

## 6

# BOILING AND CONDENSATION

### 6.1 INTRODUCTION

The fundamentals of bubble growth or collapse during boiling or condensation were described in chapter 4 and particularly in the sections dealing with thermally-inhibited growth or collapse. This chapter deals with a number of additional features of these processes. In many industrial contexts in which boiling or condensation occurs, the presence of a nearby solid surface is necessary for the rapid supply or removal of the latent heat inherent in the phase change. The presence of this wall modifies the flow patterns and other characteristics of these multiphase flows and this chapter will address those additional phenomena.

In all cases the heat flux per unit area through the solid surface is denoted by  $\dot{q}$ ; the wall temperature is denoted by  $T_w$  and the bulk liquid temperature by  $T_b$  (or  $T_L$ ). The temperature difference  $\Delta T = T_w - T_b$  is a ubiquitous feature of all these problems. Moreover, in almost all cases the pressure differences within the flow are sufficiently small that the saturated liquid/vapor temperature,  $T_e$ , can be assumed uniform. Then, to a first approximation, boiling at the wall occurs when  $T_w > T_e$  and  $T_b \leq T_e$ . When  $T_b < T_e$  and the liquid must be heated to  $T_e$  before bubbles occur, the situation is referred to as sub-cooled boiling. On the other hand condensation at the wall occurs when  $T_w < T_e$  and  $T_b \geq T_e$ . When  $T_b > T_e$  and the vapor must be cooled to  $T_e$  before liquid appears, the situation is referred to as super-heated condensation.

The solid surface may be a plane vertical or horizontal containing surface or it may be the interior or exterior of a circular pipe. Another factor influencing the phenomena is whether there is a substantial fluid flow (convection) parallel to the solid surface. For some of the differences between these various geometries and imposed flow conditions the reader is referred

to texts such as Collier and Thome (1994), Hsu and Graham (1976) or Whalley (1987). In the next section we review the phenomena associated with a plane horizontal boundary with no convection. Later sections deal with vertical surfaces.

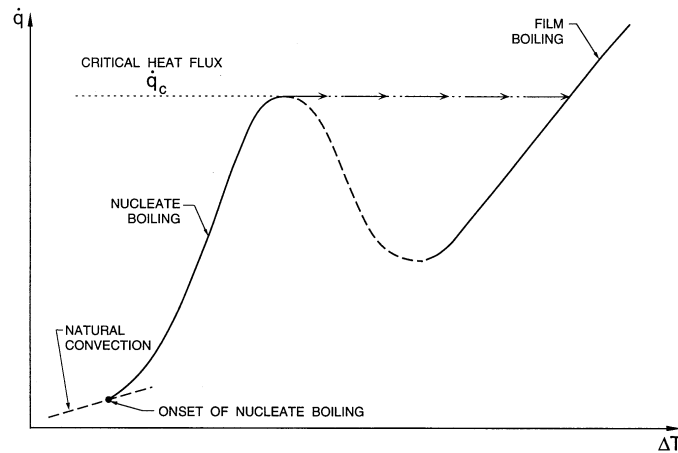
## 6.2 HORIZONTAL SURFACES

### 6.2.1 Pool boiling

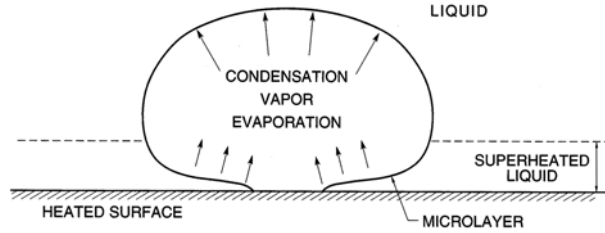
Perhaps the most common configuration, known as *pool boiling* is when a pool of liquid is heated from below through a horizontal surface. For present purposes we assume that the heat flux,  $\dot{q}$ , is uniform. A uniform bulk temperature far from the wall is maintained because the mixing motions generated by natural convection (and, in boiling, by the motions of the bubbles) mean that most of the liquid is at a fairly uniform temperature. In other words, the temperature difference  $\Delta T$  occurs within a thin layer next to the wall.

In pool boiling the relation between the heat flux,  $\dot{q}$ , and  $\Delta T$  is as sketched in figure 6.1 and events develop with increasing  $\Delta T$  as follows. When the pool as a whole has been heated to a temperature close to  $T_e$ , the onset of nucleate boiling occurs. Bubbles form at nucleation sites on the wall and grow to a size at which the buoyancy force overcomes the surface tension forces acting at the line of attachment of the bubble to the wall. The bubbles then break away and rise through the liquid.

In a steady state process, the vertically-upward heat flux,  $\dot{q}$ , should be the same at all elevations above the wall. Close to the wall the situation is



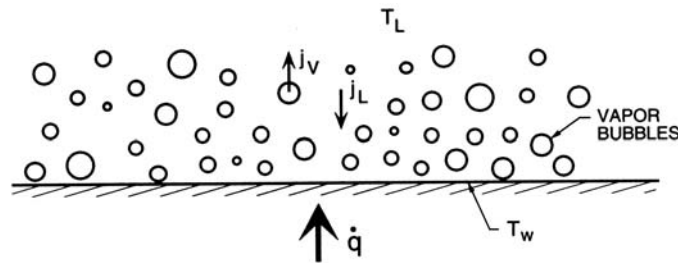
**Figure 6.1.** Pool boiling characteristics.



**Figure 6.2.** Sketch of nucleate boiling bubble with microlayer.

complex for several mechanisms increase the heat flux above that for pure conduction through the liquid. First the upward flux of vapor away from the wall must be balanced by an equal downward mass flux of liquid and this brings cooler liquid into closer proximity to the wall. Second, the formation and movement of the bubbles enhances mixing in the liquid near the wall and thus increases heat transfer from the wall to the liquid. Third, the flux of heat to provide the latent heat of vaporization that supplies vapor to the bubbles increases the total heat flux. While a bubble is still attached to the wall, vapor may be formed at the surface of the bubble closest to the wall and then condense on the surface furthest from the wall thus creating a heat pipe effect. This last mode of heat transfer is sketched in figure 6.2 and requires the presence of a thin layer of liquid under the bubble known as the *microlayer*.

At distances further from the wall (figure 6.3) the dominant component of  $\dot{q}$  is simply the enthalpy flux difference between the upward flux of vapor and the downward flux of liquid. Assuming this enthalpy difference is given approximately by the latent heat,  $\mathcal{L}$ , it follows that the upward volume flux of vapor,  $j_V$ , is given by  $\dot{q}/\rho_V \mathcal{L}$ , where  $\rho_V$  is the saturated vapor density at the prevailing pressure. Since mass must be conserved the downward mass flux of liquid must be equal to the upward mass flux of vapor and it follows



**Figure 6.3.** Nucleate boiling.

that the downward liquid volume flux should be  $\dot{q}/\rho_L\mathcal{L}$ , where  $\rho_L$  is the saturated liquid density at the prevailing pressure.

To complete the analysis, estimates are needed for the number of nucleation sites per unit area of the wall ( $N^*$   $m^{-2}$ ), the frequency ( $f$ ) with which bubbles leave each site and the equivalent volumetric radius ( $R$ ) upon departure. Given the upward velocity of the bubbles ( $u_V$ ) this allows evaluation of the volume fraction and volume flux of vapor bubbles from:

$$\alpha = \frac{4\pi R^3 N^* f}{3u_V} \quad ; \quad j_V = \frac{4}{3}\pi R^3 N^* f \quad (6.1)$$

and it then follows that

$$\dot{q} = \frac{4}{3}\pi R^3 N^* f \rho_V \mathcal{L} \quad (6.2)$$

As  $\Delta T$  is increased both the site density  $N^*$  and the bubble frequency  $f$  increase until, at a certain critical heat flux,  $\dot{q}_c$ , a complete film of vapor blankets the wall. This is termed *boiling crisis*. Normally one is concerned with systems in which the heat flux rather than the wall temperature is controlled, and, because the vapor film provides a substantial barrier to heat transfer, such systems experience a large increase in the wall temperature when the boiling crisis occurs. This development is sketched in figure 6.1. The increase in wall temperature can be very hazardous and it is therefore important to be able to predict the boiling crisis and the heat flux at which this occurs. There are a number of detailed analyses of the boiling crisis and for such detail the reader is referred to Zuber *et al.* (1959, 1961), Rohsenow and Hartnett (1973), Hsu and Graham (1976), Whalley (1987) or Collier and Thome (1994). This important fundamental process is discussed in chapter 14 as a classic example of the flooding phenomenon in multiphase flows.

### 6.2.2 Nucleate boiling

As equation 6.2 illustrates, quantitative understanding and prediction of nucleate boiling requires detailed information on the quantities  $N^*$ ,  $f$ ,  $R$  and  $u_V$  and thus knowledge not only of the number of nucleation sites per unit area, but also of the cyclic sequence of events as each bubble grows and detaches from a particular site. Though detailed discussion of the nucleation sites is beyond the scope of this book, it is well-established that increasing  $\Delta T$  activates increasingly smaller (and therefore more numerous) sites (Griffith and Wallis 1960) so that  $N^*$  increases rapidly with  $\Delta T$ . The cycle of events at each nucleation site as bubbles are created, grow and detach is termed the *ebullition cycle* and consists of

1. a period of bubble growth during which the bubble growth rate is directly related to the rate of heat supply to each site,  $\dot{q}/N^*$ . In the absence of inertial effects and assuming that all this heat is used for evaporation (in a more precise analysis some fraction is used to heat the liquid), the bubble growth rate is then given by  $dR/dt = C\dot{q}/4\pi R^2 \rho_V \mathcal{L}N^*$  where  $C$  is some constant that will be influenced by complicating factors such as the geometry of the bubble attachment to the wall and the magnitude of the temperature gradient in the liquid normal to the wall (see, for example, Hsu and Graham 1976).
2. the moment of detachment when the upward buoyancy forces exceed the surface tension forces at the bubble-wall contact line. This leads to a bubble size,  $R_d$ , upon detachment given qualitatively by

$$R_d = C \left[ \frac{S}{g(\rho_L - \rho_V)} \right]^{\frac{1}{2}} \quad (6.3)$$

where the constant  $C$  will depend on surface properties such as the contact angle but is of the order of 0.005 (Fritz 1935). With the growth rate from the growth phase analysis this fixes the time for growth.

3. the waiting period during which the local cooling of the wall in the vicinity of the nucleation site is diminished by conduction within the wall surface and after which the growth of another bubble is initiated.

Obviously the sum of the growth time and the waiting period leads to the bubble frequency,  $f$ .

In addition, the rate of rise of the bubbles can be estimated using the methods of chapters 2 and 3. As discussed later in section 14.3.3, the downward flow of liquid must also be taken into account in evaluating  $u_V$ .

These are the basic elements involved in characterizing nucleate boiling though there are many details for which the reader is referred to the texts by Rohsenow and Hartnett (1973), Hsu and Graham (1976), Whalley (1987) or Collier and Thome (1994). Note that the concepts involved in the analysis of nucleate boiling on an inclined or vertical surface do not differ greatly. The addition of an imposed flow velocity parallel to the wall will alter some details since, for example, the analysis of the conditions governing bubble detachment must include consideration of the resulting drag on the bubble.

### 6.2.3 Film boiling

At or near boiling crisis a film of vapor is formed that coats the surface and substantially impedes heat transfer. This vapor layer presents the primary resistance to heat transfer since the heat must be conducted through the

layer. It follows that the thickness of the layer,  $\delta$ , is given approximately by

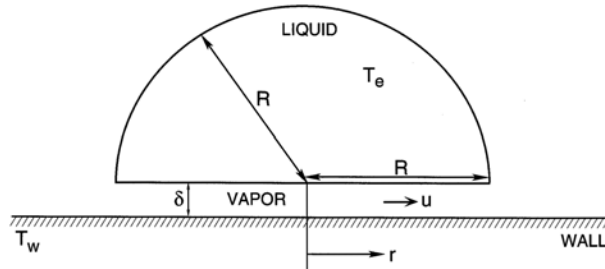
$$\delta = \frac{\Delta T k_V}{\dot{q}} \quad (6.4)$$

However, these flows are usually quite unsteady since the vapor/liquid interface is unstable to Rayleigh-Taylor instability (see sections 7.5.1 and 14.3.3). The result of this unsteadiness of the interface is that vapor bubbles are introduced into the liquid and travel upwards while liquid droplets are also formed and fall down through the vapor toward the hot surface. These droplets are evaporated near the surface producing an upward flow of vapor. The relation 6.4 then needs modification in order to account for the heat transfer across the thin layer under the droplet.

The droplets do not normally touch the hot surface because the vapor created on the droplet surface nearest the wall creates a lubrication layer that suspends the droplet. This is known as the Leidenfrost effect. It is readily observed in the kitchen when a drop of water is placed on a hot plate. Note, however, that the thermal resistance takes a similar form to that in equation 6.4 though the temperature difference in the vicinity of the droplet now occurs across the much thinner layer under the droplet rather than across the film thickness,  $\delta$ .

#### 6.2.4 Leidenfrost effect

To analyze the Leidenfrost effect, we assume the simple geometry shown in figure 6.4 in which a thin, uniform layer of vapor of thickness  $\delta$  separates the hemispherical droplet (radius,  $R$ ) from the wall. The droplet is assumed to have been heated to the saturation temperature  $T_e$  and the temperature difference  $T_w - T_e$  is denoted by  $\Delta T$ . Then the heat flux per unit surface area across the vapor layer is given by  $k_V \Delta T / \delta$  and this causes a mass rate



**Figure 6.4.** Hemispherical model of liquid drop for the Leidenfrost analysis.

of evaporation of liquid at the droplet surface of  $k_V \Delta T / \delta \mathcal{L}$ . The outward radial velocity of vapor at a radius of  $r$  from the center of the vapor layer,  $u(r)$  (see figure 6.4) must match the total rate of volume production of vapor inside this radius,  $\pi r^2 k_V \Delta T / \rho_V \delta \mathcal{L}$ . Assuming that we use mean values of the quantities  $k_V$ ,  $\rho_V$ ,  $\mathcal{L}$  or that these do not vary greatly within the flow, this implies that the value of  $u$  averaged over the layer thickness must be given by

$$u(r) = \frac{k_V \Delta T}{2 \rho_V \mathcal{L}} \frac{r}{\delta^2} \quad (6.5)$$

This connects the velocity  $u(r)$  of the vapor to the thickness  $\delta$  of the vapor layer. A second relation between these quantities is obtained by considering the equation of motion for the viscous outward radial flow of vapor (assuming the liquid velocities are negligible). This is simply a radial Poiseuille flow in which the mean velocity across the gap,  $u(r)$ , must be given by

$$u(r) = -\frac{\delta^2}{12 \mu_V} \frac{dp}{dr} \quad (6.6)$$

where  $p(r)$  is the pressure distribution in the vapor layer. Substituting for  $u(r)$  from equation 6.5 and integrating we obtain the pressure distribution in the vapor layer:

$$p(r) = p_a + \frac{3 k_V \mu_V \Delta T}{\rho_V \mathcal{L}} \frac{(R^2 - r^2)}{2 \delta^4} \quad (6.7)$$

where  $p_a$  is the surrounding atmospheric pressure. Integrating the pressure difference,  $p(r) - p_a$ , to find the total upward force on the droplet and equating this to the difference between the weight of the droplet and the buoyancy force,  $2\pi(\rho_L - \rho_V)R^3/3$ , yields the following expression for the thickness,  $\delta$ , of the vapor layer:

$$\frac{\delta}{R} = \left[ \frac{9 k_V \mu_V \Delta T}{8 \rho_V (\rho_L - \rho_V) g \mathcal{L} R^3} \right]^{\frac{1}{4}} \quad (6.8)$$

Substituting this result back into the expression for the velocity and then evaluating the mass flow rate of vapor and consequently the rate of loss of mass of the droplet one can find the following expression for the lifetime,  $t_t$ , of a droplet of initial radius,  $R_o$ :

$$t_t = 4 \left[ \frac{2 \mu_V}{9 \rho_V g} \right]^{\frac{1}{4}} \left[ \frac{(\rho_L - \rho_V) \mathcal{L} R_o}{k_V \Delta T} \right]^{\frac{3}{4}} \quad (6.9)$$

As a numerical example, a water droplet with a radius of  $2mm$  at a saturated temperature of about  $400K$  near a wall with a temperature of  $500K$  will have a film thickness of just  $40\mu m$  but a lifetime of just over  $1hr$ . Note that as  $\Delta T$ ,  $k_V$  or  $g$  go up the lifetime goes down as expected; on the other hand increasing  $R_o$  or  $\mu_V$  has the opposite effect.

### 6.3 VERTICAL SURFACES

Boiling on a heated vertical surface is qualitatively similar to that on a horizontal surface except for the upward liquid and vapor velocities caused by natural convection. Often this results in a cooler liquid and a lower surface temperature at lower elevations and a progression through various types of



**Figure 6.5.** The evolution of convective boiling around a heated rod, reproduced from Sherman and Sabersky (1981) with permission.



boiling as the flow proceeds upwards. Figure 6.5 provides an illustrative example. Boiling begins near the bottom of the heated rod and the bubbles increase in size as they are convected upward. At a well-defined elevation, boiling crisis (section 14.3.3 and figure 6.1) occurs and marks the transition to film boiling at a point about 5/8 of the way up the rod in the photograph. At this point, the material of the rod or pipe experiences an abrupt and substantial rise in surface temperature as described in section 14.3.3.

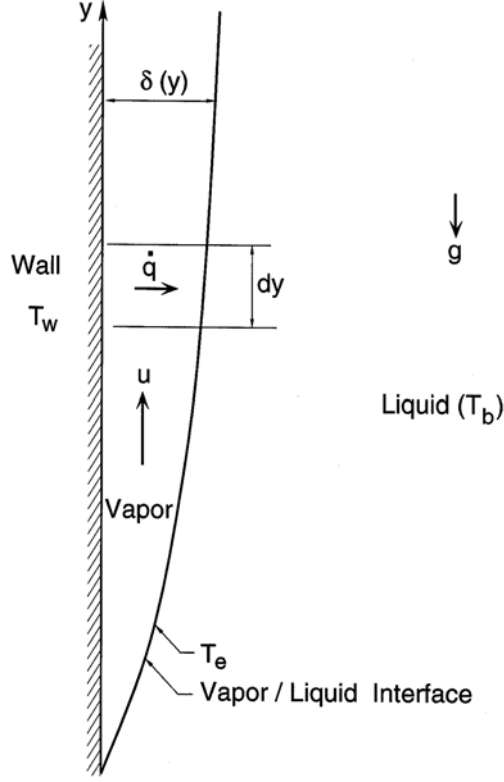
The nucleate boiling regime was described earlier. The film boiling regime is a little different than that described in section 6.2.3 and is addressed in the following section.

### 6.3.1 Film boiling

The first analysis of film boiling on a vertical surface was due to Bromley (1950) and proceeds as follows. Consider a small element of the vapor layer of length  $dy$  and thickness,  $\delta(y)$ , as shown in figure 6.6. The temperature difference between the wall and the vapor/liquid interface is  $\Delta T$ . Therefore the mass rate of conduction of heat from the wall and through the vapor to the vapor/liquid interface per unit surface area of the wall will be given approximately by  $k_V \Delta T / \delta$  where  $k_V$  is the thermal conductivity of the vapor. In general some of this heat flux will be used to evaporate liquid at the interface and some will be used to heat the liquid outside the layer from its bulk temperature,  $T_b$  to the saturated vapor/liquid temperature of the interface,  $T_e$ . If the subcooling is small, the latter heat sink is small compared with the former and, for simplicity in this analysis, it will be assumed that this is the case. Then the mass rate of evaporation at the interface (per unit area of that interface) is  $k_V \Delta T / \delta \mathcal{L}$ . Denoting the mean velocity of the vapor in the layer by  $u(y)$ , continuity of vapor mass within the layer requires that

$$\frac{d(\rho_V u \delta)}{dy} = \frac{k_V \Delta T}{\delta \mathcal{L}} \quad (6.10)$$

Assuming that we use mean values for  $\rho_V$ ,  $k_V$  and  $\mathcal{L}$  this is a differential relation between  $u(y)$  and  $\delta(y)$ . A second relation between these two quantities can be obtained by considering the equation of motion for the vapor in the element  $dy$ . That vapor mass will experience a pressure denoted by  $p(y)$  that must be equal to the pressure in the liquid if surface tension is neglected. Moreover, if the liquid motions are neglected so that the pressure variation in the liquid is hydrostatic, it follows that the net force acting on the vapor element as a result of these pressure variations will be  $\rho_L g \delta dy$  per unit depth normal to the sketch. Other forces per unit depth acting



**Figure 6.6.** Sketch for the film boiling analysis.

on the vapor element will be its weight  $\rho_V g \delta dy$  and the shear stress at the wall that we will estimate to be given roughly by  $\mu_V u / \delta$ . Then if the vapor momentum fluxes are neglected the balance of forces on the vapor element yields

$$u = \frac{(\rho_L - \rho_V) g \delta^2}{\mu_V} \quad (6.11)$$

Substituting this expression for  $u$  into equation 6.10 and solving for  $\delta(y)$  assuming that the origin of  $y$  is chosen to be the origin or virtual origin of the vapor layer where  $\delta = 0$  we obtain the following expression for  $\delta(y)$

$$\delta(y) = \left[ \frac{4k_V \Delta T \mu_V}{3\rho_V (\rho_L - \rho_V) g \mathcal{L}} \right]^{\frac{1}{4}} y^{\frac{1}{4}} \quad (6.12)$$

This defines the geometry of the film.

We can then evaluate the heat flux  $\dot{q}(y)$  per unit surface area of the plate;

the local heat transfer coefficient,  $\dot{q}/\Delta T$  becomes

$$\frac{\dot{q}(y)}{\Delta T} = \left[ \frac{3\rho_V(\rho_L - \rho_V)g\mathcal{L}k_V^3}{4\Delta T\mu_V} \right]^{\frac{1}{4}} y^{-\frac{1}{4}} \quad (6.13)$$

Note that this is singular at  $y = 0$ . It also follows by integration that the overall heat transfer coefficient for a plate extending from  $y = 0$  to  $y = \ell$  is

$$\left( \frac{4}{3} \right)^{\frac{3}{4}} \left[ \frac{\rho_V(\rho_L - \rho_V)g\mathcal{L}k_V^3}{\Delta T\mu_V\ell} \right]^{\frac{1}{4}} \quad (6.14)$$

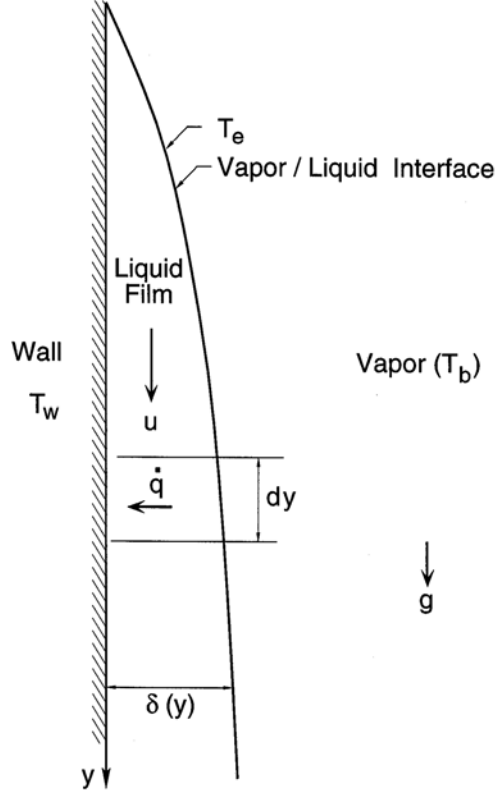
This characterizes the film boiling heat transfer coefficients in the upper right of figure 6.1. Though many features of the flow have been neglected this relation gives good agreement with the experimental observations (Westwater 1958). Other geometrical arrangements such as heated circular pipes on which film boiling is occurring will have a similar functional dependence on the properties of the vapor and liquid (Collier and Thome 1994, Whalley 1987).

## 6.4 CONDENSATION

The spectrum of flow processes associated with condensation on a solid surface are almost a mirror image of those involved in boiling. Thus drop condensation on the underside of a cooled horizontal plate or on a vertical surface is very analogous to nucleate boiling. The phenomenon is most apparent as the misting up of windows or mirrors. When the population of droplets becomes large they run together to form condensation films, the dominant form of condensation in most industrial contexts. Because of the close parallels with boiling flows, it would be superfluous to repeated the analyses for condensation flows. However, in the next section we include the specifics of one example, namely film condensation on a vertical surface. For more detail on condensation flows the reader is referred to the reviews by Butterworth (1977).

### 6.4.1 Film condensation

The circumstance of film condensation on a vertical plate as sketched in figure 6.7 allows an analysis that is precisely parallel to that for film boiling detailed in section 6.3.1. The obvious result is a film thickness,  $\delta(y)$  (where



**Figure 6.7.** Sketch for the film condensation analysis.

$y$  is now measured vertically downward) given by

$$\delta(y) = \left[ \frac{4k_L(-\Delta T)\mu_L}{3\rho_L(\rho_L - \rho_V)g\mathcal{L}} \right]^{\frac{1}{4}} y^{\frac{1}{4}} \quad (6.15)$$

a local heat transfer coefficient given by

$$\frac{\dot{q}(y)}{\Delta T} = \left[ \frac{3\rho_L(\rho_L - \rho_V)g\mathcal{L}k_L^3}{4(-\Delta T)\mu_L} \right]^{\frac{1}{4}} y^{-\frac{1}{4}} \quad (6.16)$$

and the following overall heat transfer coefficient for a plate of length  $\ell$ :

$$\left( \frac{4}{3} \right)^{\frac{3}{4}} \left[ \frac{\rho_L(\rho_L - \rho_V)g\mathcal{L}k_L^3}{(-\Delta T)\mu_L\ell} \right]^{\frac{1}{4}} \quad (6.17)$$

Clearly the details of film condensation will be different for different geometric configurations of the solid surface (inclined walls, horizontal tubes,

etc.) and for laminar or turbulent liquid films. For such details, the reader is referred to the valuable review by Collier and Thome (1994).