Effects of Vertical Vibration on Hopper Flows of Granular Material

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Abstract

This paper examines the flow of a granular material through a wedge-shaped hopper subject to vertical, sinusoidal oscillations. Experiments and discrete element computer simulations were conducted to investigate particle trajectories within and mass discharge rates from the hopper. With the hopper exit closed, side wall convection cells are observed in both the experiments and simulations. The convection cells are oriented such that particles move up along the inclined walls of the hopper and down along the centerline. Results from the computer simulation indicate that the convection cells are a result of the dilation of the granular bed during free fall and interaction with the hopper walls. Measurements of the mean mass discharge rate for various vibration parameters were also made in both the experiments and simulations. The ratio of the mass discharge rate for a vibrating hopper to the mass discharge rate for a non-vibrating hopper scales with the oscillation velocity amplitude and exhibits a maximum value just greater than one for oscillation velocity amplitudes less than 0.5. The ratio is less than one for larger velocity amplitudes. A simple model taking into account the change in the effective gravity acting on the granular material over an oscillation cycle is examined. A significant deficiency in the model is that it assumes no material discharges from the hopper during part of each oscillation cycle for acceleration amplitudes greater than gravitational acceleration. Data from the simulations indicate that although the discharge rate from the hopper varies throughout an oscillation cycle, it never equals zero. The simulation was also used to examine particle horizontal position and velocity profiles at the hopper exit. Lastly, preliminary observations of the effects of localized vibration on a granular material in a closed hopper are presented.

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Introduction

Hoppers are commonly used in the handling and storage of granular materials. Often, however, difficulties such as funnel flows, rat-holes, and bridging occur as granular material flows through the hopper (Nedderman, 1992). In an attempt to alleviate these problems, the walls of the hopper are sometimes subject to forced vibrations with the expectation that this will allow the material to flow more readily. Although this technique is commonly used, there remains a poor understanding of the effects of applied vibration to hopper flows.

Much of the previous work investigating hopper flows has concentrated on stress and velocity fields and discharge rates through non-vibrating hoppers (refer to the reviews by Nedderman *et al.*, 1982 and Tüzün, *et al.*, 1982). Perhaps the first works including the effects of vibration were performed by Takahashi *et al.* (1968) and Suzuki *et al.* (1968) in their experiments examining particle trajectories and discharge rates in vertically oscillating wedge hoppers and flat bottom bins. They found that convection cells appear near the inclined wall boundaries of the hopper and that the discharge rate from the hopper decreases significantly at high oscillation acceleration amplitudes. More recently, Evesque and Méftah (1993) examined the time required to discharge a known quantity of sand from a sealed vertically vibrating hourglass. They also found that the discharge rate decreases with increasing acceleration amplitude. Knight *et al.* (1993), although not investigating hopper flows specifically, reported the appearance of convection cells in experiments using vertically oscillating conical containers. The effects of horizontal vibration on granular materials is investigated in the recent experimental work of Weathers *et al.* (1997). They also examine the particle trajectories and discharge rates and find that the qualitative velocity profile from a horizontally vibrating hopper is significantly different than that observed in a vertically vibrating hopper. Furthermore, they found that the discharge rate increases with increasing oscillation velocity amplitude.

This paper examines the particle trajectories and discharge rates from a vertically oscillating, wedge-shaped hopper using both experiments and two-dimensional discrete element computer simulations. First the experiments are described and qualitative and quantitative results are presented. Next, a simple model originally proposed by Suzuki *et al.* (1968) to predict the mass discharge rate from a vertically oscillating hopper is examined. The assumptions used in the model are then examined more closely using a discrete element computer simulation of a vertically oscillating hopper flow. Lastly, preliminary observations from an experiment of a hopper with a single oscillating wall are presented.

Experiment

The experimental apparatus consists of a symmetric, wedge-shaped hopper mounted on a Ling electromagnetic shaker that subjects the entire hopper apparatus to vertical, sinusoidal oscillations. The vibration frequency and acceleration are monitored using an accelerometer mounted to the base plate of the shaker. The frequencies, f, examined in the experiments range from 20 to 60 Hz and the dimensionless acceleration amplitude of the oscillations, $\Gamma=a\omega^2/g$ where a is the oscillation amplitude, ω is the radian frequency of the oscillations ($\omega=2\pi f$), and g is the acceleration due to gravity, ranges from 0 to 4.0.

The front and rear walls of the hopper are lined with smooth window glass and the inclined and vertical walls are made of plexiglass with a smooth, milled finish. Static electricity effects were not problematic during the experiments. The distance between the front and back walls of the hopper, C, is 12.7 mm (approximately ten particle diameters), the exit width, B, is 4.0 mm, and the wall angle measured from the centerline is 45 degrees.

The hopper was filled with approximately 200 gms of clear, 1.3 mm diameter soda lime glass spheres giving an average free surface height above the exit plane of $24B$. The total time of discharge from the non-vibrating hopper is 21.8 seconds resulting in an average mass discharge rate, W_0, of 9.2 gm/sec. The average discharge rate is determined by recording the time required for the hopper to completely discharge a known mass of material. From previous work by other investigators (see, for example, Nedderman *et al.*, 1982), the instantaneous discharge rate from the hopper remains approximately equal to the mean discharge rate except for a short initial transient when the hopper exit is first opened, and a final transient when the free surface height of the material above the exit is approximately equal to the exit width. Thus, the measured average discharge rate from the hopper is expected to give a good approximation of the instantaneous discharge rate for the non-vibrating hopper.

In order to follow the trajectories of particles within the hopper, some of the spheres are dyed black. The non-vibrating hopper displays a typical funnel flow pattern as it discharges - a region of stagnant material occurs adjacent to each inclined hopper wall. The free surface of the material forms a V-shaped valley with particles continuously avalanching down the sloped surfaces toward the centerline of the hopper. As the height of the free surface from the exit plane decreases, the stagnant regions become smaller until all of the material in the hopper flows.

The remaining experiments were conducted for a hopper subject to vertical, sinusoidal oscillations. First the particle trajectories within the hopper were observed with the hopper exit closed. When $\Gamma>1$, two convection cells appear with particles moving up in a narrow boundary layer along the inclined walls of the hopper and down at the centerline of the hopper. These convection cells are similar to, but in the opposite direction, to the convection cells observed in granular beds subject to vertical oscillations in containers with vertical walls (Wassgren, 1997; Knight *et al.*, 1993). Surface waves forming at one-half the oscillation frequency (referred to as $f/2$ waves) also appear on the free surface of the bed when the dimensionless acceleration amplitude, Γ, is greater than 2.5. Surface waves have also been observed in other experiments concerning granular beds subject to vertical, sinusoidal oscillations (Melo *et al.*, 1995; Wassgren *et al.*, 1996).

When the exit of the hopper is opened, the convection cells are no longer observed and mass-flow behavior appears instead of the funnel flow behavior observed for the non-vibrating hopper. The slope of the V-shaped free surface decreases with increasing oscillation velocity amplitude, $a\omega$. The $f/2$ surface waves continue to form as the material discharges from the hopper until the height of the free surface of the bed from the hopper exit is approximately five exit widths above the exit plane.

The mean discharge rate from the hopper, W, for various vibration parameters was also measured. The results are non-dimensionalized by the mean discharge rate for the non-vibrating hopper, W_0, and are presented as both a function of dimensionless oscillation acceleration amplitude, $\Gamma = a\omega^2/g$, and dimensionless oscillation velocity amplitude, $a\omega/(gD_h)^{1/2}$, where D_h is the hydraulic diameter of the exit opening: $D_h=(BC)^{1/2}$. Each experimental data point in the figures represents the average of five measurements.

In figure 1, the dimensionless discharge rate, W/W_0, is plotted as a function of the dimensionless acceleration amplitude of the oscillations, $\Gamma = a\omega^2/g$. The discharge rate at high frequencies (60 Hz for example) does not deviate from the discharge rate for a non-vibrating hopper as significantly as it does at lower frequencies (for example at 20 Hz) for a particular value of Γ. Note that W/W_0 is slightly greater than one for f=60 Hz over the range of Γ investigated while for f=20 Hz W/W_0 is significantly less than one over the same range of Γ.

The dimensionless discharge rate is also presented in figure 2 as a function of the dimensionless oscillation velocity amplitude, $a\omega/(gD_h)^{1/2}$. The grouping of the data points in this figure indicates that the discharge rate scales more closely with the velocity amplitude than with the acceleration amplitude. At low velocity amplitudes the discharge rate increases slightly and reaches a maximum of approximately 1.05 for $a\omega/(gD_h)^{1/2}$ less than 0.5. For larger velocity amplitudes the discharge rate is less than the discharge rate from a non-vibrating hopper. At the largest dimensionless velocity amplitude investigated, $a\omega/(gD_h)^{1/2} = 1.2$, the discharge rate ratio, W/W_0, is approximately 0.7.

Analysis

In order to predict the observed mean discharge rate trend with the vibration parameters, Suzuki *et al.* (1968) proposed a simple model that includes the variation of the "effective gravity" acting on the granular material over an oscillation cycle. From dimensional analysis, the discharge rate from a hopper is proportional to the bulk density of the bed near the hopper exit, ρ_b, the square root of the acceleration acting on the bed, g (typically the acceleration due to gravity), and the hydraulic diameter of the hopper exit, D_h, raised to the 5/2 power:

$$W \alpha \rho_b g^{1/2} D_h^{5/2} \tag{1}$$

Since the hopper is oscillating, the effective gravity the bed experiences relative to the hopper walls, g_{eff}, will vary throughout an oscillation cycle. The hopper oscillates with a vertical trajectory given by: $y(t) = a\sin(\omega t)$ and the effective gravity the bed of material experiences if $\Gamma \leq 1$ is:

$$g_{eff} = g(1 - \Gamma\sin(\omega t)) \tag{2}$$

If the acceleration amplitude of the oscillations is greater than one ($\Gamma > 1$) the bed leaves the hopper walls during some portion of the oscillation cycle and re-impacts the walls at some later time. The effective gravity over the cycle is given by:

$g_{eff} = g(1-\Gamma\sin(\omega t))$	for $0 \leq t < t_0$	the bed is in contact with the hopper walls	
0	for $t_0 \leq t < t_1$	the bed is in flight	(3)
$d\dot{y}_{bed}/dt$	for $t_1 \leq t < t_1+\Delta t$	the bed impacts the hopper walls	
$g(1-\Gamma\sin(\omega t))$	for $t_1+\Delta t \leq t < T$	the bed is in contact with the hopper walls	

where t is time, t_0 is the time at which the granular material loses contact with the hopper walls, t_1 is the time at which the bed re-contacts the walls, $d\dot{y}_{bed}/dt$ is the acceleration of the bed during impact with the hopper walls, and Δt is the duration of the impact with the hopper walls. The mean acceleration of the bed during impact is given by

$$\overline{d\dot{y}_{bed}/dt} = (\dot{y}_{bed}(t_1+\Delta t) - \dot{y}_{bed}(t_1)) / \Delta t \tag{4}$$

where $\dot{y}_{bed}(t_1+\Delta t)$ is the vertical velocity of the bed after contacting the hopper walls, $\dot{y}_{bed}(t_1+\Delta t) = a\omega\cos(\omega(t_1+\Delta t))$, and $\dot{y}_{bed}(t_1)$ is the velocity of the bed just before impacting the walls of the hopper, $\dot{y}_{bed}(t_1) = -g(t-t_0) + a\omega\cos(\omega t_0)$. Note that this analysis assumes that the bed behaves as a totally inelastic material and does not rebound after impacting the hopper walls.

If the time of impact is assumed to be small such that, $t_1+\Delta t \cong t_1$, the mean discharge rate over an oscillation cycle, W, non-dimensionalized by the mean discharge rate for a non-vibrating hopper, W_0, for $\Gamma \leq 1$ is given by

$$\frac{W}{W_0} = \frac{1}{2\pi}\left(\int_0^{2\pi} \sqrt{1 - \Gamma\sin(\phi) d\phi}\right) \tag{5}$$

where $\phi = \omega t$. For $\Gamma > 1$ the ratio W/W_0 is given by:

$$\frac{W}{W_0} = \frac{1}{2\pi}\left(\int_0^{\phi_0} \sqrt{1 - \Gamma\sin\phi d\phi} + \sqrt{\Gamma\cos\phi_1 - \Gamma\cos\phi_0 + \phi_1 - \phi_0}\sqrt{\Delta\phi} + \int_{\phi_1}^{2\pi} \sqrt{1 - \Gamma\sin\phi d\phi}\right) \tag{6}$$

The phase angles at which the bed leaves the hopper walls, ϕ_0, and re-contacts the walls after free-falling, ϕ_1, are determined from the dynamics of a single, totally inelastic ball bouncing on a sinusoidally oscillating table and are functions of Γ (Wassgren *et al.*, 1996).

The equations originally derived by Suzuki *et al.* (1968) also included an empirically derived expression for the bulk density of the bed as a function of Γ. In the present analysis, however, ρ_b is assumed to be constant for simplicity. Furthermore, as in the analysis by Suzuki *et al.*, the impact duration, $\Delta\phi$, is assumed to remain constant for all vibration parameters. The resulting mean discharge rate ratio given in equations 5 and 6 depend only on the dimensionless acceleration amplitude, Γ. The experimental data for W/W_0 as a function of Γ for the frequencies examined are shown in figure 1. The solid line in the figure is equations 5 and 6 using a value of $\Delta\phi$ determined from the experimental data for Γ=2 and f=20 Hz. As is evident in the plot, the experimental data clearly does not collapse to a single curve when plotted against Γ as suggested by the model. Additionally, the model equations are not smooth at Γ=1 due to the contribution to the discharge rate of the bed impact with the hopper walls. Several assumptions in the model may account for these discrepancies. First, the model assumes that no particles discharge from the hopper while the bed is not in contact with the hopper walls. This assumption is investigated in more detail in the following section. Second, the dimensionless impact time, $\Delta\phi$, is assumed to be a constant and small compared to the flight time of the bed. At high frequencies, however, the impact time may comprise a significant portion of the oscillation period. Lastly, the model presented here does not include variations in the bulk density of the granular material as a function of the oscillation parameters. As is discussed in the next section, the bulk density of the material decreases when the bed is in flight implying that the discharge rate will also decrease (refer to equation 1).

The mechanisms causing the slight increase in the discharge rate at low velocity amplitudes are unclear. One possible mechanism that may account for this trend is that the bulk density of the bed may increase at low vibration velocities thus increasing the discharge rate from the hopper (refer to equation 1). Another mechanism that may be significant is the change in the effective viscosity of the granular material as a result of the oscillations. The vibrations cause particles to oscillate in their local neighborhoods - continuously forming and breaking contacts with surrounding particles. This results in a material with a lower effective viscosity. Indeed, the experimental measurements by Zik *et al.* (1992) show that the mobility of a particle in a vibrating bed is much greater when Γ>1 than for Γ<1.

<u>Discrete Element Computer Simulation</u>

A two dimensional discrete element simulation was also used to study the flow of granular material through a vertically oscillating hopper. The granular material in the simulation consists of circular particles with a uniform distribution of diameters. Each particle has two translational and one rotational degree of freedom. The contact model uses a damped linear spring in the normal direction and a linear spring in series with a frictional sliding element in the tangential direction (Cundall and Strack, 1979).

The first series of simulations were performed for a container with inclined walls and a closed exit. The simulation parameters for these simulations are given in table 1. Two convection cells similar to those observed in the experiments appear (figure 3). Particles move up along the inclined walls of the container and down at the centerline. The mechanism causing the observed motion is clearly observed in the simulation. When the particle bed leaves the hopper floor, few particle collisions occur with the hopper walls since the walls slope away from the bed and the bed remains closely packed (figure 4 - left). Thus, the walls do no work on the particle bed as the bed moves up relative to the hopper. While the bed is in flight it dilates since it is no longer constrained by the container walls. As a result of the dilation, when the bed falls back toward the hopper floor it contacts the inclined walls of the container prior to impacting the base (figure 4 - right). The work that the walls do on the particles decelerates the downward motion of particles near the wall and these particles are retarded in their downward movement compared to the remainder of the bed. Thus, the net motion of particles near the container walls over an oscillation cycle will be up along the inclined walls. This same mechanism also explains the downward motion of particles along vertical walls in vertically vibrated granular beds (Wassgren, 1997).

The flow of granular material from the simulated hopper was also examined. The parameters for these simulations are given in table 2. Note that particles are not "recycled" in the simulations. Once a particle is five hopper widths below the hopper exit plane it is neglected. The simulation uses a sufficient number of particles (N=10,000) to ensure that a steady flow is achieved.

Several quantities are recorded when a particle crosses the horizontal plane of the hopper exit including the time, the horizontal position of the particle, the particle's translational and rotational velocities, and the particle's radius and mass. The mean discharge rate from the hopper is measured by counting the total number of particles discharged from the hopper between a specified start and end time. The start and end times are chosen such that the initial and final transient periods are avoided.

The ratio of the mass discharge rate to the mass discharge rate for the non-vibrating hopper, W/W_0, is plotted in figure 2 as a function of the oscillation velocity amplitude, $a\omega/(gD_h)^{1/2}$, for various vibration parameters. The results from the simulation are similar to those observed in the experiments.

The discharge rate is also presented as a function of oscillation phase angle in figure 5 for a frequency of 20 Hz. Each data point consists of the average discharge rate over a small range of phase angles ($\Delta\phi=2\pi/10$) over 70 oscillation cycles. For a non-vibrating hopper the discharge rate remains nearly constant for all phase angles. As the acceleration amplitude increases, however, the fluctuations in the discharge rate ratio also increase. Note, however, that the discharge rate never equals zero as is assumed to occur in the model discussed in the previous section. Particles discharge continuously from the hopper for all oscillation phase angles.

Measurements of the horizontal position of particles and their velocities as they discharge from the hopper are also made with the simulations. The vertical velocity of particles relative to the hopper exit are shown in figure 6 for a hopper oscillating at 20 Hz with $\Gamma=2.0$. The data points are mass-averaged over 70 oscillation cycles. The discharge profile for a non-vibrating hopper is also included in the plot and is indicated by the thick solid line. Several observations are evident from the data. First, the vertical velocity relative to the hopper at discharge fluctuates in a manner consistent with the measurements in figure 5. Second, while the granular bed is in flight, the discharge profiles are more uniform than when the bed is in contact with the hopper walls. This is occurs because the walls shear the particle bed at the discharge plane giving a less uniform velocity profile.

Localized Vibration

The results of a preliminary investigation of the motion of granular material in a hopper subject to localized vibration is presented in this section. The motivation for this work is to model more closely the vibration system used in "live wall" hoppers where a vibration device such as an unbalanced motor or pneumatic vibrator is attached to one inclined wall of the hopper.

The experimental apparatus for this experiment is similar to that used in the vertically vibrating hopper experiment. A wedge-shaped hopper with smooth plexiglass walls, exit dimensions of 33 mm by 35 mm, and a wall angle inclined 45 degrees from the centerline of the hopper was constructed. The granular material used in the experiment consists of 3 mm diameter soda lime glass spheres. The large size of the particles reduces the effects of static electricity generated from rubbing contact with the plexiglass hopper walls.

In order to simulate the vibration occurring in a "live-wall" hopper, one of the hopper walls is constructed out of 0.32 cm (1/8") thick plexiglass and is pinned at the hopper exit and simply supported at the intersection with the bin portion of the hopper. One end of a metal rod is attached to the center of the flexible plexiglass wall and the other end is attached to a Labworks LW-140 electromagnetic shaker. The shaker oscillates the center of the flexible plexiglass wall with a prescribed frequency, f, and amplitude, a. An accelerometer mounted to the shaker monitors the acceleration and frequency of the oscillations.

Experiments have only been conducted for hoppers with closed exits. A single convection cell is observed adjacent to the oscillating wall. Particles move down along the wall and back up within the bulk of the granular material. The velocity of particles in the convection cells increases with increasing oscillation velocity amplitude. The oscillations also cause the hopper to discharge asymmetrically with particles discharging more readily adjacent to the oscillating wall. Quantitative measurements of the discharge rate as a function of the oscillation parameters are currently being conducted.

Conclusions

Granular flow from a vertically vibrating hopper was investigated. Particles within a closed hopper exhibit convection cells and surface waves. The convection cells are oriented such that particles move up along the inclined hopper walls and down at the hopper's centerline. Observations from a discrete element simulation indicate that the convective motion is a result of the dilation of the particle bed during free fall and interaction with the hopper walls. The ratio of the discharge rate from a vibrating hopper to the discharge rate from a non-vibrating hopper scales with the vibration velocity amplitude. This ratio has a maximum of approximately 1.05 for dimensionless oscillation velocity amplitudes less than 0.5 and is less than one for higher velocity amplitudes. The simple model originally proposed by Suzuki *et al.* (1968) to predict the discharge rate from vertically vibrating hoppers was examined. Using discrete element computer simulations, however, indicates that an important assumption in the model, that particles do not discharge from the hopper when the hopper acceleration is less than that of gravitational acceleration, is incorrect. The simulations indicate that although the discharge rate does fluctuate during an oscillation cycle, it is never zero. Additional measurements using the simulation indicate that the walls of the hopper influence discharge velocity profiles. Lastly, a preliminary investigation of particle trajectories in a hopper with one oscillating wall has shown that a single convection cell with particles moving down along the oscillating wall occurs. Additionally, granular material discharges more readily from the side of hopper with the oscillating wall. Future work will include modeling of the discharge velocity through discharging hoppers subject to vertical oscillations and quantitative measurements of the discharge rate from the hopper subject to localized oscillations.

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dimensionless acceleration amplitude, Γ	2.0
frequency, f	5 Hz
exit width / mean particle diameter	25.0
wall angle measured from the centerline	65^0
number of particles in the simulation	513
coefficient of restitution for particle/particle contacts	0.90
normal spring constant for particle/particle contacts	$5.90*10^3$ N/m
dashpot coefficient for particle/particle contacts	$4.17*10^{-2}$ N/(m/s)
tangential spring constant for particle/particle contacts	$5.90*10^3$ N/m
friction coefficient for particle/particle contacts	0.5
coefficient of restitution for particle/wall contacts	0.90
normal spring constant for particle/wall contacts	$1.18*10^4$ N/m
dashpot coefficient for particle/wall contacts	$8.33*10^{-3}$ N/(m/s)
tangential spring constant for particle/wall contacts	$1.18*10^4$ N/m
friction coefficient for particle/wall contacts	1.0
particle diameter distribution	0.8-1.2 mm
particle density	2500 kg/m^3
simulation time step	$3.31*10^{-6}$ sec

Table 1. The parameters used in the discrete element computer simulation for a vertically oscillating closed hopper.

exit width / mean particle diameter	11.0
wall angle measured from the centerline	45^0
number of particles in the simulation	10000
coefficient of restitution for particle/particle contacts	0.95
normal spring constant for particle/particle contacts	$1.68*10^5$ N/m
dashpot coefficient for particle/particle contacts	$5.63*10^{-2}$ N/(m/s)
tangential spring constant for particle/particle contacts	$1.68*10^5$ N/m
friction coefficient for particle/particle contacts	0.5
coefficient of restitution for particle/wall contacts	0.95
normal spring constant for particle/wall contacts	$3.36*10^5$ N/m
dashpot coefficient for particle/wall contacts	$1.13*10^{-1}$ N/(m/s)
tangential spring constant for particle/wall contacts	$3.36*10^5$ N/m
friction coefficient for particle/wall contacts	0.5
particle diameter distribution	2.8-3.2 mm
particle density	2500 kg/m^3
simulation time step	$2.35*10^{-6}$ sec

Table 2. The parameters used in the discrete element computer simulation for granular flow through a vertically oscillating hopper.

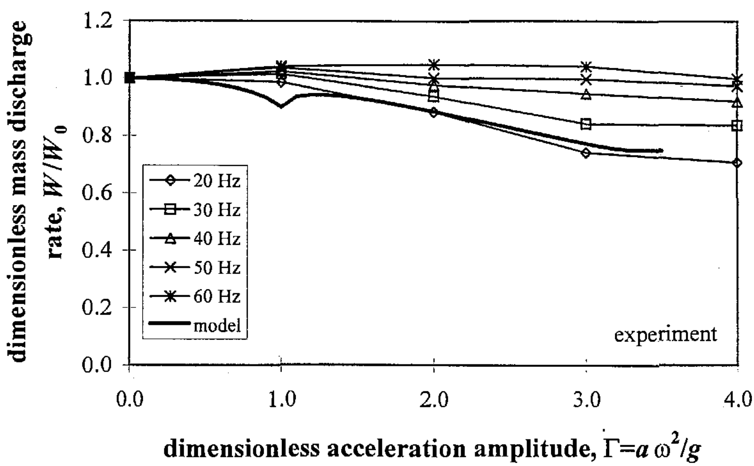

Figure 1. The ratio of the mass discharge rate from an oscillating hopper, W, divided by the mass discharge rate from a non-vibrating hopper, W_0, plotted as a function of the oscillation dimensionless acceleration amplitude, $\Gamma=a\omega^2/g$. The data points are from experiments and the solid line is the model described in the Analysis section of the text.

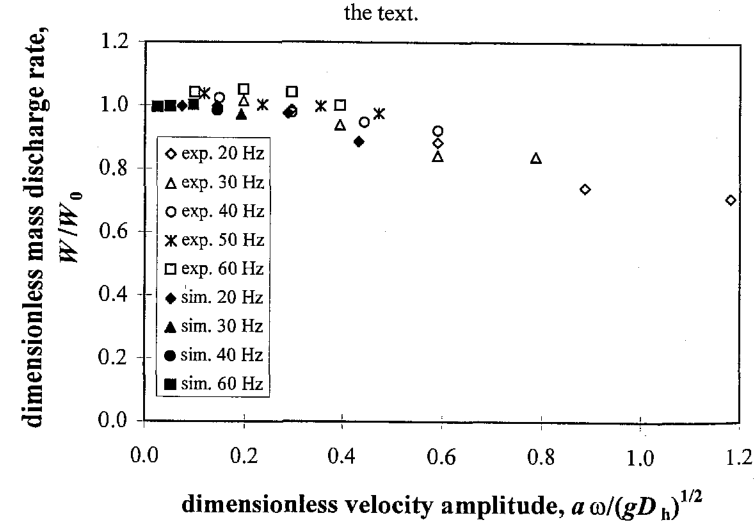

Figure 2. Mass discharge rate from an oscillating hopper divided by the mass discharge rate from a non-vibrating hopper, W/W_0, plotted as a function of the oscillation dimensionless velocity amplitude, $a\omega/(gD_h)^{1/2}$. The data points consist of experimental and simulation results.

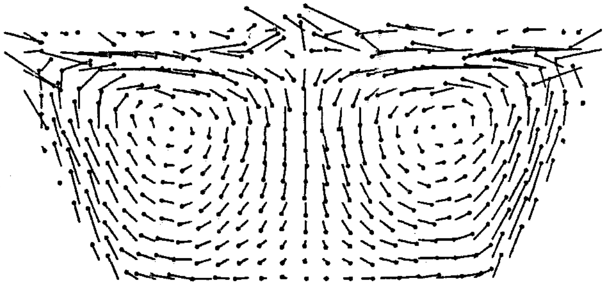

Figure 3. Side wall convection cells in a simulated vertically oscillating hopper with a closed exit. The vectors indicating the net displacement of particles per oscillation cycle are shown. The displacements are averaged over 20 oscillation cycles and the circles are the vector arrowheads. The parameters used in the simulation are given in table 1.

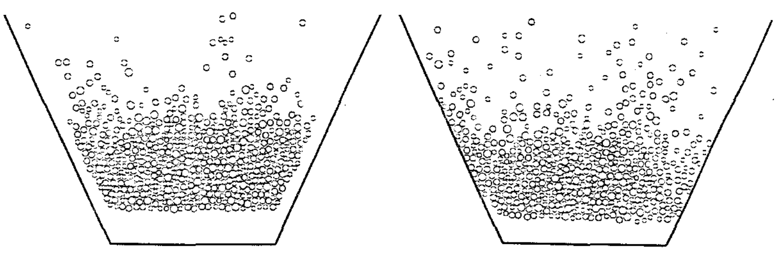

Figure 4. Snapshots from a simulation of a vertically oscillating hopper with a closed exit at a phase angle of $\phi/(2\pi) = 0.30$ (left) and $\phi/(2\pi) = 0.80$ (right). The simulation parameters are given in table 1.

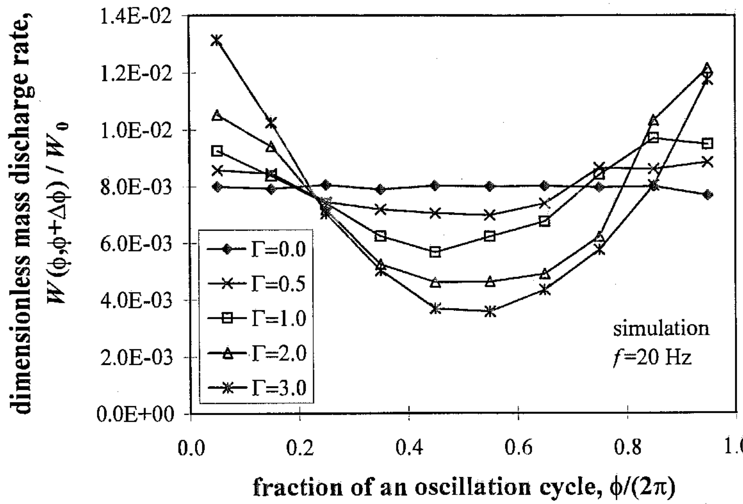

Figure 5. The ratio of the mass discharge rate for a hopper oscillating at a frequency of 20 Hz to the mass discharge rate from a non-vibrating hopper, W/W_0, plotted against fraction of an oscillation cycle, $\phi/(2\pi)$. The simulation parameters are given in table 2.

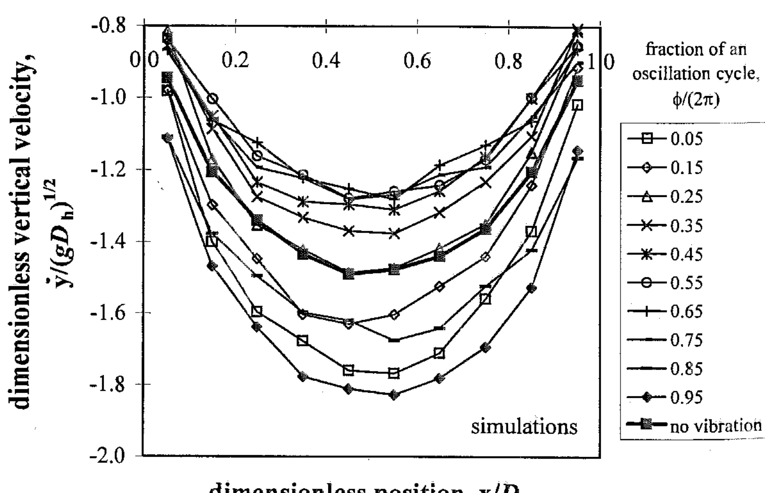

Figure 6. Vertical discharge velocity profile relative to the hopper exit for a hopper oscillating at a frequency of 20 Hz and Γ=2.0. The simulation parameters are given in table 2.