EVAPORATION FROM LAKES

By N. W. Cummings and Burt Richardson

Abstract

Evaporation can be determined by the aid of the first law of thermodynamics in such a way that wind velocity need not enter the calculation. Air temperature and humidity enter only as terms in a correction which can have a relatively small average value under typical conditions. The complete equation is

$$E=(H-S-C)/L(1+R)$$

where E is the evaporation, H the difference between the incoming and outgoing radiation, S the heat stored in a column of water having unit cross-section, C a correction for heat carried by flowing water and leakage of heat through the walls of the vessel, L the latent heat of water, and R is Bowen's ratio. A method of finding the difference between the incoming and outgoing radiation, by means of observations on a well insulated pan is described, and also a method of finding the ratio of sensible heat to latent heat transmitted through the air-water surface. Bowen's theoretical conclusions respecting this ratio were found to be consistent with observations.

The above formula can be used in estimating evaporation from an actual lake whenever the requisite data can be obtained. Although from a physical standpoint the quantity of sensible heat passing through the air-water surface is not strictly negligible, nevertheless is most meteorological and engineering applications, the error caused by neglecting this quantity will be negligible in comparison with other errors that enter the main problem of which evaporation is a part.

Introduction

IT HAS long been customary to estimate evaporation from lakes on the basis of observations on pans in the vicinity, but there has been no means of showing conclusively that the error thus involved is not excessive. Thus, pans of different sizes are known to have different depths of evaporation[1] and this obviously means that the depth of evaporation from a lake cannot in general be the same as that from any particular pan. If the theory described in this paper proves to be correct, substantial progress in the evaluation of evaporation from lakes will have been made.

Theory

Evaporation must obviously be uniquely determined by the energy that is rendered available for that purpose. Expressed as an equation this is

$$E=(I-B-S-K-C)/L \tag{1}$$

where E is the evaporation expressed in centimeters of depth, I is the incoming radiation per square centimeter of surface, B the back-radiation to the sky, S the heat consumed in warming the water, K a correction due to heat derived from the air or given up to the air as sensible heat, C a combined correction

[1] R. B. Sleight, Jour. Ag. Res. **10**, 209 (1917).

for heat leakage through the walls of the container, expansion of the water, and in the case of a lake, for heat carried by flowing water. It is supposed that all the quantities in the numerator are expressed in gram calories per square centimeter of open water surface and are to be integrated over the same time interval as E.[2] The heat of vaporization of water expressed in gram calories per cubic centimeter is denoted by L.

If it is possible to observe all the elements on the right side of Eq. (1) except K, then to compute K by means of a certain equation derived by Bowen,[3] which will presently be stated, and finally to compare the right side with the observed evaporation placed on the left side, the agreement between the right and left sides will not only constitute a check on Bowen's equation, but it will also show whether or not evaporation can be computed without making any measurements of wind velocities, as the general theory demands. The elimination of wind from evaporation calculations is one of the fundamental ideas previously emphasized by one of the present writers.[4]

Such a comparison of the right hand side of Eq. (1) with the observed evaporation has virtually been made, though in order to avoid the necessity of measuring I and B directly, a slight variant on the above procedure was adopted, as follows:

We first introduce the quantity R as defined by the relation

$$LER=K \tag{2}$$

This means that R is the ratio of sensible heat swept away by the wind to latent heat carried off by the vapor. This ratio has been discussed by Anders Angstrom,[5] the Swedish meteorologist, who finds that it averaged less than 0.10 for the North Atlantic during the forepart of September, 1905. Angstrom concluded that R is in general relatively small, but until now it has been extremely difficult to obtain extensive and reliable information with regard to this quantity. However, Bowen has recently given us an equation which makes it possible to compute R from ordinary atmospheric data, without knowing either the sensible heat or the latent heat separately. His equation is

$$R=0.46\frac{t_1-t_2}{p_1-p_2}\,\frac{P}{760} \tag{3}$$

where (t_1-t_2) is the difference between the temperature of the air and that of the water surface, and (p_1-p_2) is the difference between the vapor

[2] It may not be obvious that S and C can be thus expressed, but if the total amount of heat stored in the entire lake or entering by the methods to which C refers be divided by the lake surface-area, then a definite part of each of these total quantities of heat will be allocated to a column under each square centimeter. By thus centering attention on a column of water having a square centimeter cross-sectional area and a depth equal to the average depth of the lake we make all the quantities in the numerator comparable and can express them in the same units.

[3] I. S. Bowen, Phys. Rev. **27**, 779 (1926).

[4] N. W. Cummings, Jour. of Electricity **46**, 491 (May 15, 1921).

[5] A. Angstrom, Geografiska Annaler **3**, 13 (1920).

pressure of the moisture in the air and the pressure this would have if the air were saturated at the temperature of the water surface, while P denotes the barometric pressure expressed in millimeters. Eqs. (2) and (3) together serve to eliminate K from Eq. (1) and to express the quantity R in terms of easily measurable magnitudes.

Next, after substituting $L\ E\ R$ for K in Eq. (1), we solve for $(I-B)$, which will hereafter be denoted by H. This is really the difference between the solar and sky radiation which penetrates the water surface, and the energy which the water body radiates back to the sky. For convenience it will be called the *heat budget per square centimeter of surface.*

$$H = I - B = S + LE(1+R) + C \qquad (4)$$

Now I must be the same for all water surfaces exposed to the same external conditions whether the water bodies are shallow and heat up rapidly or deep and therefore change slowly in temperature. This equality does not necessarily hold for B, however, because if two bodies of water have different temperatures they must radiate energy at different rates. For two surfaces exposed to the same conditions we can evidently write from Eq. (4) on account of the equality of I_1 and I_2

$$B_1 - B_2 + S_1 + LE_1(1+R_1) + C_1 = S_2 + LE_2(1+R_2) + C_2 \qquad (5)$$

Since Bowen has pointed out that water radiates practically like a black body[6] because it almost completely absorbs low temperature radiation we may compute $B_1 - B_2$ by means of Stefan's law, thus

$$B = 49.5 \times 10^{-10} T^4 \text{ gram calories per sq cm per hour.} \qquad (6)$$

Because $T_2 - T_1$ in our application does not exceed 15°C and T_2 and T_1 are always between 0°C and 35°C we may calculate $B_1 - B_2$ with sufficient accuracy if we differentiate Eq. (6) with respect to T and substitute 290 for T, obtaining

$$dB/dT = 198 \times 10^{-10} \times (290)^3 = 0.5 \text{ calories per sq cm per hour} \qquad (7)$$

and then multiply by the difference between the temperatures of the two surfaces. These considerations make it possible to subject Eq. (5) to experimental test.

Experimental Procedure

Some of the experimental work was done in Pasadena and some in Fort Collins, Colorado. At both places two bodies of water were used, one being contained in a tank and the other in a pan, the same pan being used in both cases. The dimensions of the vessels are as follows:

	Pan	Tank Pasadena	Tank Fort Collins
Area	0.37 sq. m.	2.2 sq. m.	520.0 sq. m.
Depth of vessel	0.19 m.	1.47 m.	2.0 m.
Depth of water below rim	0.01 m.	0.02 m.	0.07 m.

[6] See also Abbot, Annals of the Astrophysical Observatory, vol. IV, p. 291.

Both vessels used in Pasadena were thoroughly insulated, but corrections were applied to take account of such small quantities of heat as did leak through the walls. At Fort Collins the tank was so large as to render insulation unnecessary

The evaporation loss was determined by observing the amount of make-up water required to bring the level back to a predetermined height defined by a wire projecting upward from below the surface The error of this measurement lay between one tenth and two hundredths of a millimeter. The surface temperature of each body of water was measured by means of a mercury in glass thermometer graduated in tenths, but for the change in temperature, a Beckman instrument readable to one thousandth of a degree centigrade was used. The water was stirred before reading the volume temperature. The air temperature and absolute humidity were obtained by an Assmann psychrometer.

Since Eq. (3) was derived for instantaneous values of the variables, continuous records for the entire time interval should be used, but in this work hourly readings were used, which prove to be a good substitute.

Bowen's value of K was checked experimentally by many runs: one especially interesting run was made Oct. 24, 1925 when the pan was maintained for seven hours at 30°C by a measured amount of electric energy, and when this injected energy was deducted a value of H was obtained which agreed well with the one computed from the tank by the aid of the theory.

TABLE I.

The number of gram calories per square centimeter per day making up each term of Eq. (1).

	Pasadena, California June 23, 1926–July 3, 1926.									
Day	1	2	3	4	5	6	7	8	9	10
Tank Evaporation (E)	376.8	339.3	343.4	379.1	426.1	439.0	470.8	446.9	436.1	425.5
" Warming (S)	40.1	−8.3	100.2	154.3	15.3	108.0	−31.0	−69.7	−85.0	15.4
" Convection (K)	64.4	80.3	54.0	−2.8	5.4	15.7	51.5	91.4	83.4	80.4
" Heat Budget ($I-B$)	481.3	411.3	497.6	530.6	446.8	562.7	491.3	468.6	434.5	521.3
Pan Evaporation	393.6	394.2	399.0	507.3	489.1	436.6	500.2	359.8	343.8	445.8
" Warming	34.8	−7.5	−32.1	−56.7	18.6	−9.8	46.8	65.9	−28.9	−64.6
" Convection	60.9	57.8	60.7	45.3	16.1	2.6	2.1	50.9	62.5	72.5
B_1-B_2	−27.7	−37.8	−32.7	−20.7	−8.3	−9.4	−1.0	−17.0	−23.5	−10.2
Pan Leakage (C)	30.1	27.5	29.8	24.8	14.1	4.8	10.2	28.1	30.8	34.8
" Heat budget	491.7	434.2	424.7	500.0	529.8	424.8	558.3	487.4	384.7	478.3
	Fort Collins, Colorado September 15–25, 1926.									
Day	1	2	3	4	5	6	7	8	9	10
Tank Evaporation	171.7	145.3	127.0	134.1	124.3	231.1	224.6	155.4	291.9	246.1
" Warming	85.0	79.9	13.6	83.3	−57.8	44.3	78.2	11.8	−168.3	−333.2
" Convection	3.5	4.6	29.3	27.6	40.1	−10.2	−7.9	39.3	69.2	142.5
" Heat budget	260.2	229.8	169.9	245.0	106.6	265.2	294.9	206.5	192.8	55.4
Pan Evaporation	272.3	237.5	166.9	191.3	119.7	345.5	322.1	223.8	309.5	131.9
" Warming	2.4	41.2	−21.8	49.7	−19.6	5.4	27.3	−58.1	−130.5	−80.0
" Convection	7.2	−22.4	13.1	15.8	28.2	−46.0	−37.5	41.7	26.2	66.5
B_1-B_2	−22.5	−23.8	−20.7	−24.7	−41.5	−16.1	−20.3	−15.4	−62.6	−140.2
Pan Leakage	10.0	−0.7	16.1	18.1	18.8	−14.6	−7.9	33.7	31.4	48.8
" Heat budget	269.4	231.8	153.6	250.2	105.6	274.2	283.7	225.7	174.0	26.8

Two runs of ten days each were made under quite dissimilar circumstances, the first in Pasadena June, 1926 and the second in Fort Collins September, 1926. The results of these experiments indicate the relative importance of each factor in the evaporation equation.

Comparison with Theory

Eq. (4) was used to compute H, the heat budget, or the resultant radiation for each body of water. The correction $B_1 - B_2$ was calculated by means

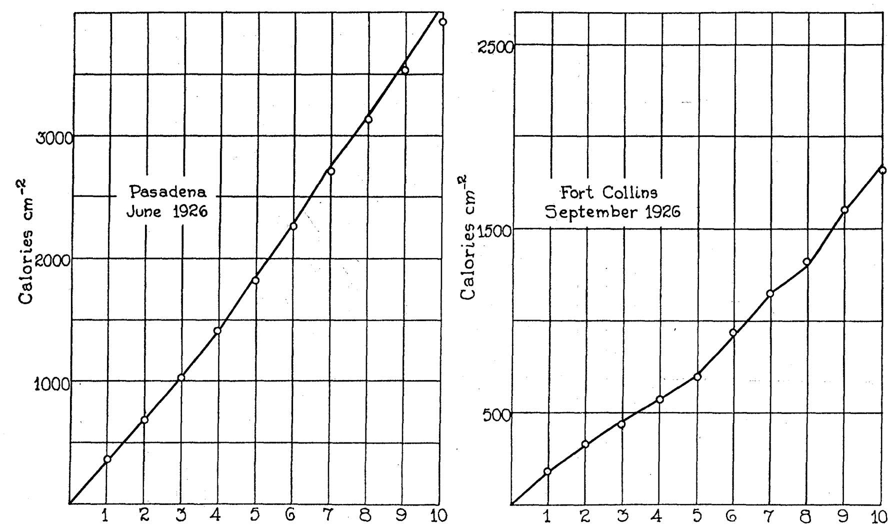


Fig. 1. Ten day summation curves for Pasadena, June, 1926, and Fort Collins, September, 1926, where solid lines represent the observed tank evaporation in calories per sq cm and the lines of circles indicate the tank evaporation calculated from the pan data.

of Eq. (7) and applied to the budget of the pan in order to reduce H of the pan to the temperature of the tank as indicated by Eq. (5). This process

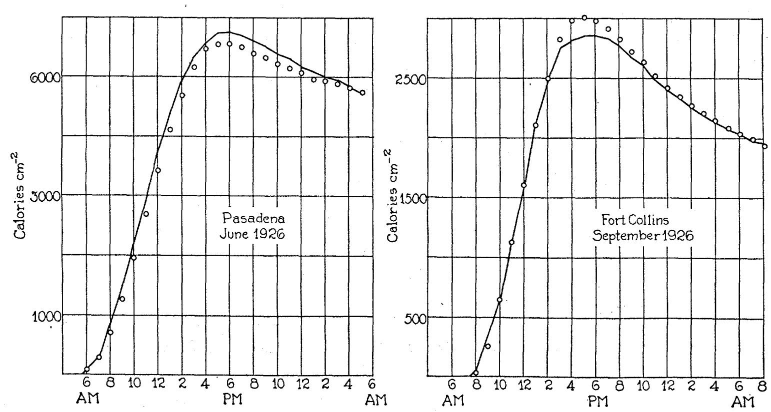
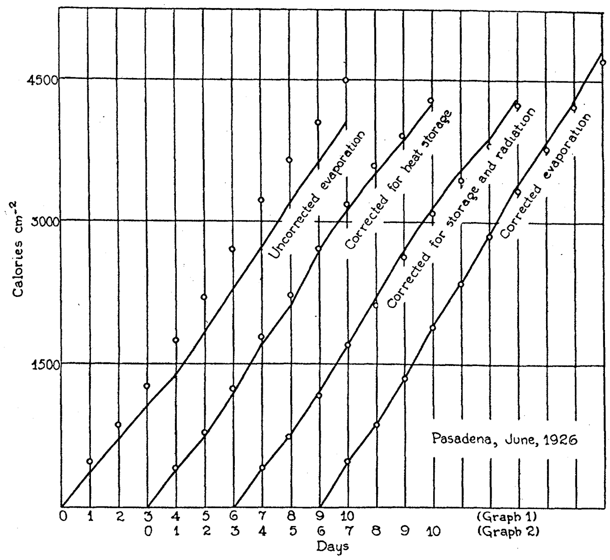


Fig. 2. Summation curves for Pasadena, June, 1926, and Fort Collins, September, 1926. The solid lines represent the heat budget of the tank for a particular hour measured on ten consecutive days, and the lines of circles indicate the same for the pan.

should then have brought the two budgets into agreement, which it did. For the purpose of comparison, cumulative heat budgets (obtained by a process of continued summation for each body of water) were used. Table I gives the numerical values of the various terms of the evaporation equation as they were determined by the ten day runs at Pasadena and Fort Collins.



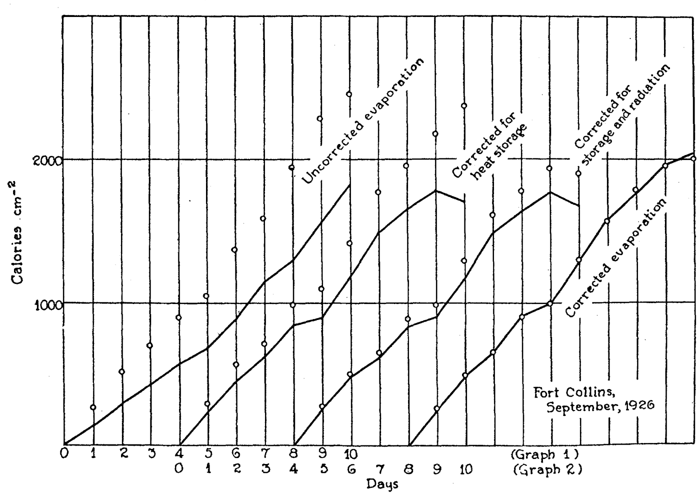


Fig. 3. Curves which indicate the marked improvement in the correspondence of the tank (solid lines) and pan (lines of circles) evaporations upon the introduction of corrections for heat storage, back radiation and convection losses.

Fig. 1 presents the Pasadena and Fort Collins runs where the observed and calculated evaporations have been plotted against time.

Fig. 2 presents Pasadena and Fort Collins summation curves of the tank and pan heat budgets of a particular hour of the day; here the budgets of such an interval of time—say from 9 to 10 A.M.—have been computed on ten consecutive days and added. The slopes of these curves show that radiant energy starts to produce an effect upon a body of water at an early hour in the morning, increases until noon (point of maximum slope), and then decreases until evening, after which time and during the night the body of water radiates to the sky.

Fig. 3 shows for Pasadena and Fort Collins respectively the relative value of each of the correction terms in the evaporation equation. It is evident that the uncorrected evaporations disagree widely in all cases. In Pasadena the most important correction was heat-storage while in Fort Collins it was back-radiation.

Fig. 4 shows the record of a thermo-electric pyranometer and the heat budgets of the tank and pan plotted against time over a ten day period in Pasadena.

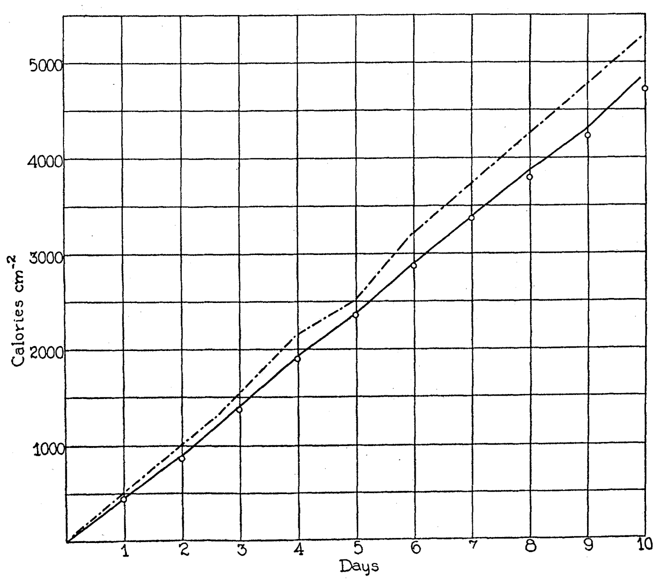


Fig. 4. Pasadena, June, 1926. Curves show the record of a thermo-electric pyranometer (broken line) and the summation heat budgets of the tank (solid line) and pan (line of circles) over a ten day period.

From a study of the graphs and Table I it is seen that by using Bowen's theory we bring into agreement the two heat budgets which we believe on the basis of the conservation of energy ought to agree. In other words, the theory works, and on this ground it is justified. The foregoing is therefore a correct method of computing H. It follows that we can take the pan to a lake, compute H as has been done here, and, after making the B_1-B_2 correction, apply this H to the lake. We may then determine S, R, and C for the lake and compute E by the following equation

$$E=(H-S-C)/L(1+R) \tag{8}$$

which is obtained from Eq. (1) on substituting $L\ E\ R$ for K and solving for E. It will be seen that wind-velocity does not enter this equation.

This can be taken as the primary standard for calibrating all other methods of determining lake evaporation. Other equations such as that of Dalton contain certain empirical constants which can only be evaluated when a series of real lake evaporations are known. The procedure outlined herein furnishes the only method thus far devised for ascertaining these required lake evaporations with satisfactory precision and reliability. Like most primary standards this method is difficult to apply because of the necessity of hourly readings. However, for many purposes where great accuracy is not required the work can be shortened considerably by adopting empirical constants.

On superficial examination it might appear that this research is nothing but a comparison of evaporation from two vessels of different dimensions, with certain corrections applied. As a matter of fact it is more than that. While it is true that H has been determined by means of observations on the pan, nevertheless this was merely an expedient that had to be resorted to because as yet no technique has been developed for measuring B with satisfactory precision and convenience. If such a technique ever is developed, then lake evaporation can be found without using any pan and without making measurements of wind velocity. Moreover, if it turns out that the effect of the atmosphere in causing a transfer of sensible heat really is small under conditions that might be called normal—which is not unlikely—then air temperature and humidity need enter the calculations only as terms in a relatively small correction which for many purposes may be neglected entirely. Obviously then it is worth while to accumulate as much information as possible in regard to the value which R assumes at various places and various times of the year. This can be done by means of Bowen's equation, when the requisite wet and dry bulb and water temperatures become available.

In conclusion the writers wish to express their appreciation of the many helpful suggestions they have received from Dr. Millikan and other members of the California Institute faculty, and of the assistance given by Paul Richardson in carrying out the observations. Also thanks are extended to R. L. Parshall of the U. S. Department of Agriculture for making possible the use of certain tanks at Fort Collins, Colorado.

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