UNSTEADY FLOWS IN AXIAL TURBOMACHINES*

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ABSTRACT

Of the various unsteady flows that occur in axial turbomachines certain asymmetric disturbances, of wave length large in comparison with blade spacing, have become understood to a certain extent. These disturbances divide themselves into two categories : self-induced oscillations and forced disturbances. A special type of propagating stall appears as a self-induced disturbance ; an asymmetric velocity profile introduced at the compressor inlet constitutes a forced disturbance.

Both phenomena have been treated from a unified theoretical point of view in which the asymmetric disturbances are linearized and the blade characteristics are assumed quasi-steady. Experimental results are in essential agreement with this theory wherever the limitations of the theory are satisfied. For the self-induced disturbances and the more interesting examples of the forced disturbances, the dominant blade characteristic is the dependence of total pressure loss, rather than the turning angle, upon the local blade inlet angle.

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I - INTRODUCTION

The occurrence of stall propagation in axial compressor blade rows is now well known and has been widely observed. It is understood to take place when the flow inlet angle to a compressor stage lies in a certain range in the neighborhood of the angle for which steady stall should be observed. As the inlet angle is increased a stall region occurs covering a group of blades and, *instead of remaining with the same group of blades, moves circumferentially* about the blade row. The properties of this stall are self induced in the sense that they are in no way a response to other disturbances introduced into the compressor. Of particular interest here are i) the conditions under which such a self-induced disturbance is possible and ii) the propagation speed, amplitude and wave length of the stall region.

A second type of unsteady stall that exists in axial compressors is of different origin and, superficially, is quite distinct from stall propagation. This type of stall takes place in a compressor rotor in response to a strong circumferential distortion of the axial velocity profile introduced at the inlet. Here the stall region is forced to remain stationary with respect to the compressor casing. The points of particular interest here are i) the magnitude of disturbance required to induce stall and ii) the progression of this stall through succeeding blade rows with special regard to whether the distortion is smoothed out or retains a finite amplitude through the compressor.

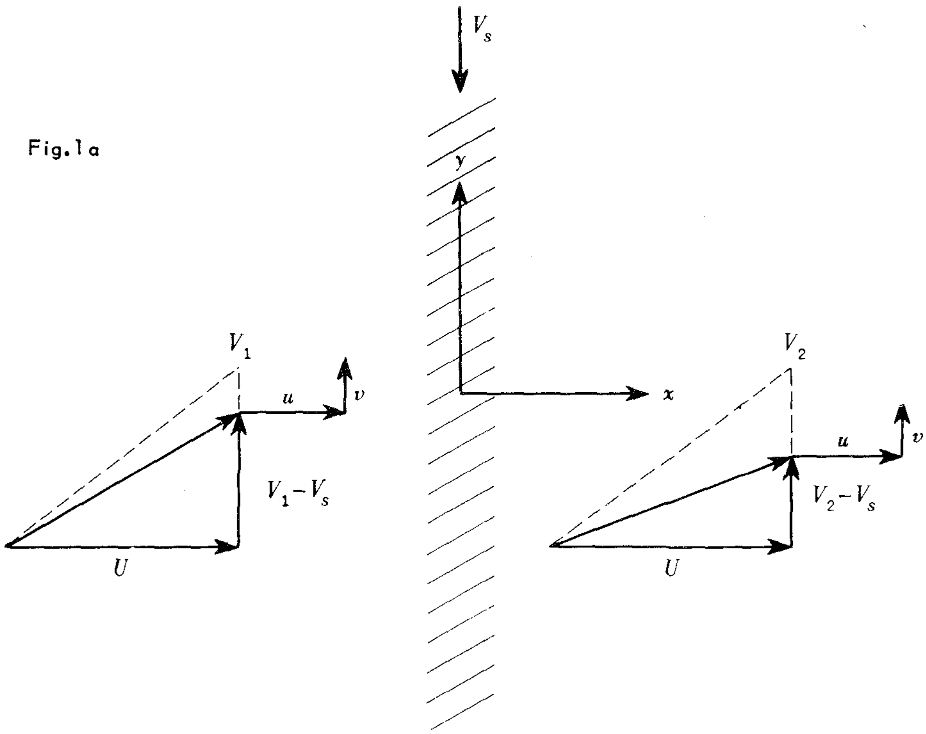
The mechanism of stall propagation has been investigated extensively from both theoretical and experimental points of view. The analytical work has, for the most part, been confined to small disturbance theories in which the amplitude of the stall and of flow angle perturbations are assumed small. Various approaches to the small perturbation theory have been proposed by H.W. EMMONS [1], W.R. SEARS [2], and F.E. MARBLE [3] ; these theories have been reviewed, and to some extent compared by W. R. SEARS , reference [4]. In all compressor experiments reported to the present time the stall amplitude appears to be outside the realm of linearized theory. Interesting and bold approaches to a non-linear theory have been proposed recently by A. R. KRIEBEL [5] and J. FABRI and R. SIESTRUNCK [6]. In contrast to the linearized theories that assume infinitesimal disturbance in the stall wake, these investigators assume the fluid in the wake to be nearly stagnant, an approximation that appears to be reasonably well confirmed by experiments. The price paid for non-linearity is, of course, the inability to compute details of the flow field. FABRI and SIESTRUNCK [6], in particular, seem to have reduced the statement of the problem to a minimum of assumptions.

The effect of circumferential distortion to the inlet velocity profile, although of considerable practical interest in connection with installation of aircraft gas turbines, has not yet been the subject of any analytical publications. The only work known to the authors is that of Dr. F.F. EHRICH contained in a personal communication.

It is the purpose of the present paper to show that these two diverse phenomena, stall propagation and circumferential distortion of the inlet velocity profile, may be treated within a common framework. A small perturbation theory is developed whose foundations differ in one essential aspect from previous theories : the mean turning angle through the blade row may be large. In this theory the propagating stall appears as a self-induced disturbance or "natural oscillation", the propagation speed is the characteristic value or "natural frequency", and the effect of an inlet distortion appears as a forced disturbance or a "forced oscillation". Experiments were made with propagating stall in a stationary annular cascade in which blade solidity and stagger angle were varied. Using this equipment small amplitude stall was observed, satisfying the assumptions of the linearized theory. These experimental results are compared in detail with theoretical calculations. Experiments on the effect of inlet distortion were made using a three stage low speed compressor. The experimental results are compared qualitatively with the corresponding theoretical calculations.

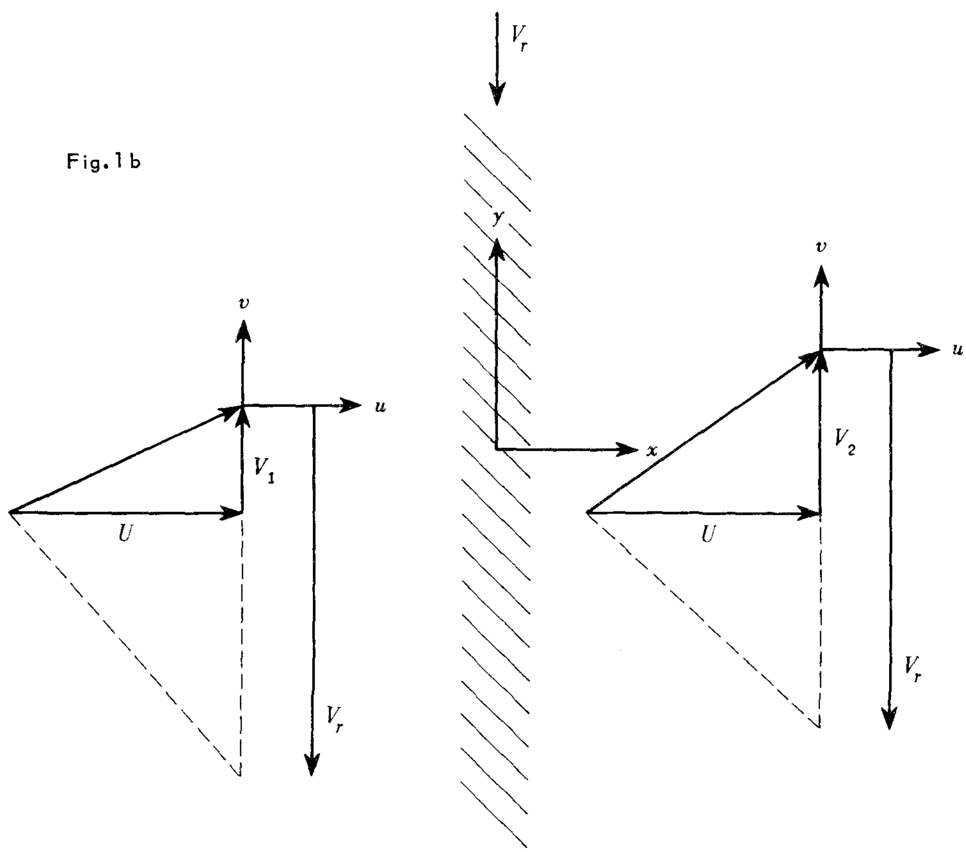
The experiments on stall propagation quoted herein were performed by M. David BENENSON and the experiments on inlet distortion were performed by M. Robert KATZ, both in connection with their doctoral thesis at the California Institute of Technology. The authors are grateful for permission to quote these data at this time.

Fig. 1a



PROPAGATING STALL

Fig. 1b



INLET DISTORTION

FIG. 1 - DIAGRAM INDICATING COORDINATES AND NOTATION FOR SELF INDUCED OR FORCED DISTURBANCE FIELD NEAR A SINGLE BLADE ROW.

II - THE DISTURBANCE FLOW FIELD

Let x, y be rectangular coordinates with the axis of y parallel to the cascade axis, Figure 1. The average velocity components and pressure are U, V and P, all constant between any blade rows. The disturbance velocity components and pressure are u, v and p, all with average values zero. The disturbance is assumed periodic in the y-direction and travels with a velocity V_s in the y-direction. The wave length of the disturbance is assumed large compared with the blade gap so the distortions produced by individual blades on the disturbance pattern are ignored even when there is relative motion between the disturbance and the cascade. Under these circumstances the disturbance flow is steady in a coordinate system moving with the disturbance. For small disturbances the equations of motion in linearized form are

$$U\frac{\partial u}{\partial x} + (V - V_s)\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x}$$

$$U\frac{\partial v}{\partial x} + (V - V_s)\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} \tag{2.1}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0.$$

The equations above are equivalent to the following three, more significant equations :

$$\left[U\frac{\partial}{\partial x} + (V - V_s)\frac{\partial}{\partial y}\right]\left[\frac{p}{\rho} + Uu + (V - V_s)v\right] = 0. \tag{2.2}$$

$$\frac{\partial}{\partial y}\left[Uv - (V - V_s)u\right] = \frac{\partial}{\partial x}\left(\frac{p}{\rho}\right). \tag{2.3}$$

$$\frac{\partial}{\partial x}\left[Uv - (V - V_s)u\right] = -\frac{\partial}{\partial y}\left(\frac{p}{\rho}\right). \tag{2.4}$$

The first of these is the condition that the perturbation of total pressure is constant along the mean stream lines in the moving system. The second two equations show that the combination $Uv-(V-V_s)u$, (proportional to flow angle perturbation) and the static pressure divided by density are potential functions and satisfy the Cauchy-Riemann conditions. For simplicity of analysis it is convenient to introduce the definitions

$$\frac{p}{\rho} + Uu + (V - V_s)v = H. \tag{2.5}$$

$$Uv - (V - V_s)u = F. \tag{2.6}$$

$$\frac{p}{\rho} = G. \tag{2.7}$$

Then the following relations hold

$$H(x, y) = H\left(y - \frac{V - V_s}{U}x\right). \tag{2.8}$$

$$\frac{\partial F}{\partial y} = \frac{\partial G}{\partial x}, \quad \frac{\partial F}{\partial x} = -\frac{\partial G}{\partial y} \tag{2.9}$$

and two functions only are required to define the disturbance flow completely. For a semi-infinite flow field upstream or downstream of a blade row the functions can be given in particularly simple terms. F and G approach zero far from the cascade and if the blade row is at $x=0$, then for $F(0, y)$ given, $G(0, y)$ can be found from

$$G(0, y) = \pm\int_0^1 F(0, \eta)\cot\pi(y - \eta)\,d\eta = \pm F^*(0, y), \quad x \gtrless 0 \tag{2.10}$$

and similarly

$$F(0,y) = \mp G^*(0,y) \qquad x \gtrless 0$$

where the wave length is unity. F and G may be found for all values of x and y if $F(0,y)$ or $G(0,y)$ is given. Hence the disturbance flow is described completely by $H(0,y)$ and $F(0,y)$ or $G(0,y)$.

For a finite region between two blade rows the prescription of the disturbance flow is somewhat more involved, but it can be done in terms of values of H, F and G along the sides of the region.

III - MATCHING CONDITIONS OVER THE BLADE ROW

The wave length of the disturbance has been assumed to be large compared with the blade gap so that the local inlet conditions at any particular blade channel can be assumed constant across the channel at any instant and to vary slowly with time if the velocity of the disturbance relative to the cascade is not very large. The flow is assumed locally quasi-steady. Three conditions relating to conservation of mass, momentum and energy can be prescribed to give the local outlet flow in terms of local inlet flow. For finite axial projection of the cascade there is a shift of the mean stream lines from inlet to outlet, but this is constant along the cascade and can be ignored unless actual stream lines are required.

Let U, V_1, P_1 be the average velocity components and pressure upstream of the cascade and U, V_2, P_2 the corresponding values downstream. Let the perturbation values immediately upstream and downstream be u_1, v_1, p_1 and u_2, v_2, p_2 respectively. The continuity condition for stator or rotor is simply

$$u_2 = u_1 \tag{3.1}$$

since the average axial velocities are the same upstream and downstream.

Let the inlet angle of the flow relative to the cascade be β_1 and the outlet angle β_2 both angles measured from the axial direction. Then very generally

$$\tan\beta_2 = A + R\,(\tan\beta_1) \tag{3.2}$$

where R is a prescribed function of $\tan\beta_1$ as indicated in Figure 2. For high solidity (constant leaving angle), $R=0$; for local flow satisfying the conditions for irrotationality $R=K\ \tan\beta_1$. The momentum equation (3.2) can be given in terms of lift coefficient as well although the present form is usually more convenient.

The energy condition over the cascade can be taken as a total pressure loss, proportional to the square of the local inlet velocity and to a function of $\tan\beta_1$, i.e.

$$\Delta p_t = \rho\,(U+u_1)^2\, P(\tan\beta_1) \tag{3.3}$$

where P is a prescribed function shown schematically also in Figure 2. It is assumed that the functions $R(\tan\beta_1)$ and $P(\tan\beta_1)$ are single-valued functions, i.e. no time lags or hysterisis loops.

In most of the calculations to follow, the outlet angle will be taken as constant; under these circumstances

$$\frac{V_2 + v_2}{U + u_2} = \tan\beta_2 = \text{const.} \tag{3.4}$$

for a stator blade row, and

$$\frac{V_r - V_2 - v_2}{U + u_2} = \tan\beta_2 = \text{const.} \tag{3.5}$$

for a rotor blade row. Hence

(3.6) $$v_2 = \frac{V_2}{U} u_2 \quad \text{(stator)}.$$

(3.7) $$v_2 = -\frac{V_r - V_2}{U} u_2 \quad \text{(rotor)}.$$

The right hand side of Equation (3.3) can be expanded in terms of the disturbance velocities i.e., for a stator

$$\frac{1}{\rho} \Delta p_t = U^2 P\left(\frac{V_1}{U}\right) + 2UP\left(\frac{V_1}{U}\right) u_1 + P'\left(\frac{V_1}{U}\right)(U v_1 - V_1 u_1) + \dots$$

and a very similar expression for a rotor.

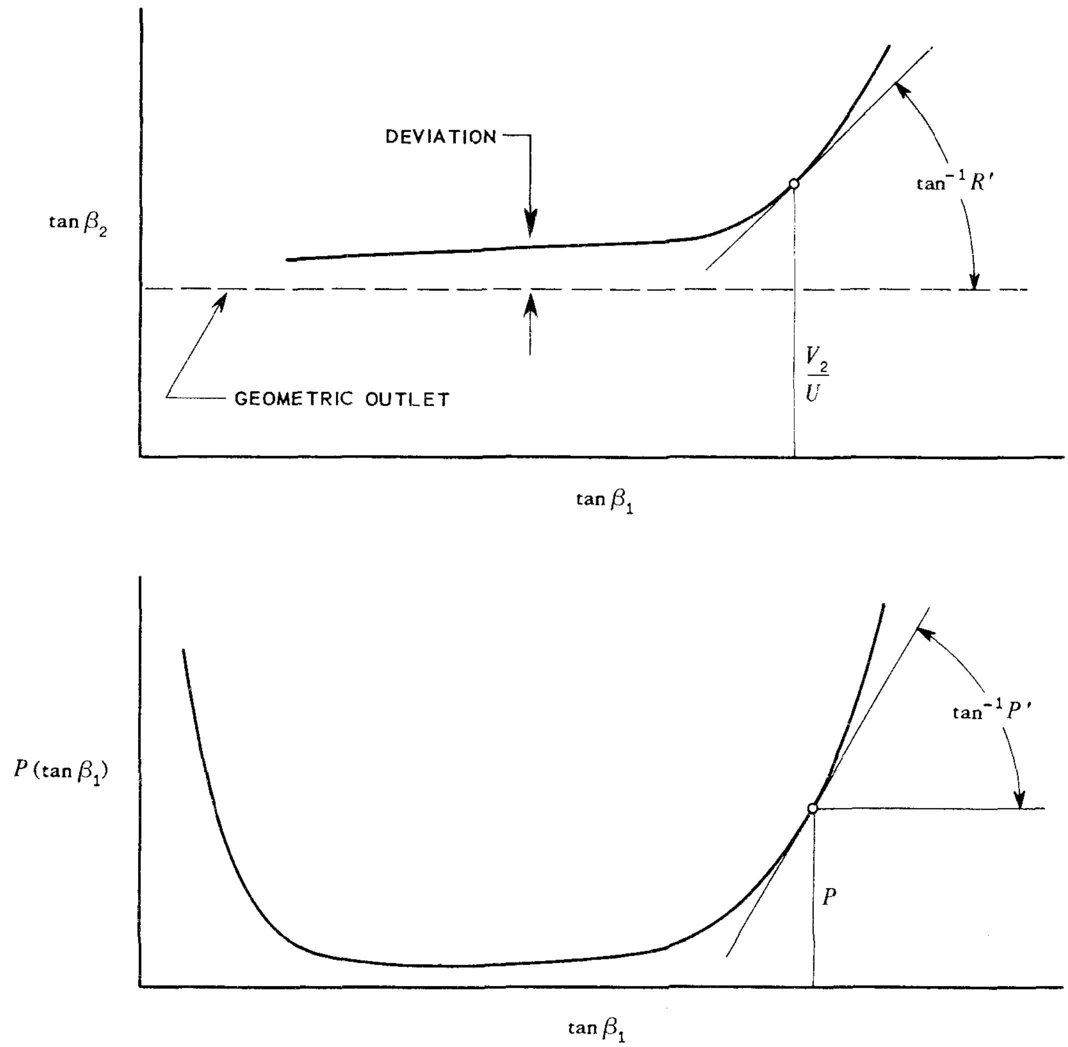


FIG. 2 - BEHAVIOR OF BLADE OUTLET ANGLE DEVIATION AND TOTAL PRESSURE LOSS AS FUNCTIONS OF BLADE INLET ANGLE.

For most calculations it is more convenient to have the matching conditions in terms of the functions F, G and H introduced previously. Expressing u_1, v_1, u_2, v_2 and Δp_t in this way, the matching conditions become, for constant leaving angle

$$
\begin{aligned}
&G_1(0,y) - G_2(0,y) + A_s F_1(0,y) + B_s F_2(0,y) = 0 \\
(3.8)\qquad &H_1(0,y) = G_1(0,y) + \alpha_s F_1(0,y) + \beta_s F_2(0,y) \\
&H_2(0,y) = G_2(0,y) + \gamma_s F_2(0,y)
\end{aligned}
$$

where

$$
\alpha_s = \frac{V_1 - V_s}{U} \qquad \beta_s = \frac{U^2 + (V_1 - V_s)^2}{U V_s} \qquad \gamma_s = \frac{U^2 + (V_2 - V_s)^2 + V_s (V_2 - V_s)}{U V_s}
$$

$$
A_s = \frac{V_1}{U} - P_s' \qquad B_s = \frac{V_1^2 - V_2^2}{U V_s} - \frac{V_1}{U} - 2\frac{U}{V_s} P_s + P_s'
$$

for a stator blade row and

$$
\begin{aligned}
&G_1(0,y) - G_2(0,y) + A_r F_1(0,y) + B_r F_2(0,y) = 0 \\
(3.9)\qquad &H_1(0,y) = G_1(0,y) + \alpha_r F_1(0,y) + \beta_r F_2(0,y) \\
&H_2(0,y) = G_2(0,y) + \gamma_r F_2(0,y)
\end{aligned}
$$

where

$$
A_r = -\frac{V_r - V_1}{U} + P_r' \qquad B_r = -\frac{(V_r - V_1)^2 - (V_r - V_2)^2}{U(V_r - V_s)} + \frac{V_r - V_1}{U} + 2\frac{U}{V_r - V_s} P_r - P_r'
$$

$$
\alpha_r = \frac{V_1 - V_s}{U} \qquad \beta_r = -\frac{U^2 + (V_1 - V_s)^2}{U(V_r - V_s)} \qquad \gamma_r = -\frac{U^2 + (V_2 - V_s)^2 - (V_r - V_s)(V_2 - V_s)}{U(V_r - V_s)}
$$

for a rotor blade row.

IV - SELF INDUCED DISTURBANCE FOR A SINGLE BLADE ROW

Let us suppose that uniform flow approaches an isolated stator blade row at the plane $x=0$. Since $H_1(x,y) \equiv 0$, the conditions that a self induced disturbance exists are given by the first two Equations (3.8) i.e.

$$
\begin{aligned}
&G_1 - G_2 + A_s F_1 + B_s F_2 = 0 \\
(4.1)\qquad &G_1 + \alpha_s F_1 + \beta_s F_2 = 0
\end{aligned}
$$

where the arguments $(0,y)$ can be dropped without confusion. Taking conjugates of these equations by the rules given in Equation (2.10) two further independent equations are found

$$
\begin{aligned}
&F_1 + F_2 - A_s G_1 + B_s G_2 = 0 \\
(4.2)\qquad &F_1 - \alpha_s G_1 + \beta_s G_2 = 0
\end{aligned}
$$

Eliminating F_2 and G_2

$$
\begin{aligned}
&(\alpha_s - \beta_s - B_s) F_1 + (1 + \beta_s A_s - \alpha_s B_s) G_1 = 0 \\
(4.3)\qquad &(1 + \beta_s A_s - \alpha_s B_s) F_1 - (\alpha_s - \beta_s - B_s) G_1 = 0.
\end{aligned}
$$

The conditions for a non trivial solution of these last two equations are that the coefficients of F_1 and G_1 vanish

$$
\begin{aligned}
&\alpha_s - \beta_s - B_s = 0 \\
(4.4)\qquad &1 + \beta_s A_s - \alpha_s \beta_s = 0.
\end{aligned}
$$

Substituting for the coefficients α_s, β_s, A_s and B_s from Equation (3.8), these last two conditions are equivalent to

$$\frac{V_1}{U} P_s' - 2P_s = \left(1 + \frac{V_2^2}{U^2}\right). \tag{4.5}$$

$$\frac{V_s}{U} = \frac{1}{2} P_s'. \tag{4.6}$$

The left hand side of Equation (4.5) is a function of V_1/U since P_s and P_s' are functions of this quantity; the right hand side is constant. One would expect the variation of P_s as a function of V_1/U to be of the form shown in Figure 2, that is P_s quite small over a limited range of inlet angle, then rises sharply as the cascade stalls positively or negatively. Under these circumstances there will be generally one and only one value of V_1/U at which Equation (4.5) is satisfied. It will occur at the positive stall condition. Hence a self induced disturbance can occur at only one value of the upstream flow angle.

The second Equation (4.6) must be satisfied simultaneously and gives a unique propagating speed if Equation (4.5) is satisfied. The amplitude and shape of the self-induced disturbance are not determined by the linearized theory, and since there is no control over the disturbance shape, the theory is applicable only for those experiments where the conditions of the theory are satisfied. The propagating speed should be compared with propagating stall observations only if the wave length of the disturbance is large compared with the blade gap, the velocity fluctuations are small in comparison with U and the leaving angle from the cascade is constant.

The theory above applies to self-induced disturbances on a fixed stator. Obviously the same theory applies to a rotor; it is merely necessary to add a uniform velocity V_r, the rotor speed, to the entire system. In the theory so far, it has been assumed that the leaving angle from the cascade is constant. If leaving angle deviations are taken into account as well as total pressure losses the two conditions for a self induced disturbance become

$$\frac{V_1}{U} P_s' - 2P_s - \frac{V_1 V_2}{U^2} R_s' = 1 + \frac{V_2^2}{U^2}. \tag{4.7}$$

$$\frac{V_s}{U} = \frac{1}{2} P_s' - \frac{1}{2} \frac{V_2}{U} R_s', \tag{4.8}$$

where R_s' is the local slope of the deviation angle curve as indicated in Figure 2. If $P_s \equiv 0$, it appears that solutions of these equations do not occur for any realistic variations of R_s with V_1/U. Hence it is unlikely that a self-induced disturbance resulting from leaving angle deviations alone can occur. It may be that leaving angle deviations can modify the self-induced disturbances that result primarily from the total pressure loss characteristics of the cascade.

V - FORCED DISTURBANCES ON AN ISOLATED ROTOR

The simplest example of an asymmetric forced disturbance results from a distorted inlet flow approaching an isolated rotor. Here the upstream disturbance is taken as a weak shear disturbance, fixed in space with the rotor passing through it. Hence the disturbance speed V_s is zero. The non-uniform inlet angles relative to the rotor produce variations on the moving rotor that induce additional regular disturbances upstream and downstream and in turn affect local inlet angles. These induced disturbances are of the type described by the F and G functions, hence satisfy Laplace's equation and die out upstream and downstream for an isolated blade row.

In the problem as formulated above, $H_1(x, y)$ is given by the conditions far upstream, and we wish to find the disturbed flow resulting from the interaction of this non-uniformity with the rotor, and in particular the total pressure variation downstream. The first two of Equations (3.9) are

$$G_1 - G_2 + A_r F_1 + B_r F_2 = 0 \tag{5.1}$$

$$G_1 + \alpha_r F_1 + \beta_r F_2 = H_1, \tag{5.2}$$

where $V_s = 0$ for a stationary disturbance. The complementary equations to these two, from the operation (2.10) are

$$F_1 + F_2 - A_r G_1 + B_r G_2 = 0 \tag{5.3}$$

$$F_1 - \alpha_r G_1 + \beta_r G_2 = H_1^*. \tag{5.4}$$

The four Equations (5.1) to (5.4) are sufficient to determine F_1, F_2, G_1 and G_2 as linear expressions in H_1 and H_1^*. Then H_2 can be found from the expression (see Equation (3.9))

$$H_2 = G_2 + \gamma_r F_2 = K_1 H_1 + K_2 H_1^*. \tag{5,5}$$

where K_1 and K_2 are rather lengthy algebraic functions of A_r, B_r, α_r and β_r. Substituting for the latter from Equation (3.9)

$$\begin{aligned}
K_1 = \frac{\cos^2\theta_1}{4}\Bigg\{ & -\frac{V_r}{U}\Bigg[\frac{V_r}{U}P_r'^2 - 2P_r'P_r - \left(\tan^2\beta_2 - \tan^2\beta_1 + 2\frac{V_r^2}{U^2}\right)P_r' \\
& \qquad + 2\frac{V_r}{U}P_r - \frac{V_r}{U}(1 - \tan^2\beta_2 - 2\tan\theta_1\tan\beta_1)\Bigg] \\
& -(1 - \tan\theta_2\tan\beta_2)\Bigg[4P_r^2 + \tan\beta_1\frac{V_r}{U}P_r'^2 - 2\left(\frac{V_r}{U} + \tan\beta_1\right)P_r'P_r \\
& \qquad - \left\{\frac{V_r}{U}(1 + \mathrm{Sec}^2\beta_2) + \tan\beta_1(\tan\theta_1\tan\beta_1 + \tan^2\beta_2)\right\}P_r' \\
& \qquad + 2\left\{\mathrm{Sec}^2\beta_2 + \tan\theta_1\tan\beta_1 + \tan^2\beta_2\right\}P_r \\
& \qquad + \left\{2\frac{V_r^2}{U^2} - 1 + \mathrm{Sec}^2\beta_2(\tan\theta_1\tan\beta_1 + \tan^2\beta_2)\right\}\Bigg]\Bigg\} \\
& \times\left\{P_r^2 + \frac{\mathrm{Sec}^2\beta_1}{4}P_r'^2 - \tan\beta_1 P_r P_r' - \left(\frac{V_r}{U} + \frac{\mathrm{Sec}^2\beta_2\tan\beta_1}{2}\right)P_r' + \mathrm{Sec}^2\beta_2 P_r + \frac{V_r^2}{U^2} + \frac{\mathrm{Sec}^4\beta_2}{4}\right\}^{-1}
\end{aligned} \tag{5.6}$$

where

$$\tan\theta_1 = \frac{V_1}{U} \qquad \tan\beta_1 = \frac{V_r - V_1}{U}$$

$$\tan\theta_2 = \frac{V_2}{U} \qquad \tan\beta_2 = \frac{V_r - V_2}{U}$$

$$K_2 = \frac{\cos^2\theta_1}{4}\left\{ \frac{V_r}{U}\left[4P_r^2 + \frac{V_r}{U}\tan\beta_1 P_r'^2 - 2\left(\frac{V_r}{U} + \tan\beta_1\right)P_r P_r'\right.\right.$$

$$- \left\{\frac{V_r}{U}(1+\mathrm{Sec}^2\beta_2) + \tan\beta_1(\tan\theta_1\tan\beta_1 + \tan^2\beta_2\right\} P_r'$$

$$+ 2\left\{\mathrm{Sec}^2\beta_2 + \tan\theta_1\tan\beta_1 + \tan^2\beta_2\right\} P_r$$

(5,6 cont'd)

$$\left.+ \left\{2\frac{V_r^2}{U^2} - 1 + \mathrm{Sec}^2\beta_2(\tan\theta_1\tan\beta_1 + \tan^2\beta_2)\right\}\right]$$

$$+ (1-\tan\theta_2\tan\beta_2)\left[-\frac{V_r}{U}P_r'^2 + 2P_rP_r' + \left(2\frac{V_r^2}{U^2} + \tan^2\beta_2 - \tan^2\beta_1\right)P_r'\right.$$

$$\left.\left. - 2\frac{V_r}{U}P_r + \frac{V_r}{U}(1 - 2\tan\theta_1\tan\beta_1 - \tan^2\beta_2)\right]\right\}$$

$$\times \left\{P_r^2 + \frac{\mathrm{Sec}^2\beta_1}{4}P_r'^2 - \tan\beta_1 P_r P_r' - \left(\frac{V_r}{U} + \frac{\mathrm{Sec}^2\beta_2\tan\beta_1}{2}\right)P_r' + \mathrm{Sec}^2\beta_2 P_r + \frac{V_r^2}{U^2} + \frac{\mathrm{Sec}^4\beta_2}{4}\right\}^{-1}$$

The coefficients P_r and P_r' occurring in Equation (5.6) for K_1 and K_2 are presumed known functions of $(V_r - V_1)/U$ and will behave as shown schematically in Figure 2. For disturbances of moderate amplitude it would be necessary to use mean values of P_r and P_r', to give the best linear representation of the curve between the amplitude limits. If the rotor were operating at the design value of V_1/U and hence close to the stall, moderate disturbance amplitudes might give large values of the mean slope of the loss curve P_r', although P_r might not be large. Hence for the most interesting applications of the theory, the slope of the loss curve appears to be the most important blade characteristic, as was found for self induced disturbances.

VI - AXISYMMETRIC DISTURBANCES IN MULTIPLE BLADE ROWS

The general theory for blade rows with finite spacing follows directly from successive applications of the results of Sections II and III. The analysis leads to such involved expressions in general that it is probably not very useful. Introduction of the trigonometric functions for F and G simplifies the analysis appreciably. Suitable forms are

(6.1)
$$F = (A\,e^{2\pi nx} + B\,e^{-2\pi nx})\cos 2\pi n y + (C\,e^{2\pi nx} + D\,e^{-2\pi nx})\sin 2\pi n y$$
$$G = (-A\,e^{2\pi nx} + B\,e^{-2\pi nx})\sin 2\pi n y + (C\,e^{2\pi nx} - D\,e^{-2\pi nx})\cos 2\pi n y.$$

Even more drastic simplification is possible for the axial gap dimensions that occur in most compressors. The theory is restricted to disturbance wave lengths large compared with the cascade blade gap. The axial gaps are ordinarily even smaller, so that the variation of the exponential factors in the expressions (6.1) from one side of the gap to the other will usually be negligible. Hence F and G are described sufficiently well by four constant factors multiplying the sine and cosine terms.

As a specific example suppose one is dealing with a rotor followed by a stator far away from other blade rows, and that a stationary sinusoidal shear disturbance is introduced far upstream. If subscripts 1, 2 and 3 represent the regions upstream of the rotor, between rotor and stator, and downstream of the stator respectively, and if the rotor is at $x=0$, the stator at $x=a$, the F and G upstream of the rotor are of the form

$$\begin{aligned} F_1 &= A_1 e^{2\pi nx} \cos 2\pi ny + C_1 e^{2\pi nx} \sin 2\pi ny \\ G_1 &= -A_1 e^{2\pi nx} \sin 2\pi ny + C_1 e^{2\pi nx} \cos 2\pi ny \end{aligned} \tag{6.2}$$

and downstream of the stator

$$\begin{aligned} F_3 &= B_3 e^{-2\pi n(x-a)} \cos 2\pi ny + D_3 e^{-2\pi n(x-a)} \sin 2\pi ny \\ G_3 &= B_3 e^{-2\pi n(x-a)} \sin 2\pi ny - D_3 e^{-2\pi n(x-a)} \cos 2\pi ny. \end{aligned} \tag{6.3}$$

Between the blade rows the F and G are of the form (6.1) with subscript 2. In addition $H_2(0,y)$, $H_2(a,y)$ and $H_3(a,y)$ each require two coefficients for complete description of their sinusoidal character. Hence there are fourteen unknown coefficients. The three matching conditions over each blade row, Equations (3.8) and (3.9) supply twelve equations, six for sine terms and six for cosine terms. The condition that the total pressure disturbance is constant along the mean stream lines gives

$$H_3(a,y) = H_3\left(0,\, y - \frac{V_2}{U} a\right) \tag{6.4}$$

and this gives two more equations, one for the sine term and one for the cosine term. Hence the entire disturbance flow field can be found. Addition of successive blade rows downstream introduces as many new equations as new unknown coefficients.

The form of the results for even two blade rows is very involved and clumsy unless the blade gap is very small, when considerable simplification occurs. The importance of an axial gap that is small compared with the disturbance wave length is that the stator rows can play an important role in the total pressure variation through successive rows. If a stator row is far downstream of a rotor row, the angle fluctuations of the disturbed flow will have vanished, since F approaches zero. Hence no appreciable change in total pressure will occur through the stator. If the stator is close to the rotor, it will be within the range of influence of the angle disturbances downstream of the rotor and may produce large variations of total pressure if near the stall.

The possibility of self-induced disturbances in multiple blade rows has been given preliminary examination. For a single blade row it has been shown that a self-induced disturbance is possible only at a particular upstream inlet angle. For multiple blade rows the self-induced disturbance can occur only at a particular approach angle for the stator and at a particular approach angle for the rotor simultaneously. In general such coincidence would not occur, although it should be possible to arrange it by appropriate blade setting. Evidently for multiple blade rows the forced disturbance is of much greater practical importance than the self induced.

VII - MEASUREMENTS OF SELF-INDUCED DISTURBANCES

The theory of small amplitude self-induced disturbances as given above was developed some time ago. It is somewhat more general than that given by MARBLE [3] and SEARS [2] but is essentially the same in principe. MARBLE had developed the theory based on a particular shape of disturbance and both he and SEARS restricted the theories to small turning through the cascade. Both made comparison with experiments on propagating stall, but it was not fully realized at the time that the observed stall was of very large amplitude and in this sense did not fit the assumptions of the theory.

Apparently David BENENSON, a graduate student at the California Institute of Technology, was the first to find a clear example of a small disturbance propagating stall. He made observations of an annular stator cascade of hub ratio 0.8 with a blade solidity of about unity in the first experiments. There were 60 blades in the cascade and he found a disturbance with velocity fluctuations near the cascade of amplitude 7 to 10 per cent of the mean velocity upstream. The disturbance upstream was approximately a sine wave of wave length equal to the annulus circumference. In the experiments the upstream flow direction was fixed and the blade pitch varied. According to the theory the disturbance should occur at one particular blade angle, but it was found over a small range of blade angles.

The conditions of the linearized theory were met in these experiments, and it was not surprising that the observed propagating speed also agreed in a very satisfactory manner. In the theory it was assumed that the term $2P_s$ was small in comparison with $(V_1/U)P_s'$ in Equation (4.5) so that

$$\frac{V_s}{U} \simeq \frac{1}{2}\,\frac{1+\frac{V_2^2}{U^2}}{\frac{V_1}{U}}\,. \tag{7.1}$$

This formula is very convenient since detailed knowledge of the variation of P_s with V_1/U is not required, but it should not be used as if V_1/U were a variable. The approximate form (7.1) underestimates the propagating speed since the term $(U/V_1)P$ is neglected on the right hand side.

The observed stall speeds are compared with theoretical values from Equation (7.1) in Figure 3. The outlet angle corresponding to $\tan^{-1}(V_2/U)$ was measured as the blade angles were changed. The theoretical propagating speed is somewhat too low as would be expected. Observations were made at reduced solidity as well and it was found that the range of blade angles in which propagating stall occurred decreased and no self induced disturbances were found below a solidity of 0.5. Certainly the leaving angle fluctuations must increase as the solidity is reduced and these would tend to become more important than the losses in their influence on the self induced disturbances. It has been pointed out in Section IV that it would be difficult to explain propagating stall in terms of outlet angle variation alone. The experiments with varying solidity seem to confirm such a conclusion.

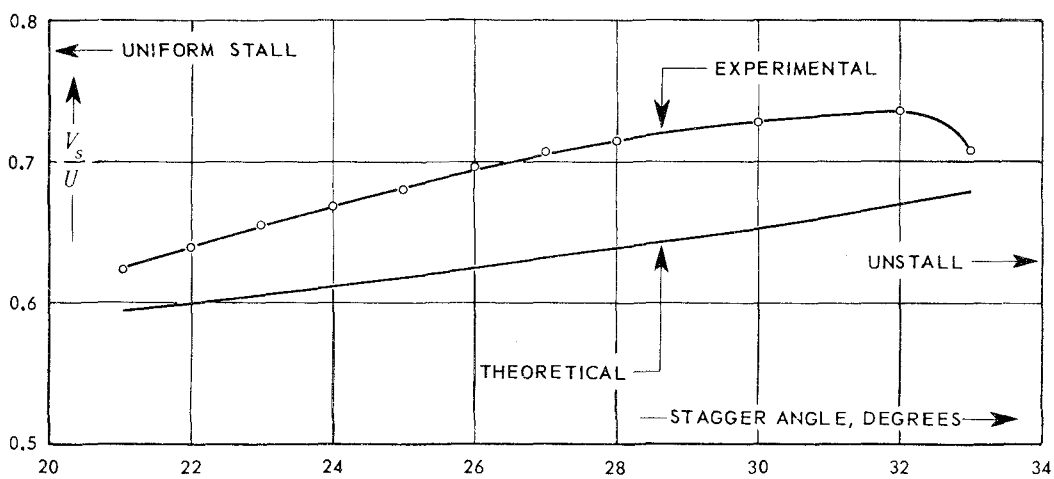


FIG. 3 - COMPARISON OF THEORETICAL STALL PROPAGATION SPEED WITH PROPAGATION SPEED OBSERVED IN ANNULAR CASCADE.

BENENSON found another example of small amplitude self induced disturbance in an isolated stator blade row is a compressor with hub ratio 0.6. The disturbance amplitude was again 7 to 10 per cent of the mean velocity but the disturbance wave length was only four or five times the blade gap. There were 32 blades with seven or eight sinusoidal waves around the annulus. The disturbances were somewhat irregular, perhaps because the appropriate conditions were not met along the entire blade length. A comparison of a theoretical point (from Equation (6.1)) with the observed stall speeds is made in Figure 4 and even here the agreement is good. Because of the short disturbance wave length, however, this example is not as convincing as the previous one.

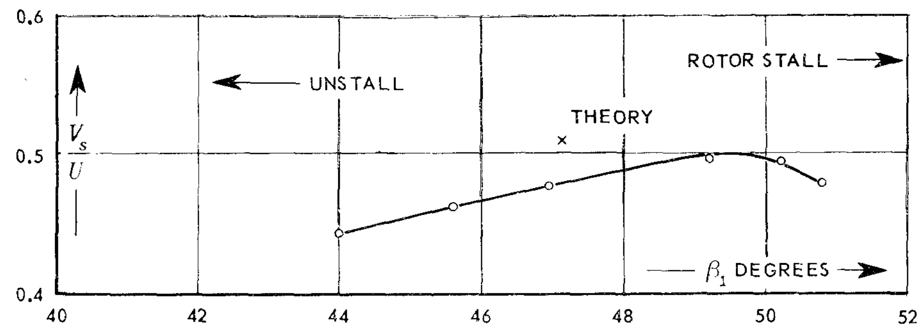


FIG. 4 - COMPARISON OF THEORETICAL STALL PROPAGATION SPEED WITH PROPAGATION SPEEDS OBSERVED IN SINGLE STAGE COMPRESSOR.

The self induced disturbances described above should *not* be confused with the large amplitude propagating stall most commonly observed in single and multistage compressors. Apparently they are two distinct phenomena, or at least are produced by quite different blade characteristics. The large amplitude propagating stall is characterized by disturbance velocities that are invariably as large as the mean stream velocity. The edges of the stalled regions are sharply defined and within the stalled region the flow is violently turbulent. As the stalled region approaches a blade the flow separates on the suction side, as it recedes from a blade the flow reattaches on the pressure side. The small amplitude self-induced disturbance involves attachment and reattachment of the boundary layer on one side only and the separated region does not cross the channel to the neighboring blade.

There is no evidence that the small amplitude disturbance represents the beginning of a large amplitude disturbance. The small amplitude disturbance first occurs at a certain inlet angle, continues at about constant amplitude over a narrow range of angle as the inlet angle is increased and then is replaced by a more or less uniform stall at higher inlet angles. Apparently the small amplitude self induced disturbance does not occur at solidities smaller than 0.5 or so. The large amplitude propagating stall, in marked contrast, occurs over a very wide range of inlet angles and over a wide range of solidities (from 1.0 down to 0.07 in one set of tests).

Of the two types of propagating stall, that of large amplitude is by far the more important in actual turbomachines. The importance of the small amplitude self-induced disturbance is quite indirect. Having demonstrated in a convincing manner that this latter type is the one that corresponds to the linearized theory, there is no further reason for confusing it with the large amplitude stall, and there is no further temptation to attempt to explain the large amplitude stall in terms of linearized theory. Considerable confusion on this matter occurred in the past and probably delayed the understanding of stall propagation appreciably. The confusion was quite natural. Self-induced disturbances were observed in turbomachines; they were difficult to describe in detail but had one easily measurable and precise characteristic, the propagating speed. The linearized theory showed the possibility of self-induced disturbances of any shape with one precise characteristic, the propagating speed. It is not surprising that the theory and experiment were compared on the basis of the characteristic speed, ignoring all other details. Occasionally the two coincided, usually not, and there was a temptation to introduce additional parameters, such as time lags, to bring the theoretical stall speed into coincidence with the observed speed.

BENENSON's observations of small amplitude self-induced disturbances lead to clarification of an unsatisfactory situation. In one sense it is disappointing; it means that the theory for the large amplitude stall will be very much more difficult than was first thought, since no linearization of the flow field or of the matching conditions will be possible. The observations of small amplitude self-induced disturbances also prove that the slope of the loss curve is the most important cascade characteristic. Until this was proved, there was some doubt as to the appropriate matching conditions for forced disturbances. Hence the experiments on small amplitude stall, in itself a phenomenon of academic importance, have had rather profound influence in helping to understand other unsteady phenomena in axial turbomachines.

VIII - MEASUREMENTS OF FORCED DISTURBANCES

Since there appeared to be no detailed measurements concerning the behavior of axial compressor blade rows when large peripheral variations of the inlet profile are introduced, an experimental investigation of this problem was begun somewhat over a year ago by Robert KATZ, a graduate student at the California Institude of Technology. He has used a large three-stage axial compressor in the investigations ; the details of this machine and some of the instrumentation are described by T. IURA and W.D. RANNIE in reference [8]. KATZ introduced the inlet disturbances by means of high solidity screens installed about 1/4 compressor diameter upstream of the inlet vanes as indicated in Figure 5. For the experimental results to be presented here the screens covered a 90 degree sector of the compressor. The compressor is so arranged that total pressure and flow angle measurements could be made downstream of each stationary or rotating blade row. Only the circumferential surveys of total pressure will be discussed in detail here.

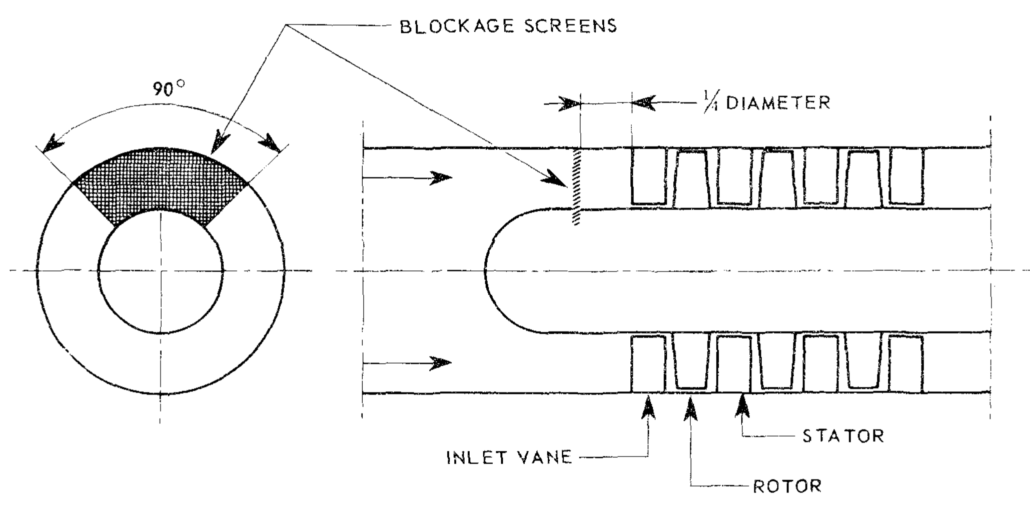


FIG. 5 - INSTALLATION OF CIRCUMFERENTIALLY NON-UNIFORM BLOCKAGE SCREENS AT INLET OF THREE-STAGE COMPRESSOR.

Utilizing all three stages of the compressor a rather severe blockage was introduced at the inlet and total pressure surveys were made downstream of each rotor to determine how the disturbance developed through the machine. Figure 6 shows plots of these measurements at the mean compressor radius and for a flow coefficient $\bar{\phi}$ corresponding approximately to the design value of the machine. The survey downstream of the screen indicates the magnitude of the disturbance; the disturbance corresponds to about one quarter of the dynamic pressure computed using the rotor tip speed, that is somewhat more than half of the normal total pressure rise across a stage. The large effect of the disturbance on inlet angles may be appreciated from the fact that this total pressure loss corresponds to a reduction in the axial velocity of more than 50 per cent.

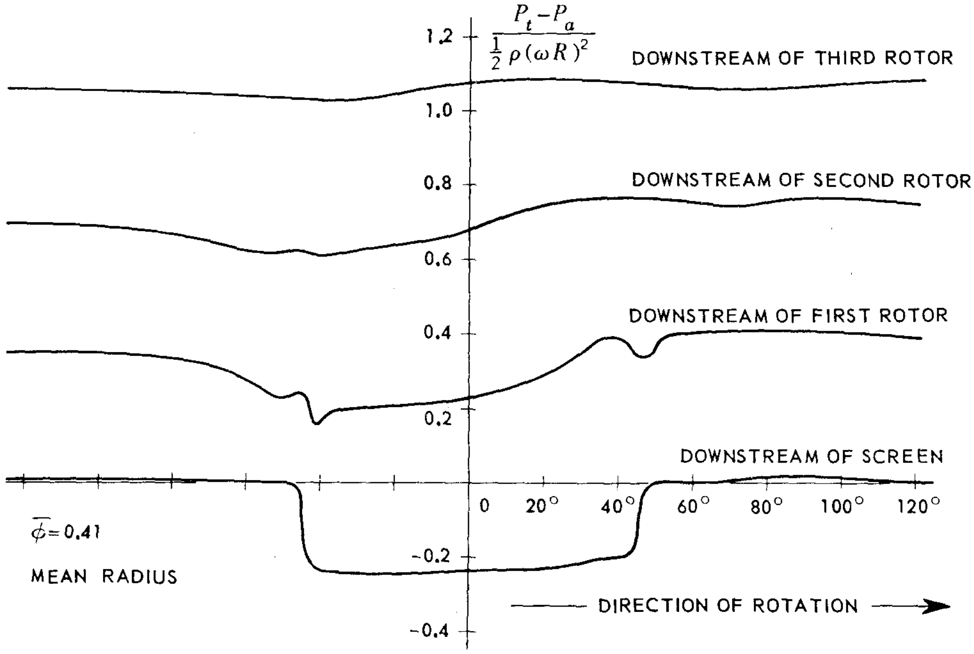


FIG. 6 - TOTAL PRESSURE VARIATIONS BEHIND EACH ROTOR OF THREE-STAGE COMPRESSOR RESULTING FROM WAKE OF BLOCKAGE SCREENS.

Downstream of the first rotor the pattern of the total pressure fluctuation is considerably modified but the amplitude is not appreciably reduced. By the time the second rotor is traversed the total pressure profile is distinctly smoothed out and it has very nearly vanished downstream of the third rotor. With the input total pressure profile given, corresponding total pressure profiles may be calculated by applying the single blade row theory successively to each blade row. While the results are in qualitative agreement with experiments the significance of such a comparison is questionable for two reasons : i) The proper loss and leaving angle deviation values to be used are not yet known with sufficient certainty and ii) The effect of mutual interference between adjacent blade rows may be quite strong, as later results will show, and this factor invalidates application of single blade row theory to successive stalling blade rows.

To observe the influence of mutual blade row interference upon the total pressure, KATZ obtained total pressure profiles behind the rotor and stator using only one stage of the compressor with both normal stage spacing and with the stator located several blade chords downstream of the rotor. With the expanded stage the losses across the stator were normal and negligible in comparison with the input total pressure variation. In contrast, large total pressure losses across the stator are shown by the surveys of the normal stage shown in Figure 7 for root, mean and tip radii. This very significant influence of blade spacing comes about in the following manner : When the stator is far downstream of the rotor, most of the local effects are smoothed out by the time the flow reaches the stator. The flow enters the stator at very nearly the same uniform relative angle (although not the same velocity) as it would in the absence of inlet disturbance. Consequently normal stator losses appear for the expanded stage. However when the rows are spaced closely, the flow angles into the stator are still severely distorted due to the flow field set up by the non-uniform flow passing through the rotor. The large losses of Figure 7 reflect this variation in inlet angle. It is to be noted also that the effect is considerably more severe at the blade root than either the mean or tip radii.

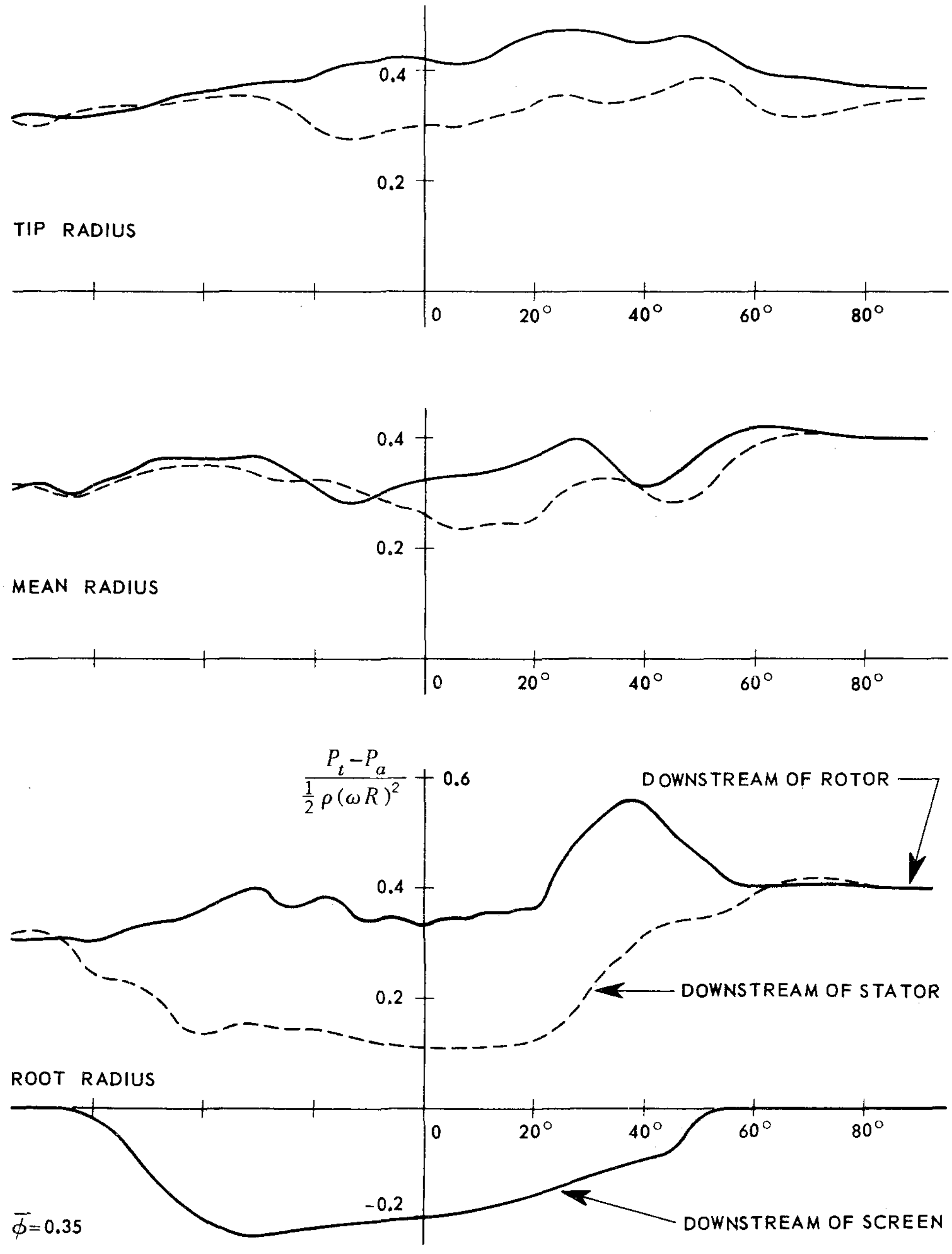


FIG. 7 - EFFECT OF STATOR LOSS AT ROOT, MEAN AND TIP RADII. TOTAL PRESSURE VARIATIONS BEHIND ROTOR AND STATOR OF SINGLE STAGE COMPRESSOR RESULTING FROM WAKE OF BLOCKAGE SCREEN.

Concentrating for the moment on losses near the stator root, Figures 8 and 9 respectively show the influence of mean flow rate and amplitude of input disturbance of the loss profile. As might be anticipated, increasing the mean flow coefficient to a value reasonably near the design point reduced the stator stall loss but not markedly. This fact confirms, in a way, the indication that the flow angles induced by the blade row interference are quite large, since a significant change in the mean angle of attack does not modify the situation appreciably.

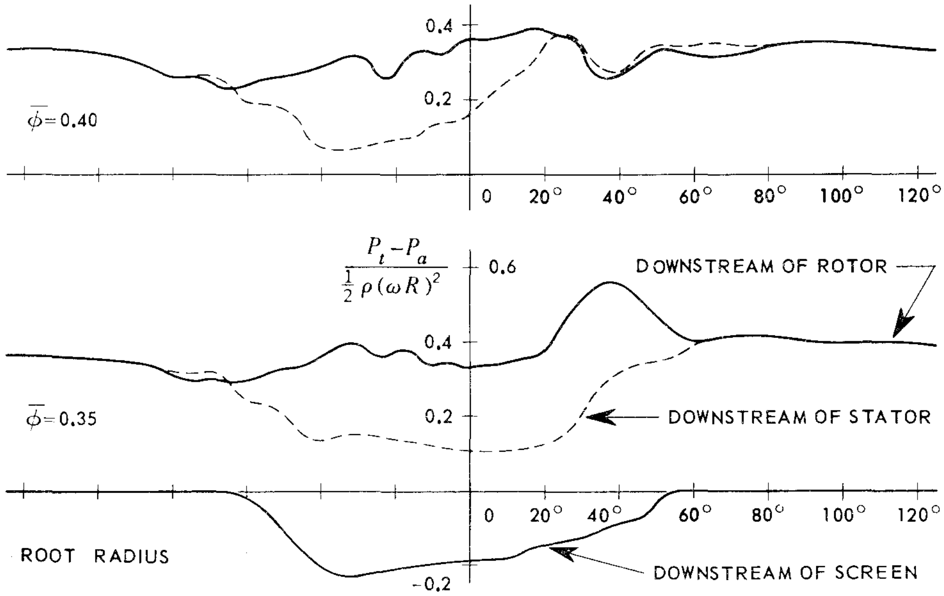


FIG. 8 - EFFECT OF STATOR LOSS AT TWO FLOW RATES. TOTAL PRESSURE VARIATIONS BEHIND ROTOR AND STATOR OF SINGLE STAGE COMPRESSOR RESULTING FROM WAKE OF BLOCKAGE SCREEN.

The variation of input profile, shown in Figure 9, exerts a quite large but linear influence on stator loss although the loss profiles retain a similar shape as their magnitude changes. It is somewhat surprising that this linear dependence upon input amplitude holds in spite of the fact the losses and perturbation angles are really of significant size.

To indicate the magnitude and origin of the stator losses, the forced disturbance flow through a typical rotor-stator combination has been calculated in several ways, utilizing the foregoing theory. Assuming a total pressure disturbance of unit amplitude at the compressor inlet the amplitude of the total pressure disturbance was computed downstream of the stator and the flow angle disturbances were computed at the rotor and stator inlet. The computations

EFFECT OF TYPICAL GUIDE VANE. ROTOR. STATOR COMBINATION ON TOTAL PRESSURE PERTURBATION INTRODUCED AT INLET

	δp_t Downstream of Stator / δp_t Input		$\delta\beta$ Rotor Inlet / $\delta\beta$ Rotor Inlet, $a=\infty$, No Loss		$\delta\beta$ Stator Inlet / $\delta\beta$ Rotor Inlet, $a=\infty$, No Loss	
	$a=\infty$	$a=0$	$a=\infty$	$a=0$	$a=\infty$	$a=0$
No Loss	0.83	0.59	1.00	1.27	0 (1.61)	2.03
Rotor Loss	0.73	0.65	1.20	0.85	0 (1.41)	2.21
Rotor Loss and Stator Loss	0.65	1.28	1.20	1.04	0 (1.41)	4.18

were carried out for infinite spacing of the rotor and stator, $a=\infty$, and for infinitesimal spacing of rotor and stator, $a=0$. In each case the disturbances were computed assuming no loss in either rotor and stator, losses in the rotor only, and losses in both rotor and stator. In each reasonable values were chosen for the important parameter, the slope of the pressure loss curve although the proper values were not known. The results of these computations are summarized in the accompanying table.

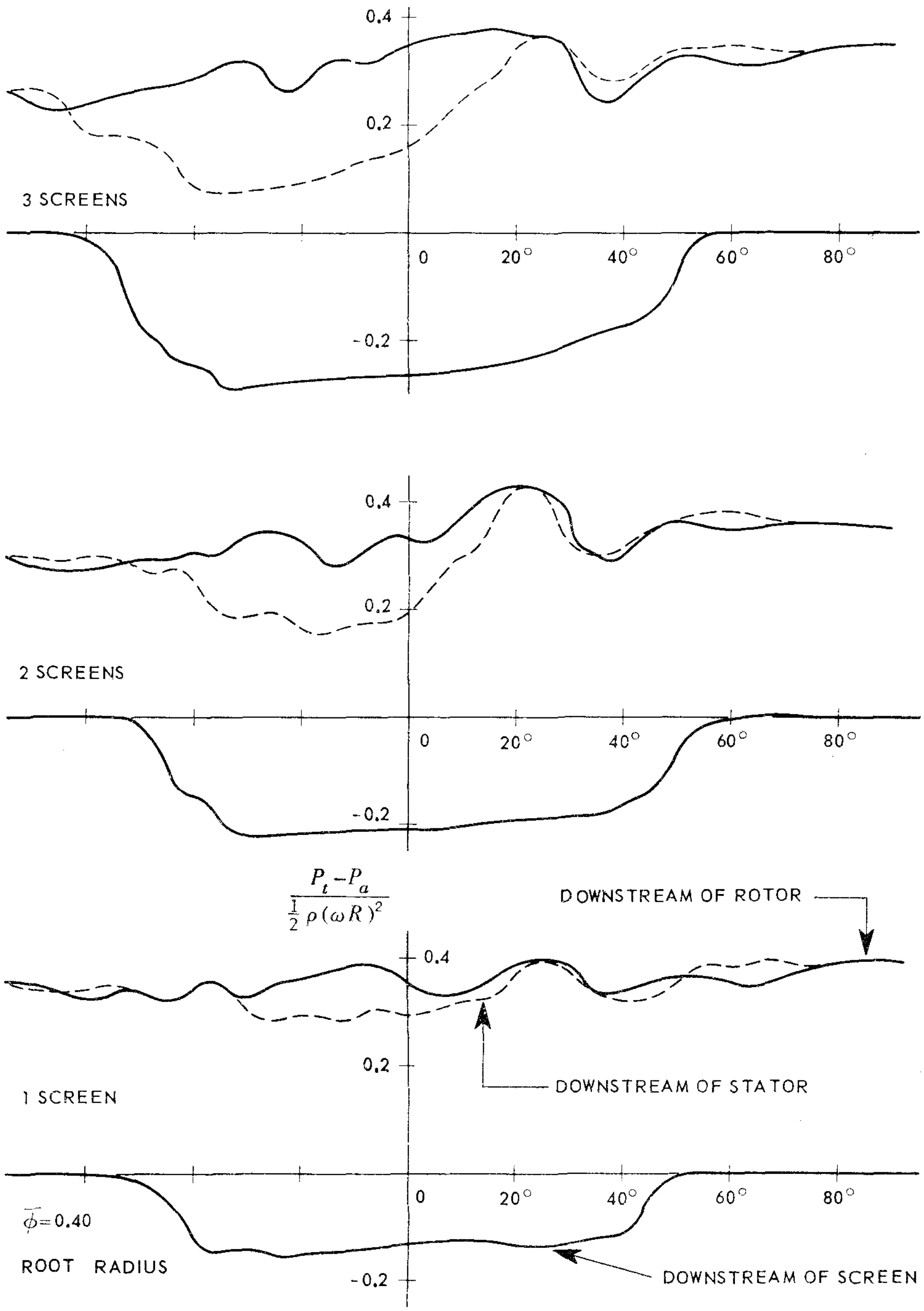


FIG. 9 - INFLUENCE OF INPUT AMPLITUDE ON LOSS DUE TO STATOR STALL. TOTAL PRESSURE VARIATIONS ROTOR AND STATOR OF SINGLE STAGE COMPRESSOR RESULTING FROM WAKE OF THREE DIFFERENT BLOCKAGE SCREENS.

Consider first the total pressure variation downstream of the stator. In the absence of losses the amplitude of losses falls across the stage due to work added in the rotor. The smoothing influence of the stage is more pronounced when the blade rows are closely spaced. The reason for this is that the pressure field of the stator increases local angles of attack on the rotor, as shown in the next column of the table, and consequently the local total pressure rise is augmented when the spacing is close. With losses in both rotor and stator components, the trend is clearly reversed. There are two reasons for this change : i) The stator losses are very large when the stages are closely spaced due to the unfavorable inlet angles induced on them by mutual interference. ii) The proximity of the stator tends to decrease local rotor inlet angles rather than increase them because high stator losses, which vary as the square of the approach velocity, encourage the fluid to "funnel" into low velocity regions. With the loss coefficients assumed, the loss profile for the closely spaced stage is worse when the fluid leaves the stage than when it enters, and the preponderant fraction of this loss may be attributed to stator stall. Because of its importance, the factors that induce high inlet angles to the stator may be analyzed somewhat. When the rotor and stator are widely spaced there is no induced angle at the stator inlet for the reasons discussed previously. Then the induced stator inlet angles for close spacing consist in two parts, the flow angle perturbations that exist immediately downstream of the rotor in the absence of the stator and the flow angle perturbations induced by the stator and by any mutual effects. The stator inlet angle perturbations that would exist irrespective of the stator are given in parentheses in the table. The stator inlet angle perturbation attributable to the stator and to mutual effects corresponds to the difference between the numbers in the last column and the corresponding numbers in parenthesis. It is clear that the major part of the stator inlet angle perturbation is caused by mutual interference and that the stator losses themselves play an essential role in inducing these perturbations. In this sense one may think of the stator losses as being "self induced" to a certain extent.

The analytical and experimental investigations carried out so far concerning forced disturbance confirm the observation made previously that the slope of the blade loss characteristic is the most influential physical parameter in the system. Moreover these results demonstrate that the induced stall loss in stator blade rows is an essential factor in deciding the rate at which a disturbance, introduced at a compressor inlet, will be smoothed out as the flow progresses through successive stages.

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