Temporal behavior of radiation-pressure-induced RF oscillation of an optical micro-cavity phonon mode.

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Abstract: We analyze experimentally and theoretically mechanical oscillation at RF frequency within an optical cavity stimulated by the pressure of circulating optical radiation. Oscillations and chaos were observed in output power.

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There are well-known examples where radiation pressure provides a dissipative response to motion [1,2]. Here, we present a different situation in which radiation pressure (of circulating photons inside a cavity) acts to stimulate regenerative oscillation at radio frequencies of an optical microcavity mechanical structure (i.e., phonon mode). We present here a simple device consisting only of an ultra-high-Q (UHQ) toroid cavity and a CW pump wave. Circulating radiation pressure induces a mechanical flex of the cavity structure; this motion, in return, takes the optical cavity out of resonance with the CW pump wave, thereby lowering radiation pressure. Upon restoration of the mechanical flex, the process resumes, leading to a periodic motion of the cavity as well as the circulating power. It should be emphasized that this oscillation is requiring no external temporal modulation of the pump wave. Recently, transmission oscillations were reported to occur exactly at the mechanical eigenfrequencies of the cavity structure [3]. Here, we explore and explain experimentally and theoretically the system's temporal behavior [4]. In particular, it is shown that the output intensity of the system evolves from sinusoidal to a train of decaying peaks and finally to chaos as input CW pump power increases. It is also shown here that when the size of a silica cavity is small, oscillations can occur well before other nonlinear effects. Namely, we demonstrate these regenerative vibrations to occur at pump powers lower than optical parametric oscillation Raman- and Erbium-lasing, (when the cavity was made to be small enough).

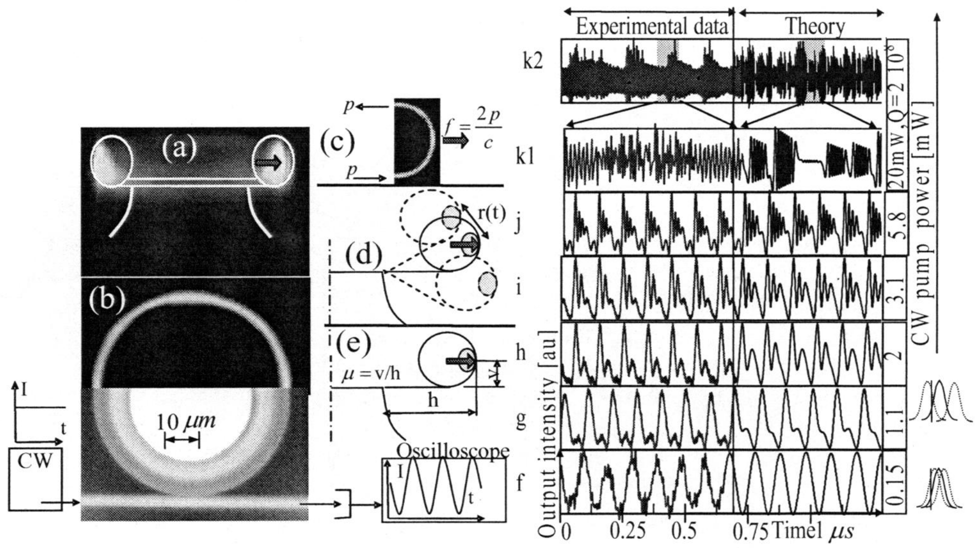

Figure 1: Experimental setup. Side view (a) and top view (b) of a toroidal cavity. A CW pump laser is coupled and builds a circulating intensity that applies pressure on the micro-cavity walls. In (c) circulating power induces a radial force which causes modulation of the cantilever (d) and the pump wave as illustrated in (b). Green luminescence here is the result of Erbium up conversion and is used here to illustrate the location of the optical mode. Note that the optical mode (green) is higher than the "cantilever beam" (a, e) holding the microtoroid. (f-k): Transmitted pump power for various values of CW input power. Left of each plate: experimental results. Right: calculation. Here the optical-mode radius R=29, and refraction index n=1.46. The ring's mass 5 10^{-11} kg, mechanical dissipation b=1.4 10^{-6} Kg/s and the oscillation frequency was measured to be 5.4 MHz. The optical quality factor was measured to be $Q_o = 5\,10^7$ [except in panel (k), where $Q_o = 2\,10^8$ and $\lambda_0 = 1550nm$]. k1 is also presented in k2 with a time scale 8 times longer.

As radiation increases quadratically with miniaturization, it should be considered in any resonant optical device of small dimension and high Q factor. The experimental system (Fig 1) consists of an UHQ toroid [5], fabricated from silica on a silicon wafer. Optical coupling of the CW pump laser to the microresonator is performed via a tapered fiber [6]. Oscillations in transmitted pump power appear above a threshold power level and evolve with increasing power from sinusoidal [Fig 1f, left] to a train of decaying ripples [Fig 1g-m] and finally to chaotic behavior [Fig 1k]. The oscillations are observed to continue as long as CW pump power is maintained. In modeling the ring cavity we note that each circulating photon changes it's propagation direction twice every revolution (Fig 1c). Therefore, a photon transfers 4 times it's linear momentum to the cavity's walls every time it completes a round trip. If the cavity is not infinitely rigid, the walls will deform in response to the resulting pressure (Fig 1d). In particular, the position of the cavity ring, *r*(*t*) (Fig 1d), will evolve according to the equation of motion for a mechanical oscillator:

$$m\ddot{r}(t)+b\dot{r}(t)+k\,r(t)=\mu f(t)=\mu 2\pi\,|A(t)|^2\,n/c \tag{1}$$

where m is the ring mass, b is the mechanical dissipation, k is a spring constant, f is the horizontal force applied by action of radiation pressure, and μ is the force efficiency defined in fig 1d that gives the conversion of horizontal to lateral force through the moment arm of the disk. Based on photo in Fig 1a, μ was estimate here to be 1/3. $A(t)$ is the slowly varying field amplitude normalized so that $|A(t)|^2$ is the circulating optical power, and c/n is the velocity of light in the cavity. The calculated radiation force on the RHS is the fundamental consequence of total momentum conservation (Fig 1c). As the optical resonance shifts with structure expansion, the wavelength difference (between the input beam and the moving-cavity resonance) changes as:

$$\Delta\omega=\Delta\omega_0-\omega\mu r(t)2\pi\;n/(\lambda_0 N) \tag{2}$$

where $\Delta\omega_0$ is the preset deviation between cavity resonance (when no light is in) and the input pump beam at frequency ω, $N=2\pi Rn/\lambda_0$,and μr is the deviation in the radius of the optical mode. The circulating optical power, $|A(t)|^2$, evolves according to the following dynamical equation [7]:

$$\dot{A}(t)+A(t)[\frac{\alpha\,c}{n}-i\,\Delta\omega]=i\,B\sqrt{\frac{\alpha\,c}{n\,\tau_0}} \tag{3}$$

where α gives the optical loss in the cavity ($\alpha=2\pi n/Q_o\lambda_0$), B is the input pump field and $\tau 0=nL/c$ is the circulation time for a photon traveling inside the cavity where $L=2\pi R$ is the circumference of the cavity. Applying the Adams' method on Eqs. 1-3 reveals the system dynamical behavior (A(t), r(t)). Oscillations typically evolve to their full scale (starting from mechanical rest) within 5 time-constants of the acoustical-cavity ($10m/b$). Having A(t), the calculated oscillating output power ($|(1-\tau_0\alpha c/2n)B+i\sqrt{\tau_0\alpha c/n}A(t)|^2$ as explained in [7]) is presented in Fig 2 (R.H.S.). The system response for low input pump power resembles a laser line modulated by a sinusoidally moving Lorentzian lineshape [8] (Fig 1g. sketch on R.H.S.). In particular, for small oscillation a nearly linear amplitude relation exists between mechanical motion and power modulation, because of the offset in pump frequency from the microcavity resonance. As the Lorentzian deviation becomes larger (Fig 1h. sketch on R.H.S.) the system response becomes nonlinear. As pumping further increases, output power transforms into a train of decaying peaks (Fig 1 h-m). This decaying peaks behavior is the result of interference between the stationary pump light and the light emitted from the cavity upon discharging. Output intensity, here, reflects the fact that light discharged from cavity is decaying exponentially with time and is frequency shifted due to cavity vibrations. We note that while the system motion is predictable for small enough pump levels (Fig1 g-j) it becomes sensitive to infinitesimal noise in the initial condition when a specific threshold in the pump power is exceeded (Fig 1k). In this aspect, this system resembles the chaotic behavior of the famous, damped, driven pendulum. Going to Fourier space, the calculated field spectrum (Fig 2a) versus pump power reveals a clear oscillation threshold at input power of about 0.15mW. As input power increases, the system response is large enough to induce a nonlinear amplitude modulation of the incident pump field. This manifests itself though the appearance of higher-order side-bands in the field spectrum.

The threshold for mechanical vibration scaled very differently with size than threshold for optical parametric oscillations Raman- and Erbium-lasing. While threshold for optical effects is scaled as l^2/Q_o^2 [9] (l here is size), the threshold for acoustical RF oscillations is scaled as l^4/Q_o^3 [3]. One reason for this difference is that optical effects depend by intensity (W/m^2) while radiation pressure is governed by the circulating energy (W). In contrary with optical effects, radiation pressure is not affected by the cross section area of the mode. The fact that RF oscillation threshold is quadratic with size while threshold for optical effects is square with size implies that when cavity size is reduced, RF oscillation threshold will go down much faster than threshold for optical effects.

Miniaturization hence will cause RF oscillation threshold to (cross and) be lower than threshold for other optical effects. For this reason we were motivated to experimentally demonstrate that when cavity is made to be small enough, RF oscillations are the first to occur; optical parametric oscillation as well as Raman- and Erbium-lasing are observed only after when input intensity is increased. In each of these cases, both signal and pump waves are observed (Fig 2 b-d) to be modulated due to the RF oscillations. As RF oscillation prevent CW signal of these 3 optical effects, radiation pressure acts here as the practical miniaturization limit for this type of cavity. Practical consequences of these oscillations as well as methods for quenching oscillations are currently under investigation.

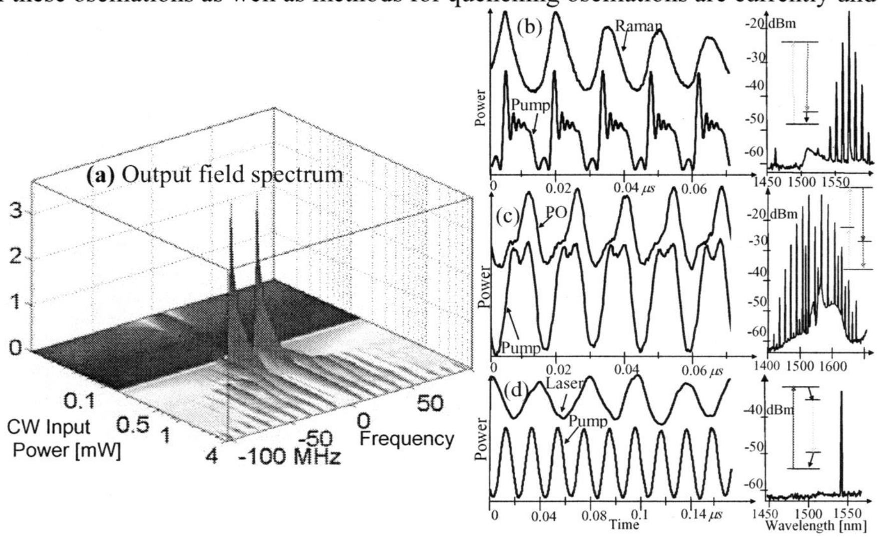

Figure 2: (a) Calculated Fourier transform of the emerging field as a function of the CW pump power. Frequency is plotted relative to frequency of the input pump beam. The field at the laser frequency was omitted here so as to not obscure the other peaks. Field is normalized to field at laser frequency. Parameters are as in Fig 1 f-m. (b-d) Cavity miniaturization lead mechanical oscillation to appear before (b) Raman lasing, (c) Parametric Oscillations and (d) Erbium lasing. The microcavity in (b-c) is the one from fig. 1 f-j. The micro cavity in (d) is similar but has a 39 outer diameter and is doped with Erbium. The optical spectrum of the emission for each distinct nonlinear effect is provided in the panels to the R.H.S. the visibility of the oscillating intensities is 0.37, 0.96, 0.19, 0.11, 0.12 and 0.29 (from above) the pump wavelength is 1462, 1555, and 1458 nm (from above) and the pump power (not shown in the spectra) is 4, 20 and 1 mW (from above)

To conclude, radiation pressure induced stimulation of regenerative (RF-frequency) mechanical oscillations of a silica micro-cavity have been demonstrated experimentally and analyzed theoretically. The vibrational mode and the optical mode act as (parametrically) coupled oscillators despite of their very different frequencies. The oscillations in output optical power grow from a continuous-wave input beam and evolve with increasing pump power from sinusoids; to a train of decays; and finally to chaos. Scaling arguments suggest that radiation pressure should be considered in other micro-resonant optical devices where high intensities are confined to a small volume. As miniaturization can improve key features of many optical devices and leads to significant economies of scale for manufacturing, there is much effort aimed at bringing devices to the micron- and nano-size levels. We hence expect that more and more optical devices will encounter radiation pressure related effects, whether as a limiting floor in miniaturization, or as a means to stimulate oscillations at RF frequencies.

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