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Analysis Of Aircraft Pitch Axis Stability Augmentation System Using Sum Of Squares Optimization

Kailash Krishnaswamy, George Papageorgiou, Sonja Glavaski, Antonis Papachristodoulou

I. INTRODUCTION

Design and stability analysis of an aircraft pitch axis control system has been a well studied problem, [1], [2]. Some researchers, [1], determined analytically the stability regions of a F-14 in terms of the engine thrust and pitch attitude and presented a global stability result for nonlinear modeled pitch axis of a F-14 modulated by a nonlinear dynamic inversion control law. Recently, a new computationally tractable nonlinear system analysis method was proposed, [3]. From a computation perspective, the method relaxes searching for positive semi definite stability certificate functions (eg. Lyapunov functions) to searching for sum of squares (of appropriate polynomials) certificate functions. Solving for the relaxed requirement leads to solving a SDP (Semi Definite Program) which is computationally tractable. This approach was used for Lyapunov function synthesis, [4] and estimating the stability region of SDRE (State Dependant Riccati Equation) systems, [5]. A recently developed software, SOSTOOLS (v1.01), [6] was used to convert the required sum of squares conditions to an appropriate SDP which was then solved using SeDuMi, [7]. In this paper we determine, numerically, the region of attraction of a trim point for the pitch axis of a nonlinear modeled aircraft modulated via a linear dynamic inversion based controller. The model incorporates uncertainty in the position of center of gravity along X-body axis. The stability regions are computed using SOSTOOLS.

II. ROBUST STABILITY OF NONLINEAR SYSTEMS

We consider the nonlinear dynamics of the pitch axis of a fictitious Aircraft - the Robust Civil Aircraft Model (RCAM), [8]. The state space is given by $[q,\ V,\ \alpha,\ \theta]^T$, which stand for the pitch rate (rad/s), velocity (m/s), angle of attack (rad) and pitch angle (rad), respectively. The control inputs are $[\delta_e,\ \delta_{TH}]^T$ which stand for the elevator deflection (rad) and throttle lever deflection (rad), respectively. The throttle input is help at a constant trim and is not dynamically varied. We designed a LTI dynamic inversion based control law for the short period dynamics of the AC pitch axis at a particular trim configuration which is given by a particular flight condition. The controller regulates the pitch angle error, $\widetilde{\theta} := \theta - \theta^{trim}$ to the origin.

Robust stability of the nonlinear A/C pitch axis model when controlled via LTI controller is the subject of this paper. Stability is analyzed in the presence of parametric uncertainty in the center of gravity (cg) of the A/C along the body x-axis. From a computation perspective three levels of A/C model complexity are considered in the paper.

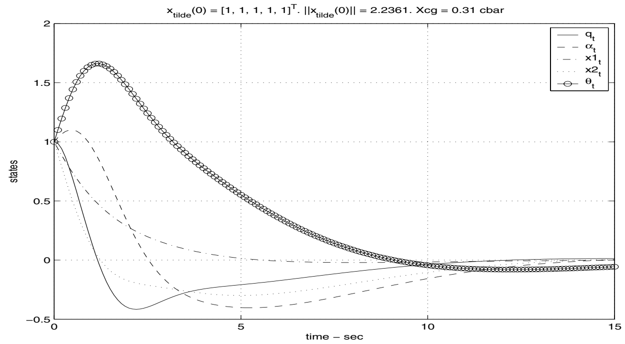

Fig. 1. Initial conditions initialized to indicate simultaneous perturbations in all states.

- Prog 1: We determine the stability region for the LTI controlled short period dynamics model of the A/C. This model does not incorporate any parametric uncertainty. The key issue here is to understand / validate the stability of a linear controller (designed for a linearized plant) operating on a nonlinear model of the plant.
- Prog 2: We determine the stability regions for the LTI controlled short period + phugoid dynamics model of the A/C. This model, again, does not incorporate parametric uncertainty and the analysis is geared towards validating the performance of the same linear controller as used in the previous case, but operating on a nonlinear longitudinal model of the A/C.
- Prog 3: We determine the stability regions for the LTI controlled short period dynamics model of the A/C in the presence of parametric uncertainty in the X-axis cg position of the A/C. The key issue here is to validate the robust stability properties of the controller.

In all 3 cases, the stability regions are computed using SOSTOOLS which solves a Sum-Of-Squares Optimization problem (equivalently a Semi-Definite Program). Once the stability regions are computed, simulations are performed to validate the computed stability regions, Fig. 1.

III. CONCLUSION AND FUTURE WORK

In all the three cases, it was shown that there existed initial conditions which did not lie within the stability regions that were stabilized by the controller. Such a result

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K. Krishnaswamy (`kailash.krishnaswamy@honeywell.com`), G. Papageorgiou and S. Glavaski are with Honeywell Labs, 3660 Technology Dr, Minneapolis, MN 55418.

A. Papachristodoulou is with the Department of Control and Dynamical Systems, California Institute of Technology, Pasadena, CA 91125.

poly	n (Prog 1)	n (Prog 3)	n (Prog 2)
V	15	30	21
ψ	5i	5i	6i
p_1	6	7	7
p_3	-	7	28
p_2	21	28	7
p_4	-	28	7
C_1	21	28	28
C_2	56	84	84
t_1 (s)	6.64	28.59	19.71
t_2 (s)	13.43	71.96	98.21

TABLE I

Computation times

is probably due to the fact that the SOS program we solve, verifies only a sufficient condition for stability. We showed that the closed loop system's ability to remain stable to perturbations depends on the direction of the perturbation in the state space. It was also found that the size of a stability region was also a function of the trim velocity of the Aircraft. Hence, an A/C flying at higher speed could sustain larger perturbations in the pitch axis states than an A/C flying at lower speeds.

The analysis presented in this paper does not assume actuator limits. We believe, that including tailplane actuator saturation limits is not only a more realistic problem but can also shrink the stability regions significantly. However, it may be challenging for SOSTOOLS (v1.01) to solve a problem with 1 or 2 additional states / higher degree vector fields. It has been known that the largest dimension of the Positive Semi Definite cone (Z-axis) which contains the candidate Positive Semi Definite matrix certifying the positive definiteness of a polynomial of degree m, with p variables grows exponentially with the arguments. The time-taken to solve the programs, Prog 1,2,3 in this paper are tabulated in Tab. I. In the table, t_1 is approximately the time it takes SOSTOOLS(v1.01) to formulate the problem, t_2 is the time is takes SeDuMi to solve the formulated SDP, n is the dimension of the PSD cone and OOM is Out Of Memory. $(V,\ ,\psi,\ ,p_i,\ ,C_j)$ are 'certificate' polynomials that are computed by SOSTOOLS. The problems were solved on a Pentium 4, 1GB RAM, 2GHz machine. In our experience, it was possible to reduce computation time and problem complexity (dimension of the psd cone) for certain problems, by choosing the right 'basis' polynomials which constitute the lyapunov function and the 'multiplier' polynomials. However, such a method would need the control engineer to exploit the explicit structure of the polynomial vector fields. In this paper, the focus was on using SOS programming to determine approximately the stability regions of a well-studied nonlinear modeled controlled aircraft pitch axis problem. Accurate estimates of the stability regions and practical interpretation of the size of destabilizing perturbations as output by SOSTOOLS will be significant complements to the present work.

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