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Undergraduate Texts in Mathematics

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- Hijab:** Introduction to Calculus and Classical Analysis.
- Hilton/Holton/Pedersen:** Mathematical Reflections: In a Room with Many Mirrors.
- Iooss/Joseph:** Elementary Stability and Bifurcation Theory. Second edition.
- Isaac:** The Pleasures of Probability. *Readings in Mathematics.*
- James:** Topological and Uniform Spaces.
- Jänich:** Linear Algebra.
- Jänich:** Topology.
- Jänich:** Vector Analysis.
- Kemeny/Snell:** Finite Markov Chains.
- Kinsey:** Topology of Surfaces.
- Klambauer:** Aspects of Calculus.
- Lang:** A First Course in Calculus. Fifth edition.
- Lang:** Calculus of Several Variables. Third edition.
- Lang:** Introduction to Linear Algebra. Second edition.
- Lang:** Linear Algebra. Third edition.
- Lang:** Undergraduate Algebra. Second edition.
- Lang:** Undergraduate Analysis.
- Lax/Burstein/Lax:** Calculus with Applications and Computing. Volume 1.
- LeCuyer:** College Mathematics with APL.
- Lidl/Pilz:** Applied Abstract Algebra. Second edition.
- Logan:** Applied Partial Differential Equations.
- Macki-Strauss:** Introduction to Optimal Control Theory.
- Malitz:** Introduction to Mathematical Logic.
- Marsden/Weinstein:** Calculus I, II, III. Second edition.
- Martin:** The Foundations of Geometry and the Non-Euclidean Plane.
- Martin:** Geometric Constructions.
- Martin:** Transformation Geometry: An Introduction to Symmetry.
- Millman/Parker:** Geometry: A Metric Approach with Models. Second edition.
- Moschovakis:** Notes on Set Theory.
- Owen:** A First Course in the Mathematical Foundations of Thermodynamics.
- Palka:** An Introduction to Complex Function Theory.
- Pedrick:** A First Course in Analysis.
- Peressini/Sullivan/Uhl:** The Mathematics of Nonlinear Programming.
- Prenowitz/Jantosciak:** Join Geometries.
- Priestley:** Calculus: A Liberal Art. Second edition.
- Protter/Morrey:** A First Course in Real Analysis. Second edition.
- Protter/Morrey:** Intermediate Calculus. Second edition.
- Roman:** An Introduction to Coding and Information Theory.
- Ross:** Elementary Analysis: The Theory of Calculus.
- Samuel:** Projective Geometry. *Readings in Mathematics.*
- Scharlau/Opolka:** From Fermat to Minkowski.
- Schiff:** The Laplace Transform: Theory and Applications.
- Sethuraman:** Rings, Fields, and Vector Spaces: An Approach to Geometric Constructability.
- Sigler:** Algebra.
- Silverman/Tate:** Rational Points on Elliptic Curves.
- Simmonds:** A Brief on Tensor Analysis. Second edition.
- Singer:** Geometry: Plane and Fancy.
- Singer/Thorpe:** Lecture Notes on Elementary Topology and Geometry.
- Smith:** Linear Algebra. Third edition.
- Smith:** Primer of Modern Analysis. Second edition.

Undergraduate Texts in Mathematics

Stanton/White: Constructive Combinatorics.

Stillwell: Elements of Algebra: Geometry, Numbers, Equations.

Stillwell: Mathematics and Its History.

Stillwell: Numbers and Geometry. *Readings in Mathematics.*

Strayer: Linear Programming and Its Applications.

Thorpe: Elementary Topics in Differential Geometry.

Toth: Glimpses of Algebra and Geometry. *Readings in Mathematics.*

Troutman: Variational Calculus and Optimal Control. Second edition.

Valenza: Linear Algebra: An Introduction to Abstract Mathematics.

Whyburn/Duda: Dynamic Topology.

Wilson: Much Ado About Calculus.

29. $\int \operatorname{csch} x \, dx = \ln \left| \tanh \frac{x}{2} \right| = -\frac{1}{2} \ln \frac{\cosh x + 1}{\cosh x - 1}$
30. $\int \sinh^2 x \, dx = \frac{1}{4} \sinh 2x - \frac{1}{2} x$
31. $\int \cosh^2 x \, dx = \frac{1}{4} \sinh 2x + \frac{1}{2} x$
32. $\int \operatorname{sech}^2 x \, dx = \tanh x$
33. $\int \sinh^{-1} \frac{x}{a} \, dx = x \sinh^{-1} \frac{x}{a} - \sqrt{x^2 + a^2} \quad (a > 0)$
34. $\int \cosh^{-1} \frac{x}{a} \, dx = \begin{cases} x \cosh^{-1} \frac{x}{a} - \sqrt{x^2 - a^2} & \left[\cosh^{-1} \left(\frac{x}{a} \right) > 0, a > 0 \right] \\ x \cosh^{-1} \frac{x}{a} + \sqrt{x^2 - a^2} & \left[\cosh^{-1} \left(\frac{x}{a} \right) < 0, a > 0 \right] \end{cases}$
35. $\int \tanh^{-1} \frac{x}{a} \, dx = x \tanh^{-1} \frac{x}{a} + \frac{a}{2} \ln |a^2 - x^2|$
36. $\int \frac{1}{\sqrt{a^2 + x^2}} \, dx = \ln(x + \sqrt{a^2 + x^2}) = \sinh^{-1} \frac{x}{a} \quad (a > 0)$
37. $\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a} \tan^{-1} \frac{x}{a} \quad (a > 0)$
38. $\int \sqrt{a^2 - x^2} \, dx = \frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \quad (a > 0)$
39. $\int (a^2 - x^2)^{3/2} \, dx = \frac{x}{8} (5a^2 - 2x^2) \sqrt{a^2 - x^2} + \frac{3a^4}{8} \sin^{-1} \frac{x}{a} \quad (a > 0)$
40. $\int \frac{1}{\sqrt{a^2 - x^2}} \, dx = \sin^{-1} \frac{x}{a} \quad (a > 0)$
41. $\int \frac{1}{a^2 - x^2} \, dx = \frac{1}{2a} \ln \left| \frac{a+x}{a-x} \right|$
42. $\int \frac{1}{(a^2 - x^2)^{3/2}} \, dx = \frac{x}{a^2 \sqrt{a^2 - x^2}}$
43. $\int \sqrt{x^2 \pm a^2} \, dx = \frac{x}{2} \sqrt{x^2 \pm a^2} \pm \frac{a^2}{2} \ln |x + \sqrt{x^2 \pm a^2}|$
44. $\int \frac{1}{\sqrt{x^2 - a^2}} \, dx = \ln |x + \sqrt{x^2 - a^2}| = \cosh^{-1} \frac{x}{a} \quad (a > 0)$
45. $\int \frac{1}{x(a+bx)} \, dx = \frac{1}{a} \ln \left| \frac{x}{a+bx} \right|$
46. $\int x\sqrt{a+bx} \, dx = \frac{2(3bx-2a)(a+bx)^{3/2}}{15b^2}$
47. $\int \frac{\sqrt{a+bx}}{x} \, dx = 2\sqrt{a+bx} + a \int \frac{1}{x\sqrt{a+bx}} \, dx$
48. $\int \frac{x}{\sqrt{a+bx}} \, dx = \frac{2(bx-2a)\sqrt{a+bx}}{3b^2}$
49. $\int \frac{1}{x\sqrt{a+bx}} \, dx = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a+bx} - \sqrt{a}}{\sqrt{a+bx} + \sqrt{a}} \right| \quad (a > 0)$
 $= \frac{2}{\sqrt{-a}} \tan^{-1} \sqrt{\frac{a+bx}{-a}} \quad (a < 0)$
50. $\int \frac{\sqrt{a^2 - x^2}}{x} \, dx = \sqrt{a^2 - x^2} - a \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$
51. $\int x\sqrt{a^2 - x^2} \, dx = -\frac{1}{3} (a^2 - x^2)^{3/2}$
52. $\int x^2 \sqrt{a^2 - x^2} \, dx = \frac{x}{8} (2x^2 - a^2) \sqrt{a^2 - x^2} + \frac{a^4}{8} \sin^{-1} \frac{x}{a} \quad (a > 0)$

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53. $\int \frac{1}{x\sqrt{a^2 - x^2}} dx = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - x^2}}{x} \right|$
54. $\int \frac{x}{\sqrt{a^2 - x^2}} dx = -\sqrt{a^2 - x^2}$
55. $\int \frac{x^2}{\sqrt{a^2 - x^2}} dx = -\frac{x}{2} \sqrt{a^2 - x^2} + \frac{a^2}{2} \sin^{-1} \frac{x}{a} \quad (a > 0)$
56. $\int \frac{\sqrt{x^2 + a^2}}{x} dx = \sqrt{x^2 + a^2} - a \ln \left| \frac{a + \sqrt{x^2 + a^2}}{x} \right|$
57. $\int \frac{\sqrt{x^2 - a^2}}{x} dx = \sqrt{x^2 - a^2} - a \cos^{-1} \frac{a}{|x|}$
 $= \sqrt{x^2 - a^2} - a \sec^{-1} \left(\frac{x}{a} \right) \quad (a > 0)$
58. $\int x\sqrt{x^2 \pm a^2} dx = \frac{1}{3} (x^2 \pm a^2)^{3/2}$
59. $\int \frac{1}{x\sqrt{x^2 + a^2}} dx = \frac{1}{a} \ln \left| \frac{x}{a + \sqrt{x^2 + a^2}} \right|$
60. $\int \frac{1}{x\sqrt{x^2 - a^2}} dx = \frac{1}{a} \cos^{-1} \frac{a}{|x|} \quad (a > 0)$
61. $\int \frac{1}{x^2\sqrt{x^2 \pm a^2}} dx = \mp \frac{\sqrt{x^2 \pm a^2}}{a^2 x}$
62. $\int \frac{x}{\sqrt{x^2 \pm a^2}} dx = \sqrt{x^2 \pm a^2}$
63. $\int \frac{1}{ax^2 + bx + c} dx = \frac{1}{\sqrt{b^2 - 4ac}} \ln \left| \frac{2ax + b - \sqrt{b^2 - 4ac}}{2ax + b + \sqrt{b^2 - 4ac}} \right| \quad (b^2 > 4ac)$
 $= \frac{2}{\sqrt{4ac - b^2}} \tan^{-1} \frac{2ax + b}{\sqrt{4ac - b^2}} \quad (b^2 < 4ac)$
64. $\int \frac{x}{ax^2 + bx + c} dx = \frac{1}{2a} \ln |ax^2 + bx + c| - \frac{b}{2a} \int \frac{1}{ax^2 + bx + c} dx$
65. $\int \frac{1}{\sqrt{ax^2 + bx + c}} dx = \frac{1}{\sqrt{a}} \ln |2ax + b + 2\sqrt{a} \sqrt{ax^2 + bx + c}| \quad (a > 0)$
 $= \frac{1}{\sqrt{-a}} \sin^{-1} \frac{-2ax - b}{\sqrt{b^2 - 4ac}} \quad (a < 0)$
66. $\int \sqrt{ax^2 + bx + c} dx = \frac{2ax + b}{4a} \sqrt{ax^2 + bx + c} + \frac{4ac - b^2}{8a} \int \frac{1}{\sqrt{ax^2 + bx + c}} dx$
67. $\int \frac{x}{\sqrt{ax^2 + bx + c}} dx = \frac{\sqrt{ax^2 + bx + c}}{a} - \frac{b}{2a} \int \frac{1}{\sqrt{ax^2 + bx + c}} dx$
68. $\int \frac{1}{x\sqrt{ax^2 + bx + c}} dx = \frac{-1}{\sqrt{c}} \ln \left| \frac{2\sqrt{c} \sqrt{ax^2 + bx + c} + bx + 2c}{x} \right| \quad (c > 0)$
 $= \frac{1}{\sqrt{-c}} \sin^{-1} \frac{bx + 2c}{|x|\sqrt{b^2 - 4ac}} \quad (c < 0)$
69. $\int x^3 \sqrt{x^2 + a^2} dx = \left(\frac{1}{5} x^2 - \frac{2}{15} a^2 \right) \sqrt{(a^2 + x^2)^3}$
70. $\int \frac{\sqrt{x^2 \pm a^2}}{x^4} dx = \frac{\mp \sqrt{(x^2 \pm a^2)^3}}{3a^2 x^3}$
71. $\int \sin ax \sin bx dx = \frac{\sin(a - b)x}{2(a - b)} - \frac{\sin(a + b)x}{2(a + b)} \quad (a^2 \neq b^2)$

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$$72. \int \sin ax \cos bx \, dx = -\frac{\cos(a-b)x}{2(a-b)} - \frac{\cos(a+b)x}{2(a+b)} \quad (a^2 \neq b^2)$$

$$73. \int \cos ax \cos bx \, dx = \frac{\sin(a-b)x}{2(a-b)} + \frac{\sin(a+b)x}{2(a+b)} \quad (a^2 \neq b^2)$$

$$74. \int \sec x \tan x \, dx = \sec x$$

$$75. \int \csc x \cot x \, dx = -\csc x$$

$$76. \int \cos^m x \sin^n x \, dx = \frac{\cos^{m-1} x \sin^{n+1} x}{m+n} + \frac{m-1}{m+n} \int \cos^{m-2} x \sin^n x \, dx \\ = -\frac{\sin^{n-1} x \cos^{m+1} x}{m+n} + \frac{n-1}{m+n} \int \cos^m x \sin^{n-2} x \, dx$$

$$77. \int x^n \sin ax \, dx = -\frac{1}{a} x^n \cos ax + \frac{n}{a} \int x^{n-1} \cos ax \, dx$$

$$78. \int x^n \cos ax \, dx = \frac{1}{a} x^n \sin ax - \frac{n}{a} \int x^{n-1} \sin ax \, dx$$

$$79. \int x^n e^{ax} \, dx = \frac{x^n e^{ax}}{a} - \frac{n}{a} \int x^{n-1} e^{ax} \, dx$$

$$80. \int x^n \ln ax \, dx = x^{n+1} \left[\frac{\ln ax}{n+1} - \frac{1}{(n+1)^2} \right]$$

$$81. \int x^n (\ln ax)^m \, dx = \frac{x^{n+1}}{n+1} (\ln ax)^m - \frac{m}{n+1} \int x^n (\ln ax)^{m-1} \, dx$$

$$82. \int e^{ax} \sin bx \, dx = \frac{e^{ax} (a \sin bx - b \cos bx)}{a^2 + b^2}$$

$$83. \int e^{ax} \cos bx \, dx = \frac{e^{ax} (b \sin bx + a \cos bx)}{a^2 + b^2}$$

$$84. \int \operatorname{sech} x \tanh x \, dx = -\operatorname{sech} x$$

$$85. \int \operatorname{csch} x \coth x \, dx = -\operatorname{csch} x$$

Greek Alphabet

α	alpha	ι	iota	ρ	rho
β	beta	κ	kappa	σ	sigma
γ	gamma	λ	lambda	τ	tau
δ	delta	μ	mu	υ	upsilon
ϵ	epsilon	ν	nu	ϕ	phi
ζ	zeta	ξ	xi	χ	chi
η	eta	\omicron	omicron	ψ	psi
θ	theta	π	pi	ω	omega