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INTEGRAL SOLUTION OF A BUOYANT DIFFUSION FLAME

by

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Introduction:

Measurements of the rate of ambient air entrainment by axisymmetric diffusion flames suggest that entrainment occurs at a wrinkled laminar flame front in the lower regions of visible flame (Ref. 1). Entrainment of such flames requires a solution of the axisymmetric steady laminar diffusion flame which does not yield a self-similar solution. If one then considers the simple case of steady plane diffusion flame in semi-infinite fuel and oxidizer media separated by a flame sheet, an exact similarity solution can be obtained from equations of motion, energy and species conservation equations. This solution can also incorporate the differences in fuel and oxidizer densities resulting from either molecular weight differences or the temperature differences of oxidizer and fuel media. This problem was treated by G. C. Fleming and F. E. Marble to investigate the stability of such a flame front to periodic disturbances (Ref. 2). Inspired by their study, we chose to develop an integral solution to the same problem by appropriate selection of velocity, temperature and species profiles.

Governing Equations:

The coordinate system is oriented as shown in Figure 1. x-axis is taken to be along the dividing streamline whereas the y-axis is perpendicular to the x-axis. The equations of motion with usual boundary layer approximation are

$$\text{Continuity:} \quad \frac{\partial}{\partial x}(\rho u) + \frac{\partial}{\partial y}(\rho v) = 0 \tag{1}$$

$$\text{x-momentum:} \quad \frac{\partial}{\partial x}(\rho u^2) + \frac{\partial}{\partial y}(\rho uv) = -\frac{\partial P}{\partial x} - \rho g + \frac{\partial}{\partial y}\left(\mu\frac{\partial u}{\partial y}\right) \tag{2}$$

$$\text{y-momentum:} \quad \frac{\partial P}{\partial y} = 0 \tag{3}$$

Energy and species conservation equations are,

$$\frac{\partial}{\partial x}(\rho u h) + \frac{\partial}{\partial y}(\rho v h) = \frac{\partial}{\partial y}\left(\frac{k}{c_p}\frac{\partial h}{\partial y}\right) - \dot{\omega}_f h_c \tag{4}$$

$$\frac{\partial}{\partial x}(\rho u Y_i) + \frac{\partial}{\partial y}(\rho v Y_i) = \frac{\partial}{\partial y}\left(\rho \mathcal{D}\frac{\partial Y_i}{\partial y}\right) + \dot{\omega}_i \tag{5}$$

where h_c is the specific heating value of fuel. (For nomenclature, see list of symbols)

Pressure Variation:

The y-momentum equation implies that pressure is independent of y and given by static pressure outside the boundary layer. In our problem it will be assumed that the oxidizer medium is more dense than the fuel and the oxidizer mass is stationary for points far from the flame. Thus the pressure in fuel and oxidizer media are given by

$$\begin{aligned} P_o &= \tfrac{1}{2}\rho_{f\infty}u_{f\infty}^2 + \rho_{f\infty} g x + P \\ P_o &= \rho_\infty g x + P \end{aligned} \tag{6}$$

where P_o is the pressure at x = 0 and $u_{f\infty}$ (x = 0) = 0. The case where $u_{f\infty}$ (x = 0) $\neq$ 0 does not yield a similarity solution as will be discussed later. Combining these two equations gives an expression for fuel velocity,

$$u_{f\infty} = \left[2\left(\frac{\rho_\infty - \rho_{f\infty}}{\rho_{f\infty}}\right) g x\right]^{\frac{1}{2}} \tag{7}$$

Also the second of equation (6) is used to replace the pressure term in x-momentum equation (2) as

$$\frac{\partial P}{\partial x} = -\rho_\infty g \tag{8}$$

Equation of State:

The equation of state used is that for an ideal gas,

$$P = \rho RT = \rho \frac{\mathcal{R}}{M} T \tag{9}$$

where $\mathcal{R}$ is the universal gas constant and M is the local molecular weight. We also assume that the specific heat ratio $\gamma \equiv C_p/C_v$ varies so little across the flame that

$$\rho h = \rho_f h_f = \rho_\infty h_\infty = \text{constant} \tag{10}$$

See Appendix A for more details.

Chemical Reaction:

The fuel mixture is assumed to be composed of a gaseous fuel and a diluent gas. The term "fuel mixture" refers to the complete mixture whereas "fuel" is reserved for the combustible fuel component of the mixture.

A simple, one step chemical reaction is used to approximate the combustion process which is assumed to be confined to a thin flame of negligible thickness. The stoichiometric reaction between ν_o moles of oxidizer and ν_f moles of fuel is of the form

$$\nu_f \text{ (Fuel)} + \nu_o \text{ (Oxidizer)} \rightarrow \nu_p \text{ (Products)} + H_c \tag{11}$$

where ν_p is the number of moles of product formed and H_c is the heat of combustion per ν_f moles of fuel.

Conservation of mass in reaction (11) leads to relations for the reaction rates of species in terms of molecular weights, M_i, and stoichiometric coefficients ν_i.

$$\frac{\dot{\omega}_f}{\nu_f M_f} = \frac{\dot{\omega}_o}{\nu_o M_o} = -\frac{\dot{\omega}_p}{\nu_p M_p} \tag{12}$$

one then gets,

$$\dot{\omega}_o = \frac{\nu_o M_o}{\nu_f M_f} \dot{\omega}_f \equiv \phi_{of} \dot{\omega}_f \tag{13}$$

Solution:

The production terms in equations (4) and (5) can be eliminated by linear combination of dependent variables h and Y_i, when Lewis number, $L_e \equiv \rho \mathcal{D} C_p / k$ is unity. The resulting homogeneous equations are identical in so-called Schwab-Zeldovich variables and given by

$$\frac{\partial}{\partial x}(\rho u \Theta) + \frac{\partial}{\partial y}(\rho v \Theta) = \frac{\partial}{\partial y}\left(\rho \mathcal{D} \frac{\partial \Theta}{\partial y}\right) \tag{14}$$

where

$$\Theta = (h - h_\infty + h_c Y_f)/(h_{f\infty} - h_\infty + h_c Y_{f\infty}) \tag{15a}$$

or

$$\Theta = (Y_{o\infty} - Y_o + \phi_{of} Y_f)/(Y_{o\infty} + \phi_{of} Y_{f\infty}) \tag{15b}$$

with proper normalization of variables such that $\Theta = 0$ far from the flame on oxidizer side and $\Theta = 1$ away from the flame on the fuel side. The Schwab-Zeldovich variables are particularly convenient to use since they are continuous and smooth through the flame front.

Integral momentum equation is obtained by integrating equation (2) along the y-axis, first from $y = -\infty$ to $y = 0$ then from $y = 0$ to $y = +\infty$, adding and making use of

i) x-axis being the dividing streamline

ii) Integral form of continuity equation from equation (1)

iii) Equation (8) to replace the pressure term

$$\frac{d}{dx}\left[\int_{-\infty}^{0} \rho u^2 dy + \int_{0}^{\infty} \rho u(u-u_{f\infty}) dy\right] + \frac{du_{f\infty}}{dx}\int_{0}^{\infty} (\rho u - \rho_{f\infty} u_{f\infty})\, dy$$

$$= g\left[\int_{-\infty}^{0} (\rho_{\infty} - \rho)\, dy + \int_{0}^{\infty} (\rho_{f\infty} - \rho)\, dy\right] \qquad (16)$$

Similarly, integrating equation (4) along the y coordinate and rewriting the production term as the product of fuel flux into the flame front due to diffusion and the heating value per mass of fuel gives,

$$\frac{d}{dx}\left[\int_{-\infty}^{0} \rho u\ (h-h_{\infty})\ dy + \int_{0}^{\infty} \rho u\ (h-h_{f\infty})\ dy\right] = \rho\mathcal{D}\left(\frac{\partial Y_f}{\partial y}\right)_{y\,=\,y_{f1}} h_c \qquad (17)$$

The solutions of equations (16) and (17) follow by assuming suitable profiles for velocity and Θ in terms of a width parameter δ and a characteristic velocity u_o both of which depend on x only. No temperature parameter is required because the maximum of the temperature profile is given by the adiabatic flame temperature. Furthermore, Howarth transformation,

$$\rho dy = \rho_{\infty}\, dy' \qquad (18)$$

is used to reduce the equations (16), (17) into their incompressible form. Selected profiles are expressed in terms of a dimensionless horizontal length as

$$\eta = \frac{y'}{\delta} \qquad (19)$$

The characteristic width scales for velocity and Θ profiles, δ_u and δ_Θ, are assumed to be the same implying $Pr \equiv \frac{\mu C_p}{k}$ is unity. The assumed profiles are,

$$\frac{u}{u_o} = (1 - \frac{\Lambda}{2})\, e^{-\eta^2} + \frac{\Lambda}{2}\,(1 + \text{erf}\ \eta) \tag{20}$$

and

$$\Theta = \tfrac{1}{2}\,(1 + \text{erf}\ \eta) \tag{21}$$

where $\Lambda = u_{f\infty}/u_o$

and $\text{erf}\ \eta \equiv \frac{2}{\sqrt{\pi}} \int_o^{\eta} e^{-\xi^2}\, d\xi$ with $\text{erf}\ (-\infty) = -1,$

$\text{erf}\ (\infty) = 1$.

As sketched in Figure 2, the velocity profile has a bulge at $\eta = 0$ which occurs due to flame buoyancy. The flame position is fixed by the expression

$$\text{erf}\ \eta_{fl} = \frac{Y_{o\infty} - \phi_{of}\, Y_{f\infty}}{Y_{o\infty} + \phi_{of}\, Y_{f\infty}} \tag{22}$$

which is obtained from equations (15b) and (21). The flame lies on either side of x axis depending upon the fuel and oxidizer concentrations far from the flame and stoichiometry of reaction. The oxidizer and fuel diffuse in stoichiometric proportions at the flame front.

Θ-profile is chosen to be a solution of equation (14) with smooth variation through the flame front. Furthermore equations (20) and (21) satisfy the boundary conditions

$$\left.\begin{aligned} u &= 0 \\ \Theta &= 0 \end{aligned}\right\} \quad \text{at } \eta = -\infty$$

$$\left.\begin{aligned} u &= u_{f\infty} \\ \Theta &= 1 \end{aligned}\right\} \quad \text{at } \eta = +\infty$$

Upon substitution of assumed profiles of equations (20) and (21) into the integral momentum and energy equations, integrations are performed by making use of the equations (18) and (19) as well as the assumption $\rho^2 \mathcal{D} = \rho_\infty^2 \mathcal{D}_\infty = \text{const}$ in energy integral equation (for ideal gases, $\rho \sim \frac{1}{T}$ and $\mathcal{D} \sim T^{7/4}$).

The results are most conveniently expressed in terms of three nondimensional parameters which represent the density ratio of oxidizer and fuel media far from the flame front, heat of combustion per mass of fuel mixture and heat of combustion per mass of required fuel mixture for total consumption of oxidizer far from the flame front. These are,

$$\chi_1 = \frac{h_{f\infty}}{h_\infty} = \frac{\rho_\infty}{\rho_{f\infty}} \quad , \quad \chi_2 = \frac{h_c Y_{f\infty}}{h_\infty} \quad , \quad \chi_3 = \frac{h_c Y_{o\infty}}{h_\infty \phi_{of}} \tag{23}$$

Equations (16) and (17) take the forms

$$\frac{d}{dx}\left[\rho_\infty u_o^2 \delta A\right] + \rho_\infty u_o \delta \frac{du_{f\infty}}{dx} B = g \rho_\infty \delta C \tag{24}$$

and

$$\frac{d}{dx}\left[\rho_\infty u_o \delta D\right] = \rho_\infty \mathcal{D}_\infty \frac{E}{\delta} \tag{25}$$

where

$$A = \left(1 - \frac{\Lambda}{2}\right)^2 \sqrt{\frac{\pi}{2}} + \frac{\Lambda}{2}\left(1 - \frac{\Lambda}{2}\right)\sqrt{\pi} - \frac{\Lambda^2}{4}\frac{\sqrt{2}}{\pi}\left(2 - \sqrt{2}\right)$$

$$B = \left(1 - \frac{\Lambda}{2}\right)\frac{\sqrt{\pi}}{2} - \frac{\Lambda}{2}\frac{1}{\sqrt{\pi}}\frac{1+\chi_3}{\chi_1}$$

$$C = \frac{e^{-\eta_{fl}^2}}{2\sqrt{\pi}}\left(\chi_2 + \chi_3\right) + \frac{(\chi_1 - 1)(\chi_1 - \chi_3 - 1)}{2\sqrt{\pi}\,\chi_1}$$

$$D = \chi_2 \left\{\left(1 - \frac{\Lambda}{2}\right)\frac{\sqrt{\pi}}{8}\left(1 + \text{erf}\,\eta_{fl}\right)^2 + \frac{\Lambda}{4}\left[\eta_{fl}\left(1 + \text{erf}\,\eta_{fl}\right)^2\right.\right.$$

$$+ \frac{2e^{-\eta_{f1}^2}}{\sqrt{\pi}} (1+\text{erf}\ \eta_{f1}) - \sqrt{\frac{2}{\pi}}\ (1+\text{erf}\ (\sqrt{2}\ \eta_{f1}))\Bigg]\Bigg\} \quad +$$

$$\chi_3 \left\{(1-\frac{\Lambda}{2}) \frac{\sqrt{\pi}}{8} (1 - \text{erf}\ \eta_{f1})^2 + \frac{\Lambda}{4} \left[- \eta_{f1}(1 - \text{erf}^2\ \eta_{f1})\right.\right.$$

$$\left.\left. + \frac{2e^{-\eta_{f1}^2}}{\sqrt{\pi}} \text{erf}\ \eta_{f1} + \sqrt{\frac{2}{\pi}}\ (1 - \text{erf}\ (\sqrt{2}\ \eta_{f1}))\right]\right\} - (\chi_1 - 1)\frac{\sqrt{2}-1}{\sqrt{\pi}} \frac{\Lambda}{2}$$

$$E = (\chi_2 + \chi_3)\ \frac{e^{-\eta_{f1}^2}}{\sqrt{\pi}}$$

Also equation (22) can be written as

$$\text{erf}\ \eta_{f1} = \frac{\chi_3 - \chi_2}{\chi_2 + \chi_3} \tag{26}$$

Equations (24) and (25) have a similarity solution if $u_o \sim x^{\frac{1}{2}}$, $\delta \sim x^{\frac{1}{4}}$ and $u_{f\infty}$ at $x = 0$ is zero so that Λ is independent of x. Writing

$$K_u = \frac{u_o}{\sqrt{gx}} \tag{27}$$

and

$$K_\delta = \frac{\delta}{x} \left[\frac{gx^3}{\mathcal{D}_\infty^2}\right]^{\frac{1}{4}} \tag{28}$$

Substituting these relations into equations (24) and (25) we get,

$$K_u = \left[\frac{4C}{5A+2\Lambda B}\right]^{\frac{1}{2}} \tag{29}$$

$$K_\delta = \left[\frac{4E}{3DK_u}\right]^{\frac{1}{2}} \tag{30}$$

Squaring equation (29) and combining it with the expression for fuel velocity, $\dot{u}_{f\infty}$, from equation (7) rewritten as

$$\Lambda K_u = \sqrt{2(\chi_1 - 1)} \tag{31}$$

gives a simple quadratic equation in Λ whose coefficients are known functions of χ_1, χ_2, χ_3. Only one root is meaningful and used to evaluate K_u and K_δ.

Entrainment:

Entrainment can easily be calculated by a mass balance on the oxidizer side of the flame sheet as shown in Figure 3, we define the two mass flow rates of oxidizer $\dot{m}_{oL}$, $\dot{m}_{oc}$ which are expressed as

$$\dot{m}_{oL} = \int_{-\infty}^{y_{fl}} \rho u Y_o \, dy \tag{32}$$

and

$$\dot{m}_{oc} = \int_{o}^{x} \rho \left(\frac{\partial Y_o}{\partial y}\right)_{y=y_{fl}} dx \tag{33}$$

Under the assumptions which led to equations (24) and (25) we can show that,

$$\dot{m}_{oL} = \rho_\infty \, u_o \, \delta \, Y_{o\infty} \, F \tag{34}$$

$$\dot{m}_{oc} = \rho_\infty \, u_o \, \delta \, Y_{o\infty} \, G \tag{35}$$

where

$$F = (1 + \text{erf } \eta_{fl}) \left[(1 - \frac{\Lambda}{2}) \frac{\sqrt{\pi}}{2} + \frac{\Lambda}{2} \eta_{fl}\right] + \frac{\Lambda}{2} \frac{e^{-\eta_{fl}^2}}{\sqrt{\pi}}$$
$$- \left(1 + \frac{\chi_2}{\chi_3}\right) \left\{(1 - \frac{\Lambda}{2}) \frac{\sqrt{\pi}}{8} (1 + \text{erf } \eta_{fl})^2 + \frac{\Lambda}{4} \left[\eta_{fl} (1 + \text{erf } \eta_{fl})^2\right.\right.$$
$$\left.\left. + \frac{2e^{-\eta_{fl}^2}}{\sqrt{\pi}} (1 + \text{erf } \eta_{fl}) - \sqrt{\frac{2}{\pi}} (1 + \text{erf } (\sqrt{2} \, \eta_{fl}))\right]\right\}$$

and

$$G = \frac{4}{3K_\delta} \left(1 + \frac{\chi_2}{\chi_3}\right) \frac{e^{-\eta_{f1}^2}}{\sqrt{\pi}}$$

Since there is no oxidizer generation in the control volume, we can balance the oxidizer mass flow rates by the equation,

$$\dot{m}_{oE} = \dot{m}_E Y_{O\infty} = \dot{m}_{oc} + \dot{m}_{oL} \tag{36}$$

and hence by

$$\dot{m}_E = \rho_\infty u_o \delta (F + G) \tag{37}$$

The latter equation gives the entrainment rate per unit width of the flame, $\dot{m}_E$.

Results:

In figures (4) through (8) the quantities of primary interest, entrainment, characteristic velocity and width scales, velocity ratio $\Lambda = u_{f\infty}/u_o$, adiabatic flame temperature are plotted as a function of the density ratio of oxidizer and fuel mixture for three values of parameters χ_2 and χ_3. The values $\chi_2 = 120$ and $\chi_3 = 6.96$ correspond to the case of pure methane as fuel and air as oxidizer with $Y_{O\infty} = 0.232$. The other two curves show the variations in these quantities when the values of χ_2 and χ_3 are both changed by a factor of 2.

All quantities were made nondimensional so as to minimize the variations with respect to the density ratio of oxidizer and fuel. This involved the introduction of modified gravitational constant $g' = g\,(h_{f1}/h_\infty - 1)$ in normalizing entrainment, velocity and width scales. The adiabatic flame temperature was nondimensionalized by the heating value per mass of fuel (combustible component). The change in the adiabatic flame temperature is then only due to the differences of enthalpies of fuel and oxidizer media as shown in Figure 8.

Nondimensional entrainment constant and characteristic width scale have almost constant values of 1.85 and 2.13 respectively up to oxidizer densities 6 times that of the fuel mixture for all values of χ_2 and χ_3 as shown in Figures (4) and (6). For fuel mixture densities very much lower than that of oxidizer, velocity profile approaches to that of a free shear layer with a small modification around the flame sheet due to flame buoyancy. The fuel mixture density far from the flame can be decreased by either increasing the temperature of the mixture or diluting the combustible fuel component with a lower density diluent. In the first case, if the fuel mixture temperature exceeds the flame temperature by a large amount, the left hand side term in equation (17) can become zero resulting in a singularity in equation (30). A typical example is the divergence of results for low heating value and large density ratios (e.g. $h_c Y_{f\infty}/h_\infty = 60$, $h_c Y_{o\infty}/h_\infty \phi_{of} = 3.48$, $\rho_\infty/\rho_{f\infty} > 5$) as shown in Figures (4) and (6). In the second case, the mass fraction of fuel, $Y_{f\infty}$ decreases forcing the flame front to move towards the fuel mixture. Thus, the left hand side of equation (17) does not vanish since the contribution from the second integral becomes smaller compared to the first term. The variables χ_1 and χ_2 are not independent because of the variation in $Y_{f\infty}$ due to dilution. The dimensionless characteristic velocity which is taken to be the velocity at $y = 0$ normalized with respect to buoyant velocity $\sqrt{\Delta\rho g x/\rho}$ is given in Figure (5). The fuel mixture velocity far from the flame sheet normalized with respect to characteristic velocity U_o is shown in Figure (7) for the similar case of $u_{f\infty}(x=0) = 0$.

Table I below gives some numerical values of entrainment per unit area as well as the magnitude of the velocity at $y = 0$. This velocity is the maximum in the profile for the CH_4 - air and 50% dilution $H_2 + N_2$ - air flames. Pure H_2 - air flame has a free shear layer-like velocity profile due to the very low density of H_2.

TABLE 1

	CH_4 + Air		H_2 + Air	
	$Y_{f\infty} = 1.00$	$Y^{+}_{f\infty} = 0.50$	$Y_{f\infty} = 1.00$	$Y^{+}_{f\infty} = 0.50$
	$\dot{m}^{*}_{E} = 0.0243\ x^{3/4}$ kg/m²-s $U_o = 8.3297\ x^{1/2}$ m/s	$0.0220\ x^{3/4}$ $7.5003\ x^{1/2}$	$0.0163\ x^{3/4}$ $12.238\ x^{1/2}$	$0.0207\ x^{3/4}$ $6.2590\ x^{1/2}$
0.1 m	0.004 2.63	0.004 2.37	0.003 3.87	0.004 1.98
0.3 m	0.010 4.56	0.009 4.11	0.007 6.70	0.008 3.43
1.0 m	0.024 8.33	0.022 7.50	0.016 12.24	0.021 6.26
3.0 m	0.055 14.43	0.050 12.99	0.037 21.19	0.047 10.84

(*) Calculated with $\mathcal{D}_\infty = 1.5 \times 10^{-5}$ m²/s, $\rho_\infty = 1.17$ kg/m³

(+) Dilution with N_2

Appendix A:

The sensible enthalpy of an ideal gas can be written as,

$$h(T) = \int_{T_{ref}}^{T} C_p(T)\, dT \tag{A1}$$

where T_{ref} is the reference temperature at which the enthalpy of formation is evaluated.

For calorically perfect gas, if T_{ref} is taken to be 0° K then

$$h = C_p T \tag{A2}$$

Using equation of state for ideal gas

$$P = \rho R T \tag{A3}$$

the density ratio can be written as

$$\frac{\rho_\infty}{\rho} = \frac{C_{p\infty}}{C_p} \frac{R}{R_\infty} \frac{h}{h_\infty} \tag{A4}$$

where it is known that $P = P_\infty$ across the flame front.

Specific heat for a mixture of fuel, oxidizer and products (assuming C_p for products and air are approximately equal) is,

$$\frac{C_p}{C_{p\infty}} = \left(\frac{C_{pf}}{C_{p\infty}} - 1\right) Y_f + 1 \tag{A5}$$

Since $C_{pi} = \Gamma_i R_i$ where $\Gamma = \dfrac{C_p}{C_p - C_v}$ and $R = \dfrac{\mathcal{R}}{M}$, equation A5 can also be written as,

$$\frac{C_p}{C_{p\infty}} = 1 + \left(\frac{\Gamma_f M_\infty}{\Gamma_\infty M_f} - 1\right) Y_f \tag{A6}$$

Assuming the mixture of products have the same molecular weight as oxidizer (air) (e.g. for Paraffin family, C_nH_{2n+2}, $1 \leqq n \leqq 15$ $27.62 \leqq M_p \leqq 28.66$ and for Olefin family, C_nH_{2n}, $M_p = 28.78$ for complete combustion) we can write

$$\frac{R}{R_\infty} = 1 + \left(\frac{M_\infty}{M_f} - 1\right) Y_f \tag{A7}$$

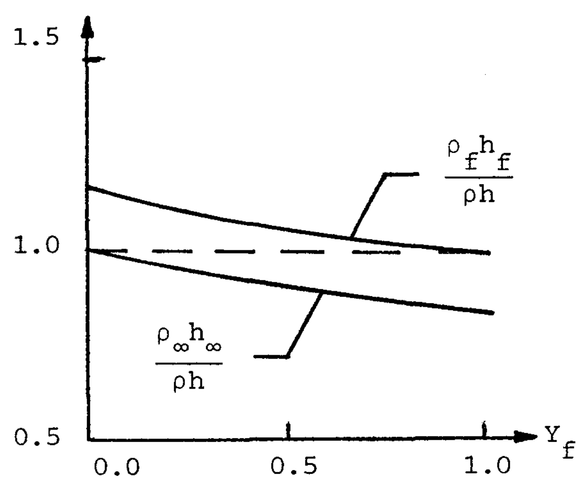
Combining A4, A6 and A7 one gets,

$$\frac{\rho_\infty h_\infty}{\rho h} = \frac{1 + \left(\frac{M_\infty}{M_f} - 1\right) Y_f}{1 + \left(\frac{\Gamma_f M_\infty}{\Gamma_f M_\infty} - 1\right) Y_f} \tag{A8}$$

Similarly,

$$\frac{\rho_f h_f}{\rho h} = \frac{\frac{M_f}{M_\infty} + \left(1 - \frac{M_f}{M_\infty}\right) Y_f}{\frac{\Gamma_\infty M_f}{\Gamma_f M_\infty} + \left(1 - \frac{\Gamma_\infty M_f}{\Gamma_f M_\infty}\right) Y_f} \tag{A9}$$

For methane-air $\Gamma_f/\Gamma_\infty \cong 1.179$ and $M_\infty/M_f = 1.813$



In our treatment we used,

$$\frac{\rho_f}{\rho} = \frac{h}{h_f} \text{ and } \frac{\rho_\infty}{\rho} = \frac{h}{h_\infty}$$

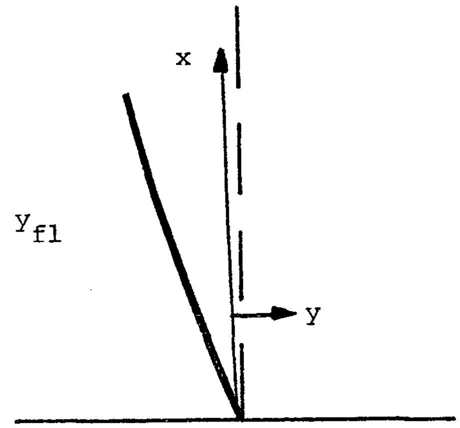


FIGURE 1

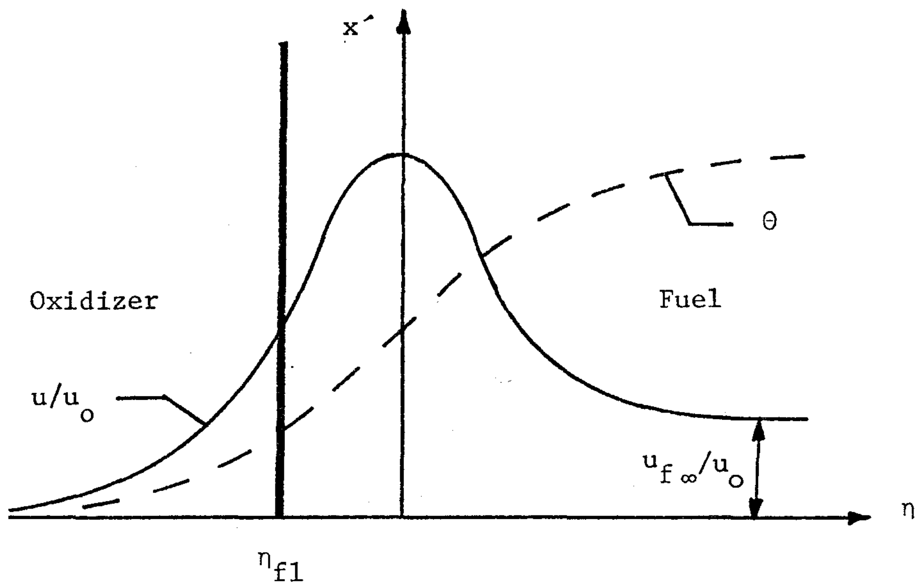


FIGURE 2

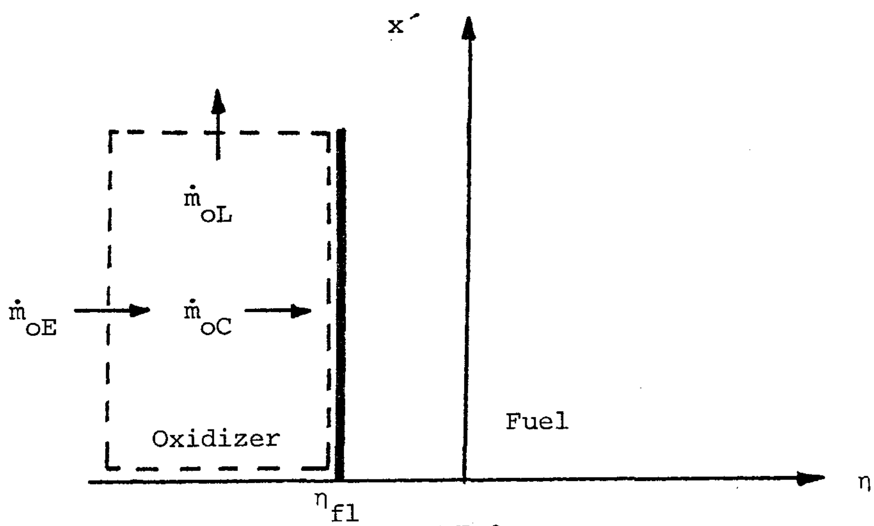
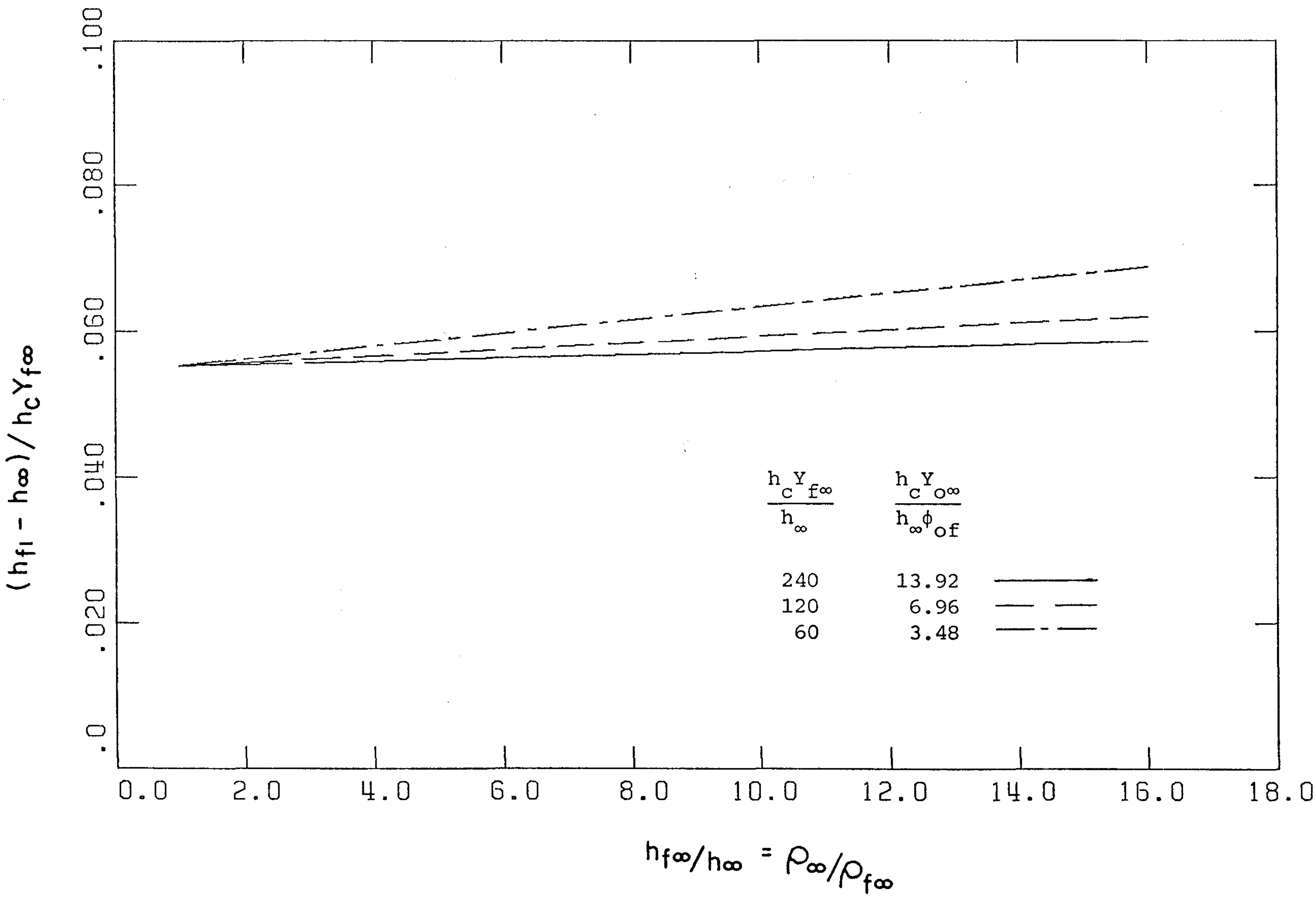
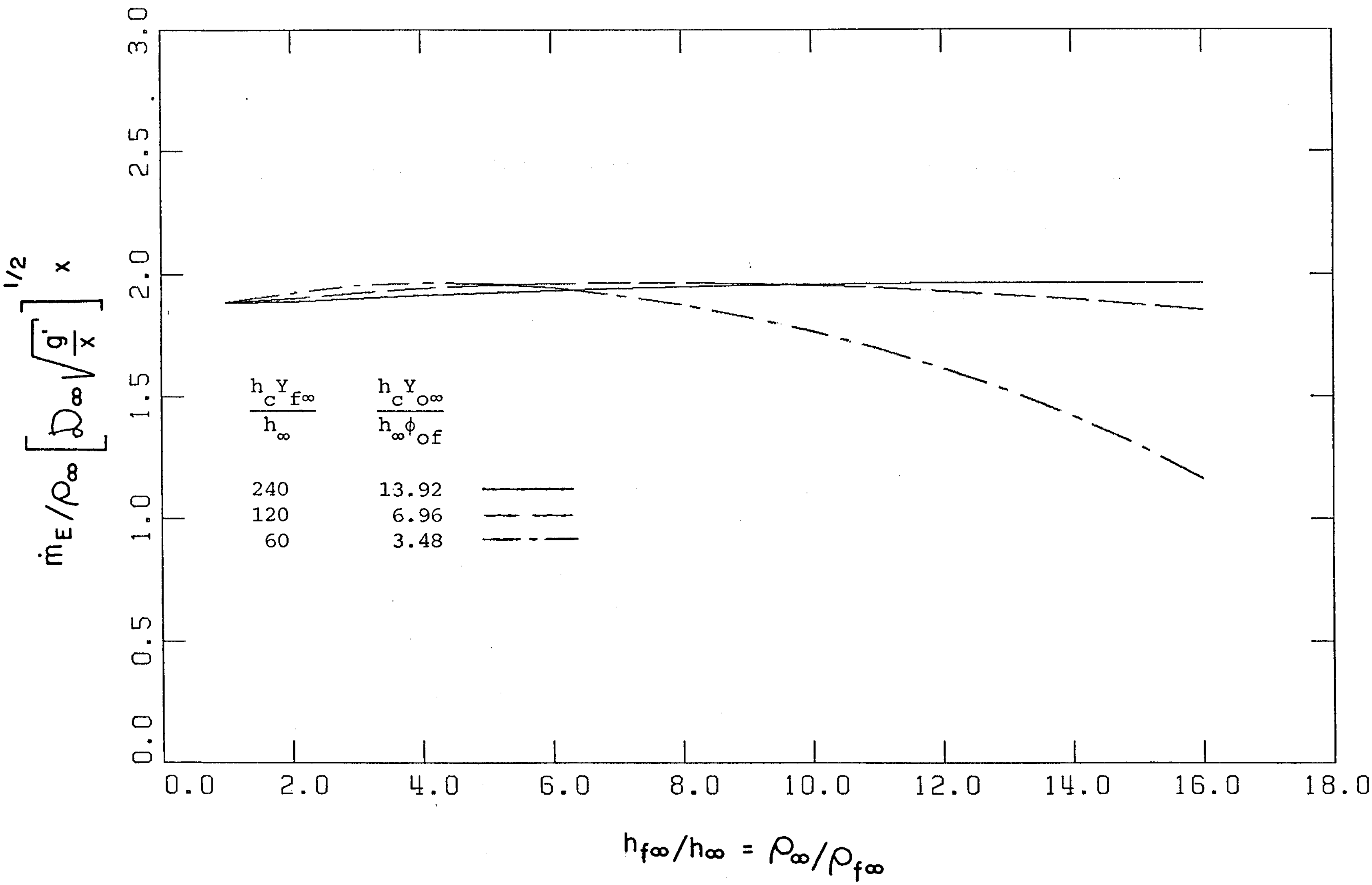


FIGURE 3



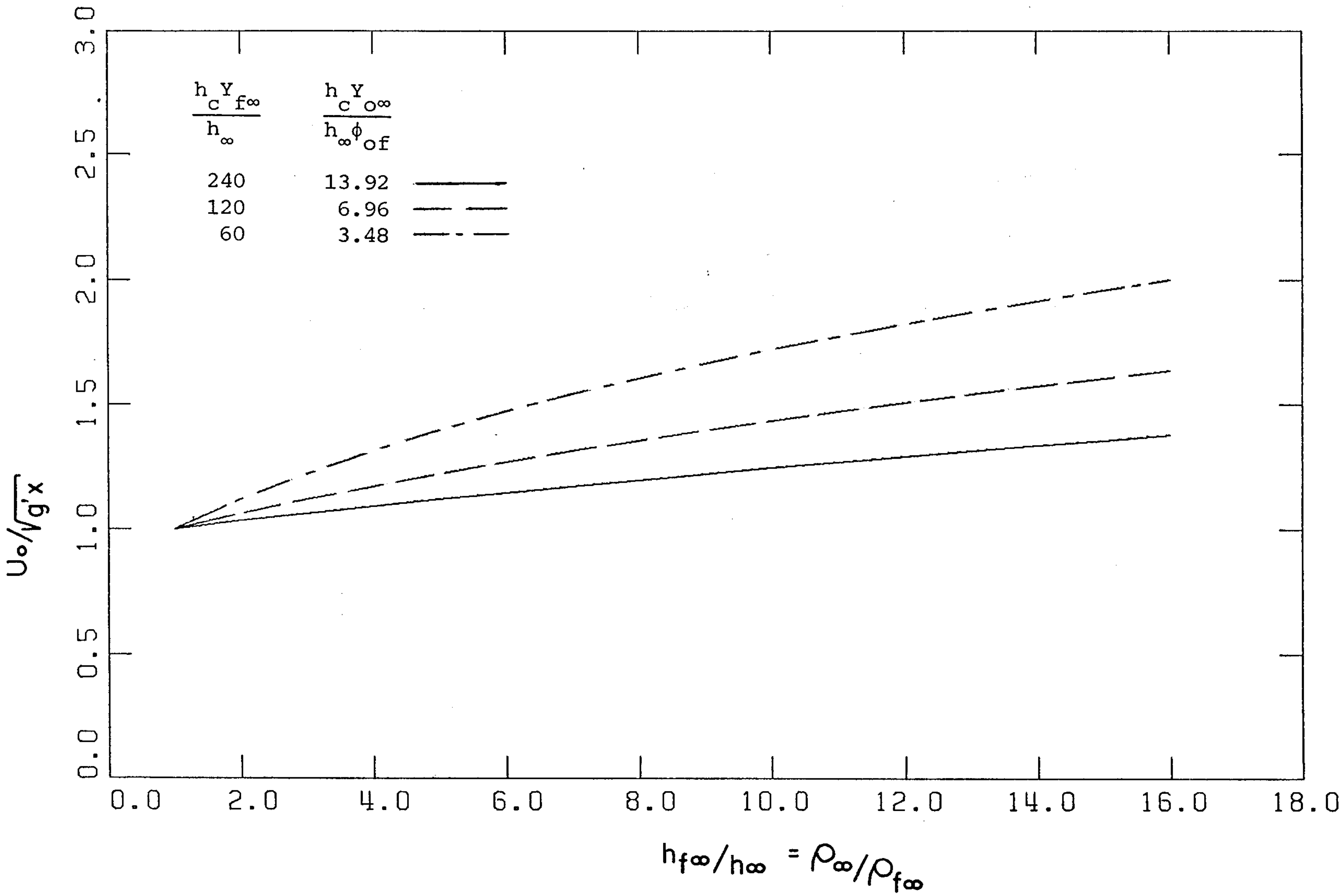
.0
.020
.040
.060
.080
.100
$(h_{fl} - h_\infty)/h_c Y_{f\infty}$
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
$h_{f\infty}/h_\infty = \rho_\infty/\rho_{f\infty}$
$\frac{h_c Y_{f\infty}}{h_\infty}$
$\frac{h_c Y_{o\infty}}{h_\infty \phi_{of}}$
240 13.92
120 6.96
60 3.48

FIGURE 8



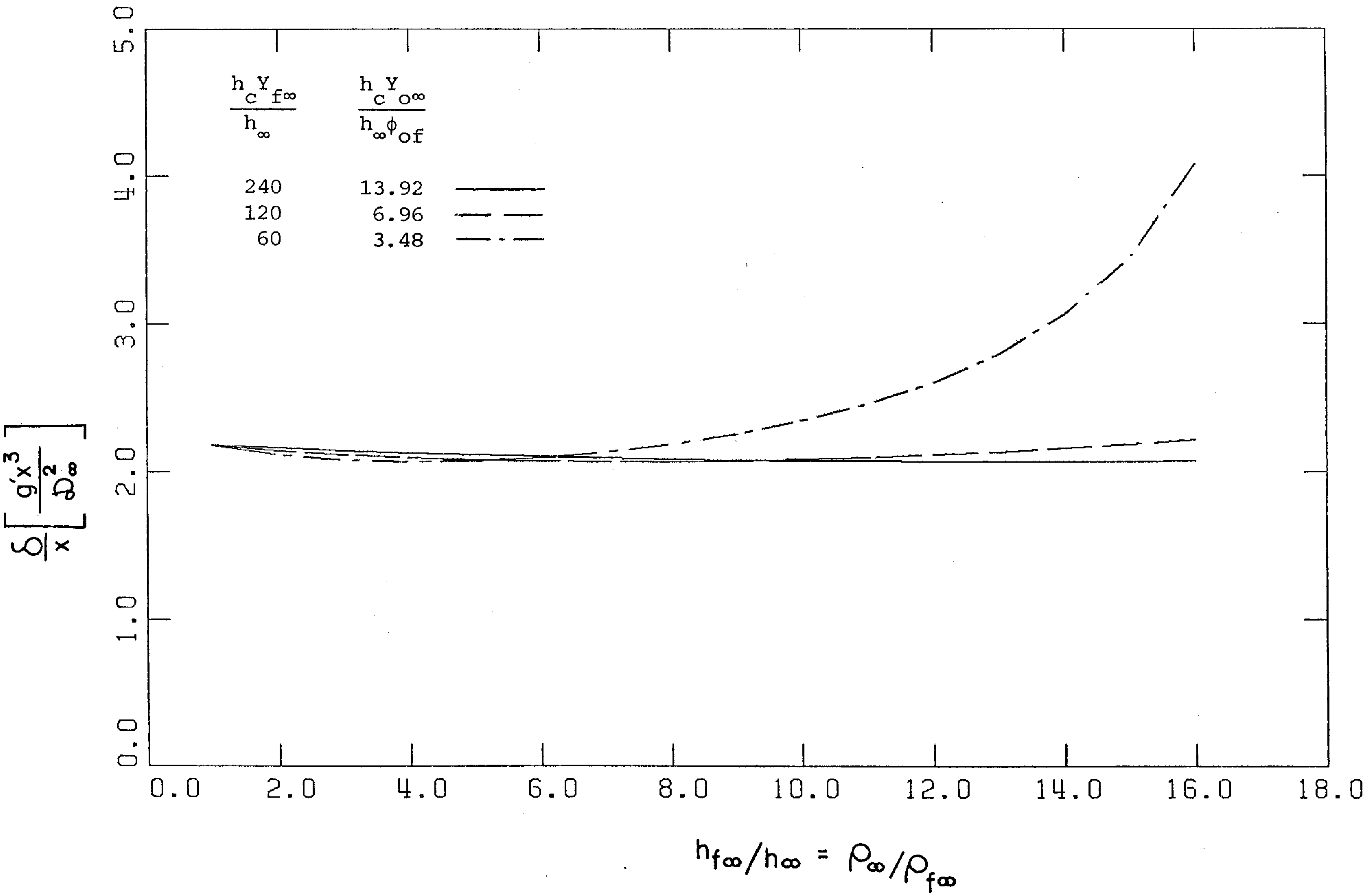
$\dot{m}_E / \rho_\infty \left[D_\infty \sqrt{\frac{g'}{x}} \right]^{1/2} x$
0.0
0.5
1.0
1.5
2.0
2.5
3.0
$\frac{h_c Y_{f\infty}}{h_\infty}$
$\frac{h_c Y_{o\infty}}{h_\infty \phi_{of}}$
240 13.92
120 6.96
60 3.48
0.0 2.0 4.0 6.0 8.0 10.0 12.0 14.0 16.0 18.0
$h_{f\infty}/h_\infty = \rho_\infty/\rho_{f\infty}$

FIGURE 4



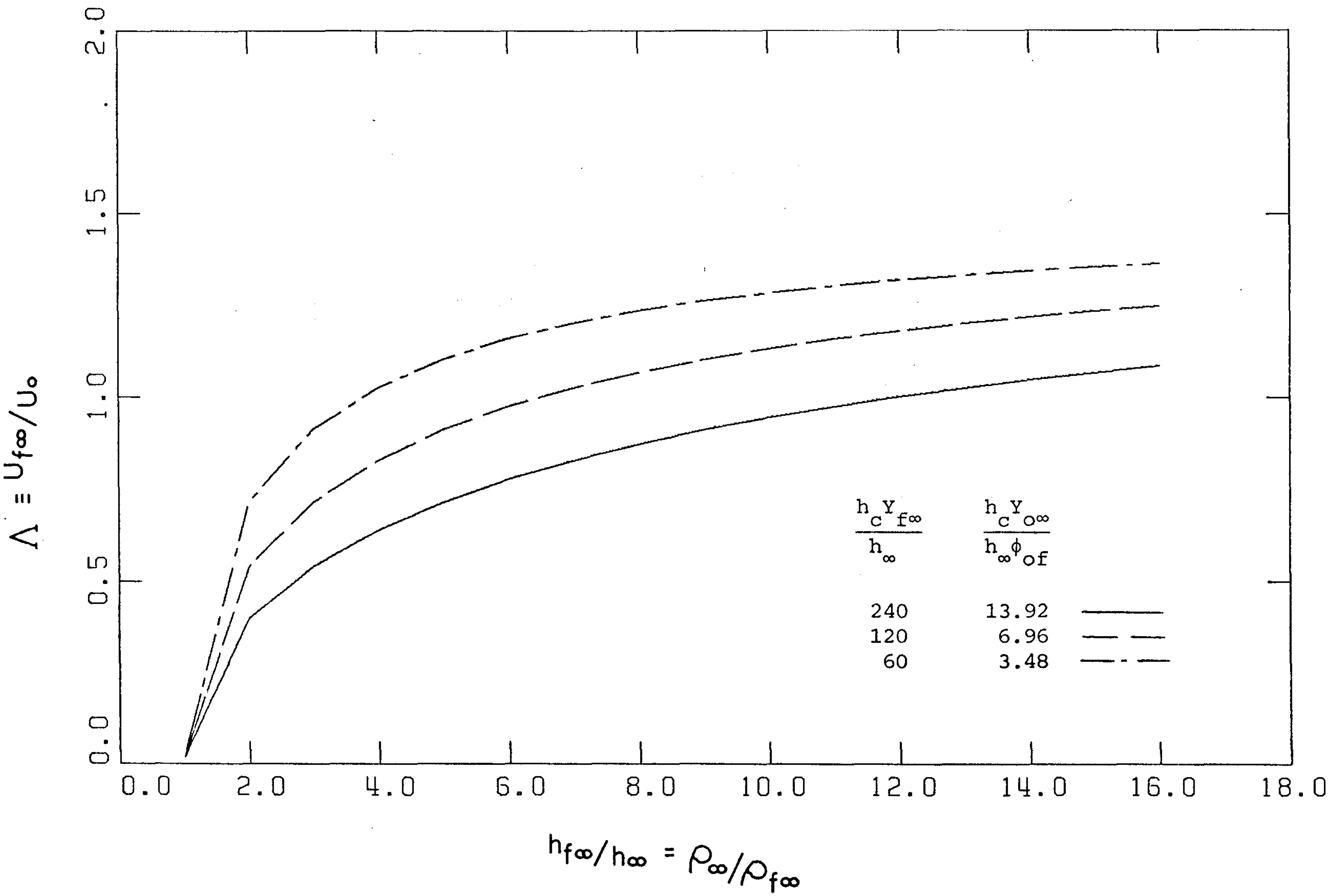
3.0
2.5
2.0
1.5
1.0
0.5
0.0
$U_o/\sqrt{g'x}$
$\frac{h_c Y_{f\infty}}{h_\infty}$
$\frac{h_c Y_{o\infty}}{h_\infty \phi_{of}}$
240 13.92
120 6.96
60 3.48
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
$h_{f\infty}/h_\infty = \rho_\infty/\rho_{f\infty}$

FIGURE 5



h_c Y_f∞ / h_∞
h_c Y_o∞ / h_∞ φ_of
240
13.92
120
6.96
60
3.48
0.0
1.0
2.0
3.0
4.0
5.0
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
δ/x [g′x³/𝒟∞²]^(1/4)
h_f∞/h_∞ = ρ_∞/ρ_f∞

FIGURE 6



0.0
0.5
1.0
1.5
2.0
$\Lambda \equiv U_{f\infty}/U_o$
0.0
2.0
4.0
6.0
8.0
10.0
12.0
14.0
16.0
18.0
$h_{f\infty}/h_\infty = \rho_\infty/\rho_{f\infty}$
$\frac{h_c Y_{f\infty}}{h_\infty}$
$\frac{h_c Y_{o\infty}}{h_\infty \phi_{of}}$
240 13.92
120 6.96
60 3.48

FIGURE 7

LIST OF SYMBOLS

A, B, C, D, E, F, G	Constants in equations 24, 25, 34, 35
C_p	Specific heat at constant pressure
$\mathcal{D}$	Mass diffusion coefficient
g	Gravitational constant
h	Specific enthalpy, see equation (A1)
h_c	Heat of combustion per mass of fuel
k	Heat conductivity
K_u, K_δ	Parameters defined in equations 27, 28
Le	Lewis Number
$\dot{m}$	Mass flux
M	Molecular weight
P	Pressure
Pr	Prandtl Number
R	Gas constant
$\mathcal{R}$	Universal gas constant
T	Temperature
u	Velocity in x direction
v	Velocity in y direction
x	Vertical coordinate
y	Transverse coordinate
Y	Species concentration

Greek Symbols

γ	Specific heat ratio C_p/C_v
δ	Characteristic length scale
η	Similarity coordinate
Θ	Schwab-Zeldovich variable

Greek Symbols (cont.)

Λ	Velocity ratio u_{f_∞}/u_o
μ	Dynamic viscosity
ν	Stoichiometric coefficient
ρ	Density
ϕ_{of}	Oxidizer-fuel mass ratio
$\chi_{1,2,3}$	Dimensionless parameters
$\dot{\omega}$	Species production rate

Subscripts

f	Fuel
fl	Flame
i	i^{th} species
o	Values at y = 0, oxidizer
p	Products
∞	Values at infinity

REFERENCES

1. E. E. Zukoski, Toshi Kubota and Baki Cetegen, "Entrainment in the Near Field of a Fire Plume," Report, Daniel and Florence Guggenheim Jet Propulsion Center, California Institute of Technology, Pasadena, (1981).

2. G. C. Fleming and F. E. Marble, Personal Communication.