claim to independence, seemed to me a less courteous step. No question of priority of publication did or could arise, as this was assured to him by his abstract of 1927 December.

The height at which mixing ceases is important for the constitution of the upper atmosphere, but it remains speculative because the degree of mixing by winds at great heights is not known. For some years prior to 1920 the height was generally assumed to be about 10 km; E. A. Milne and I then broke away from that idea (Q. J. R. Meteor. Soc. **46,** 357, 1920), but not to the extent that Dr. Maris and I have since thought necessary. In 1926 (Proc. R. Soc. **A111,** 4, lines 24–26) I mentioned the possibility that mixing extends up to auroral heights, though without giving my reasons. Early in 1927 Mr. T. W. Dickson and I made calculations as to the rate of diffusive stratification of the air at great heights, but in view of the abstract and paper by Dr. Maris we have not published them. Such calculations were made as early as 1914 by Gouy (Comptes rendus **158,** 664).

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Evidence for the Richtmyer Double Jump Hypothesis of X-Ray Satellites

Richtmyer[1] has recently proposed the hypothesis that many of the x-ray satellite lines may be due to double transitions in which two electron transitions cooperate to emit one quantum. I should like to call attention to a number of known facts about x-ray satellites which are very satisfactorily explained on this hypothesis several of which facts have not yet been mentioned by Richtmyer.

The main facts known and published to date about x-ray satellites may be roughly summarized as follows:

1. Definition: An x-ray satellite is a line whose frequency is not directly derivable from the known system of x-ray absorption levels. (Lines which *can* be so derived are called "diagram" lines.)

2. Satellites are generally very close to some "parent" diagram line and on the short wave-length side.

3. The same satellite can be identified for different elements and the frequency difference $\Delta\nu$ between the satellite and the parent line follows a Moseley diagram when $(\Delta\nu/R)^{1/2}$ is plotted against atomic number Z.

4. Satellites are much less intense than the parent line. They appear abruptly in the series of increasing atomic numbers about at the point where a new electron shell two levels higher (at least) than the terminal level of the parent line starts to form. Thus K satellites appear about when the M level starts to form.

5. They are intense near this abrupt beginning point and fall off in intensity relative to the parent line as the atomic number increases.

6. Richtmyer believes he has observed something like a continuous spectrum associated with the satellites of a given parent line.

As additional information to the above facts the author in collaboration with Mr. A. Hoyt has just finished an investigation with the double crystal spectrometer soon to be published in this Journal on the excitation potential of the satellite $K\alpha_3$ of copper. We find

7. That the satellite Cu $K\alpha_3$ is excited at not more (if at all) than 200 or 300 volts higher critical potential than the parent line (89 K.V.).

8. Taht the satellite intensity is rigorously proportional to the exciting cathode-ray current.

9. That the ratio of the intensities $K\alpha_3/K\alpha_1$ is about 1/120 based on the areas of the lines α_3 and α_1 (integrated over their breadth), but only about 1/440 based on the maximum ordinate values. In other words, α_3 is nearly 4 times as broad as α_1.

10. $K\alpha_3$ of Cu is a doublet. This is in accord with recent observations of Richtmyer and with the doublet structure of $K\alpha_3$ for lighter elements.

Let us now consider how many of the above facts are explained and correlated by the Richtmyer double jump hypothesis. For definitness and lucidity let us consider the line $K\alpha_3$ of copper as explained by the double transition $N\rightarrow M$ and $L\rightarrow K$. As Richtmyer has pointed out facts (1) to (3) inclusive are explained since the extra energy of the satellite as compared to its parent represented by

[1] Richtmyer, Jour. Frank. Inst. **208,** 325–361 (1929).

the frequency difference $\Delta\nu$ is interpreted as the energy of the transition $N\rightarrow M$ which is of the right order of magnitude and would of course follow a Moseley diagram.

Fact (4) is explained because unless there were at least two levels above the terminal level of the parent line any double transition that might occur (e.g. $M\rightarrow L$; $L\rightarrow K$) would be equivalent to a diagram line ($M\rightarrow K$). The fact that the satellites do not appear at exactly the point in the atomic table where the third level higher up starts to form (sometimes one or two atomic numbers lower) is not to be taken as contrary evidence because it must be recollected that x-ray satellites always come from targets in the *solid* state and it is highly likely that the peripheral electrons of atoms in a crystal lattice behave differently from those of free atoms as in the vapor state.

Fact 6 needs no comment that Richtmyer has not already made.

Facts 7 and 8 seem to invalidate the Wentzel-Druyvesteyn spark line hypothesis which requires for say a K satellite that the atom be doubly ionized in deep levels. This would have to be done either (a) in a single collision in which case the excitation potentials of satellites would be much greater (about double) than the excitation potentials of the parent, or (b) in two successive collisions in which case the intensity of the satellite would vary as the square of the current. Facts 7 and 8 are in accord with the double jump hypothesis since only a few extra volts would be necessary to ionize the M level at the same time as the K level in the case of K satellites.

Facts 5 and 9 are in accord with information given the writer by J. R. Oppenheimer on the theory of the probability of double excitation and double emission. Quoting from a letter of Oppenheimer's to the writer he says,

"If there were no coupling between the M, N electrons on the one hand with the K, L electrons, then both the probability of a double excitation and that of a double emission would vanish; because of the coupling in each case the ratio of the probability of the double jump to that of the single jump involving only the more tightly bound electron has a finite value; the value of the ratio r is very nearly the same for the excitation and the emission so that $R\sim r^2$. Nor r is given by the overlapping, or quantitatively by the so-called scalar product of the wave functions for an electron in the M and N shells, when the interaction of all electrons is considered, and this in turn is given, as one sees, by working out the perturbation of the electrons on each other, roughly by the square of the mean interaction of an L and an M electron divided by the square of the energy difference $M-N$. If one takes your value of 30 volts for the $M-N$ difference and your value for $R=1/240$, [assuming a doublet $K\alpha_3$ of two equal members] then one gets for the mean interaction energy about 8 volts, which is not at all unreasonable for an L and an M electron of copper. (The apparent neglect of the K and N shells comes from this, that the mean interaction of electrons in these states is negligibly smaller than the $L-M$ interaction.) I believe, therefore, that Richtmyer's interpretation is applicable to this satellite."

Dr. W. V. Houston has pointed out that the progressive decrease of intensity of a given satellite with increase of atomic number is beautifully explained by the above remark of Oppenheimer for since the ratio

$$R=r^2=\left[\frac{(\text{mean interaction energy of } L \text{ and } M)^2}{(\text{energy difference } M-N)^2}\right]^2$$

the decrease of R with atomic number is seen to be due to the Moseley increase of the energy difference $M-N$ entering to the fourth power in the denominator of the above expression.

Finally the writer believes there may be some significance in the apparently greater breadth of the satellite $K\alpha_3$ of copper. As mentioned under (9), the ratio of $K\alpha_3/K\alpha_1$ for areas (1/120) disagrees with the same ratio for maximum ordinates (1/440) by a factor of almost 4. If the satellite were a poorly resolved doublet of two equally intense lines the disagreement would be by a factor of two only. If the lines were not equally intense the factor would be less than two. To explain a factor of more than two, it is necessary to suppose that each of the two members of $K\alpha_3$ are broader than the parent line $K\alpha_1$, indeed almost twice as broad. The writer wishes to suggest that this extra breadth may be precisely attributed to the almost continuous dense distribution of energy levels for the peripheral (N) electrons required in the solid target by the Fermi statistics shown by A. Sommerfeld.[2] The energy of the $N \rightarrow M$ transition would be indefinite to just the extent defined by the energy breadth of the Fermi distribution of the conduction electrons in copper. The excess broadening of $K\alpha_3$ of copper over $K\alpha_1$ corresponds to an energy of about 12 volts a value which is in qualitative agreement with the probable energy value of the Fermi distribution as well as the experimental and theoretical uncertainties warrant.

The field of x-ray satellites may therefore prove to have an important bearing on the field of the electron theory of metals and metallic conductors making possible a study of the behavior of peripheral electrons *in the solid state* which would be impossible with optical spectra.

It would seem that about all the known facts of x-ray satellites are explained qualitively by Richtmyer's double-jump hypothesis and only await further experimental work for a quantitative verification.

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[2] A. Sommerfeld, Zeits. f. Physik **47**, 1–60 (1928).

Hyperfine Structure of X-Ray Lines

In the June 15 (1930) issue of The Physical Review Professor G. Breit discusses the possible effect of nuclear spin on x-ray terms. By straightforward calculation from the Dirac equation for a Coulomb field of force, and by use of only minor approximations, he shows that K terms of the heavier elements should be split into two components with a frequency difference $\Delta\nu$ given by

$$\Delta\nu = \left(\frac{8\pi^2 mc^2}{h^2}\right)^3 \cdot Z^3 \cdot \left(k + \frac{1}{2}\right) \cdot \frac{1840\mu}{2k\mu_0(2\rho^2 - \rho)} \mathrm{cm}^{-1}$$

where m, c, h, Z have their usual meanings; k and μ are respectively the angular momentum and magnetic moment of the nucleus; μ_0 is the Bohr magneton; and ρ is $(1-\alpha^2)^{1/2}$, where $\alpha \equiv (2\pi e^2/ch)Z$. Taking $k=9/2$ and assuming $1840\mu/2k\mu_0 = 1$, Breit finds that the $WK\alpha_1$ line should contain two components separated by 7.3 volts—or by 1 part in 8092. In wave-length, this is a separation of $\Delta\lambda = 0.026$ X.U., taking $\lambda K\alpha_1$ of tungsten as 208.8 X.U; in angle, it is a separation of 0.86 seconds of arc, for a calcite crystal, first order.

With the direct-reading two-crystal spectrometer developed in this laboratory (Phys. Rev. **35**, 1428, June 1, 1930) we have searched for this predicted fine structure of $WK\alpha_1$. We were unsuccessful in detecting any certain evidence in first order, but on going to the higher resolution available in fifth order we find distinct evidence that there are two peaks separated, in that order, by some 5″ of arc—corresponding to a $\Delta\lambda$ in first order of about 0.03 X.U., substantially as predicted by Breit. A qualitative check was obtained by observations in fourth order. The long wavelength component appears to be the more intense by some 25 or 50 percent. However, the measurements as yet are not sufficiently precise to warrant reporting more than qualitative data as regards separation or relative intensity.

To check whether this separation might possibly be due to some peculiarity of the crystals used we very carefully repeated the observations on first order; and also measured the $\mathrm{Mo}K\beta$ doublet. Crystal imperfections, e.g., twinning, should cause the same apparent angular separation into components, regardless of wave-length. In these cases no separations as great as 1″ of arc could be found.

A detailed report will be made later after we have had opportunity to obtain more precise and complete data.

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August 15, 1930.