# Microring coupled-resonator optical waveguides

**Joyce K. S. Poon, Jacob Scheuer, and Amnon Yariv**
*Department of Electrical Engineering and Department of Applied Physics,*
*California Institute of Technology, MC 136-93, Pasadena, CA 91125.*
*tel: 626-395-4413, e-mail: poon@caltech.edu*

**Abstract:** We use transfer matrices to obtain the dispersion relations of microring Coupled-Resonator Optical Waveguides (CROWs). We also analyze pulse propagation through finite and semi-infinite microring CROWs. The results agree well with FDTD simulations.

**OCIS codes:** (230.5750) Resonators; (230.7370) Waveguides; (230.3120) Integrated optics devices

## 1 Introduction

A Coupled-Resonator Optical Waveguide (CROW) consists of a chain of resonators in which light is guided through the coupling between adjacent resonators [1]. Due to the reduced group velocity inside a CROW, this type of waveguide may be useful for optical delay lines, pulse storage, or enhancing nonlinear optical effects [2, 3]. Our previous analysis of light propagation through a CROW has been based on a tight-binding approach (see for example, [4]).

Rings constitute an important class of resonators since they can be readily fabricated and can be designed to support a single radial mode, making them the simplest to analyze and use. Optical filters composed of coupled ring resonators have been extensively studied theoretically and experimentally [5, 6], and their potential use for optical delay lines and "slow light" propagation has been proposed and analyzed [7, 8, 9].

Using the transfer matrix approach introduced by Oda *et al.* and Orta *et al.*[10, 11], we analyze the dispersion relation of and pulse propagation through microring CROWs.

## 2 Dispersion Relation

To calculate the dispersion relation of the microring CROW, we consider both forward and backward propagating waves in an individual ring resonator, as shown in Fig. 1(a). We assume phase-matched coupling between two adjacent rings and no coupling between the forward and backward propagating waves in the resonators, so

$$\begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}_{n+1} = \begin{bmatrix} 0 & PQ \\ PQ & 0 \end{bmatrix} \begin{bmatrix} a \\ b \\ c \\ d \end{bmatrix}_{n} \tag{1a}$$

$$P = \frac{1}{\kappa}\begin{bmatrix} -t & 1 \\ -1 & t^* \end{bmatrix}; \quad Q = \begin{bmatrix} 0 & \exp(i\pi\beta R) \\ \exp(-i\pi\beta R) & 0 \end{bmatrix}. \tag{1b}$$

where $t$ is the transmission coefficient of the coupler, $\kappa$ is the dimensionless coupling coefficient, $R$ is the ring radius, and $\beta$ is the propagation constant in the ring such that $\beta = n(\omega)\omega/c$. For lossless coupling, $|t|^2 + |\kappa|^2 = 1$. $\beta$ may be complex to account for loss or gain in the ring.

To obtain the dispersion relation, we apply Bloch's theorem to Eq. (1). We assume lossless propagation and $\mathrm{Im}(\kappa) \gg \mathrm{Re}(\kappa)$ for phase-matched coupling. At the resonant frequency of a resonator, $\Omega$, the phase accumulated over a round trip is $2\pi R\Omega n(\Omega)/c = 2m\pi$ where $m$ is an integer. Approximating $n(\omega) \approx n(\Omega)$, the CROW dispersion relation is

$$\sin(\frac{\omega}{\Omega}m\pi) = \pm\mathrm{Im}(\kappa)\cos(K\Lambda), \tag{2}$$

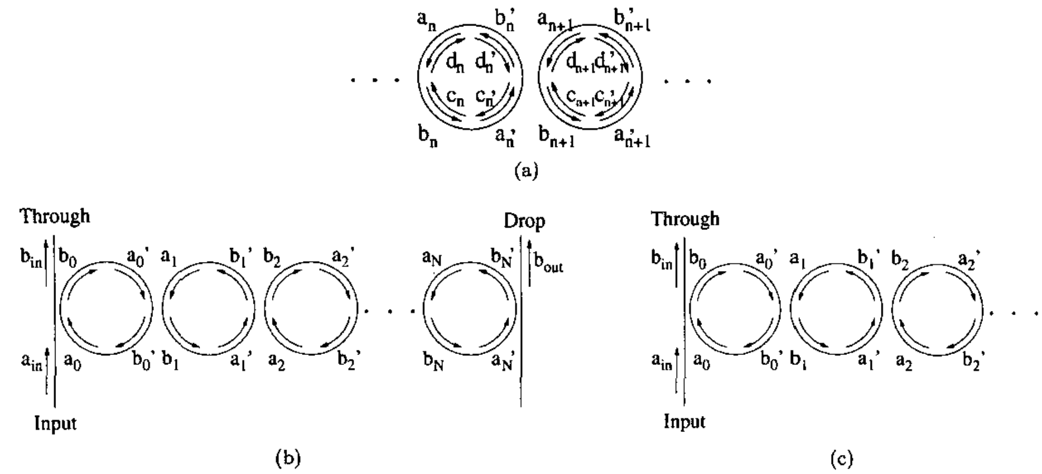


Fig. 1. (a) an infinite CROW, (b) a finite CROW, (c) a semi-infinite CROW

where $\Lambda$ is the CROW periodicity. If we expand Eq. (2) in the parameter $\Delta\omega m\pi/\Omega$, $\Delta\omega = \omega - \Omega$, for $|\kappa| \ll 1$, we obtain to first order

$$\frac{\omega}{\Omega} = 1 \pm \kappa_2 \cos(K\Lambda), \tag{3}$$

where $\kappa_2 \equiv \mathrm{Im}(\kappa)/(m\pi)$. The two dispersion relations coexist to allow for forward and backward wave propagation. The result is identical to that from the tight-binding approach.

## 3 Pulse Propagation

Physically, for a structure with input and output waveguides as in Fig. 1(b), only the travelling wave with the matching group velocity as the input will be excited in the CROW. The travelling wave can be expressed as a superposition of the Bloch modes, which are standing waves. The $4 \times 4$ transfer matrix can be reduced to a $2 \times 2$ matrix in this basis.

For a finite structure, by cascading the transfer matrices, we obtain the transfer functions for the through and drop ports. The transfer functions completely describe the output pulse shape given an arbitrary input pulse. Fig. 2(a) compares pulse propagation through 2 coupled rings using the matrix method with FDTD simulations. The matrix analysis is in excellent agreement with the numerical simulation. By selecting appropriate coupling constants between the waveguides and the CROW, we may obtain a sufficiently flat filter response [12], such that the finite CROW mimics the semi-infinite CROW as in Fig. 1(c).

For the semi-infinite CROW in Fig. 1(c) with weak inter-resonator coupling, there exists an analytic expression for the pulse evolution. Assuming that the input pulse bandwidth is within the bandwidth of the CROW, then $b_{in} = 0$ and $b_0(\omega) = -1/\kappa a_{in}(\omega)$. If the field where $b_0$ is taken is $\mathcal{E}(t, z = 0) = \int_{band} \mathrm{d}\omega b_0(\omega) \exp(-i\omega t)$, then the field at $z = N\Lambda$ where $b_N$ is taken is

$$\mathcal{E}(t, z = N\Lambda) = \int_{band} \mathrm{d}\omega \exp(-i\omega t) \int \frac{\mathrm{d}t'}{2\pi} \mathcal{E}(t', z = 0) \exp[i(\omega t' + K(\omega) N\Lambda)]. \tag{4}$$

However, $K(\omega)$ is given by Eq.(3), the dispersion relation of the CROW. Therefore, integrating over the half of the Brillouin zone that gives the appropriate group velocity instead of frequency in Eq.(4) and invoking the Jacobi-Anger relation, we obtain a convolution integral for the pulse envelope $E(t, z) = \mathcal{E}(t, z) \exp(i\Omega t)$

$$E(t, z = N\Lambda) = -\frac{\kappa_2 \Omega}{2\pi} \sum_m c_m \alpha_{m,n} \int \mathrm{d}t' J_m[\Omega \kappa_2 (t' - t)] E(t', z = 0), \tag{5a}$$

$$c_m = \begin{cases} 1 & \text{if } m = 0 \\ 2i^m & \text{if } m > 0 \end{cases} ; \quad \alpha_{m,n} = \begin{cases} -\frac{i\pi}{4} & \text{for } N = m-1 \text{ and } N = -m-1 \\ \frac{i\pi}{4} & \text{for } N = m+1 \text{ and } N = -m+1 \\ \frac{-2(m^2+n^2-1)}{(m^2+n^2-1)^2-4m^2n^2} & \text{for } N+m = \text{even} \end{cases} \tag{5b}$$

Fig.2(b) shows the pulse evolution of a Gaussian input pulse computed using (5). In this example, pulse distortion only becomes prominent at large distances.

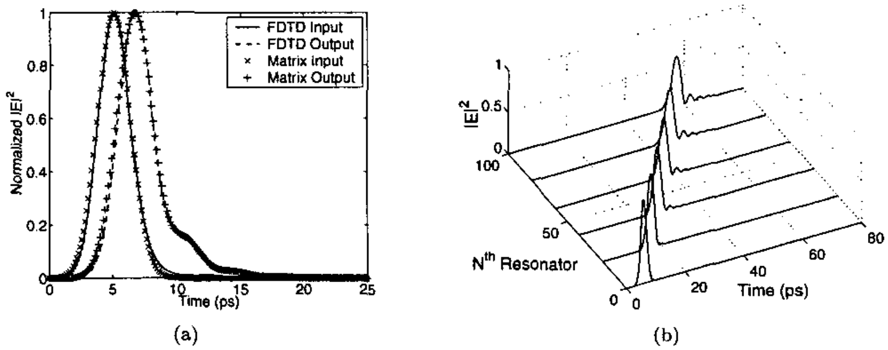


Fig. 2. (a) Propagation of a 3ps (FWHM) pulse centered about 1.55$\mu$m through 2 coupled ring resonators as computed by a FDTD simulation and the transfer matrix method ($\kappa_{ring-ring} = 0.32i$, $\kappa_{wg-ring} = 0.4i$). (b) Evolution of a 2.4ps (FWHM) Gaussian pulse centered about 1.5$\mu$m in a semi-infinite CROW with $\kappa_2 = 0.01i$.

## 4 Conclusion

The transfer matrix method is a powerful tool for analyzing microring CROWs. The formalism is particularly suited to physically realizable, finite systems. Pulse propagation can also be accurately modelled using this approach.

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