BRIEF REPORTS

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Comparison of lattice and dual QCD results for heavy quark potentials

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Lattice results and dual QCD results for all heavy quark potentials through order (quark mass)$^{-2}$ are exhibited and compared. The agreement on the whole is quite good, confirming the validity of dual QCD. [S0556-2821(97)04719-X]

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Bali, Schilling, and Wachter [1] have recently calculated from lattice theory all of the heavy quark potentials—the central potential, all spin-dependent potentials, and all velocity-dependent potentials—through order velocity squared or, equivalently, through order (quark mass)$^{-2}$. We have previously computed all of these same potentials from the dual superconducting model of QCD, i.e., dual QCD [2,3]. Our purpose in this Brief Report is to compare this new lattice data with dual QCD predictions.

The definitions of the potentials by Bali, Schilling, and Wachter [1] are the same as in dual QCD, except for those proportional to velocity squared [4]. (Bali, Schilling, and Wachter include in their calculation some numbers called c_2, c_3, c_4 etc., which represent ratios of the running coupling α_s at various energies. We have set all these ratios equal to 1 because in dual QCD the coupling constant, in the classical approximation used to derive the potentials, does not run.) The comparison of the potentials is given in Table I.

In dual QCD, the potential V_a can also be broken up into an electric and a magnetic part:

$$V_a = V_a^E - V_a^B, \tag{1}$$

where [3]

$$\nabla^2 V_a^E = -\nabla^2 V_0^{\rm NP} \tag{2}$$

and

TABLE I. Comparison of the potentials of Bali, Schiller, and Wachter and dual QCD.

Bali, Schiller, and Wachter	Dual QCD
$V_0 + \frac{1}{8}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right)(\nabla^2 V_0 + \nabla^2 V_a^E - \nabla^2 V_a^B)$	$V_0 + \frac{1}{8}\left(\frac{1}{m_1^2}+\frac{1}{m_2^2}\right)(\nabla^2 V_0 + \nabla^2 V_a)$
V_1'	V_1'
V_2'	V_2'
V_3	V_3
V_4	V_4
V_b	$\frac{1}{3}(-V_+ + V_- + \frac{1}{2}V_\parallel - \frac{1}{2}V_L)$
V_c	$\frac{1}{2}(-V_+ + V_- - V_\parallel + V_L)$
V_d	$\frac{1}{6}(V_+ + V_- + \frac{1}{2}V_\parallel + \frac{1}{2}V_L)$
V_e	$\frac{1}{2}(V_+ + V_- - V_\parallel - V_L)$

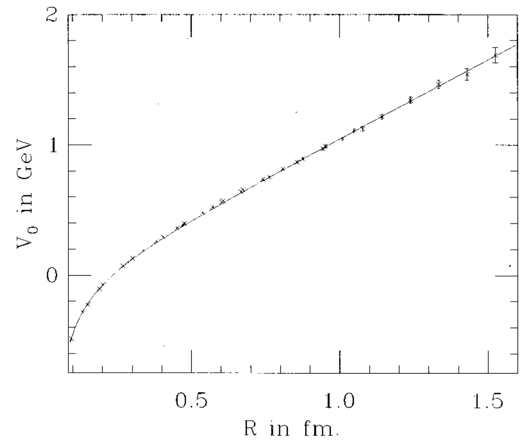

FIG. 1. Comparison of the dual QCD central potential (solid line) with the lattice central potential (points) for $\beta = 6.2$. The dual QCD parameters are $\alpha_s = 0.2048$ and $\sigma = -0.2384\ \mathrm{GeV}^2$. The lattice string tension, in contrast, is $\sigma = 0.2190\ \mathrm{GeV}^2$.

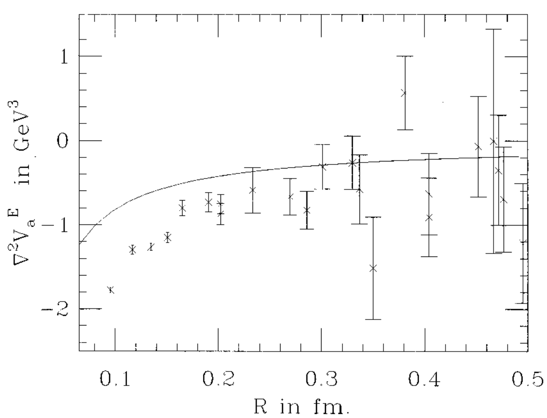

FIG. 2. A similar comparison (using the same parameters) for the quantity $\nabla^2 V_a^E$.

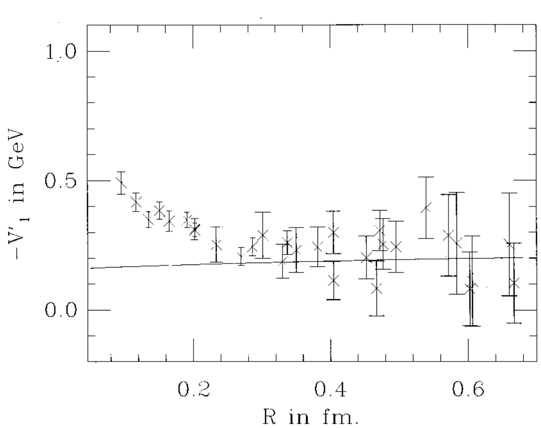

FIG. 3. Same as for Fig. 2 but for $-V'_1$. (Note the minus sign.)

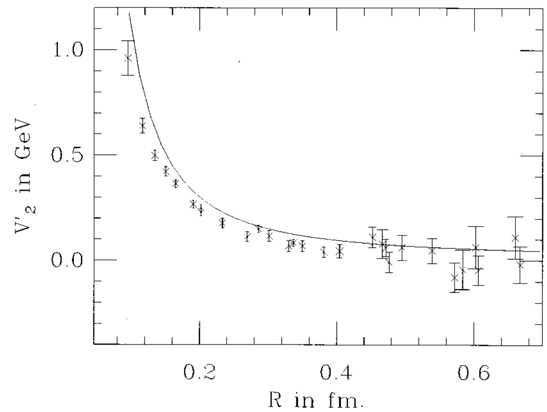

FIG. 4. Same as Fig. 2 but for V'_2.

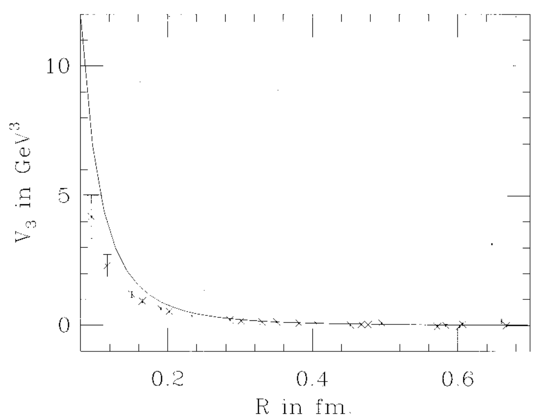

FIG. 5. Same as Fig. 2 but for V_3.

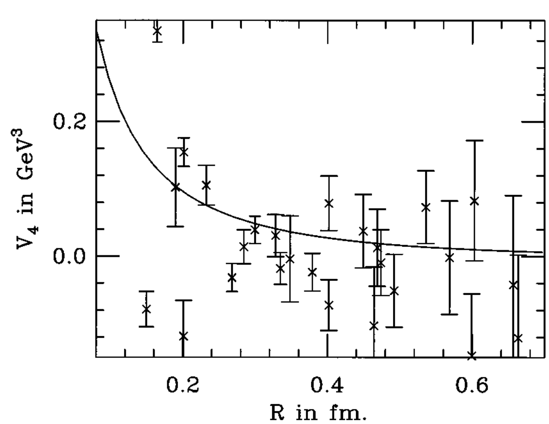

FIG. 6. The lattice result for the lowest value of R has been omitted as it contains the δ function contribution to the potential. Our curve is the contribution of dual QCD other than the delta function.

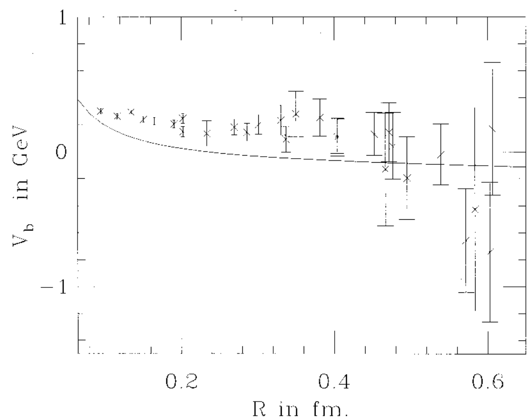

FIG. 7. Same as Fig. 6 but for V_b.

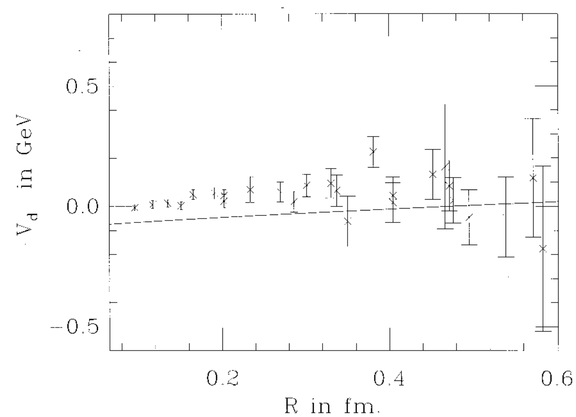

FIG. 9. Same as Fig. 6 but for V_d.

$$\nabla^2 V_a^B = -\tfrac{4}{3}\, e^2 \vec{\nabla}\cdot\vec{\nabla}' G^{\mathrm{NP}}(\vec{x},\vec{x}')|_{\vec{x}=\vec{x}'=z_j}. \qquad (3)$$

(Here the superscript ''NP'' stands for nonperturbative.) The dual QCD result for V_a^B is weakly singular and requires a cutoff [5]. The same result obtains for the lattice calculation of V_a^B [6]. The spin-spin potential V_4 has a δ function term and the term $\nabla^2(V_0+V_a^E)$ in dual QCD is simply proportional to a δ function at the origin [3], though these naturally do not show up cleanly in the lattice calculation. All of the remaining potentials are finite and well behaved in both approaches.

The comparison of the two sets of results are shown in Figs. 1–10. Figure 1 shows the lattice and the dual QCD calculations of the central potential $V_0(R)$. The units are GeV and Fermis. The dual QCD parameters given in Ref. [2] have been changed to produce a best fit to the lattice V_0 for $\beta=6.2$. The new parameters are $\alpha_s=0.2048$, the string tension $\sigma=0.2384\ \mathrm{GeV}^2$, and the dual gluon mass is 950 MeV. These changes significantly worsen the fits for the $c\bar{c}$ and $b\bar{b}$ spectra given in Ref. [2]. The resulting effective χ^2 is 11.4, about 6 times that of our earlier fit. The average error increases from 13 MeV to 29 MeV. While our method of calculation differs considerably from that used by Bali, Schiller, and Wachter, the quality of our fit described here is comparable to theirs.

Figure 2 shows the comparison of the quantity $\nabla^2 V_a^E$. The agreement, evidently, is not bad.

We recall, however, as mentioned before, that in dual QCD $\nabla^2(V_0+V_a^E)$ is simply a δ function. This result does not hold on the lattice, and so some discrepancy in $\nabla^2 V_a^E$, especially at small R, is not surprising. There is no figure for $\nabla^2 V_a^B$, because, also as mentioned above, in both dual QCD and on the lattice this quantity is weakly divergent but is not very sensitive to the required cutoff. A detailed analysis and comparison of $\nabla^2 V_a$ in dual QCD and on the lattice is given in Ref. [5].

The remaining figures (Figs. 3–10) show the parameter-free dual QCD predictions and lattice data for the rest of the potentials, namely, V_1', V_2', V_3, V_4, V_b, V_c, V_d, and V_e. All of these agree remarkably well (within the lattice calculation uncertainties), with a few relatively minor exceptions. The short distance behavior of $-\nabla^2 V_a^E$ and V_1' is above the lattice data for distances less than 0.2 fermis. In this domain radiative corrections giving rise to running coupling constants and asymptotic freedom become important, and those are not included in dual QCD. Also, at small R, Figure 5

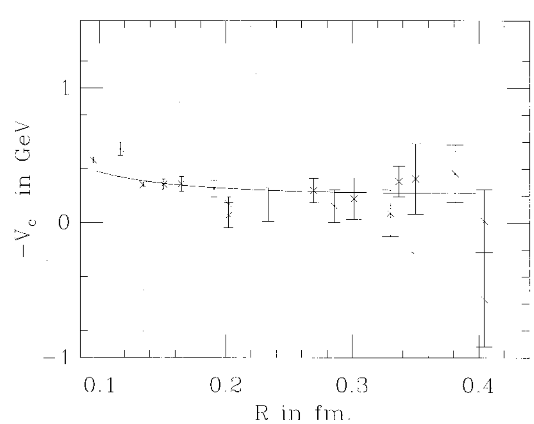

FIG. 8. Same as Fig. 6 but for V_c.

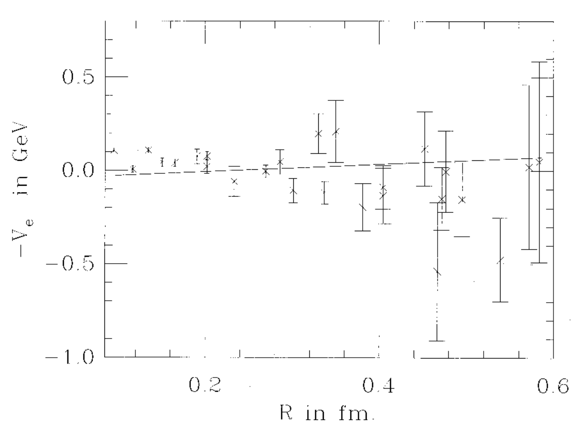

FIG. 10. Same as Fig. 6 but for V_e.

shows the dual QCD spin-spin potential to be well above the lattice points. To understand the possible origin of this difference, consider the interaction of a point magnetic dipole with a sphere of constant magnetization in which dipole and magnetization directions are determined by the two spin directions. For the dipole outside of the sphere the interaction potential is of the form of V_3 and produces the usual perturbative QCD result. For the dipole inside the sphere the interaction is a constant and has the spin dependence of V_4. If one takes the radius of the sphere to zero, holding its magnetic moment constant, this potential becomes a δ function at the origin. Because of the fact that the finite lattice size represents a granularity in space, one might expect a modification of the small R behavior of both of these potentials in a lattice calculation.

To summarize, the lattice data for the central potential $V_0(R)$ were used to determine the parameters α_s and σ of dual QCD. The resulting fit is shown in Fig. 1. All the remaining potentials are then uniquely predicted (Figs. 2–10). Overall the agreement of these dual QCD predictions with the lattice data is remarkably good, and we feel that it provides evidence for the validity of the dual picture of long distance Yang-Mills theory.

We would like to thank Gunnar Bali for making their lattice results available to us in the numerical form necessary for the detailed fits and comparison.

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