Chaotic Motions of F-Ring Shepherds

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ABSTRACT

Recent HST images of the Saturnian satellites Prometheus and Pandora show that their longitudes deviate from predictions of ephemerides based on Voyager images. Currently Prometheus is lagging and Pandora leading these predictions by somewhat more than 20°. We show that these discrepancies are fully accounted for by gravitational interactions between the two satellites. These peak every 24.8 d at conjunctions and excite chaotic perturbations. The Lyapunov exponent for the Prometheus-Pandora system is of order $0.35\,\mathrm{yr}^{-1}$ for satellite masses based on a nominal density of $1.3\,\mathrm{g\,cm}^{-3}$. Interactions are strongest when the orbits come closest together. This happens at intervals of 6.2 yr when their apses are anti-aligned. In this context we note the sudden changes of opposite signs in the mean motions of Prometheus and Pandora at the end of 2000 occured shortly after their apsidal lines were anti-aligned.

Key Words: Satellites of Saturn, Orbits, Chaos

1 INTRODUCTION

Orbits for Pandora and Prometheus in the form of precessing ellipses of fixed shape were fit to Voyager data by Synnott *et al.* (1981, 1983) and Jacobson (personal communication). Mean motions were determined from images and precession rates were calculated to be consistent with the gravity field of the Saturnian system (Nicholson and Porco 1988; Campbell and Anderson 1989).

Observations with HST made during the 1995-1996 Sun and Earth ring plane crossings led to the discovery that Prometheus was lagging its predicted longitude based on the Voyager ephemeris by about 20° (Bosh and Rivkin 1996, Nicholson *et al.* 1996). Subsequently McGhee (2000) found that Pandora was leading the Voyager ephemeris prediction by a similar amount. These discrepancies have been confirmed by French *et al.* (1999, 2000, 2001, 2002), Murray *et al.* (2000), McGhee *et al.* (2001), and Evans (2001).

These and other researchers looked for a dynamical origin of the longitude discrepancies. Several hypotheses were investigated including: perturbations exerted by an undetected coorbital satellite of Prometheus (see French *et al.* 1998); interactions with clumps in the F ring, or 1 to 5 km objects in the F ring, or the F ring itself (Showalter *et al.* 1999a, 1999b); long-term resonance dynamics (Dones *et al.* 1999), and chaos (Dones *et al.* 2001). However, none of these attempts provide a clear resolution of this puzzle. That is the goal of our paper.

We focus on direct interactions between Pandora and Prometheus because their longitude discrepancies have comparable magnitudes and opposite signs (French *et al.* 2002). This suggests that the satellites are exchanging angular momentum and energy and that their orbits are chaotic. Results from orbit integrations presented in §2 of the paper confirm this suspicion. Implications of these findings for estimates of the age of Saturn's rings are discussed in §3. This section also reproduces evidence from French *et al.* (2002) that supports our finding that sudden changes in mean motion tend to occur around times when the satellites' apses are anti-aligned.

2 Confirmation of Chaos

2.1 Calculational Method

Working in a planet centered coordinate system and adopting conventional notation, the vector equation of motion for each satellite reads

$$\begin{aligned} \frac{d^2\mathbf{r}_i}{dt^2} = & - \frac{GM\mathbf{r}_i}{r_i^3}\left[\left(1+\frac{m_i}{M}\right)\frac{3}{2}J_2\left(\frac{R}{r_i}\right)^2 - \frac{15}{8}J_4\left(\frac{R}{r_i}\right)^4 + \frac{35}{16}J_6\left(\frac{R}{r_i}\right)^6\right] \\ & - Gm_j\left(\frac{\mathbf{r}_i-\mathbf{r}_j}{|\mathbf{r}_i-\mathbf{r}_j|^3}+\frac{\mathbf{r}_j}{r_j^3}\right), \end{aligned} \tag{1}$$

where i and j ($i \neq j$) assume values 1 and 2. We integrate only four first order scalar differential equations for each body since to observational accuracy the orbits of Prometheus and Pandora lie in Saturn's equatorial plane.

Equations (2) admit energy and angular momentum integrals given by

$$
\begin{aligned}
E &= \frac{1}{2}\left[m_1\left|\dot{\mathbf{r}}_1\right|^2 + m_2\left|\dot{\mathbf{r}}_2\right|^2 - \frac{\left|m_1\dot{\mathbf{r}}_1 + m_2\dot{\mathbf{r}}_2\right|^2}{M + m_1 + m_2}\right] - \frac{Gm_1m_2}{|\mathbf{r}_1 - \mathbf{r}_2|} \\
&- \frac{GMm_1}{r_1}\left[1 - J_2\left(\frac{R}{r_1}\right)^2 - J_4\left(\frac{R}{r_1}\right)^4 - J_6\left(\frac{R}{r_1}\right)^6\right] \\
&- \frac{GMm_2}{r_2}\left[1 - J_2\left(\frac{R}{r_2}\right)^2 - J_4\left(\frac{R}{r_2}\right)^4 - J_6\left(\frac{R}{r_2}\right)^6\right],
\end{aligned} \tag{2}
$$

and

$$
H = m_1\left(\mathbf{r}_1 \times \dot{\mathbf{r}}_1\right) + m_2\left(\mathbf{r}_2 \times \dot{\mathbf{r}}_2\right) - \frac{(m_1\mathbf{r}_1 + m_2\mathbf{r}_2) \times (m_1\dot{\mathbf{r}}_1 + m_2\dot{\mathbf{r}}_2)}{M + m_1 + m_2}. \tag{3}
$$

Numerical integrations of the equations of motion are carried out using the algorithm of Bulirsch and Stoer (1980) which offers the luxury of a variable time step. Fractional changes in total energy and angular momentum are of order 10^{-10} for integrations of 10^3 yr. For comparison, jumps in these quantities are of order 10^{-5} at each conjunction.

Initial conditions are computed from Jacobson's equinoctial elements (Jacobson, personal communication) and the transformation between cylindrical elements and epicyclic elements derived in Borderies-Rappaport and Longaretti (1994). The same transformation is applied to compute the epicyclic eccentricity and the epicyclic mean longitude at each output step. We also output values of the angular momentum and energy (neglecting the interaction term) for each satellite, and the total angular momentum and energy (including the interaction term).

To compute Lyapunov exponents we integrate the orbits of two shadow bodies whose initial conditions differ slightly from those of Prometheus and Pandora. We reset the state vector of the shadow bodies to reduce the magnitude of their phase space separation from the physical bodies whenever it exceeds a preset tolerance. In practice rescaling is done when the longitudinal separations that dominate the phase space separations are slightly less than 10^{-4} radians.

2.2 Results

Variations over 20 years of orbital longitudes for Prometheus and Pandora are displayed in Figs. 1 and 2. Initial values for the satellites' epicyclic eccentricities and mean motions obtained following the prescription described in §2.1 are presented in Table 1; this table also contains the ratios of the satellites' masses to Saturn's mass. The simulation begins with Prometheus at periapse and Pandora at apoapse and with the satellites' apsidal lines in phase. To emphasize the chaotic irregularities of the mean motions, we subtract a drift rate based on the initial mean motion from the longitude of each satellite. These figures reproduce the characteristics of the puzzling longitude discrepancies reported in the papers referenced in §1. Line widths are due to epicyclic longitude oscillations which have full amplitudes of $4e$ radians.

Table I. Initial values of Prometheus and Pandora eccentricities and mean motion, and masses scaled to Saturn's mass.

Satellite	e	n (rd/s)	m_i/M
Prometheus	2.29×10^{-3}	1.1864×10^{-4}	1.19×10^{-9}
Pandora	4.37×10^{-3}	1.1571×10^{-4}	7.65×10^{-10}

Figure 3 displays the difference between the apsidal angles over 20 years, and Fig. 4 shows the behavior of the magnitude of the relative eccentricity vector. The two orbits come closest together and the magnitude of the relative eccentricity peaks when the apsidal lines are anti-aligned. This occurs at about $t = 3.1\,\mathrm{yr}$, $9.3\,\mathrm{yr}$, and $15.5\,\mathrm{yr}$. Comparison with Figs. 1 and 2 reveals that these mark the times at which abrupt changes in the satellites' mean motions take place. Note that net changes in mean motion around $t = 3.1\,\mathrm{yr}$ and $15.5\,\mathrm{yr}$ years are of comparable magnitude but opposite sign, but that the net change in mean motion that takes place around $t = 9.3\,\mathrm{yr}$ is much smaller. This is another indication of chaos.

Additional evidence for chaos is found in plots of epicyclic eccentricity vs. time shown in Figs. 5 and 6. Two distinct types of eccentricity variation are apparent. Small jumps occur at conjunctions separated by about 24.8 days. These have magnitudes of order $\mu\,(a/\Delta r)^2 \sim 10^{-5}$, where μ is the mass of the perturbing satellite divided by the mass of Saturn, a is the mean orbit radius, and Δr is the radial distance between the satellites at conjunction. As expected, the largest jumps occur when the satellites' apses are near anti-alignment. Quasi-periodic variations of eccentricity are associated with the relative apsidal precession period of 6.2 years. They arise from secular perturbations which promote the exchange of angular momentum but not of energy. Since secular eccentricity variations are entirely due to angular momentum exchanges, Prometheus's and Pandora's are 180 degrees out of phase. Although the secular variations are somewhat larger than the jumps, they are small in comparison to the mean eccentricity. Their small size is a consequence of the dominance of Saturn's oblate gravitational equipotentials in forcing the differential precession; secular terms in the satellites' interaction potential contribute only a small fraction of the differential precession rate. Eccentricity jumps and secular eccentricity variations are not the entire story, nor even the most important part of it. That distinction goes to the lack of periodicity over the differential precession cycle which is a clear signature of chaos.

To prove that the mean motion variations arise from chaos, we compute the Lyapunov exponent for the Prometheus-Pandora system. Figure 7 illustrates its behavior over an interval of 3000 years. The figure also includes a dashed line showing a constant plus $(\log t)/t$ fit to the final point of the solid curve. This is the behavior that would be expected in the absence of chaos. Evidence for chaos is overwhelming. The Lyapunov exponent is of order $0.35\,\mathrm{yr}^{-1}$.

Figures 8-11 derived from data spaced by $0.1\,\mathrm{d}$ show perturbations near conjunctions in greater detail. Variations of epicyclic eccentricity are depicted in Figs. 8 and 9 while Figs. 10 and 11 provide data on energy and angular momentum accrued during the numerical integration. Each panel covers an interval of one year centered either on $t = 3.1\,\mathrm{yr}$, when the apses are anti-aligned, or on $t = 6.1\,\mathrm{yr}$, when they are aligned. Perturbations are noticeably larger during the former than during the latter. Fractional jumps of the energy and angular momentum of each satellite at conjunctions are $\sim \mu\,(a/\Delta r)^2$ and $\sim \mu\Delta e\,(a/\Delta r)^3$, respectively.[1] Each of these is of order 10^{-5}. That our estimates are reasonable can be seen by noting that the energy and angular momenta of Prometheus and Pandora are $\sim 10^{33}\,\mathrm{g\,cm^2\,s^{-2}}$

[1] Δe is the magnitude of the relative eccentricity vector.

and $\sim 10^{37}\,\mathrm{g\,cm^2\,s^{-1}}$, and that their jumps are $\sim 10^{27}\,\mathrm{g\,cm^2\,s^{-2}}$ and $\sim 10^{31}\,\mathrm{g\,cm^2\,s^{-1}}$. Spikes seen in the plots of energy and angular momentum are arise from the strong interactions near conjunctions. Their widths of a few hours are marginally resolved.

3 Discussion

The suggestion that interactions between Saturn's F-ring shepherds make their motions chaotic is not new. It was raised long ago in an article we wrote with Scott Tremaine (Borderies *et al.* 1984). For us its confirmation is almost like a dream come true. Below we quote from this earlier work because a full 20 years after it was written we could scarcely improve upon it.[2] We have, however, added a footnote to introduce modern notation.

"As external satellites extract angular momentum from the rings, their orbits expand. Calculations based on the formula for the linear satellite torque predict remarkably short time scales for the recession of close satellites from the rings (Goldreich and Tremaine 1982)." "Thus these short time scales remain perhaps the most intriguing puzzle in planetary ring dynamics." "The severe nature of the problem is well-illustrated by the system composed of the F-Ring and its two shepherd satellites, 1980S26 and 1980S27, hereafter called S26 and S27."[3] "The outward movement of the system could be reduced if S26 were involved in a resonance with a more massive outer satellite." "No resonance has been found linking S26 with one or more outer satellites. What might this imply? Are the F ring and its shepherd

[2] Our article was prepared for a conference held in the summer of 1982.

[3] 1980S26 and 1980S27 were renamed Pandora and Prometheus by the International Astronomical Union.

satellites very young? Does the long sought resonance exist awaiting detection? A most interesting possibility is that S26 is transferring angular momentum to Mimas even though the two bodies are not in an exact orbital resonance. This could be accomplished if the motion of S26 were chaotic, i.e., if the value of its mean motion were undergoing a slow random walk. We have proven that if the mean longitude of S26 were subject to a significant random drift, in addition to its dominant secular increase, angular momentum transfer to Mimas would take place by virtue of the near resonance between S26 and Mimas. By significant drift, we mean of order one radian on the circulation time scale of the critical argument associated with the near resonance. To check this hypothesis, we must first determine whether the orbital motion of S26 is chaotic. To do so we need to investigate the perturbations of its orbit produced by S27. Solution of this and the other outstanding theoretical problems will await results of ongoing research."

Proving that the F-ring shepherds move chaotically as the result of their mutual interactions is an important step. What it implies about the lifetime of Saturn's rings remains to be determined.

We would be remiss if we failed to mention that chaotic motions of Prometheus and Pandora are discussed in Poulet and Sicardy (2001), Dones *et al.* (2001), and French *et al.* (2002). Poulet and Sicardy (2001) investigate the long term evolution of the system and find intervals of chaos. Dones *et al.* (2001) suggest that chaos might account for unexplained motions of the satellites but do not identify the specific mechanism responsible for creating it. French *et al* (2002) raise the possibility that changes of opposite sign in the mean motions of

Prometheus and Pandora may signal the exchange of energy between their orbits. However, they do not simulate the effects of interactions between the shepherds. Instead they present evidence that external satellites can excite chaotic motions of test particles in a portion of a region of 2×10^3 km width covering the semimajor axes of both shepherds.

We close this paper by displaying evidence in support of our finding that abrupt changes in mean motions tend to occur around times during which the satellites' apses are anti-aligned. Figure 12 reproduces, with embelishments, panels from Figs. 3 and 5 of French *et al.* (2000). It shows that Prometheus and Pandora underwent oppositely directly changes of their mean motions around the end of year 2000, shortly after the time at which their apsidal longitudes differed by 180°.

4 Acknowledgments

We thank R. French for providing us with a copy his preprint which contains the latest results on the motions of Prometheus and Pandora, and for allowing us to reproduce from it the material in our Fig. 12. We are grateful to R. Jacobson for advice on initializing our integrations in a manner compatible with the Voyager ephemerides. Research by PG was supported by NSF grant AST-0098301 and that by NR by NASA Planetary Geology and Geophysics grant 344-30-53-02.

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6 Figure Captions

FIGURE 1: Prometheus longitude from numerical integration as a function of time. A drift rate based on the initial mean motion is subtracted from the longitude. Units are degrees and years.

FIGURE 2: Pandora longitude from numerical integration as a function of time. A drift rate based on the initial mean motion is subtracted from the longitude. Units are degrees and years.

FIGURE 3: Difference between the epicyclic apsidal longitudes (in degrees) of Prometheus and Pandora over 20 years.

FIGURE 4: Magnitude of the Prometheus and Pandora relative eccentricity vector. Note that the peaks correspond to anti-aligned apses.

FIGURE 5: Prometheus epicyclic eccentricity as a function of time. The epicyclic frequency is computed from the state in rectangular coordinates following Borderies-Rappaport and Longaretti (1994).

FIGURE 6: Pandora epicyclic eccentricity as a function of time. The epicyclic frequency was computed from the state in rectangular coordinates following Borderies-Rappaport and Longaretti (1994).

FIGURE 7: Lyapunov exponent for the Prometheus-Pandora system over a period of $3 \times 10^3\,\mathrm{yr}$ (solid line). The dashed line depicts a constant $+ (\log t)/t$ fit to the final point of the solid curve. The unit for the Lyapunov exponent is yr^{-1}.

FIGURE 8: Prometheus and Pandora epicyclic eccentricities during an interval of one year centered on $t = 3.1\,\mathrm{yr}$ when the apses are anti-aligned.

FIGURE 9: Prometheus and Pandora epicyclic eccentricities during an interval of one year centered on $t = 6.2\,\mathrm{yr}$ when the apses are aligned.

FIGURE 10: Prometheus (solid lines) and Pandora (dotted lines) variations in energy (in $\mathrm{g\,cm^2\,s^{-2}}$) and angular momentum (in $\mathrm{g\,cm^2\,s^{-1}}$) during an interval of one year centered on $t = 3.1\,\mathrm{yr}$ when the apses are anti-aligned. The dot-dashed lines display differences between current and initial total energies and angular momenta.

FIGURE 11: Prometheus (solid lines) and Pandora (dotted lines) variations in energy (in $\mathrm{g\,cm^2\,s^{-2}}$) and angular momentum (in $\mathrm{g\,cm^2\,s^{-1}}$) during an interval of one year centered on $t = 6.2\,\mathrm{yr}$ when the apses are aligned. The dot-dashed lines display differences between current and initial total energies and angular momenta.

FIGURE 12: Evidence for sudden jumps in the mean motions of Prometheus and Pandora at the end of year 2000. Reproduced, with permission, from French *et al.* (2002). The solid vertical lines mark the time at which the satellites' apses were anti-aligned.

FIGURE 1
Prometheus longitude from numerical integration including Pandora minus cirular motion longitude (

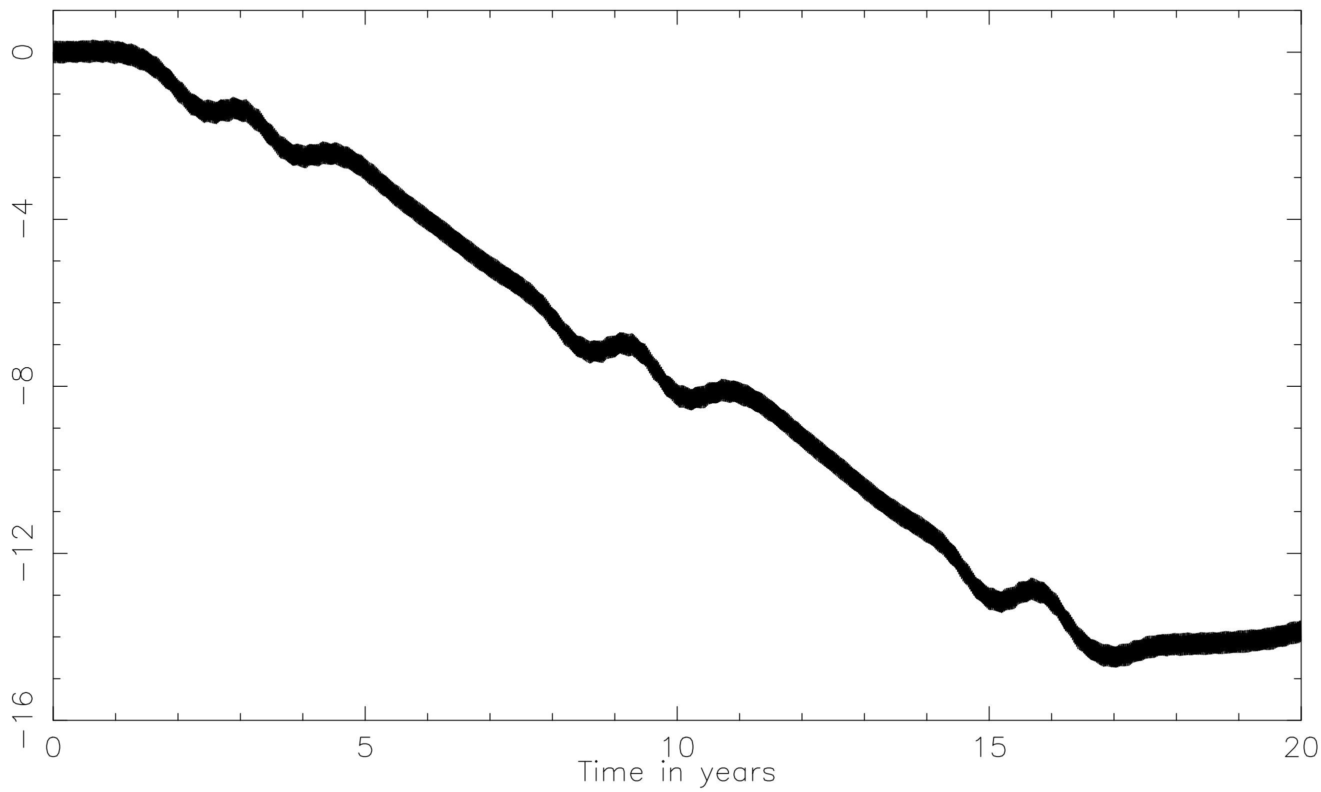


FIGURE 2
Pandora longitude from numerical integration including Prometheus minus circular motion longitude

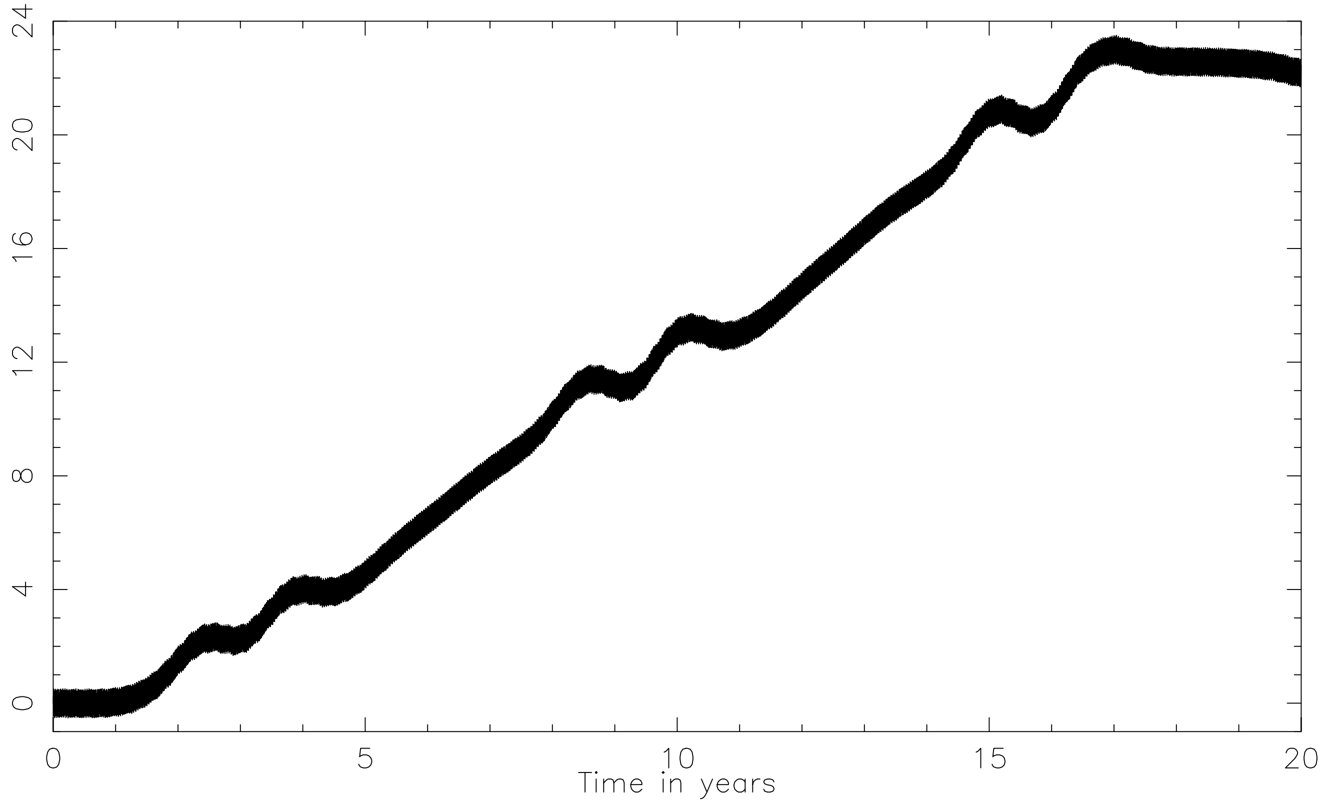


FIGURE 3
Longitude of Prometheus periapsis minus longitude of Pandora periapsis (degrees)

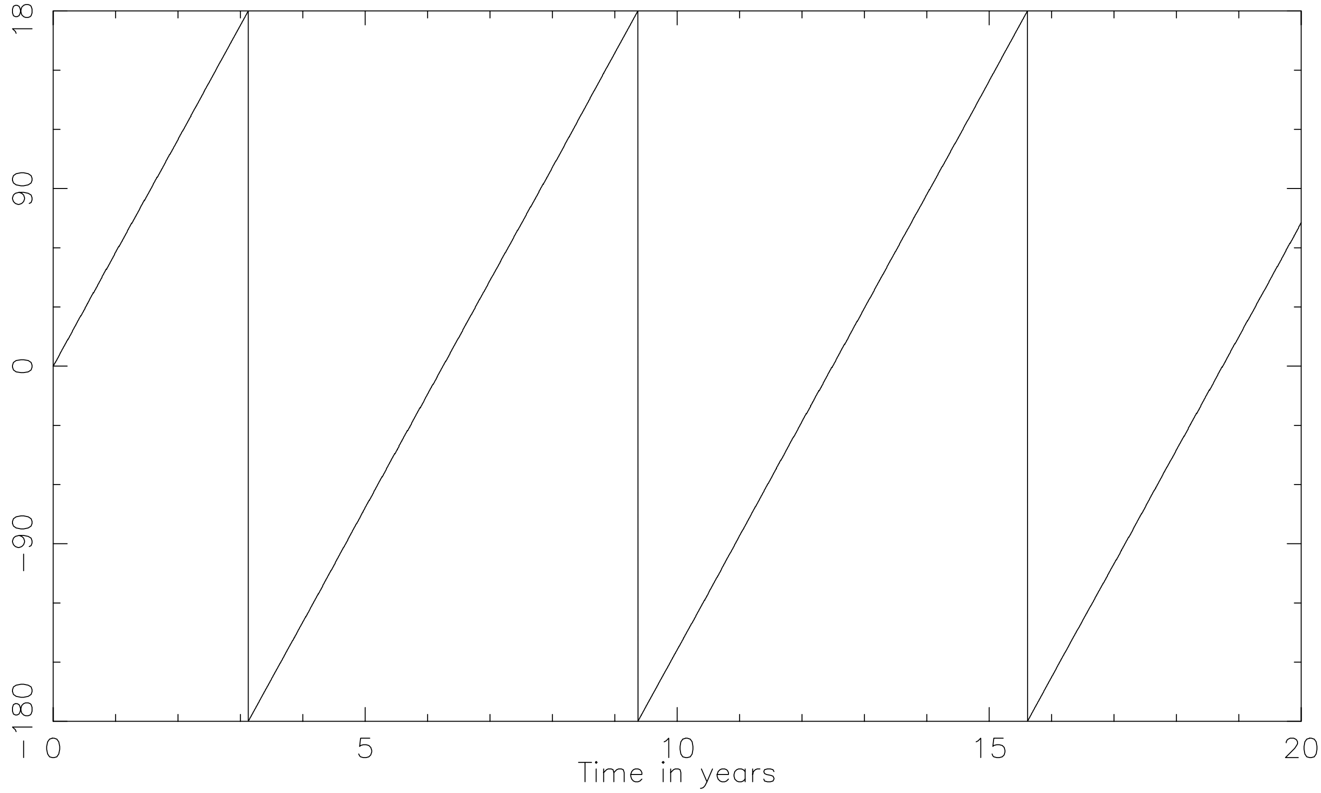


FIGURE 4
Magnitude of the relative eccentricity vector

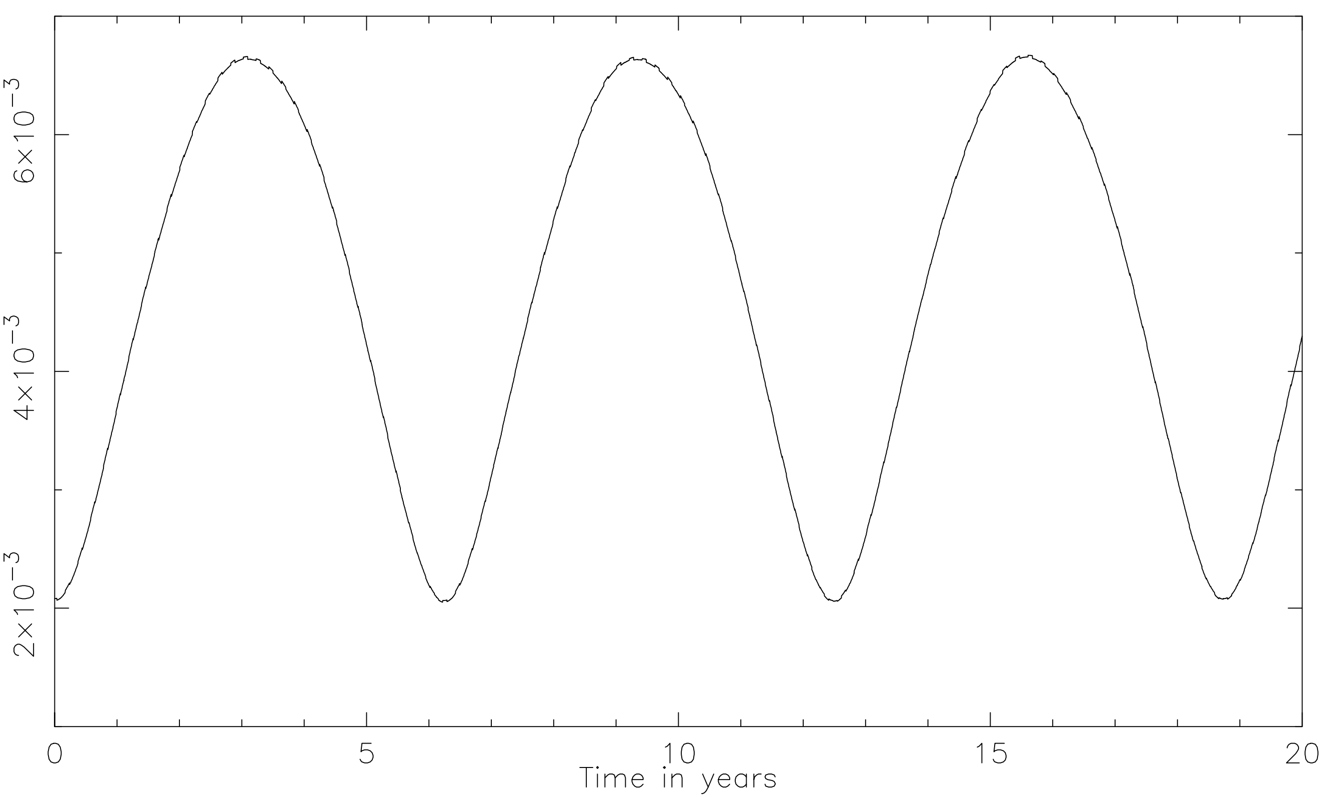


FIGURE 5
Prometheus epicyclic eccentricity

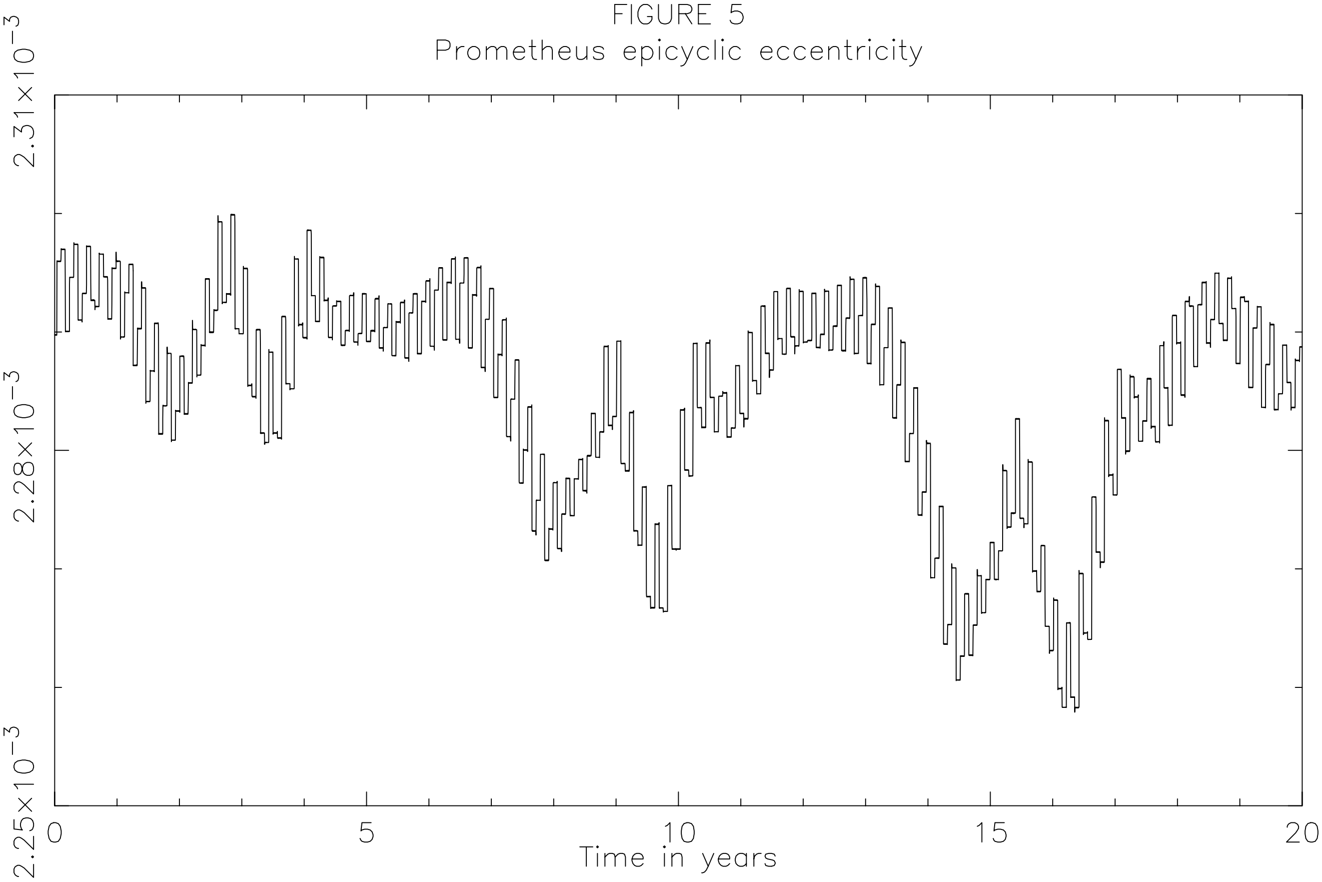


FIGURE 6
Pandora epicyclic eccentricity

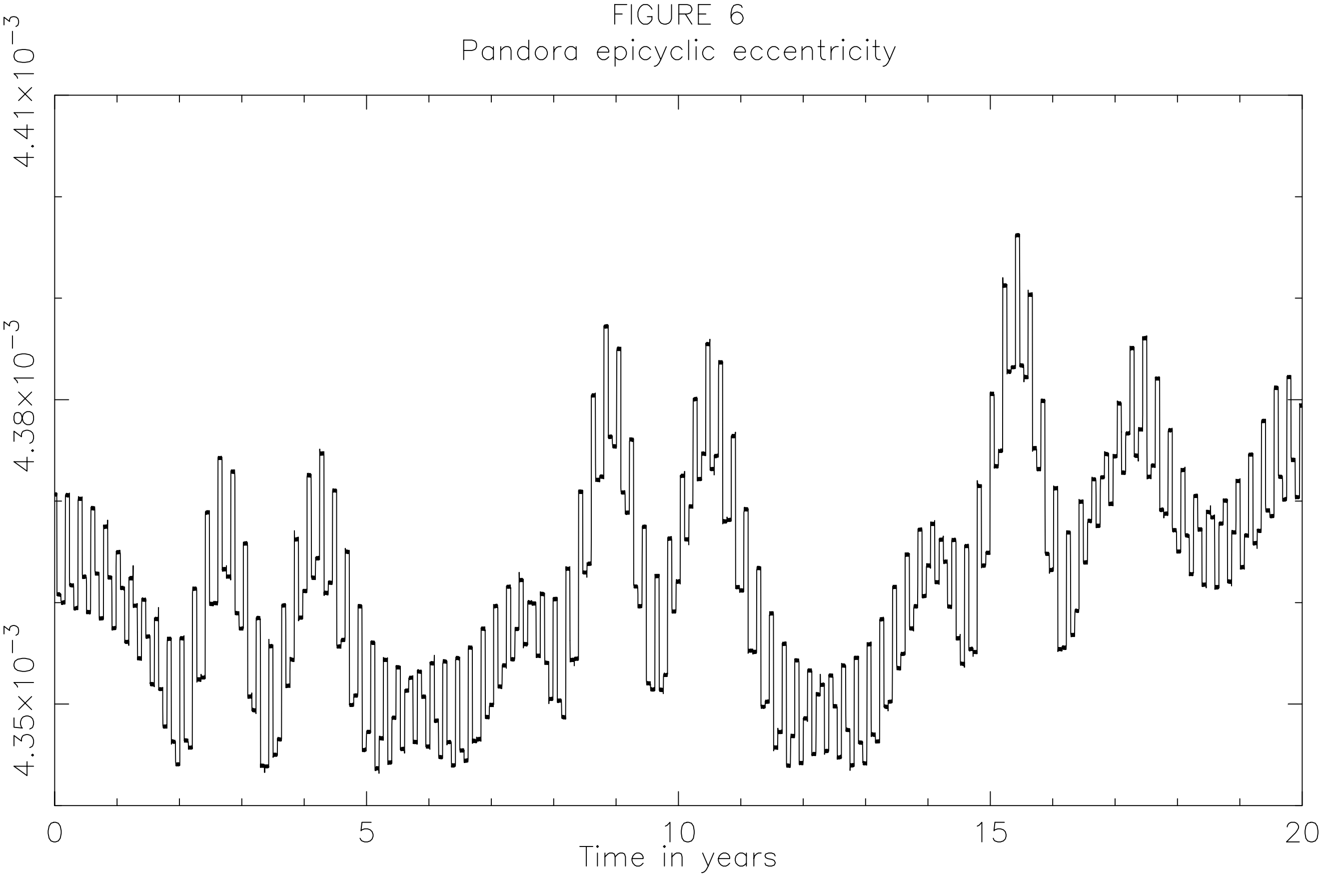


FIGURE 7
Lyapunov exponent of the Prometheus–Pandora system in 1/year

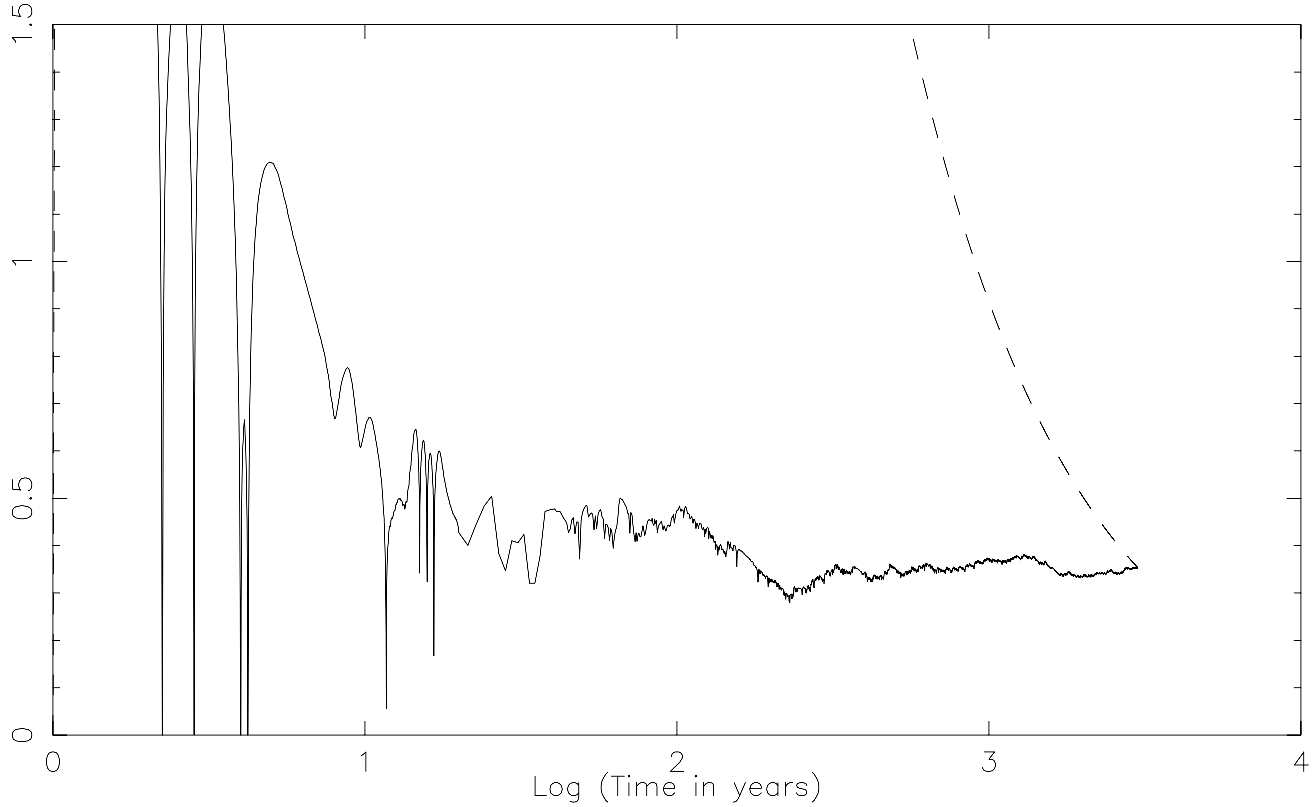
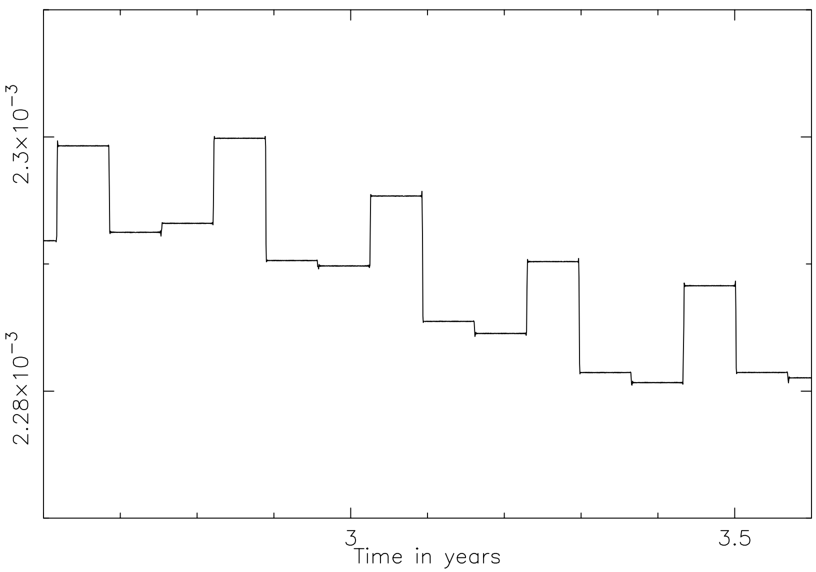


FIGURE 8

Prometheus epicyclic eccentricity



Pandora epicyclic eccentricity

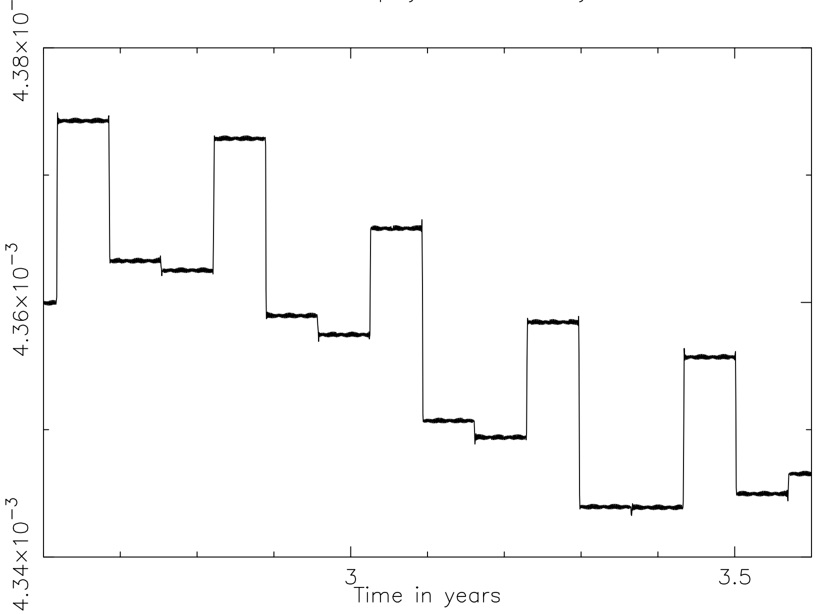
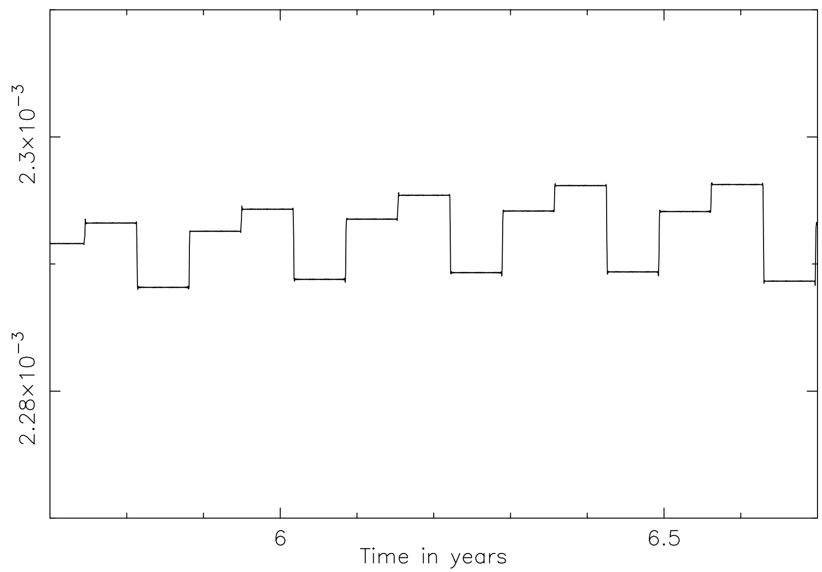


FIGURE 9

Prometheus epicyclic eccentricity



Pandora epicyclic eccentricity

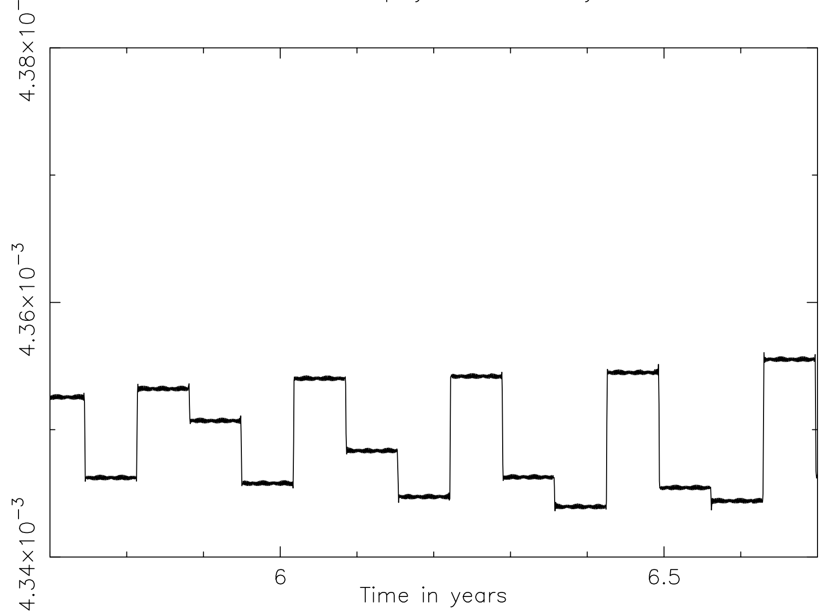


FIGURE 10
ENERGY VARIATIONS: Prometheus (solid line), Pandora (dotted line), Total (dash-dot line)

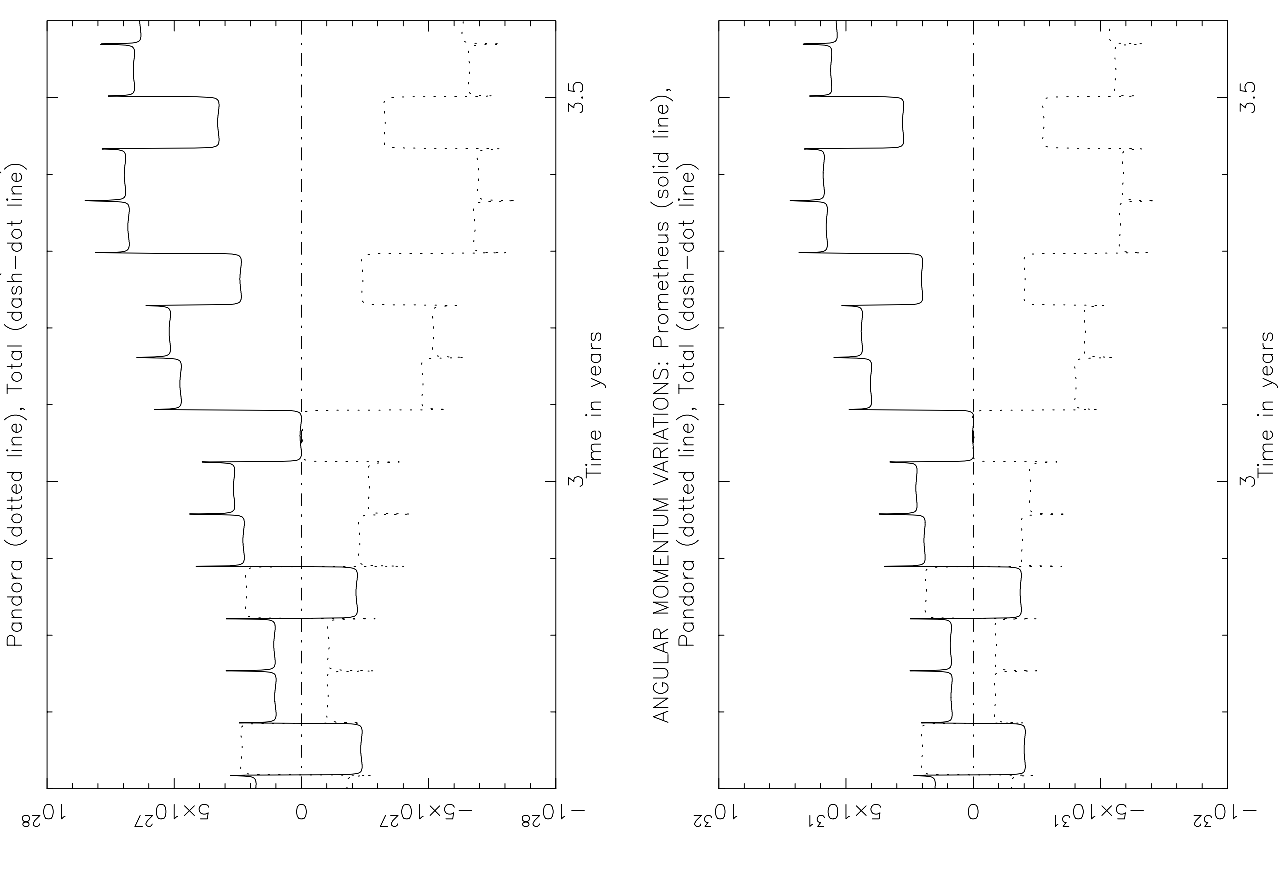


FIGURE 11
ENERGY VARIATIONS: Prometheus (solid line), Pandora (dotted line), Total (dash-dot line)

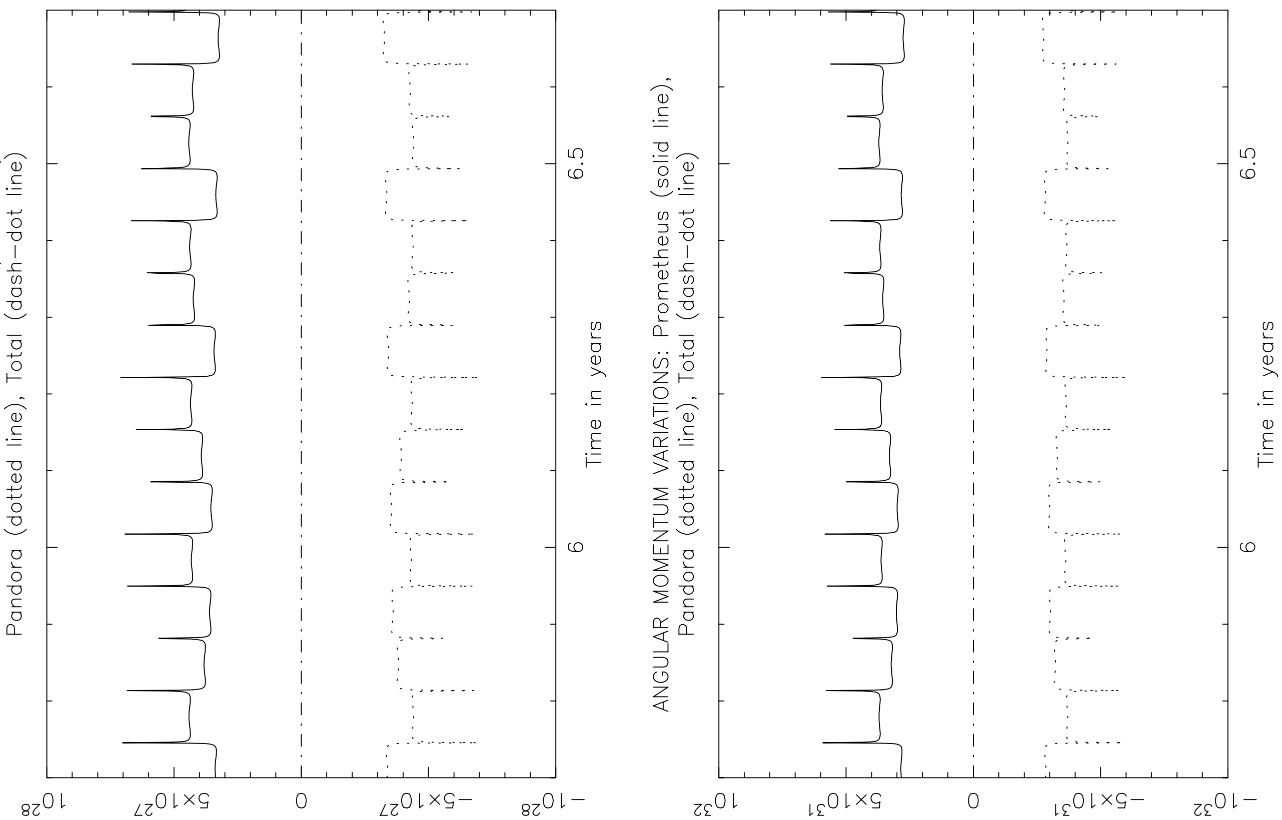
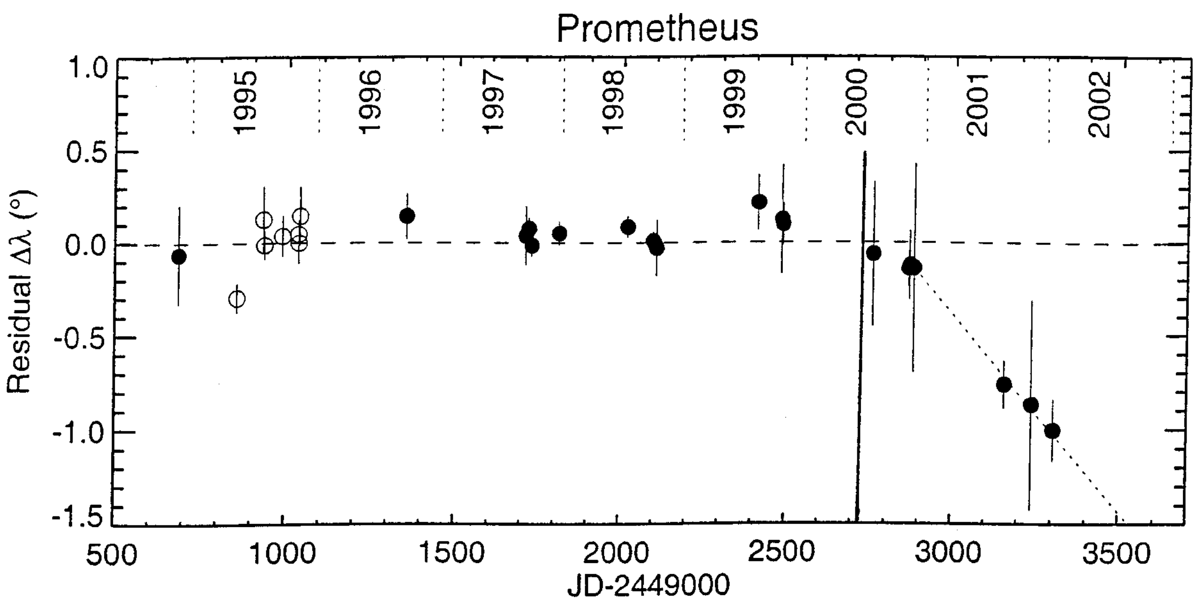
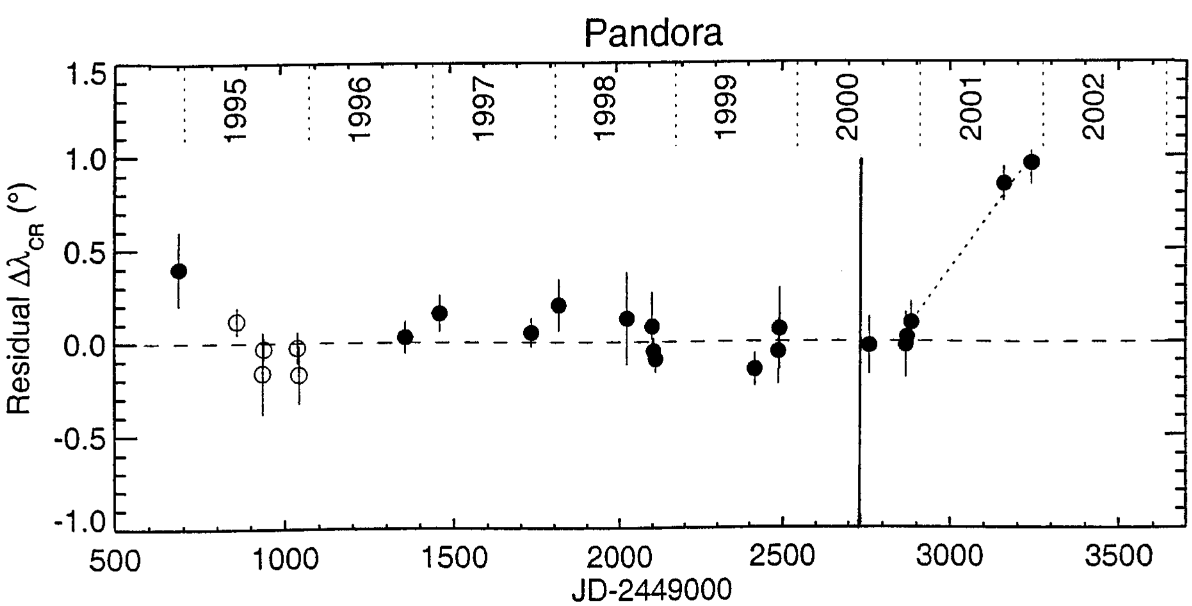


FIGURE 12


Prometheus
Residual Δλ (°)
JD-2449000
1995
1996
1997
1998
1999
2000
2001
2002


Pandora
Residual Δλ_CR (°)
JD-2449000
1995
1996
1997
1998
1999
2000
2001
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