Interstitial fluid effects in hopper flows of granular material

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NOMENCLATURE

Ba	Bagnold number.
d	Particle diameter.
D	Hopper opening diameter.
g	Acceleration due to gravity.
r	Radial position measured form conical hopper vertex.
r_E	Value of r at the hopper discharge, $r_E = \frac{1}{2} D \sin\theta$.
u	Velocity
u_E	Average hopper discharge velocity.
U	Dimensionless hopper flow rate.
β	Mass fraction
δ	Shear rate or extension rate.
ρ_s	Particle density.
ρ_F	Interstitial fluid density.
μ_F	Interstitial fluid viscosity.
θ	Hopper half angle.

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1. INTRODUCTION

In recent years a number of theoretical, experimental and computational research programs (Refs. [5], [8] and [3] for example) have substantially increased our fundamental understanding of the mechanics of flowing granular material. However most of these studies have concentrated on the simplest type of flow namely that of uniform size particles in the absence of any interstitial fluid effects or other complicating factors. The purpose of the present paper is to investigate the effects of interstitial fluid. In his classic study of granular flows Bagnold (1954) observed from his Couette flow studies that viscous effects of the interstitial fluid became significant when a number (which is now termed the Bagnold number, Ba) defined as

$$Ba = \rho_S d^2 \delta / \mu_F \qquad [1]$$

becomes less than about 450. Here δ is the velocity gradient or shear rate. (We have chosen to omit from the definition of Ba a volume fraction parameter which is usually of order unity and is therefore not important qualitatively). In the Couette flow experiments the appropriate shear rate, δ, is clearly defined; in other flows (such as the very practical flow in a hopper) the corresponding condition (or shear rate) is not known. The purpose here is to investigate the effects of the interstitial fluid in the primarily extensional flows which occur in the flow of a granular material in a hopper.

2. HOPPER FLOW EXPERIMENTS

The granular materials used in the present experiments are listed in Table I. All of these materials have a fairly narrow size distribution. With the exception of the Celanex and the polyester particles they consist of nearly spherical particles. The Celanex particles are roughly cylindrical in shape having a diameter approximately equal to their length. The polyester particles have a shape between a cylinder and a sphere.

The five different conical hoppers made of sheet metal had the following dimensions:

Cone No.	Half-angle θ(degs.)	Exit diameter D(cm.)
1	35°	3.217
2	25°	3.274
3	15°	3.111
4	12.5°	2.888
5	10°	2.832

A piece of duct tape was placed over the hopper opening prior to the hopper being filled with 3000 ccs of granular material which had been accurately weighed.

TABLE I. GRANULAR MATERIALS

Type	Mean Diameter d (cm.)	Specific Gravity ρ_s	Remarks
A-285 Glass Beads	0.292	2.5	Spherical
A-135 Glass Beads	0.139	2.5	Spherical
BI-B Glass Beads	0.051	2.5	Spherical
BT-6 Glass Beads	0.026	2.5	Spherical
P-0280 Glass Beads	0.061	2.5	Spherical
Lead Shot	0.226	11.3	Spherical
Polystyrene Plastic Stock	0.325	1.036	Close to Spherical
Polyethylene Plastic Stock	0.427	0.98	Spherical cylindrical shape
Celanex (P.V.C.) Plastic Stock	0.272	1.436	Cylindrical shape

A stopwatch was started when the tape was removed and flow began. The time for complete discharge was noted and a mass flow rate calculated from this. We recognize that although the flow rate remains constant during most of the discharge it does change slightly at the end. However the duration of this final transient was sufficiently short that the measured flow rate must be very close to the constant value exhibited during the majority of the discharge. Furthermore our purpose here is to examine changes in the flow rate rather than the absolute value and the simplicity of the present measurements facilitated the conduct of the investigation. The principal limitation to the accuracy of the results was the ability to measure the time of discharge by means of the stopwatch (for which the scatter was about ±0.05 secs). Thus each experiment was performed at least three times and usually five or more times. The quoted values use arithmetic means of discharge durations and vertical error bar on the dimensionless flow rates is of the order of ±1%.

The experiments in water were conducted in a large tank of water with the hopper completely submerged so that there was negligible resistance to the flow of water entering the top of the hopper as it is displaced by the material leaving the hopper. The object was to eliminate as far as possible any global relative motion between the granular material and the water in the flow within the hopper.

Specific measured masses of granular material were used in the experiment. The volumes of these masses were measured for the loosely packed state obtained when this mass was poured into a graduated cylinder. This density was used to convert the mass flow rate to a volume flow rate which was, in turn, converted to an average discharge velocity, u_E, using the exit area of the hopper. A dimensionless discharge, U, for either the experiments in air or those in water was obtained as

$$U = u_E / [g(1 - \rho_F/\rho_S)\, D]^{1/2} \qquad [2]$$

This dimensionless discharge is consistent with that which emerges from all of the more recent theoretical analyses of granular material flows in hoppers (see, for example, ref. [2] and [6]). The factor $(1 - \rho_F/\rho_S)$ accounts for the effective gravitational acceleration in the underwater hopper experiments; it is clearly unity for the flows in air.

3. HOPPER FLOW RATES IN AIR

The dimensionless flow rates of most of the granular materials in air are plotted against the half-angle of the hopper in figure 1. They are consistent with previous measurements of flow rates in conical hoppers (Refs. [6],[7]). They are also qualitatively consistent with the recent theoretical analyses of granular material flows in hoppers (Refs. [2] and [6]). We did note, however, that the flow rate is quite sensitive to the geometry of the hopper exit. Thus the flow variation from hopper to hopper may represent not only a functional dependence on the hopper half-angle but also minor differences in the exit geometries. However, since the primary purpose of the present investigation is to compare flow rates for different materials, different mixtures and different interstitial fluids we will not dwell on this sensitivity to hopper exit geometry except to state that care was taken not to alter or damage this exit geometry during the experiments.

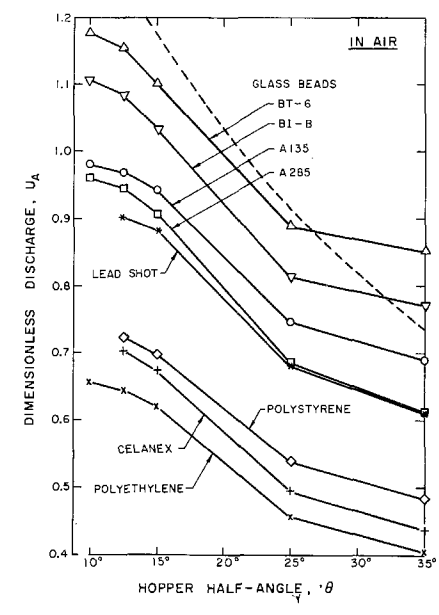

Figure 1. Dimensionless hopper flow rates of the granular materials in air presented as a function of the hopper half-angle.

The differences with material exhibited in Figure 1 represent two separate effects, namely the dependence of the flow rate on the ratio of exit diameter to particle diameter, D/d, and the dependence on the frictional properties of the material. The four sizes of glass beads have similar frictional properties and hence the differences between the four upper curves are probably due to differences in the diameter ratio. To illustrate this the flow rates for the 15° cone are plotted versus diameter ratio in Figure 2. This exhibits a somewhat similar kind of asymptotic behaviour observed by Nguyen et al (Ref. 6); namely that for sufficiently large diameter ratios the flow rate is independent of the diameter ratio. However Nguyen's experiments were conducted with different hopper openings rather than different sized particles and they suggested little change with flow rate above a

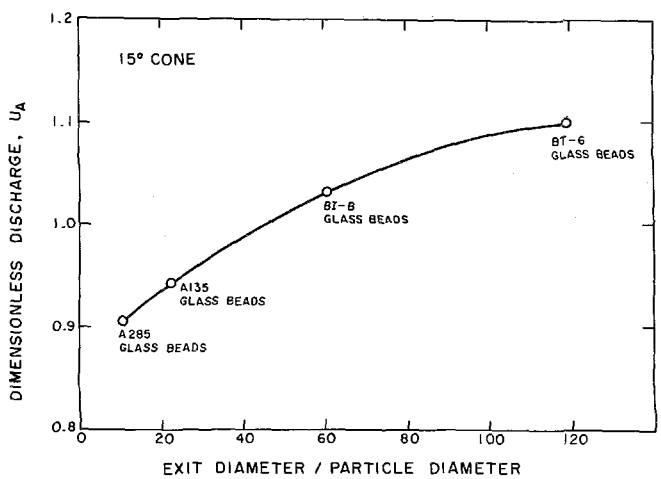

Figure 2. Dimensionless discharge from the 15° cone as a function of the ratio of hopper exit diameter to particle diameter for four types of glass beads (in air).

diameter ratio of 30. We are inclined to believe that this is still an appropriate value; the differences between the BT-6 and BI-B flow rates in Figure 2 are more likely due to different frictional properties than to diameter ratio effects. The BI-B glass beads are somewhat more irregular in shape and size than the BT-6 glass beads and therefore the former is expected to have somewhat greater frictional properties.

The remaining differences in Figure 1. are due to differences in the frictional properties of the materials. We refer not only to the internal friction between particles but also to the friction between the particles and the hopper walls. The harder materials (glass beads and lead shot) yield flow rates which are in fairly good agreement with each other (the lead shot is similar in size to the A285 glass beads) despite substantial differences in material density. On the other hand the softer plastics exhibit substantially reduced flow rates. Note that the non-spherical shape of the celanex does not appear to yield a large effect on the flow rate.

4. HOPPER FLOW RATES IN AIR; MIXTURES

Hopper flow rates of mixtures of two different sizes of glass beads were also measured over the full range of mass fractions and yielded some unexpected results. Various combinations of glass beads allowed examination of the effect of the ratio of the diameters of the particles. Typical results for the 15° cone are presented in Figure 3. The surprising feature of these results is that mixtures of particles having substantially different diameters exhibit a greater flow rate for certain intermediate mass fractions than is exhibited by either of the granular materials tested alone. This effect is greatest for the mixtures characterized by the greatest ratio of particle diameters (11.1); on the other hand the mixture with a

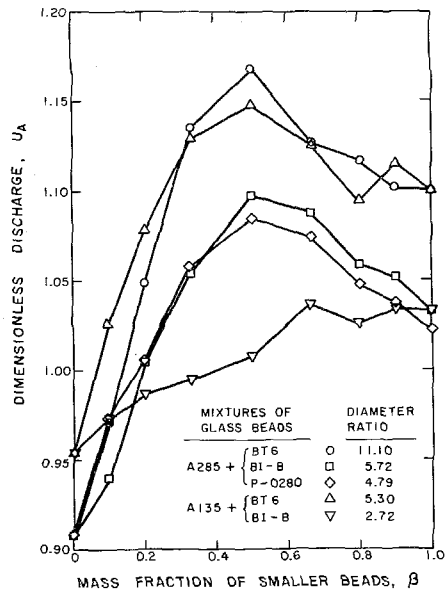

Figure 3. Dimensionless discharge (in air) for different mixtures of glass beads as a function of the mass fraction of the smaller beads, β. Tests in the 15° cone.

particle diameter ratio of 2.7 exhibits a variation with mass fraction which is close to monotonic. Though only results for the 15° cone are shown in Figure 3, similar results were obtained for the other hoppers.

This phenomenon was not only unexpected but is difficult to explain. It clearly indicates that the mixtures have smaller friction angles than either of the constituents. This is contrary to the common belief that the presence of different sizes causes a greater degree of interlocking of particles and therefore larger friction angles. It should also be stressed that Figure 3 presents dimensionless volume flow rates. It might be anticipated that peaks in the mass flow rates would result from higher maximum solid fractions for the mixtures than for the constituents alone. Such an effect was indeed observed in our measurements of the static solid fractions of the mixtures and might be expected to result in peaks in the mass flow rates. The phenomena observed here is an effect on the volume flow rate; the mass flow rates exhibit even larger peaks.

5. HOPPER FLOW RATES IN WATER; INTERSTITIAL FLUID EFFECTS

Typical dimensionless flow rates in water are presented in Figure 4 which should be compared with the results of Figure 1. Note that the flow rates for some materials such as lead shot are the same in water as in air implying that the viscous effects of the interstitial fluid are inconsequential in those flows. On the

other hand the flow rates of other materials are substantially different (for example the BT-6 glass beads).

In order to examine further the interstitial fluid effect, the ratio of the dimensionless flow rate in water to that in air is plotted in Figure 5 against an appropriate Bagnold number as defined by Equation (1).

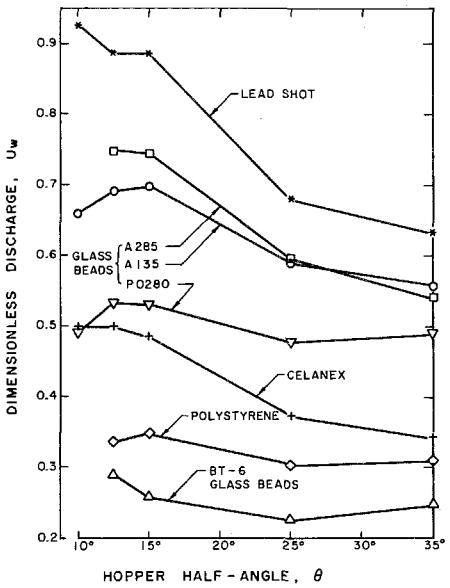

Figure 4. Dimensionless hopper flow rates under water.

We have assumed that the appropriate velocity gradient, δ, is that in the direction of flow at the hopper exit since it is this extensional deformation which dominates hopper flows. In the interior of the hopper near the discharge the velocity, u, will vary with radial position, r, according to $u = u_E(r_E/r)^2$, and it follows that

$$\delta = \left(\frac{du}{dr}\right)_{r=r_E} = 4\, u_E \sin\theta/D \qquad [3]$$

The ratios of dimensionless flow rates in water and air are plotted in Figure 5 against a Bagnold number, Ba, which uses this expression for δ. In the authors opinion it is remarkable that all of this data closely defines a single curve which asymptotes to unity at a value of Ba of the order of 500. In other words a value essentially the same as that noted by Bagnold (i.e. 450). For each material, data for each of the hoppers is shown connected by a line and it would appear that the variation from hopper to hopper is consistent with the variation from material to material.

Thus the present data confirms Bagnolds conclusion that the interstitial fluid effects are determined by an appropriately chosen Bagnold number and that the value of the Bagnold number above which the viscous effects of the interstitial fluid are negligible is about 500. As a postscript we note that the flows of the BT-6 glass beads <u>in air</u> have Bagnold numbers in the range 150 → 350 and therefore are subject to some small interstitial fluid effects. All the other flows in air have Bagnold numbers considerably greater than the critical.

Finally a set of mixtures of A285 and BT-6 glass beads were also tested underwater in the 15° cone and the results are presented in Figure 6. Though the of A285 and BT-6 glass beads. (tests in the 15° cone)

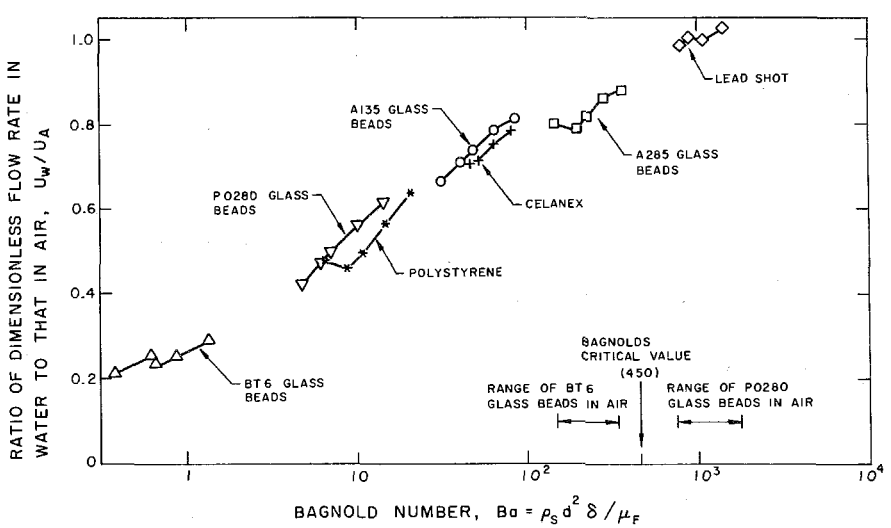

Figure 5. The ratio of the dimensionless flow rate in water to that in air plotted as a function of Bagnold number, Ba, for the flows in water. For a given material, the date from individual hoppers are connected by lines. Also shown are the critical Bagnold number and the ranges of Bagnold number for the test of BT-6 and P0280 glass beads in air.

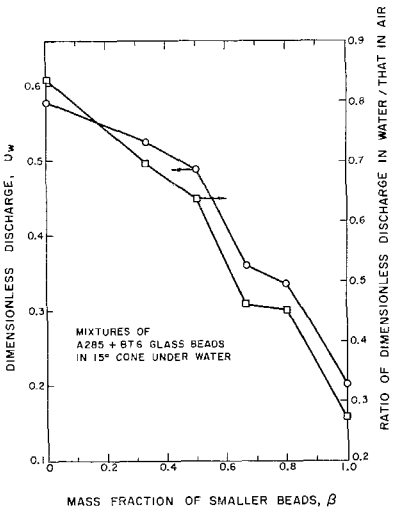

Figure 6. The dimensionless discharge in water and the ratio of this to the dimensionless discharge in air for various mixtures of A285 and BT-6 glass beads. (tests in the 15° cone)

curves have an upwardly convex curvature similar to the results in Figure 3, it is clear that the variation with mass fraction, β, is dominated by an increase in the interstitial fluid effects with increasing β. From a practical point of view it would be valuable to know how to extend the Bagnold number criterion to mixtures of different sizes of particles. To do so would require the definition of some effective particle diameter for use in the Bagnold number. Using Figures 5 and 6 it is possible to speculate on the appropriate effective particle diameter. For example at $\beta = 0.8$ since the dimensionless discharge ratio is 0.45 the effective Bagnold number from Figure 5 is about 5.5 and the effective particle diameter calculated from this is 0.063 cm. The constituent particle diameters being 0.292 and 0.026 cm it is clear that the effective diameter is not a mass-weighted mean (.171 cm at $\beta = 0.8$) but is closer to simple linearly-weighted mean diameter ($0.292(1 - \beta) + 0.026\beta = 0.058$ cm at $\beta = 0.8$).

6. CONCLUSION

It has been demonstrated experimentally that the effect of the interstitial fluid on the flow of a granular material in a conical hopper is governed by the magnitude of a Bagnold number, Ba, defined as $Ba = \rho_s d^2 \delta / \mu_F$ where d, ρ_s are respectively the diameter and density of the solid particles and μ_F is the viscosity of the interstitial fluid. The quantity, δ, is a velocity gradient which in the case of a shear flow is simply the shear rate. However in the primarily extensional flow in a hopper the appropriate δ is the maximum velocity gradient in the direction of flow as given by Equation (3). It is demonstrated that for five different hoppers and nine different granular materials the effect of the interstitial fluid on the flow rate correlates with the Bagnold number defined in this way. Furthermore, the critical value of Ba above which the interstitial fluid effect is negligible is about 500, a value identical with the critical Ba recognized for Couette flow by Bagnold himself.

Data on the flow rates for mixtures of two different sizes of particles is also presented. This exhibits the surprising feature that the volume flow rate for the mixture can be larger than that for either of the constituents; the peak is more pronounced the larger the ratio of the particle diameters of the constituents. Insterstitial fluid effects on these mixtures are also investigated and it is demonstrated that the appropriate size, d, for use in defining Ba is a mean diameter linearly- weighted according to the mass fractions of the constituents.

7. ACKNOWLEDGMENTS

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