EURODYN 2005, C. Soize & G.I. Schuëller (eds)
© 2005 Millpress, Rotterdam, ISBN 90 5966 033 1

Analytical reliability calculation of linear dynamical systems in higher dimensions

A.A.Taflanidis & J.L. Beck
Division of Engineering and Applied Science, California Institute of Technology, USA

ABSTRACT: The recent application of reliability analysis to controller synthesis has created the need for a computationally efficient method for the estimation of the first excursion probabilities for linear dynamical systems in higher dimensions. Simulation methods cannot provide an adequate solution to this specific application, which involves numerical optimization of the system reliability with respect to the controller parameters, because the total computational time needed is still prohibitive. Instead, an analytical approach is presented in this paper. The problem reduces to the calculation of the conditional upcrossing rate at each surface of the failure boundary. The correlation between upcrossings of the failure surface for the different failure events may be addressed by the introduction of a multi-dimensional integral. An efficient algorithm is adopted for the numerical calculation of this integral. Also, the problem of approximation of the conditional upcrossing rate is discussed. For the latter there is no known theoretical solution. Three of the semi-empirical corrections that have been proposed previously for scalar processes are compared and it is shown that the correction should be based on the bandwidth characteristics of the system. Finally, examples that verify the validity of the analytical approximations for systems in higher dimensions are discussed.

1 INTRODUCTION

Calculation of system reliability is one of the most difficult problems in stochastic analysis of dynamical systems. This problem is defined as the determination of the probability of failure; i.e. the probability that within some given time duration, *any* of the output states of a system out-crosses the boundary of a safe region.

Our interest in the problem stems from the recent application of reliability-based design in controller synthesis for linear structures under stochastic excitation (May and Beck 1998; Yuen and Beck 2003; Scruggs et al. 2005). In this synthesis method, the objective is the minimization of the probability of failure, based on the stationary response of some selected performance variables. This requires an optimization over the considered set of admissible controllers, a task which involves a large number of iterations. The state-space dimension of the system and the number of response variables selected to describe the performance are typically high. Another important feature of the synthesis problem is that it becomes computationally difficult if the performance objective function does not have a smooth relationship with respect to the controller parameters.

These features create a need for a computationally-efficient method for relatively accurate estimation of the failure probability for stationary vector processes. Here "relative accuracy" means that between iterations the estimation error is consistent, so that a smooth relationship exists between the controller parameters and the corresponding performance objective function. Even though an extremely efficient stochastic simulation method exists for linear dynamical systems (Au and Beck 2001), the computational time associated with simulation algorithms is prohibitive for the required optimization process. This is because very high accuracy in the failure probability is needed in order to get a smooth objective function. Analytical calculation is therefore the only feasible solution.

Problems of higher dimensions have not been sufficiently addressed as far as analytical approximations are concerned. Even though first-excursion problems for scalar processes have received a lot of attention, (Rice 1944, 1945; Vanmarcke 1975; Winterstein and Cornell 1985), the vector process counterpart has not been addressed sufficiently. Theoretical arguments have been made (Belyaev 1968), but results have been presented only for the cases of independent processes and very small dimensions (Veneziano et al. 1977; Soong and Grigoriu 1993), since the hardware speed required for the calculations was not available at that time. Recent advances in computational speed have created the necessary conditions for this problem to be effectively addressed, but it seems that the needs of the reliability problems that have arisen in recent practice have not yet created an incentive for researchers to revisit this topic. In this paper, we address the problem of computationally efficient reliability calculations for linear dynamical systems under stationary stochastic excitation.

2 PROBABILITY OF FAILURE

2.1 *First-passage failure probability*

Let $\mathbf{x}(t) \in \mathbb{R}^n, \forall t \in \mathbb{R}$, be a stochastic vector process. Consider a convex safe region $D_s \subset \mathbb{R}^n$ that is bounded by a series of hyperplanes B_i, $i=1,\dots,k$, defined by a set of linear limit-state functions. Let S_D be the boundary of D_s. Assuming stationarity of the response, the probability of failure defined as a first passage probability, is equal to

$$P_F(D_s,t) = P\left[\mathbf{x}(t) \notin D_s \ for\ some\ \tau \in [0,t]\right] \\ = 1-\exp\left(-n_{\mathbf{x}}^+(S_D)\cdot t\right) \tag{1}$$

where the hazard function $n_{\mathbf{x}}^+(S_D)$ corresponds to the mean upcrossing rate conditioned on no previous upcrossing having occurred. This conditional upcrossing rate is difficult to evaluate. Typically, each upcrossing event is considered independent of any previous upcrossings. This corresponds to using the unconditional, or mean, up-crossing rate $v_{\mathbf{x}}^+(S_D)$. In Eq. (1), we have assumed that $\mathbf{x}(0) \in D_S$.

2.2 *Unconditional upcrossing rate for stationary vector processes*

The mean upcrossing rate for a stationary vector process may be calculated by considering upcrossings perpendicular to the boundary as discussed by Belyaev (1968)

$$v_{\mathbf{x}}^+(S_D) = \lim_{\Delta t\to 0} \frac{1}{\Delta t} \int_{S_D-\dot{x}_n\Delta t}^{S_D} \int_0^\infty p(\mathbf{x},\dot{x}_n) d\dot{x}_n d\mathbf{x} \tag{2}$$

where the scalar $\dot{x}_n = \mathbf{n}^{\mathrm{T}}\dot{\mathbf{x}}$ is equal to the velocity component perpendicular to the boundary and $\mathbf{n}$ is the unit outward normal vector at the boundary. Fig.1 shows an example in a two-dimensional space with linear limit state functions.

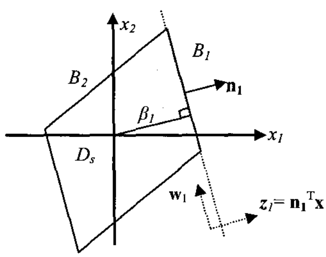

Figure 1. Two-dimensional example of failure surfaces.

Let β_i be the distance of hyperplane B_i from the origin and let Δ_i be the hyperpolyhedral corresponding to the (n-1)-dimensional intersection of S_D and that hyperplane. The scalar process perpendicular to B_i is $z_i = \mathbf{n}_i^T\mathbf{x}$. The component of the vector process $\mathbf{x}$ that is orthogonal to $\mathbf{n}_i$ is denoted by $\mathbf{w}_i$, so that $\mathbf{w}_i = \mathbf{x} - z_i\,\mathbf{n}_i$. At each hyperplane, the unit normal has the same direction everywhere which, along with the assumption of stationarity, makes z_i and $\dot{x}_{n_i}$ independent. Using this fact and that the state $\mathbf{x} = z_i\,\mathbf{n}_i + \mathbf{w}_i$, the unconditional upcrossing rate may be calculated as a sum of the upcrossing rates on each boundary (Veneziano et al. 1977):

$$v_{\mathbf{x}}^+(S_D) = \sum_{i=1}^{k} v_{z_i}^+(\beta_i) P\left[\mathbf{w}_i \in \Delta_i \mid z_i = \beta_i\right] \tag{3}$$

where $v_{z_i}^+(\beta_i)$ is equivalent to Rice's mean outcrossing rate (Rice 1944, 1945) for the scalar process z_i perpendicular to the boundary

$$v_{z_i}{}^+(\beta_i) = p_{z_i(t)}(\beta_i)\cdot E\left[\mathbf{n}_i{}^{\mathrm{T}}\dot{\mathbf{x}}(t)\right]^+ \tag{4}$$

and $P\left[\mathbf{w}_i \in \Delta_i \mid z_i = \beta_i\right]$, the correlation weighting factor, is an (n-1)-dimensional integral corresponding to the probability that $\mathbf{w}_i$, the component of the vector process $\mathbf{x}$ orthogonal to $\mathbf{n}_i$, lies in Δ_i, conditioned on the fact that an upcrossing of the $z_i=\beta_i$ boundary occurs. This term takes into account the correlation between failure events for different boundaries and is equal to

$$P\left[\mathbf{w}_i \in \Delta_i \mid z_i = \beta_i\right] = \int_{\Delta_i} p(\mathbf{w}_i \mid z_i = \beta_i) d\mathbf{w}_i \tag{5}$$

In many applications, the failure region is defined by limit-state functions which are symmetric with respect to the origin. Assuming a zero-mean process, the upcrossing rate for the double barrier $v_{z_i}^+(|\beta_i|)$ equals twice the upcrossing rate described by Eq.(4).

2.3 *Correction factor for conditional upcrossing rate*

The use of the unconditional outcrossing rate given in Eq. (4), instead of the conditional one in Eq. (1), introduces a significant error in two cases that are discussed by Lutes and Sarkani (1997): a) for smaller values of the thresholds β_i, and b) for narrowband systems. For smaller threshold values, the time for the first upcrossing is small compared to the time between upcrossings. For narrowband systems, the approximation of independent upcrossing times is unreasonable since an upcrossing of a specific level, especially if the corresponding threshold is not

© 2005 Millpress, Rotterdam, ISBN 90 5966 033 1

too large, is most probably going to be accompanied by a second upcrossing, one period later.

To address these problems, a correction factor, γ_i, has been introduced for the unconditional upcrossing rate of the scalar variable z_i

$$n_{z_i}^{+}(\beta_i) \approx v_{z_i}^{+}(\beta_i)\cdot\gamma_i \tag{6}$$

For the rest of this section, we drop the index notation for z_i. We will also assume that z is a Gaussian scalar process with standard deviations σ_z and $\sigma_{\dot{z}}$ for the process and its derivative respectively.

Using stochastic averaging and the Cramer and Leadbetter definition of the amplitude of a process, Vanmarcke (1975) has proposed the following correction factor

$$\gamma = \frac{1-\exp\left\{-q^{0.6}\left(\frac{2}{\sqrt{\pi}}\right)^{0.1}\frac{2\sqrt{2}}{n_b}\frac{\beta}{\sigma_z}\right\}}{1-\exp\left(-\beta^2/\left(2\sigma_z^{\,2}\right)\right)},\ q=\frac{\pi}{4}\left(1-\alpha^2\right) \tag{7}$$

where q is called the bandwidth parameter; for a SDOF oscillator under white noise, it is roughly proportional to the damping coefficient. For a single barrier, n_b is equal to 1 and for the symmetric double barrier, it is 2. If S_{zz} is the spectral density of z, the parameter α is given by

$$\alpha = I_{cv}/\left(\sigma_z\sigma_{\dot{z}}\right),\ I_{cv}=\int_{-\infty}^{\infty}|\omega|S_{zz}d\omega \tag{8}$$

This approximation assumes that only a small band of frequencies contributes significantly to the response. The effect of this assumption is a correction factor sensitive to high frequency dynamics.

Using the energy-fluctuation scale, Winterstein and Cornell (1985) have suggested another correction term and argued that it is less sensitive to higher order dynamics. This factor is defined as

$$\gamma=\frac{1-2\Phi_c\left(\frac{\beta}{\sigma_z}\sqrt{\frac{1-\rho}{1+\rho}}\right)}{1-exp\left(-\beta^2/\left(2\sigma_z^{\,2}\right)\right)},\Phi_c(h)=\frac{\int_h^{\infty}exp\left(-\frac{u^2}{2\sigma_z^{\,2}}\right)du}{\sigma_z\sqrt{2\pi}} \tag{9}$$

where the parameter ρ and the energy fluctuation scale θ_E are

$$\rho=exp\left(-\frac{2\pi\cdot\sigma_z}{n_b\sigma_{\dot{z}}\theta_E}\right),\ \theta_E=\frac{4\pi\cdot I_{ce}}{\sigma_z^{\,4}},I_{ce}=\int_{-\infty}^{\infty}S_{zz}^{\,2}(\omega)d\omega \tag{10}$$

A detailed discussion about other possible correction terms may be found in the report by Taflanidis and Beck (2005). In particular, it is shown that the first integral and derivative envelope corrections terms, based on stochastic averaging but with different definitions of the amplitude, have no advantages when compared to the above two corrections. In this report, another semi-empirical correction, called the equivalent energy-fluctuation approach, is suggested. This term adopts Vanmarcke's form but considers a definition of bandwidth derived through the above energy-fluctuation approach. The correction term is given by Eq. (7) with a new definition for q (which is still roughly equal to the damping coefficient for a SDOF system under white noise excitation); i.e.

$$q=\sigma_z^{\,5}/\left(4\pi\cdot I_{ce}\cdot I_{cv}\right) \tag{11}$$

3 CALCULATION FOR LINEAR SYSTEMS UNDER GAUSSIAN EXCITATION

3.1 *Problem description*

Consider a linear structural system subjected to earthquake excitation which is modeled as filtered white noise. The system's dynamic description may be augmented to model the stochastic ground acceleration input, and in the case of feedback control, the dynamics of the control law, sensors and actuators. The input vector for the system is comprised of the white noise inputs for both the ground acceleration and the sensor models.

The safe region is considered to be a hypercube and is defined by the inequality $|\mathbf{x}|\leq\boldsymbol{\beta}$, where $\mathbf{x}$ corresponds to the performance variables vector. At each hyperplane B_i of the boundary, the perpendicular z_i $\mathbf{n}_i$ corresponds to one element of the performance variables vector and the component $\mathbf{w}_i$ orthogonal to $\mathbf{n}_i$ gives the rest.

3.2 *Calculation of correlation weighting factor*

The calculation of the multi-dimensional integral in Eq. (5) corresponding to the correlation weighting factor for each hyperplane B_i, is computationally challenging. Since we require estimation with a consistent error, numerical integration is desirable but in high dimensions the time required for numerical integration is known to increase exponentially with the dimension of the integral.

For jointly Gaussian processes and hypercubic regions, Genz (1992) has introduced a series of transformations that reduce significantly the computational effort needed for the Monte Carlo integration. A subregion adaptive algorithm proposed by Genz (1992) is chosen in the current study for the calculation of the correlation weighting factor.

Note that we are mainly interested in higher probabilities because it is expected that if the correlation weighting factor is small on some hyperplane B_i, then the corresponding failure rate will also be small which implies that the dominant failure rates will be weighted by large values of the multivariate integral. This feature enables us to obtain good absolute accuracy without too much effort.

3.3 *Calculation of correction factor for upcrossing rate*

The calculation of the correction factor requires a parameter that cannot be obtained analytically. This parameter corresponds to the integrals I_{cv} and I_{ce} for Vanmarcke's and Winterstein's approaches, respectively, or to both for the equivalent energy-fluctuation approach.

For the calculation of these integrals, the spectral density $S_{z_i z_i}$ is substituted by the equivalent expression $\mathbf{H}_{z_i}(\omega)\mathbf{S}_o\mathbf{H}_{z_i}(\omega)^*$ where $\mathbf{H}_{z_i}(\omega)$ is the transfer function vector for the multiple inputs corresponding to the performance variable z_i, and $\mathbf{S}_o$ is the spectral density matrix for the multiple inputs. The frequency range over which the dynamics of the augmented structural system are important is partitioned with a desired step size, then the frequency response is obtained with the use of MATLAB's Control System Toolbox. The one-dimensional integral is then numerically calculated.

3.4 *Summary of calculations*

The performance variables have Gaussian distributions with statistics calculated based on the state space representation of the system. The probability of failure can be calculated by Eq. (1), where the conditional upcrossing rate is approximated by

$$n_{\mathbf{x}}^{+}(S_D) \approx \sum_{i=1}^{k} \gamma_i v_{z_i}^{+}(\beta_i) P[\mathbf{w}_i \in \Delta_i \mid z_i = \beta_i] \tag{12}$$

The calculation is performed separately at each boundary hyperplane B_i. In Eq. (12), the correction in Eq. (6) has been used to approximate the conditional upcrossing rate at B_i, which consists of the mean unconditional upcrossing rate multiplied by a correction factor γ_i. The latter depends on the approach adopted and involves the numerical calculation of an integral, as presented in Section 3.3. For a Gaussian process, the mean upcrossing rate is

$$v_{z_i}^{+}(\beta_i) = \sigma_{\dot{z}_i} \cdot \exp\left\{-\beta_i^2 / \left(2\sigma_{z_i}^{\;2}\right)\right\} / \left(2\pi\sigma_{z_i}\right) \tag{13}$$

The conditional upcrossing rate at each hyperplane B_i is weighted by $P[\mathbf{w}_i \in \Delta_i \mid z_i = \beta_i]$, which is a multidimensional integral that takes into account the correlation in the upcrossings between the various performance variables.

4 EXAMPLES

4.1 *Example descriptions and results*

Three examples are chosen to investigate the analytical methodology with the objective of addressing issues regarding the correlation of the performance variables and the bandwidth of the system. The analytical results are compared with those obtained using the highly efficient stochastic simulation algorithm ISEE (Au and Beck 2001).

Four different cases are considered for the analytical approximation of the probability of failure. The difference between cases is in a) the calculation of the conditional failure rate, and b) the consideration of the correlation weighing factor. Vanmarcke's correction, usually considered to be the most accurate for scalar stochastic processes, is presented for the cases with (denoted VC) and without (VU) the correlation weighting factor. Also, three cases using Winterstein's approach (WC), the equivalent energy-fluctuation approach (EEFC) and no upcrossing correction factor (RC), are presented. For these last three cases, the correlation weighting factor is included.

The Kanai-Tajimi filter is used in all cases to model the stationary part of the ground acceleration input. For the filter parameters, damping is 0.5, frequency is 1 Hz and the standard deviation of the input is 0.2g m/sec^2. The time duration of interest in each case corresponds to 10 fundamental periods of the structure of interest.

Initially, two examples involving control applications are considered. The first system is the five-story, base-isolated shear structure with an actuator at the base level discussed in Taflanidis and Beck (2005). The only feedback measurement is the filtered absolute acceleration of the base. Noise is added to the simulated measurements. The use of a second-order filter for the measurement signals is necessary in order to reduce the control effort in the high-frequency region where noise is dominant. The cut-off frequency of this filter is chosen as 0.5 Hz. The uncontrolled structure has a dominant fundamental mode with frequency 0.4 Hz and participation factor equal to 95.8 %. The performance variables (with their corresponding thresholds in parenthesis) are inter-story drifts (0.02m), displacement of base (0.4m), accelerations (2 m/sec^2) and actuator force (total weight of building).

The second system is the previous structure without base isolation. An actuator is considered in the first story and the feedback measurement in this case is the filtered absolute acceleration of the first floor. The cut-off frequency for the filter is 3.5 Hz. The frequencies and participation factors for the first two modes are 3.2 Hz and 8.72 Hz and 85.1% and 9.61%, respectively. The performance variables (and corresponding thresholds) are inter-story drifts (0.02m), accelerations (5 m/sec^2) and actuator force (total weight of building).

The probability of failure as a function of the feedback gain is presented in Figs. 2 and 3.

© 2005 Millpress, Rotterdam, ISBN 90 5966 033 1

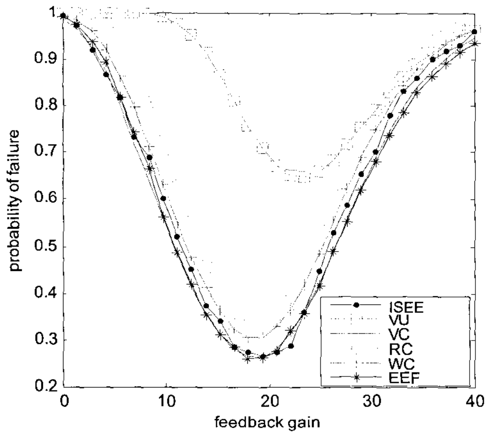

Figure 2. Probability of failure as a function of feedback gain for base-isolated five-story structure with control system.

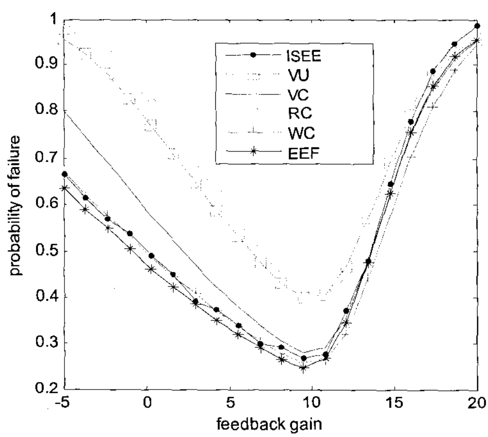

Figure 3. Probability of failure as a function of feedback gain for five-story structure with control system.

The third and last example is an eight-story 3D structure without any control system. It is the same structure considered in Scruggs et al. (2005) but without the base isolation. The influence of higher modes is greater with this selection (see Table 1). The performance variables are inter-story drifts and accelerations. The probability of failure for different levels of the threshold vector β with respect to the standard deviation is shown in Fig. 4.

Table 1. Modal characteristics of 3D structure.

Mode	Frequency (Hz)	Participation factor (%): x-direction	Participation factor (%): y-direction
1st	1.12	72.9	0.3
2nd	1.29	0.5	74.91
3rd	1.51	0.1	0.3
4th	3.55	0.2	17.17
5th	3.76	17.17	0.5

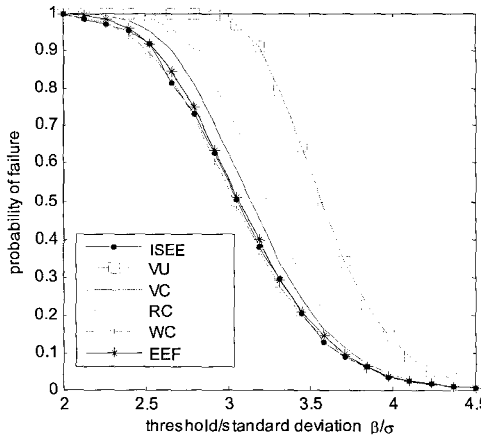

Figure 4. Probability of failure as a function of threshold level for eight-story 3D structure.

4.2 *Discussion of results*

The analytical approximation proposed for the calculation of the probability of failure for linear dynamical systems is in good agreement with the results coming from stochastic simulations using ISEE when both the upcrossing rate correction and correlation weighting factor are taken into account.

The importance of the correlation weighting factor is obvious in all examples by comparing VU and VC to the simulation results. The estimation for the probability of failure by VU is significantly in error except for the high feedback gain region (see Fig. 2 and 3). In these regions, the actuator force failure becomes dominant and the significance of the correlation is reduced. It is obvious that the approach of simply summing up the upcrossing rates, without considering the correlation of failure events, may give a highly conservative estimation of the probability of failure. Note that this approach has been the one usually adopted for reliability-based control synthesis, with the acknowledgment always that it provided only an upper bound of the probability of failure. Another relevant comment is that the optimum feedback gain might be significantly influenced by ignoring the correlation weighting factor.

The effect of including the bandwidth correction factor is important but to a smaller degree when compared to the effect of the correlation weighting factor. The different approaches considered for this correction give similar results for a structure with a dominant fundamental mode (Fig. 2). For this case, the approximations of Vanmarcke and the equivalent energy-fluctuation seem to give the best fit to the stochastic simulation results, with Vanmarcke's being slightly better. For systems with important higher-order dynamics (as in Figs. 3 and 4), the corrections of Winterstein and the equivalent energy fluctuation provide by far the best match, with the first one having a slightly better fit. Vanmarke's approximation appears to be sensitive to the higher-order dynamics and tends to significantly over-estimate the probability of failure.

The case of the uncontrolled eight-story structure with increasing threshold levels (Fig. 4) shows that the importance of the bandwidth correction decreases as the threshold values become larger. This is consistent with the fact that the significance of the bandwidth of the system, and the error in the assumption of independence of upcrossing times introduced by using the mean upcrossing rate, decreases for larger thresholds.

A final comment is warranted, regarding the time required for the reliability calculation. The only time consuming computations are the derivation of the frequency response and the numerical calculation of the multivariate integral in Eq. (5). The time needed for these two steps in the last example (3D model with 88 states, 2 inputs and 32 performance variables) was 1.0 sec and 6.1 sec, respectively, on a Pentium III 2.1GHz processor running MATLAB 7.01. Five thousand points were used for the calculation of the frequency response and one hundred samples for the Monte Carlo integration in the procedure of Genz. The Monte Carlo numerical integration in this step takes the most time. This time increases almost linearly with the dimension of the performance variables vector and with the size of the samples for the Monte Carlo integration, but the latter does not need to be large since we are mostly concerned with high probabilities. The total computational time for the calculation of each reliability estimate was 7.5 sec, which may be considered efficient for iterative calculations.

5 CONCLUSIONS

A computationally efficient analytical approximation for the probability of failure of linear dynamical systems under stationary stochastic excitation has been presented. The correlation of failure events and the correction for approximation of the conditional upcrossing rate have been addressed. Examples that verify the validity of the analytical approximation for systems in high dimensions have been presented.

The approximation of the conditional upcrossing rate through a semi-empirical correction term is the only part of the analysis that might introduce an error. An estimate of the absolute value of this error cannot be analytically provided. In any case, the selection of the correction term must be done by taking into account the bandwidth characteristics of the system. This error will always be less significant for higher threshold levels. The correction term proposed by Taflanidis and Beck (2005) seems to provide a robust estimate that is independent of the bandwidth characteristics of the system.

The proposed analytical approximation is appropriate for cases that require a large number of consistent (high relative accuracy) estimates of the probability of failure. Reliability-based controller synthesis is one of them. Another is reliability-based optimal structural design.

ACKNOWLEDGEMENTS

The authors would like to thank Dr Jeffrey Scruggs and Judith Mitrani-Reiser, both at Caltech, for helpful comments on the manuscript.

REFERENCES

Au, S.K. & Beck, J.L. 2001. First-excursion probabilities for linear systems by very efficient importance sampling. *Probabilistic Engineering Mechanics* Vol.16:193-207.

Belyaev, Y. 1968. On the number of exits across the boundary of a region by a vector stochastic process. *Theory Probabilistic Applications* Vol.13 (No.2):320-324.

Genz, A. 1992. Numerical computation of multivariate Normal probabilities. *Journal of Computational and Graphical Statistics* Vol.1:141-149.

Lutes, L.D. & Sarkani, S. 1997. *Stochastic Analysis of Structural and Mechanical Vibrations*. Upper Saddle River, NJ. Prentice Hall.

May, B.S. & Beck, J.L. 1998. Probabilistic control for the active mass driver benchmark structural model. *Earthquake Engineering and Structural Dynamic* Vol.27 (No.11): 1331-1346.

Rice, S.O. 1944, 1945. Mathematical analysis of random noise. *Bell System Technical Journal* Vol.23 and 24.

Scruggs, J.T., Taflanidis, A.A. & Beck, J.L. 2005. Reliability-based control optimization for active base isolation systems. *Journal of Structural Control* in Press.

Soong, T.T. & Grigoriu, M. 1993. *Random Vibration of Mechanical and Structural Systems*. Prentice Hall.

Taflanidis, A.A. & Beck, J.L. 2005. *Analytical Approximations for Stationary Reliability Calculations of Linear Dynamical Systems*. Earthquake Engineering Research Laboratory, California Institute of Technology, Pasadena.

Vanmarcke, E.H. 1975. On the distribution of the first passage time for Normal stationary processes. *ASME Journal of Applied Mechanics,* Vol.42:215-220.

Veneziano, D., Grigoriu, M. & Cornell, C.A. 1977. Vector-process models for system reliability. *ASCE Journal of Engineering Mechanics* Vol.103 (No.EM3):441-460.

Winterstein, S.R. & Cornell, C.A. 1985. Energy fluctuation scale and diffusion models. *ASCE Journal of Engineering Mechanics,* Vol.111 (No.2):125-142.

Yuen, K.V. & Beck, J.L. 2003. Reliability-based robust control for uncertain dynamical systems using feedback of incomplete noisy response measurements. *Earthquake Engineering and Structural Dynamics* Vol.32 (No.5):751-770.

 © 2005 Millpress, Rotterdam, ISBN 90 5966 033 1