pV}(T_\infty - T_0)/\Delta H$ exceeds unity [Fig. 11].

Since the combined thickness of the duct wall and the liquid film was relatively small (less than 0.1 in.) and the heat-transfer rate from the test section to its environment was small, the duct wall temperature was taken to be equal approximately to the film-surface temperature. Hence, the measured wall temperatures have been plotted in Fig. 12 for comparison with predicted film-surface temperatures at 1 atm (test pressures ranged from 14.2 to 16.7 psia). The data agree with the theoretical curve with a maximum error of 6° R. Inasmuch as the data were taken at relatively large partial-pressure and temperature gradients (the parameter $(p_{V_s} - p_{V_\infty})/(p - p_{V_s})$ exceeded 0.7, and the parameter $\bar{c}_{pV}(T_\infty - T_0)/\Delta H$ exceeded 0.9 in some of the tests), it is concluded that the data support the proposal of section 5-A that Equations [27] and [28], taken collectively, constitute an extension of the wet-bulb-thermometer equation to the case when relatively large temperature and partial-pressure gradients occur.

Summarizing, experimental data concerning evaporation rates from stable liquid wall films into heated turbulent gas streams were brought into agreement with calculated evaporation rates after corrections for entrance effects were made. Good agreement was realized between predicted and measured liquid wall film temperatures.

The present investigation has re-emphasized the fact that information concerning the effects of entrance conditions on the transfer of heat, mass, and momentum in pipe flows is scarce. It would be useful to extend previous studies (28, 29) of the effects of various entrance conditions on transport phenomena in pipe flows analytically and experimentally.

Reynolds-analogy extensions of the type proposed in section 5-A are useful potentially in the analyses of several important processes (e.g., combustion, evaporation, and jet mixing) which involve transport phenomena in turbulent gas streams. However, the proposed treatment of mass-addition effects on transport phenomena in the turbulent core of pipe flows has not been confirmed; i.e., the relative merits of Equation [27] and Crocco's equation have not been established. Consequently, further studies are required to determine the correctness of hypotheses of the type expressed by Equation [27].

D Condensing Versus Evaporating Films

Since this study is presented with evaporating films uppermost in mind, comments on the applicability to condensing films of results obtained from studies of evaporating films are in order.

For the parameter ranges which were investigated, it was found that the film stability does not depend on mass-transfer rate. Hence, the results of the present film-stability investigation may be applied to evaporating and condensing films alike, provided, of course, that shearing stresses are large enough to make gravity forces negligible. Caution should be exercised, however, when applying the present results to cases in which the mass-transfer rates are much larger than those which have been investigated.

The treatment given the mass-transfer process in the present paper cannot be applied to general condensing films. For evaporating films of the type which were investigated, the heat transfer from the liquid film to the duct wall is negligible compared with the heat transfer to the liquid film from the gas stream (gas-stream temperature and velocity gradients are relatively large); for general condensing films, the heat transfer from the liquid film to the duct wall is of the same order of magnitude as the heat transfer to the liquid film from the gas stream (gas-stream temperature and velocity gradients are relatively small).

6 Summary

Thin liquid wall films flowing under the influence of high-velocity turbulent gas streams were studied for the purpose of obtaining an understanding of the mechanics of film cooling. The problem was divided into three parts: (a) the determination of conditions sufficient for the attachment of liquid films to solid surfaces in the presence of high-velocity gas streams without entrainment of unevaporated liquid by the gas stream; (b) the determination of conditions sufficient for the stability of thin liquid wall films flowing under the influence of high-velocity turbulent gas streams (a stable film is a film which loses no liquid droplets to the adjacent gas stream as the result of surface disturbances); and (c) the determination of the evaporation rate from a stable inert liquid wall film into a heated turbulent gas stream.

The studies on liquid-film attachment indicated that the use of radial-hole injectors in conjunction with the effects of a high-velocity gas stream for the attachment of liquid films to solid surfaces is effective over a wide range of operating conditions. Data corresponding to the inception point of inefficient film attachment (inefficient attachment occurring when liquid droplets are entrained by the gas stream during the attachment process) were plotted in dimensionless form; the abscissa was a function of the gas-stream Reynolds number, the liquid-stream Reynolds number, and a modified cavitation parameter, and the ordinate was the ratio of the gas- and liquid-stream momenta.

The studies on liquid-film stability led to the conclusions that small disturbances with wave lengths of the order of 10 film thicknesses are present on the liquid-film surface for all liquid-flow rates; that the scale of the small disturbances decreases as the diameter Reynolds number of the gas stream increases but does not vary appreciably when the liquid-flow rate is changed; that long wave-length disturbances appear on the surface of the film for liquid-flow rates larger than some critical value; that the critical film thickness for long-wave-length disturbances depends primarily on the wall shear stress; and that liquid droplets are entrained by the gas stream from the crests (regions where relatively large quantities of liquid are collected) of the long wave-length disturbances. Obviously, the unstable long wave-length disturbances are to be avoided when designing for an efficient film-cooling system. The data corresponding to the inception point of unstable liquid-wall-film flows are presented in dimensionless form by plotting the dimensionless film thickness corresponding to the inception point of unstable liquid-wall-film flows as a function of the ratio of the gas-vapor-mixture viscosity to the liquid viscosity, where the viscosities were evaluated at the liquid-film surface temperature.

A theoretical analysis of the evaporation problem was based on an extension of the Reynolds analogy to heat, mass, and momentum transfer in the turbulent core of two-component fully developed turbulent pipe flow with undirectional radial diffusion and on subsequent extensions of the Prandtl-Taylor equation to heat transfer and mass transfer in the case of film cooling. The resulting pair of equations, taken together, permits the calculation of the evaporation rate and the surface temperature for a liquid film when the fluid properties and gas-stream parameters are known.

Experimentally determined evaporation rates were brought into agreement with calculated evaporation rates after corrections for entrance effects were made. Good agreement was realized between predicted and measured film temperatures.

Acknowledgments

The author wishes to express his appreciation to Dr. W. Duncan Rannie for his critical guidance during the course of the study described herein.

The author is grateful for the many assistances given him by the staff members of the Jet Propulsion Laboratory, California Institute of Technology, where the major part of the work was carried out. N. Van De Verg, Chief of the Liquid Rockets Section, under whose general direction the research was performed, showed a continued interest in and gave indispensable support to the investigation.

The author is also grateful for the financial assistance afforded him by the Daniel and Florence Guggenheim Foundation in the form of a Fellowship in Jet Propulsion during the 1950–1951 and 1951–1952 school years.

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