Shock waves and noise in the collapse of a cloud of cavitation bubbles

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Abstract: Calculations of the collapse dynamics of a cloud of cavitation bubbles confirm the speculations of Mørch and his co-workers and demonstrate that collapse occurs as a result of the inward propagation of a shock wave which grows rapidly in magnitude. Results are presented showing the evolving dynamics of the cloud and the resulting far-field acoustic noise.

Key words: Cloud cavitation, Shock wave, Cavitation noise

1. Introduction

In many flows of practical interest one observes the periodic formation and collapse of a "cloud" of cavitation bubbles (see, for example, Bark and Berlekom 1978, Shen and Peterson 1978). Such a structure, which has a foamy appearance and consists of numerous small bubbles, is termed "cloud cavitation." Clouds of cavitation bubbles are often shed from a cavitating foil and the coherent dynamics of these clouds can result in a collapse process which has much greater potential for noise production and damage than the individual bubbles, acting independently, would have (see, for example, Soyama et al. 1992).

Most previous theoretical studies of the dynamics of cavitating clouds have been linear or weakly nonlinear analyses but have not, as yet, shown how a cloud would behave during the massively nonlinear response in a cavitating flow. The linearized behavior of a collapsing layer of bubbly fluid next to a solid wall was first modeled by van Wijngaarden in 1964. Later d'Agostino and Brennen (1983) investigated the linearized dynamics of a spherical cloud (see also d'Agostino and Brennen 1989) and showed that the interaction between bubbles leads to a coherent dynamics of the cloud, including natural frequencies that can be much smaller than the natural frequencies of individual bubbles. Other studies of cloud dynamics are summarized by Brennen (1995).

Several years ago, Mørch and his co-workers (Mørch 1980, 1981; Hanson et al. 1981) speculated that the collapse of a cloud of bubbles involves the formation and inward propagation of a shock wave and that the geometric focusing of this shock at the center of the cloud creates the enhancement of the noise and damage potential associated with cloud collapse. The present paper extends a previous computational investigation of this flow (Wang and Brennen 1994) and examines the formation, development, and acoustic consequences of the shock wave.

2. Basic equations

Consider a spherical cloud of bubbles surrounded by an unbounded pure liquid at rest at infinity, as shown in Fig. 1. The pure liquid is assumed incompressible, with a density ρ_L. It is assumed that the population of bubbles per unit liquid volume, η, within the cloud, is uniform initially and that there is no coalescence or break-up of bubbles. Since relative motion between two phases and the mass of liquid vaporized or condensed are both neglected it follows that η remains both constant and uniform within the cloud. The basic equations (see

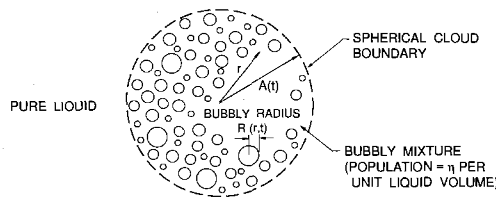

Figure 1. Schematic of a spherical cloud of bubbles.

d'Agostino and Brennen 1989) consist of the following dimensionless forms of the continuity and momentum equations for the spherical bubbly flow:

$$\frac{1}{r^2}\frac{\partial(r^2u)}{\partial r}=\frac{12\pi\eta R^2}{3+4\pi\eta R^3}\frac{DR}{Dt} \quad ; \quad r\le A(t)\ , \tag{1}$$

$$\frac{Du}{Dt}=-\frac{1}{6}(3+4\pi\eta R^3)\frac{\partial C_P}{\partial r} \quad ; \quad r\le A(t)\ , \tag{2}$$

where $\frac{D}{Dt}\equiv\frac{\partial}{\partial t}+u\frac{\partial}{\partial r}$ is the Lagrangian derivative, $u(r,t)$ is the mixture velocity, $R(r,t)$ is the individual bubble radius, $C_P(r,t)=(p(r,t)-p_0)/(\frac{1}{2}\rho_L U^2)$ is the pressure coefficient in the cloud, ρ_L is the liquid density, $p(r,t)$ is the mixture pressure, p_0 is the initial equilibrium pressure in the mixture, and $A(t)$ is the radius of the cloud. The bubble population per unit liquid volume, η, is related to the void fraction, α, by $(\frac{4}{3}\pi R^3)\eta=\alpha/(1-\alpha)$. The variables and equations are non-dimensionalized using the initial bubble radius, R_0, a reference flow velocity, U, the time scale, R_0/U, and the dynamic pressure, $\frac{1}{2}\rho_L U^2$. To incorporate the bubble/bubble interactive effects, the dynamics of the bubbles are modeled using the Rayleigh-Plesset equation which connects the local mixture pressure coefficient, C_P, to the bubble radius, R:

$$R\frac{D^2R}{Dt^2}+\frac{3}{2}\left(\frac{DR}{Dt}\right)^2+\frac{\sigma}{2}[1-R^{-3k}]+\frac{4}{Re}\frac{1}{R}\frac{DR}{Dt}+\frac{2}{We}[R^{-1}-R^{-3k}]+\frac{1}{2}C_P=0\ , \tag{3}$$

where $\sigma = (p_0 - p_v)/(\frac{1}{2}\rho_L U^2)$ is the cavitation number, p_v is the vapor pressure, $We = \rho_L U^2 R_0/S$ is the Weber number, S is the surface tension of liquid, $Re = UR_0\rho_L/\mu_E$ is the Reynolds number, μ_E is the effective viscosity of liquid which incorporates the various bubble-damping mechanisms described by Chapman and Plesset (1971), and k is the effective polytropic index of the gas inside the bubble.

The boundary condition on the surface of the cloud, $r = A(t)$, can be derived from the solution of incompressible liquid flow outside the cloud (see Wang and Brennen 1995):

$$C_P(A(t), t) = C_{P\infty}(t) + \frac{2}{A(t)} \frac{d[A^2(t)u(A(t), t)]}{dt} - u^2(A(t), t) \tag{4}$$

where $C_{P\infty}(t)$ is the known and imposed driving pressure at infinity and is chosen as a simple sinusoidal form:

$$C_{P\infty}(t) = \begin{cases} \frac{1}{2}C_{PMIN}\{\cos(\frac{2\pi}{t_G}t) - 1\} & ; \quad 0 < t < t_G \\ 0 & ; \quad t < 0 \quad \text{and} \quad t > t_G \end{cases} \tag{5}$$

where C_{PMIN} is the minimum pressure coefficient at infinity and t_G is the dimensionless duration of the low-pressure perturbation. Consequently, for a cloud flowing with velocity U past a body of size D, the order of magnitude of t_G will be D/R_0, and C_{PMIN} will be the minimum pressure coefficient of the flow. At the center of the cloud, the symmetry of the problem requires $u(0, t) = 0$. Also, at time $t \leq 0$, it is assumed that the whole flow field is in equilibrium. It is also assumed, for simplicity, that all the bubbles have the same initial radius R_0. Thus we have the following initial conditions: $R(r, 0) = 1, \frac{DR}{Dt}(r, 0) = 0, u(r, 0) = 0, C_P(r, 0) = 0$. The above equations (1), (2), and (3) with the above boundary conditions can, in theory, be solved to find the unknowns $C_P(r, t)$, $u(r, t)$, and $R(r, t)$ for any bubbly cavitating flow with spherical symmetry. However, the nonlinearities in the Rayleigh-Plesset equation and in the Lagrangian derivative, D/Dt, present considerable computational impediments. An integral method which uses the Lagrangian coordinate system, (r_0, t), has been developed (see Wang and Brennen 1995) in which r_0 is the non-dimensional initial radial position at time $t = 0$.

3. Results and discussion

We have chosen the following typical flow for the computational results presented here. A cloud of nuclei, composed of air bubbles of initial radius $R_0 = 100$ μm in water at 20°C, is convected through a low-pressure region with velocity $U = 10$ m/sec. Other parameters are: initial void fraction, $\alpha_0 = 0.5\%$; cavitation number, $\sigma = 0.45$; the minimum pressure coefficient, $C_{PMIN} = -0.75$; the

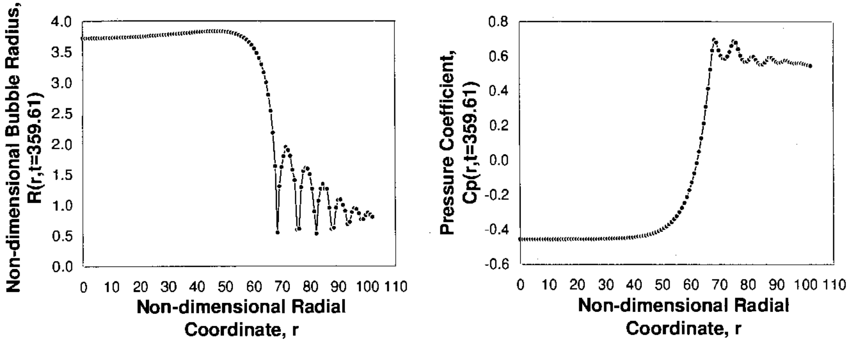

Figure 2. The structure of a shock wave accompanying the collapse of the cloud of cavitation bubbles. Left: The dimensionless bubble-size distribution in the cloud as a function of the dimensionless radial coordinate at the dimensionless time, $t = 359.61$. Right: The pressure-coefficient distribution in the cloud as a function of the dimensionless radial coordinate at the same time as on the left.

non-dimensional cloud radius, $A_0/R_0 = 100$; the non-dimensional duration of the low-pressure perturbation, $t_G = 250$, which corresponds to the ratio of the length scale of the low-pressure perturbation to the initial radius of the cloud, $D/A_0 = 2.5$. The Reynolds number, $Re = UR_0\rho_L/\mu_E$, is taken as 0.05.

The structure of the developing shock wave is illustrated by the results presented in Fig. 2. This structure is similar to that of the steady planar shocks analyzed by Noordij and van Wijngaarden (1974). The shock wave strengthens not only because of the geometric effect of an inwardly propagating spherical shock but also because of the coupling of the single-bubble dynamics with the global dynamics of the flow through the pressure and velocity fields. When the shock reaches the center of the cloud, very high pressures are produced and cause a volumetric rebound of the cloud. The time history of the radius of the cloud is shown in Fig. 3. Note that, unlike single bubbles, the cloud radius, $A(t)$, only decreases to a size marginally smaller than its equilibrium size during the collapse process. But each volumetric rebound causes an acoustic peak in the far field, as shown in Fig. 3. The normalized measure, p_a, of the acoustic noise, p_a^*, can be calculated from the volumetric acceleration of the cloud and is given by

$$p_a(t) = \frac{p_a^* r}{\frac{1}{2}\rho_L U^2 D} = \frac{2R_0}{D}\left[A^2(t)\frac{d^2A(t)}{dt^2} + 2A(t)(\frac{dA(t)}{dt})^2\right] . \qquad (6)$$

After several collapse cycles, the cloud begins to oscillate in a regular way as shown in Fig. 3 (right). This regular oscillation contributes large peaks in the noise spectrum at the natural frequency of the cloud and at higher harmonics,

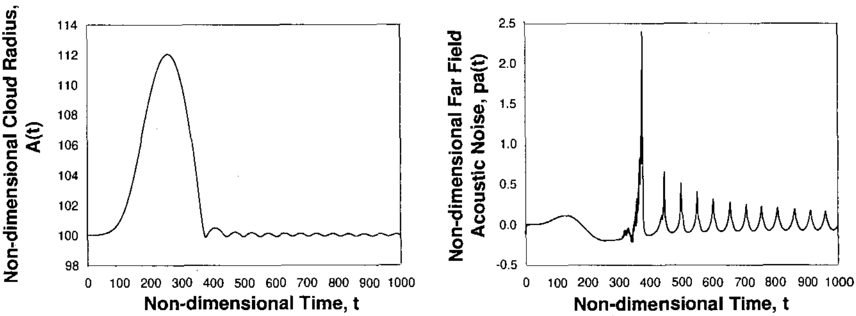

Figure 3. Left: The time history of the dimensionless cloud radius. Right: The time history of the far-field acoustic noise calculated from the data shown in the left figure.

as shown in Fig. 4. In the present example, the first natural frequency of the cloud is about 0.02 (2 kHz) and the natural frequency of bubbles in the cloud is 0.158 (15.8 kHz).

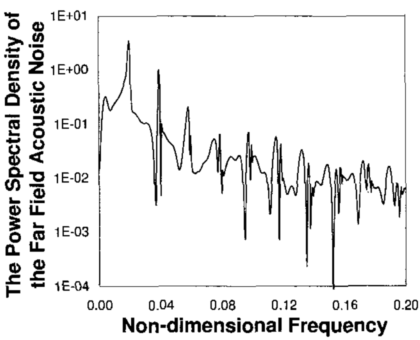

Figure 4. The non-dimensional power spectral density of the far-field acoustic noise in Fig. 3 as a function of dimensionless frequency. The lowest cloud natural frequency is about 0.02. The natural frequency of bubbles in the cloud is 0.158.

4. Conclusions

We have shown how a shock wave develops as part of the nonlinear collapse of a spherical cloud of cavitation bubbles. The focusing of the shock produces very high pressures at the center of the cloud and then causes rebound of the cloud. The volumetric acceleration induces a large pulse in the far-field noise. Moreover, there are further but weaker shocks which arrive at the center and thus produce a train of acoustic impulses which, eventually, lead to a regular oscillation of the cloud at the first cloud natural frequency. These results suggest that shock focusing may be one of the major mechanisms for the enhanced noise and damage potential associated with cloud cavitation.

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