Neutrino Mass Constraints on μ Decay and $\pi^0 \to \nu\bar{\nu}$

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In this Letter, we show that upper limits on the neutrino mass translate into upper limits on the class of neutrino-matter interactions that can generate loop corrections to the neutrino mass matrix. We apply our results to μ and π decays and derive model-independent limits on six of the ten parameters used to parametrize contributions to μ decay that do not belong to the standard model. These upper limits provide improved constraints on the five Michel parameters, ρ, ξ', ξ'', α, α', that exceed Particle Data Group constraints by at least one order of magnitude. For $\pi^0 \to \nu\bar{\nu}$ we find, for the branching ratio, $B(\pi^0 \to \nu\bar{\nu}) < 10^{-10}$.

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With the discovery of neutrino oscillations a few years ago [1–3], the neutrino mass matrix has become a subject of intensive experimental and theoretical research as it provides a unique window into physics beyond the standard model (SM). Indeed, the combination of WMAP [4], 2DFGRF [5], and neutrino oscillation data yield an upper limit of 0.23 eV for the mass of an active neutrino. The Planck mission [6], to be launched in 2007, may further improve this limit to ~0.04 eV [7]. With masses of active neutrinos at least 6 orders of magnitude smaller than those of all other SM fermions, neutrino masses are presumably generated at an energy scale that significantly exceeds the electroweak scale. At low energies, manifestations of such new physics, including neutrino masses, are suppressed by inverse powers of this heavy scale. For example, in the seesaw mechanism, neutrino masses are inversely proportional to the heavy right-handed neutrino mass, which can range from a few TeVs to 10^{13} GeV depending on the model.

The study of non-SM neutrino-matter interactions may also shed light on physics beyond the SM. However, since neutrino-matter cross sections are generally small, direct observation of these interactions is experimentally challenging. Moreover, since the number of candidates for physics beyond the SM is large, determining the most viable particle physics scenario is nontrivial. In view of this situation, model-independent constraints on non-SM neutrino-matter interactions in combination with the study of the neutrino mass matrix should prove a valuable tool in the search for new physics.

In this Letter we point out a general connection between the neutrino mass and scalar (S), pseudoscalar (P), and tensor (T) neutrino-matter interactions. In particular, we show that under minimal assumptions these chirality-changing interactions generate contributions to neutrino mass through loop effects. We do not make any assumption about the dynamical origin of the neutrino mass. Instead, we perform a phenomenological analysis and require that such contributions to the mass not exceed the physical neutrino mass. This allows us to place stringent constraints on chirality-changing neutrino-matter couplings. Our general conclusions are then applied to the SM-forbidden decay of π^0 into a neutrino and an antineutrino with the same helicity ($\pi^0 \to \nu\bar{\nu}$) and to μ decay. In the former case we show that the cosmological neutrino mass upper limit constrains the branching ratio for $\pi^0 \to \nu\bar{\nu}$ to be $\sim 10^4$ times smaller than the best current experimental limit [8,9]. For μ decay, we derive improved constraints on five out 11 Michel parameters (MPs) that exceed current experimental limits by at least one order of magnitude [10]. We also point out that a nonzero measurement by TRIUMF Weak Interaction Symmetry Test (TWIST) [11] of the MPs δ and ρ could be used to make a statement about the neutrino mass that should be consistent with cosmological limits extracted from WMAP and Planck in combination with galaxy redshift surveys (GRS). Finally, we observe that the non-SM chirality-changing interactions cannot account for the deviation from the SM value of the weak mixing angle reported by the NuTeV Collaboration [12].

General argument.—The general chirality-changing effective neutrino-fermion interaction can be written as

$$\mathcal{L} = G_{\rm F} \sum_{l,l',f,f',i} a^{ff'}_{i;ll'} \bar{f}\Gamma_i f' \bar{\nu}_l \Gamma_i \nu_{l'} + \text{H.c.}, \qquad (1)$$

where $i = S$, P, and T with $\Gamma_S = 1$, $\Gamma_P = \gamma^5$, and $\Gamma_T = \sigma^{\mu\nu}$; the sum over l, l' runs over *active* neutrino flavors while the sum over f, f' is over the SM charged fermions (this approach does not yield competitive constraints on neutrino-neutrino scattering), and $a^{ff'}_{i;ll'}$ are dimensionless constants parametrized in terms of the Fermi constant $G_{\rm F} = 1.16637(1) \times 10^{-5}$ GeV^2. The chirality-changing interaction in Eq. (1) generally contributes to the neutrino mass via diagrams shown in Fig. 1. Substituting $\bar{\nu}_l \Gamma_i \nu^c_{l'}$ in Eq. (1) induces Majorana neutrino masses.

Equation (1) is a general effective Lagrangian for neutrino-matter interactions constructed from nonrenormalizable operators (the coupling constants have negative

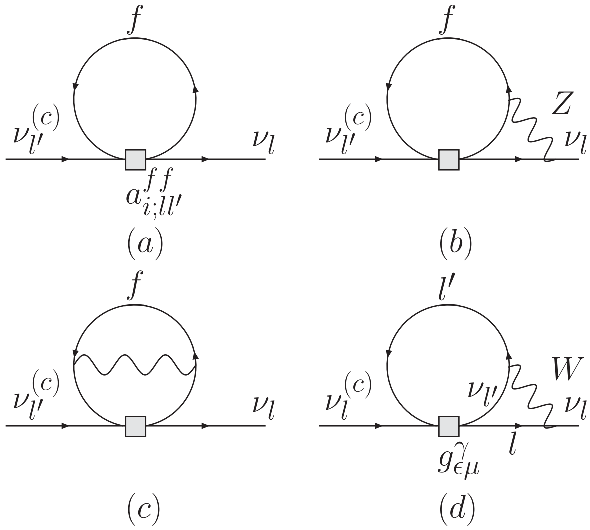

FIG. 1. One- and two-loop contributions to the neutrino mass (denoted $\delta m_\nu^{(1)}$ and $\delta m_\nu^{(2)}$, respectively) generated by chirality-changing neutrino-fermion interaction. For Majorana mass terms, $\nu_{l'} \to \nu_{l'}^c$, and there are two additional diagrams similar to and of the same order as (b) and (d), where the weak bosons interact with $\nu_{l'}^c$.

mass dimension as seen from the overall factor of G_F). Therefore, a new counterterm will be needed for each operator to cancel divergences that may appear in the evaluation of loop graphs. The unique physical content of the loop graphs resides in their nonanalytical part. Analytical contributions can change with the renormalization scheme used to make the graphs finite while nonanalytical terms remain the same. Since we are interested only in orders of magnitude, the only nonanalytical contributions we consider are logarithms.

We evaluate leading logarithmic contributions to the neutrino mass from the diagrams in Fig. 1. The pseudoscalar and tensor neutrino-matter interactions are unconstrained to one loop—the one-loop Feynman diagrams with $\Gamma_i = \gamma^5, \sigma^{\mu\nu}$ give zero—while the scalar interaction can be constrained by both the one- and two-loop contributions. Since we are interested in orders of magnitude, we do not take into account the factors of $\mathcal{O}(1)$ associated with the different Γ_i's. The result is

$$\begin{aligned} \delta m_\nu^{(1)} &\approx N_c G_F a_{S;ll'}^{ff} \frac{m_f^3}{(4\pi)^2} \ln\frac{\mu^2}{m_f^2}, \\ \delta m_\nu^{(2)} &\approx g^2 N_c G_F a_{i;ll'}^{ff'} \frac{(m_f \quad \text{or} \quad m_{f'}) M_Z^2}{(4\pi)^4} \left(\ln\frac{\mu^2}{M_Z^2}\right)^2, \end{aligned} \tag{2}$$

where the superscript in $\delta m_\nu^{(i)}$ indicates the loop order, N_c equals three for quarks and one for leptons, m_f is the mass of fermion f [in Eq. (2), it is the mass of the fermion internal line that requires the chirality flip to couple to the weak boson that is inserted], $g \cong 0.64$ is the $SU(2)_L$ coupling constant, μ is the renormalization scale, as well as where the subscripts ll' are suppressed on the left-hand side of the matrix Eq. (2). Since $\delta m_\nu^{(1)}/\delta m_\nu^{(2)} \sim 50 m_f^2/M_Z^2$, $\delta m_\nu^{(1)}$ is negligible for all fermions except the top quark. Note that the loop expansion series converges since each loop order is suppressed by a numerical factor of $[\ln(\mu^2/M_Z^2)/(4\pi)^2]^L$ where $L \geq 2$ is the loop order and the logarithm is of order ten as discussed below. Furthermore, the mass dependence of each loop diagram must be an expansion series in powers of $(m_f^2/M_Z^2)^n m_f$ with $n = 0, 1, 2, \ldots$, and where the $n = 0$ term appears only at second order with the exchange of a weak boson as in the diagrams of Fig. 1. It follows that the $L = 2$ diagrams with a weak boson are largest except for the case where $m_f = m_{\rm top}$ as mentioned above. We thus have the counterintuitive result that the one-loop graph is generally subdominant.

The $(\ln\mu^2)^2$ factors in Eq. (2) appear because the diagrams are ultraviolet divergent; they are compensated by the μ dependence of the neutrino mass counterterm and the μ dependence of the $a_{i;ll'}^{ff'}$'s deduced from the renormalization group (RG) equations they satisfy. Thus, in order to extract constraints on the $a_{i;ll'}^{ff'}$'s, one must choose a renomalization scale μ.

This value of μ should exceed the mass of the heaviest particle included in the effective field theory (EFT)—in our case m_t, the top quark mass—while at the same time take into account the scale at which the onset of new physics might be expected. We choose the renormalization scale to be about 1 TeV, a scale often associated with physics beyond the SM in many particle physics models. Since μ appears in a logarithm, our conclusions do not depend strongly on its precise value. Note that the renormalization scale μ is far above the energy scale at which processes like μ decay and $\pi \to \nu\bar{\nu}$ occur. In principle, the couplings appearing in Eq. (2) should be evolved down to $\sim$1 GeV using the appropriate RG equations, but this can at most generate factors of $\mathcal{O}(1)$. For example, the running of the coupling constant associated with the four-quark operators in kaon decay, from the weak scale down to $\mu \approx$ 1 GeV, generates only factors of 2 [13]. There is no reason to expect a more substantial change to the four-lepton or quark-lepton operators of Eq. (1) when running μ down to $\sim$100 MeV. Thus, in the model-independent analysis of this Letter, there is no need to take the RG running of coupling constants into account. We emphasize that values of μ below the weak scale cannot be substituted in Eq. (2). Below the weak scale, the dependence of the amplitude on μ becomes suppressed by inverse powers of the weak scale as required by the decoupling theorem. See the section on QCD renormalization in Ref. [10] for a more detailed discussion of this point in the case of QCD. Note that μ would not appear in a specific model where neutrino masses are calculated radiatively from finite diagrams. In that case, the logarithms would instead have arguments of the form M_Λ^2/M_Z^2 where M_Λ would be the mass of a heavy particle in the model.

Below we use Eq. (2) to constrain $a_{i;ll'}^{ff'}$ by requiring $\delta m_\nu \equiv \delta m_\nu^{(1)} + \delta m_\nu^{(2)} \lesssim m_\nu$ where m_ν is the physical neutrino mass. Since the graphs of Fig. 1 are divergent, there are counterterms that absorb the infinities. In the absence of fine-tuning and assuming perturbation theory to be valid, the leading log contributions of the loop graphs should be no larger than the physical value of neutrino masses.

We now apply our general results to non-SM π^0 and μ decays. We adopt the upper limit of 0.7 eV on the sum of the neutrino masses from Ref. [4], which translates into the limit $m_\nu < 0.23$ eV for individual neutrino masses when neutrino oscillation constraints are included.

π^0 decay.—We obtain from Eq. (2) $a_{P;ll'}^{qq} < 10^{-3}$ for $q = u, d$. For the calculation we used $m_f = m_u = m_d = (m_u + m_d)/2 = 4$ MeV [10] (constituent quark masses are inappropriate when working at $\mu \sim 1$ TeV). We can use this result to place an upper limit on the branching ratio $B(\pi^0 \to \nu\bar{\nu})$. Starting from the neutrino-quark interaction Lagrangian in Eq. (1) with $\Gamma_P = \gamma^5$, we obtain the effective interaction

$$\mathcal{L}_{\pi\nu\bar{\nu}} = -\frac{G_F}{\sqrt{2}}\frac{F_\pi m_\pi^2}{m_u + m_d}(a_{P;ll'}^{uu} - a_{P;ll'}^{dd})\pi^0\bar{\nu}_l\gamma^5\nu_{l'}, \quad (3)$$

where $F_\pi = 92.4$ MeV is the pion decay constant and m_π is the pion mass. The above equation leads to the branching ratio $B(\pi^0 \to \nu\bar{\nu}) = 10^{-4}(a_{P;ll'}^{uu} - a_{P;ll'}^{dd})^2 < 10^{-10}$, which is 4 orders of magnitude stronger than the current best experimental limit $B(\pi^0 \to \nu\bar{\nu})^{\text{Exp}} < 8.3 \times 10^{-7}$ where $l = l' = \mu$ [9]. Our limit on $B(\pi^0 \to \nu\bar{\nu})$ improves by a further 2 orders of magnitude if the possible Planck limit of $m_\nu < 0.04$ eV is used instead of $m_\nu < 0.23$ eV.

μ decay.—Muon decay can be described with the following effective interaction [10]:

$$\mathcal{L}_{\mu\to e\nu_\mu\bar{\nu}_e} = \frac{4G_F}{\sqrt{2}}\sum_{\substack{\gamma=S,V,T\\ \epsilon,\mu=R,L}} g_{\epsilon\mu}^\gamma \bar{e}_\epsilon\Gamma^\gamma\nu_e^n\bar{\nu}_{\text{muon}}^m\Gamma_\gamma\mu_\mu, \quad (4)$$

where $\gamma = S$, V, and T indicate, respectively, scalar, vector, and tensor interactions and $\epsilon, \mu = R, L$ indicate the chiralities of the charged leptons. The chiralities n and m of the neutrinos are determined by the values of γ, ϵ, and μ. The constants $g_{\epsilon\mu}^\gamma$ parametrize the strength of the corresponding phenomenological interactions and can be related to the $a_{i;\mu e}^{e\mu}$ through Fierz transformations. In the SM, $g_{LL}^V = 1$ with the rest being zero.

Limits on g_{RL}^S, g_{RL}^V, g_{RL}^T, g_{LR}^S, g_{LR}^V, and g_{LR}^T can be obtained from Fig. 1(d) and Eq. (2) with $M_Z \to M_W$ and $m_f = m_e$, the mass of the electron, for g_{RL}^S, g_{RL}^V, g_{RL}^T, and $m_f = m_\mu$, the mass of the muon, for g_{LR}^S, g_{LR}^V, and g_{LR}^T.

Our results are given in the third column of Table I; the second column displays current upper limits from Ref. [10]. Except for g_{RL}^T, our model-independent upper limits are at least 1 order of magnitude better than the ones appearing in Ref. [10].

TABLE I. Approximate upper limits on the $g_{\epsilon\mu}^\gamma$'s from Ref. [10] in comparison to the ones derived from the loop graph of Fig. 1(d) in combination with cosmological limits on the neutrino mass.

$g_{\epsilon\mu}^\gamma$	Current upper limits	Upper limits from m_ν
g_{RL}^S	0.424	10^{-2}
g_{LR}^S	0.125	10^{-4}
g_{RL}^V	0.110	10^{-2}
g_{LR}^V	0.060	10^{-4}
g_{RL}^T	0.036	10^{-2}
g_{LR}^T	0.122	10^{-4}

The limits on $g_{\epsilon\mu}^\gamma$ translate into order of magnitude upper limits on the MPs. Using the definitions in Ref. [10], and their limit on $(b + b')/A < 10^{-3}$ at 90% C.L. as well as the fact that $A \cong 16$, we obtain the limits given in Table II. The meaning of the numbers is explained in the caption. The bracketed limits on ξ' are not fully constrained by upper limits on neutrino mass. They are included in the table because the parameters with the largest uncertainties that enter its definition are here better constrained. In particular, the largest uncertainty in

$$1 - \xi' = [(a + a') + 4(b + b') + 6(c + c')]/A \quad (5)$$

TABLE II. Order-of-magnitude upper limits on the MPs. All numbers should be multiplied by 10^{-3}. Note that a, a', c, c' are not technically MPs, and instead belong to a set of parameters defined by Kinoshita and Sirlin [14]. PDG numbers are given in the second column at 95% confidence level (C.L.) and 90% C.L. (numbers with daggers). The third column shows the order-of-magnitude limits extracted from the $g_{\epsilon\mu}^\gamma$'s given in Table I and Ref. [10]. The fourth column gives expected order-of-magnitude limits from the TWIST and PSI experiments [11,15]. The fifth column refers to improved limits on the MPs due to the anticipated data from the Planck mission expected to constrain the upper limit on the neutrino mass to about $m_\nu \lesssim 0.04$ [7]; see also Refs. [16,17]. The meaning of the bracketed numbers is explained in the text.

MP	PDG	WMAP/GRS	TWIST/PSI	Planck/GRS
$\rho - 3/4$	7	1	0.1	0.1
η	33	X	0.1	X
$\delta - 3/4$	7	10	0.1	0.1
$1 - \xi\delta/\rho$	$3.2^\dagger$	X	0.1	X
$1 - \xi'$	80	[10]	X	[4]
$1 - \xi''$	58	10	X	0.1
α/A	9	0.001	X	0.0001
α'/A	8.8	0.001	X	0.0001
a/A	$15.9^\dagger$	1	X	0.1
a'/A	13	1	X	0.1
c/A	$6.4^\dagger$	0.1	X	0.01
c'/A	7.5	0.1	X	0.01

stems from the relatively large Particle Data Group (PDG) upper limits on the parameters a, a', c, c' when compared to the upper limit on $(b + b')/A$. Our limits on the former parameters are substantially better, thus improving on the PDG limit for $1 - \xi'$ even though the neutrino mass upper limit does not constrain $(b + b')/A$. With the improved limits on a, a', c, c' due to Planck data, the upper limit on $1 - \xi'$ should then be entirely due to the upper limit on $4(b + b')/A$. In the same vein, note that because our constraint on α is so strong, the measurement of $\eta = (\alpha - 2\beta)/A$ at PSI [15] to a few parts in 10^{-4} will also constitute a measurement of the MP β.

Finally, note that a similar analysis for the decay $\tau \to \mu \bar{\nu}_\mu \nu_\tau$ can be performed

$$\mathcal{L}_{\tau\to\mu\bar{\nu}_\mu\nu_\tau} = \frac{4G_F}{\sqrt{2}} \sum_{\substack{\gamma=S,V,T \\ \mu,\tau=R,L}} g^{\gamma(\tau)}_{\mu\tau} \bar{\mu}_\mu \Gamma^\gamma \nu^n_{\rm muon} \bar{\nu}^m_{\rm taon} \Gamma_\gamma \tau_\tau, \quad (6)$$

and the following limits are obtained: $g^{S(\tau)}_{RL}, g^{V(\tau)}_{RL}, g^{T(\tau)}_{RL} < 10^{-4}$ and $g^{S(\tau)}_{LR}, g^{V(\tau)}_{LR}, g^{T(\tau)}_{LR} < 10^{-6}$. In a particle physics model where the charged-lepton decay couplings are all of the same order, the $g^{\gamma(\tau)}_{\mu\tau}$ should provide the best limits on the MPs.

Non-SM contributions to neutral currents.—In light of the NuTeV result on the weak mixing angle (θ_W) [12], constraining non-SM neutral currents is particularly timely. To determine $\sin^2\theta_W$, the experiment measures the ratio of neutral to charged currents in $\nu_\mu(\bar{\nu}_\mu)$-quark interactions. Any deviation from the SM neutral or charged current can be interpreted as a deviation from the SM predictions for $\sin^2\theta_W$. For neutral currents, the relevant coupling constants are $a^{qq}_{i;ll'} < 10^{-3}$ for $q = u, d$ and $i = S, P, T$. The (axial-)vector currents of the SM do not change the chirality or the flavor of the neutrino while the chirality-changing coupling interactions considered in this work do. Thus, the final states are different and the rates—not the amplitudes—must be added. Therefore, the chirality-changing non-SM operators can at most modify the SM neutral current by 10^{-6} and cannot account for the NuTeV anomaly.

Conclusions.—Derivation of our results requires only minimal assumptions. We view the SM as an EFT valid below a certain energy scale (taken to be above 1 TeV) and assume the validity of perturbation theory. Note that the interactions of Eq. (1) are not gauge invariant under $SU(2)_L \times U(1)_Y$. From a strictly formal point of view, our EFT is not allowed since μ is above the weak scale; the operators could be embedded in a gauge-invariant structure, but the resulting Ward identities may impose relationships between the parameters that are assumed independent in this Letter. However, since the neutrino mass does not violate gauge invariance (e.g., in the SM, the neutrino mass is generated through the spontaneous breaking of a gauge symmetry), diagrams that contribute to m_ν are not forced to cancel in a gauge-invariant model. Our order-of-magnitude estimates should therefore be robust—finely-tuned cancellations not withstanding. The MPs δ and ρ will soon be constrained with improved precision by the TWIST experiment at the 10^{-4} level [11]. Although results of such measurements will be valuable whether or not a positive signal is observed, an especially interesting situation would arise in the case where TWIST measured finite deviations from the SM values of δ and ρ since that would have implications for the neutrino mass. Thus, any particle physics model that could accommodate deviations of δ and ρ at the 10^{-3}–10^{-4} level would also be challenged to simultaneously generate neutrino masses consistent with observations; for example, this could be achieved through finely tuned cancellations of the radiative corrections to the neutrino mass shown in Fig. 1 or mixing with right-handed neutrino states with masses $\gg 0.23$ eV that could lead to large contributions to the MPs. Furthermore, such a measurement would have implications for all physical processes where the magnitude of the neutrino mass plays a role, like $0\nu\beta\beta$ decay when the neutrino is a Majorana fermion.

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