diffused in furnaces with different dopant concentrations. The surface concentrations obtained for the two groups by the plasma-minima technique were $7.85 + 0.15 \times 10^{19}$ and $5.63 + 0.17 \times 10^{19}$ carriers/cm^3, which indicates a reproducibility of measurement of ±3%.

Similar infrared measurements have been made on a limited number of boron-doped silicon wafers. For shallow diffusions (approx. 0.5 μ). The infrared minima were often obscured by superimposed interference maxima and minima corresponding to the wafer thickness. Consequently no data for comparison with other techniques have been obtained to date.

The infrared plasma-minima technique is capable of measuring surface concentrations of diffusants in silicon ranging in value from 3×10^{18} to 5×10^{20} cm^{-3} with an infrared instrument covering the wavelength from 2.5 to 40 μ. The main source of error in determining C_0 in the phosphorus-doped silicon wafers was in locating the plasma minima. It is estimated that the overall error in determining C_0 is no worse than ±10%, which compares very favorably with other techniques. In addition this technique is relatively fast and nondestructive.

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NOISE SUPPRESSION IN A DOUBLE-INJECTION SILICON DIODE*

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We measured the noise spectrum between 10 kHz and 10 MHz. In the region where $I \propto V^2$, we found a noise suppression factor of 1/2 for the white noise, i.e., $\overline{I^2} = (1/2)4kT(\partial V/\partial I)^{-1}\Delta f$.

We present here the result of measurements on bipolar space-charge-limited current (sclc) in a solid which establishes that, in this case, noise can be suppressed when compared with $4kT(\partial V/\partial I)^{-1}\Delta f$, where $\partial V/\partial I$ is the differential resistance of the double-injection diode at the dc operating point considered.

The V-I characteristic and the relevant parameters of the diode are given in Fig. 1. The two circles indicate the operating points to which our noise measurements refer. Electrical contact to the doped layers of the two end surfaces of the diode was performed with a liquid mixture of Ga and In.

1. At 3.0 V/32 μA the diode has a differential resistance of approx. 80 kΩ and is essentially a resistor. The noise spectrum was measured from 10 kHz to 1.5 MHz. Excess noise extends to about 30 kHz beyond which the spectrum is white. Its value falls within 10% of the white noise measured when the diode is replaced by a resistor of 80 kΩ. We thus find at this operating point

$$\overline{I^2} = \beta \cdot 4kT(\partial V/\partial I)^{-1}\Delta f \text{ with } \beta = 1.0 \pm 0.1, \quad (1)$$

as the linear V-I characteristic suggests.

2. At 30.0 V/1.03 mA the diode has a differential resistance of 15.8 kΩ and here I is proportional to V^2. Measurements were taken from 10 kHz to

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10 MHz. The upper portion of the spectrum is shown in Fig. 2 after some corrections are applied to the raw data to account for known experimental deficiencies in the electronic system. Two preamplifiers of different design were used in an effort to eliminate systematic errors. Their results agree well. Also shown are the spectra obtained when a metal film resistor of 15.7 kΩ is substituted for the diode and when this resistor is cooled to 77°K by immersion in liquid nitrogen, everything else remaining unchanged. In addition to this relative calibration of the diode noise, all measurements were provided with an absolute calibration by shot noise (Sylvania tube 5722) to insure the reproducibility of the results. They are plotted in this normalized form in Fig. 2. Fitting white noise levels to these data gives

$$\overline{I^2}_{\text{resistor}}/\overline{I^2}_{\text{cal.}} = 0.0465 \pm 0.002 \text{ for } 15.7\text{ k}\Omega \text{ at } T = 300°\text{K}, \quad (2)$$

$$\overline{I^2}_{\text{resistor}}/\overline{I^2}_{\text{cal.}} = 0.0198 \pm 0.001 \text{ for } 15.7\text{ k}\Omega \text{ at } T = 77°\text{K}, \quad (3)$$

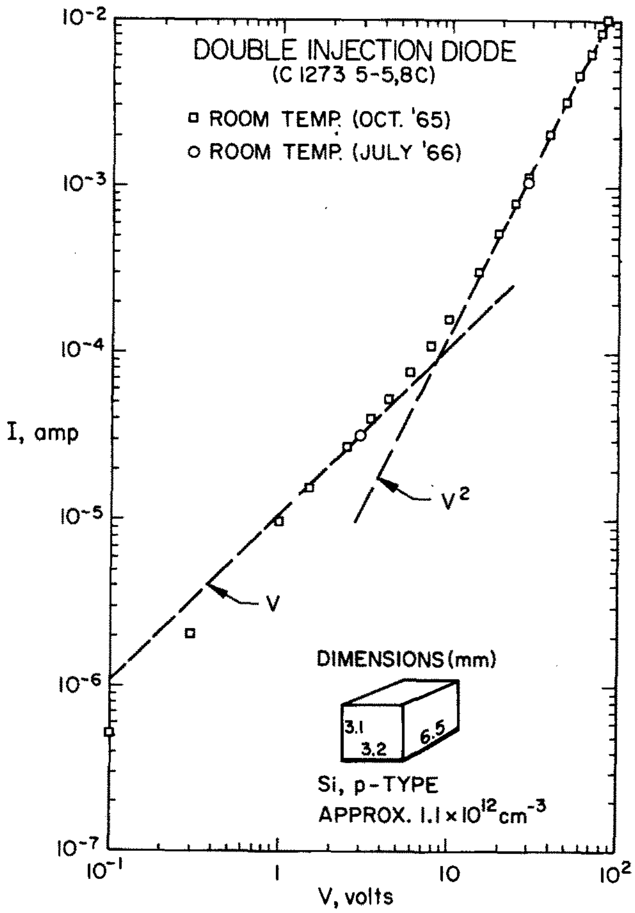

Fig. 1. V-I characteristic of double-injection diode at room temperature. The circles indicate the two operating points for which noise measurements are reported.

and about 0.029 for the double-injection diode. Below approx. 0.3 MHz the excess noise of the diode follows $1/f^2$ within a few percent (dashed line, measured points not shown) suggesting that the spectrum in Fig. 2 is composed of white noise and a $1/f^2$ component. An excellent fit is indeed obtained throughout the spectrum with

$$\overline{I^2}_{\text{diode}}/\overline{I^2}_{\text{cal.}} = 0.0292 + 0.0280/f(\text{MHz})^2 . \quad (4)$$

This dependence is shown as a solid line in Fig. 2.

To obtain the noise temperature T_d of the diode we interpolate linearly between the white noise levels of the resistor at 300°K and 77°K, thus eliminating the contributions of other noise sources in the analyzer, and obtain

$$T_d = 155°\text{K} \quad (5)$$

for the white component of the diode noise or, expressed in terms of a noise suppression factor β

$$\overline{I^2}_{\text{diode}} = \beta \cdot 4kT(\partial V/\partial I)^{-1}\Delta f \text{ with } \beta = 155/300 = 0.52 . \quad (6)$$

We estimate the uncertainty in β to ± 0.1. At room temperature this double-injection diode therefore exhibits a white noise of 1/2 the equivalent Nyquist noise of its differential resistance at this operating point.

To the best of our knowledge noise measurements on bipolar sclc have not been reported in the literature. A. G. Jordan and R. W. Knepper[1] have studied fluctuation phenomena in a double-injec-

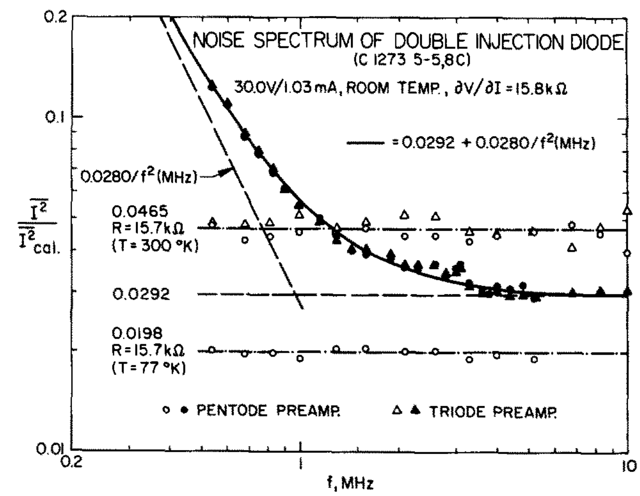

Fig. 2. Noise spectra of double-injection diode (30 V/1.03 mA, room temperature), of a 15.7 kΩ resistor at room temperature and immersed in liquid nitrogen. The spectrum of the diode is the sum of $1/f^2$ excess noise and a white component (dashed lines).

tion silicon diode in which unipolar sclc dominates at low operating points because of the presence of traps in the bulk. But their study is limited to this lower range of operation. It was brought to our attention, however, that unpublished work was performed earlier by G. S. Picus[2] on the noise of double-injection silicon diodes described by J. W. Mayer et al.[3]

These results constitute the first experimental evidence that noise in a solid can be less than the "Nyquist" noise, if this concept is defined for nonlinear elements as $4kT\Delta f/r$, where r is the differential resistance of the device at the operating point considered. Webb and Wright[4] and van der Ziel[5,6] have proposed models of noise in unipolar sclc and have also used this same formal generalization of Nyquist' formula as a point of reference for noise values. Webb and Wright argue that $\beta = 1$ may hold, while van der Ziel proposes alternately[5] $\beta = kT/qV \approx 10^{-3}$ and[6] $\beta = 2$. Insofar as these predictions apply to unipolar sclc, while the present result is obtained on bipolar sclc, a comparison is justifiable merely on a heuristic basis. It is interesting to note, however, that a formal argument presented in ref. 6 to derive $\beta = 2$ makes no detailed reference to the mechanism of current flow beyond that of eliminating diffusion. Diffusion plays no significant part in the square-law range of our device. That same argument would then predict $\beta = 2$ in the present case as well, in contradiction to the observed $\beta = 1/2$.

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ACOUSTIC QUARTER-WAVE PLATES AT MICROWAVE FREQUENCIES

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The use of birefringent acoustic media in microwave–shear-wave polarization transformers is described. Restrictions on the properties of the acoustic medium are discussed as well as the limitations imposed by the use of imperfect bonds between the transformer section and a delay medium. The principles are illustrated by application to a double-ended variable delay line.

In optical[1] and microwave[2] electromagnetic propagation the quarter-wave plate has long been used as a means for producing and detecting circularly polarized waves, and has been applied in a number of devices. For elastic waves at microwave frequencies it is well known that circular polarization can be produced without quarter-wave plates by using a ferromagnetic resonance transducer.[3] However, this requires that the applied dc magnetic field intensity be adjusted for transducer resonance at the operating frequency. In many experiments and devices this is not satisfactory because the dc magnetic field intensity must be free for other functions, and there is, therefore, need for a practically realizable acoustic quarter-wave plate.

Operation of a quarter-wave plate requires a birefringent propagating medium having normal modes linearly polarized at right angles to each other. Although a variety of crystal symmetries may be used for this purpose, a medium with cubic symmetry and normal mode axes $[1\bar{1}0]$ and $[001]$ will be chosen for illustration (Fig. 1). An input wave linearly polarized at 45° to these axes will excite the two normal modes with equal amplitude and phase. When the phase velocities $v_{[1\bar{1}0]}$ and $v_{[001]}$ are different, the normal modes will experience a relative phase shift

$$\Delta\phi = \left(\frac{\omega}{v_{[001]}} - \frac{\omega}{v_{[1\bar{1}0]}}\right) L \qquad (1)$$