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LECTURE NOTES ON SEDIMENT TRANSPORTATION AND CHANNEL STABILITY

Vito A. Vanoni, Norman H. Brooks and John F. Kennedy

W. M. Keck Laboratory of Hydraulics and Water Resources
CALIFORNIA INSTITUTE OF TECHNOLOGY
Pasadena, California

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SEDIMENT TRANSPORTATION AND CHANNEL STABILITY

by

Vito A. Vanoni, Norman H. Brooks and John F. Kennedy

Notes prepared for a series of lectures for Technicians
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FOREWORD

These notes have been prepared for a series of lectures on sediment transportation and channel stability given by the authors to a group of engineers and geologists of the U. S. Department of Agriculture assembled at Caltech on September 12-16, 1960. The material herein is not intended to serve as a complete textbook, because it covers only subjects of the one-week sequence of lectures. Due to limitation of space and time, coverage of many subjects is brief and others are omitted altogether. At the end of each chapter the reader will find a selected list of references for more detailed study.

TABLE OF CONTENTS

Section		Page
	List of Symbols	vi
CHAPTER 1.	HYDRAULICS OF OPEN CHANNELS	
	Fundamental Equations and Concepts	
1.1	Bernoulli and Continuity Equations	1-1
1.2	Concept of Head	1-2
1.3	Head Loss, Friction and Turbulence	1-2
1.4	Reynolds Number	1-3
1.5	Specific Energy, Wave Velocity and Froude Number	1-4
	Flow Formulas	
1.6	Chezy Formula	1-5
1.7	Chezy Coefficient	1-6
1.8	Equations for Two-Dimensional Flow	1-7
	Velocity Distribution	
1.9	Shear Stress	1-7
1.10	Velocity Distribution	1-8
	Non-Uniform Flow in Open Channels	
1.11	Definition	1-12
1.12	Characteristics of Non-Uniform Flow in Open Channels	1-12
	References	1-13
CHAPTER 2.	FLOW IN ALLUVIAL CHANNELS	
2.1	Introduction	2-1
2.2	Bed Forms and Their Formation	2-1
	(a) Dunes	2-1
	(b) Flat bed	2-3
	(c) Antidunes	2-5
2.3	Effect of Bed Forms on Roughness	2-6
2.4	Description of Transport and Definition of Load	2-7
2.5	Statement of the Problem of Hydraulics of Alluvial Channels	2-8
	References	2-10

Section		Page
CHAPTER 3. PROPERTIES OF SEDIMENT		
3.1	Particle Size and Shape	3-1
	(a) Sedimentation diameter	3-1
	(b) Sieve diameter	3-1
	(c) Nominal diameter	3-1
	(d) Triaxial dimensions	3-1
3.2	Density and Petrographic Analysis	3-2
3.3	Settling Velocity	3-2
	(a) Spheres	3-2
	(b) Stokes law	3-2
	(c) Sedimentation diameter	3-5
	(d) Hindered settling	3-5
	(e) Flocculation	3-6
3.4	Frequency Distributions	3-6
	(a) Definition of cumulative frequency distribution	3-6
	(b) Arithmetic probability paper for normal distribution	3-9
	(c) Logarithmic probability paper	3-12
	(d) Typical values of σ_g	3-15
3.5	Measurement of Size Distribution	3-16
	(a) Sieve analysis	3-16
	(b) Settling analysis for finer particles	3-16
	References	3-19
	Appendix 3-A. Memorandum on Procedures for Sieve Analyses of Natural Sands	3-20
CHAPTER 4. INITIATION OF SEDIMENT MOTION		
4.1	Introduction	4-1
4.2	Theory: White's Analysis	4-1
4.3	Theory: Shields' Analysis	4-3
4.4	Probable Effect of Bed Form on Results	4-6
4.5	Example Problem	4-6
	References	4-8
CHAPTER 5. THEORY OF SEDIMENT TRANSPORTATION		
5.1	Introduction	5-1
5.2	Mechanism of Sediment Suspension	5-1
5.3	Distribution of Suspended Load	5-2

Section		Page
5.4	Validity of Distribution Equation	5-5
5.5	Application of Distribution Equation	5-8
	References	5-9
CHAPTER 6. SEDIMENT DISCHARGE FORMULAS		
6.1	Introduction	6-1
6.2	Sediment Discharge Formulas	6-1
	(a) Meyer Peter Formula	6-2
	(b) Schoklitsch formula	6-2
	(c) Duboys formula	6-3
	(d) Shields formula	6-3
	(e) Einstein-Brown formula	6-3
	(f) Meyer Peter-Muller formula	6-4
	(g) Laursen formula	6-4
	(h) Einstein bed load function	6-5
6.3	Flow Formulas	6-5
	References	6-9
CHAPTER 7. COMPARISON OF SEDIMENT DISCHARGE FORMULAS		
7.1	Basis for Comparison	7-1
7.2	Results of Calculations	7-10
7.3	Recommendations	7-13
	References	7-14
CHAPTER 8. ROUGHNESS OF ALLUVIAL CHANNELS		
8.1	Need for Knowledge of Roughness of Alluvial Channels	8-1
8.2	Example of Variable Roughness; Laboratory Data	8-2
8.3	Example of Variable Roughness; Field Data	8-5
8.4	Estimating Channel Roughness	8-7
	References	8-8

Section		Page
CHAPTER 9. PRINCIPLES OF CHANNEL STABILITY		
9.1	Introduction	9-1
9.2	Definition of Stable Channel	9-1
9.3	Basic Requirements	9-1
	(a) Overall	9-1
	(b) Local	9-1
9.4	Stream Variables for Overall Stability	9-1
9.5	Solution of Equations for Overall Stability	9-5
9.6	Continuity Principle for Sediment	9-6
9.7	Bank Stability	9-7
9.8	Critical Shear Stress on Sloping Banks	9-8
9.9	Shear Distribution in Channels	9-9
9.10	Curves	9-9
9.11	Local Scour Due to Structures	9-13
	(a) Contractions such as bridge openings	9-13
	(b) Energy dissipators	9-13
	References	9-14
CHAPTER 10. REGIME OF CHANNELS		
10.1	Introduction	10-1
10.2	Definition of Regime	10-2
10.3	Blench's Regime Theory	10-2
10.4	Simons' and Albertson's Regime Method	10-5
10.5	Leopold and Maddock's Regime Analysis of Natural Streams	10-6
10.6	Example of Disturbed Regime	10-8
10.7	River Channel Patterns	10-9
	References	10-13
	Appendix 10-A	10-15

LIST OF SYMBOLS

A = area of channel cross section

a = length of longest axis of a particle

b = length of intermediate axis of a particle

b = width of channel

C = Chezy coefficient

c = length of shortest of the three mutually perpendicular axes of a particle

c = celerity of gravity wave

c = concentration of suspended sediment at distance y up from bed

c_a = concentration of suspended sediment at some reference level $y = a$

c_i = concentration of grains with settling velocity W_i

$\bar{c}$ = mean sediment discharge concentration in percent by weight

c_m = sediment discharge concentration in weight per unit volume

C = bed material load concentration in ppm

C_c = bed material load concentration in ppm at critical velocity

d = water depth or mean flow depth

D = pipe diameter

D = mean sediment size

D_{mg} = geometric mean sieve diameter

d_c = critical depth of flow

d_s = particle diameter

d_m = mean size of sediment, eq. 6.1a

d_{si} = mean size of a size fraction of the bed sediment

d_{90} = size of sediment in bed for which 90% by weight is finer

$\frac{e}{D}$ = relative roughness of boundary

F = Froude number

F = dimensionless quantity given by eq. 6.5c

f = Darcy-Weisbach friction factor

f_b = bed friction factor

F_b = V^2/d = bed factor

F_s = V^3/d = side factor

F_{bo} = bed factor for cases of small bed load material discharge

g = acceleration of gravity

g_s = sediment discharge in lbs per second per ft of channel width

h_f = head loss between two sections

h = total head

H = specific energy-head

k = von Karman constant 0.4 for clear water

L = suspended load eq. 10.11

m = exponent in eq. 7.1

n = Manning roughness coefficient

p = wetted permieter

P_i = weight percent of bed sediment within a size fraction

Q = water discharge

Q_s = sediment discharge

Q_{se} = equilibrium sediment discharge

q = water discharge per unit width of channel

q_{oi} = the value of q at which sediment motion begins

R = Reynolds number

r = hydraulic radius

S = slope of channel

s_s = specific gravity of particle

s_f = specific gravity of fluid

u_* = shear velocity

U_{*b} = bed shear velocity

V = mean velocity of flow

v' = turbulence velocity fluctuation

v'_y = instantaneous value of the vertical turbulence velocity

V_c = critical velocity

v = velocity at distance y from bed

w = settling velocity of sand grain

w = width of channel

X = random variable

z = elevation of channel bed

z = exponent in suspended load distribution eq. 5.2

ν = kinematic viscosity of fluid in ft^2/sec

τ_o = bed shear stress in lbs per sq ft

τ_c = critical bed shear stress

τ'_{oi} = bed shear stress due to grain roughness of a grain of size d_{si}

τ_{ci} = critical bed shear stress for particle of size d_{si}

γ = unit weight of water

γ_f = unit weight of water

γ_s = unit weight of sediment

λ = wave length

σ = standard deviation

σ_g = geometric standard deviation

ρ_s = density of sediment

ρ_f = density of fluid

ψ = coefficient depending on mean size of bed sediment, ft^3 per lb per second (Fig. 6.1).

ψ = function defined by eq. 6.5b

ϕ = function defined by eq. 6.5a

ϕ = angle of side slope of channel

θ = angle of repose of sediment

CHAPTER 1 - HYDRAULICS OF OPEN CHANNELS

FUNDAMENTAL EQUATIONS AND CONCEPTS

1.1 Bernoulli and Continuity Equations. In analyzing open channel flow the Bernoulli and the Continuity Equations are used. The Bernoulli equation for steady flow applied to a reach of open channel is, (see Fig. 1.1)

$$z_1 + d_1 + \frac{V_1^2}{2g} = z_2 + d_2 + \frac{V_2^2}{2g} + h_f \qquad (1.1)$$

where z is the elevation of the channel bottom in ft., d is the water depth in ft, V is the mean velocity in the cross section in fps, g is the acceleration of gravity usually taken as 32.2 ft per sec per sec and h_f is the head loss between sections in ft.

Fig. 1.1 Steady flow in an open channel

In equation 1.1 the subscripts 1 and 2 apply to the quantities at section 1 and 2 respectively. The mean velocity, V at a section is given by

$$V = \frac{Q}{A} \qquad (1.2)$$

where Q is the water discharge in cfs and A is the area of the water cross section in sq ft. Since for steady flow, Q does not vary from section to section one can write:

$$Q = A_1V_1 = A_2V_2. \qquad (1.3)$$

Equation 1.3 is known as the continuity equation, and states that the rate of flow or discharge past any section is the same as that past any

section is the same as that past any other section. The Bernoulli equation as expressed in eq. 1.1 applies only for cases where the depth of the flow changes slowly with distance along the stream. (Note: for further information on this subject see reference 1.1, pp 68-81).

1.2 Concept of Head. Each of terms in eq. 1.1 is referred to as a "head". The term $z + d$ is called the elevation head, $\frac{V^2}{2g}$ is the velocity head and h_f is the head loss. These terms are actually energies per unit weight, i.e.; foot-pounds per pound of water. Therefore eq. 1.1 is an energy equation which states that the energy per pound of water at section 1 is equal to that at section 2 plus the loss in energy per pound incurred by the water in flowing from section 1 to section 2. The total head h, at a section can be written,

$$h = z + d + \frac{V^2}{2g}. \tag{1.4}$$

The Bernoulli equation written in terms of the total head is

$$h_1 = h_2 + h_f \tag{1.5}$$

where h_1 and h_2 are respectively the total head at section 1 and 2.

1.3 Head loss, Friction and Turbulence. The head loss, h_f, experienced by the flow in going from section 1 to 2 (fig. 1.1) is due to the friction between the water and the solid boundaries of the stream channel. This loss continually reduces the total head h, of the flow which is the same as saying that the total energy of the flow is reduced. The energy expended by friction ultimately is dissipated into heat. However, much of the energy lost is first converted to turbulence before it finally is converted to heat.

Turbulence is nothing more than random motion due to the presence of eddies or swirls in a flow. These eddies cause the velocity at a point to vary with time. Because of this a particle of water in turbulent flow will have components of motion in all directions that fluctuate or pulsate with time. These pulsating velocities make up the turbulence of a flow.

The mean value of the square of the turbulence velocity fluctuations is taken as a measure of the turbulence intensity. If the fluctuation measured at any instant is denoted by v' the mean square may be denoted by $\overline{v'^2}$ where the bar denotes mean value of a large number of instantaneous observations. The ratio $\sqrt{\overline{v'^2}}/V$ is known as the relative turbulence intensity. A relative intensity of 0.10 is very high and 0.01 is low.

Very few turbulence studies have been made in water and knowledge of the subject comes mainly from studies in air flows. From these studies it appears that the relative turbulence intensities of rivers lies somewhere between 0.01 and 0.10. These turbulence velocities are responsible for the entrainment and suspension of sediment and hence are of primary interest in sedimentation work. (For further information see ref. 1.2, p 83-95).

1.4 Reynolds Number. Reynolds number is a dimensionless parameter that expresses the relative effect on the fluid motion of the forces due to the inertia or mass of the fluid and those due to viscous friction. The Reynolds number R, of the flow in a pipe used in most treatises on hydraulics is

$$R = \frac{VD}{\nu}. \tag{1.6}$$

In this equation V is the mean velocity in the pipe in fps, D is the diameter in ft and ν (Nu) is the kinematic viscosity of the fluid in square ft per sec. (Values of ν are given in texts on fluid mechanics as a function of temperatures for example see ref. 1.1 p 11).

The Reynolds number for open channels that is equivalent to the one given above for pipes is

$$R = \frac{4Vr}{\nu} \tag{1.7}$$

Where V is the mean velocity in fps, r is the hydraulic radius of the channel in ft, which is given by

$$r = \frac{A}{p} \tag{1.8}$$

where A is the area of the water cross section in sq ft and p is the

wetted perimeter in ft. It has been found that the friction resistance of any flow can be expressed in terms of Reynolds number and other quantities such as channel dimensions and velocity. Hence it is very useful in dealing with flow problems.

1.5 Specific Energy, Wave Velocity and Froude Number. Specific energy H, is defined as

$$H = d + \frac{V^2}{2g} \tag{1.8}$$

where d, V and g are as defined in equations 1.1 and 1.4. For uniform flow d and hence V does not change along the channel and the specific energy also remains unchanged. In non-uniform flow d, V and H change along the channel.

By substituting equation 1.2 into 1.8 one gets

$$H = d + \frac{Q^2}{2gA^2}. \tag{1.9}$$

A graph of H against d is given in Fig. 1.2 for a rectangular channel 20 ft wide with a discharge Q of 250 cfs. It will be seen that the curve has two branches and that the minimum value of H occurs at a depth of about 1.7 ft. This value of the depth for which

Fig. 1.2 Specific energy diagram for rectangular channel 20 ft wide with discharge of 250 cfs.

the specific energy is a minimum is called the critical depth and is denoted by d_c. The expression for d_c for rectangular channels obtained by differentiating eq. 1.9 with respect to d is

$$d_c = \sqrt[3]{\frac{Q^2}{b^2 g}} \tag{1.10}$$

where b is the width of the channel in ft. The value of d_c for the case of Fig. 1.2, given by eq. 1.10 is $d_c = 1.69$ ft.

A convenient parameter in dealing with open channel flows is the Froude number F, which for rectangular channels is

$$F = \frac{V}{\sqrt{gd}} \tag{1.11}$$

When $d = d_c$ one can show by substituting the value of d_c from eq. 1.10 for d in eq. 1.11 that $F = 1$ or that $V_c = \sqrt{gd_c}$. V_c is known as the critical velocity. The expression $\sqrt{gd}$ is also equal to the celerity c of a gravity wave of small height and large wave length in water of depth d. Therefore when $F = 1$ the flow velocity V is just equal to the celerity of a surface wave. Since a disturbance is propagated into the flow by waves this is a significant relation. When $F = 1$, $V = \sqrt{gd}$ or $V = c$ and a wave cannot propagate upstream. However if $F > 1$ a wave can move upstream. Flows for which $F > 1$ are called shooting flows and those for which $F < 1$ are called tranquil flows.

FLOW FORMULAS

1.6 <u>Chezy Formula.</u> The Chezy formula is

$$V = C\sqrt{rS} \tag{1.12}$$

where C is the Chezy coefficient, S is the slope of the channel in ft per ft, r is the hydraulic radius as defined by equation 1.8 and V is the mean velocity as in eq. 1.2. This equation is used for pipes along with other formulas and is used exclusively for open channels. It applies only for uniform steady flow. In steady uniform

flow there are no changes in discharge, depth, velocity, etc., with time or from section to section along the channel. The Chezy formula may be applied to flows that are only slightly non-uniform by using a mean value of r for the reach involved and substituting the slope of the energy grade line for S.

1.7 Chezy Coefficient. In general the coefficient C varies with the roughness of channel boundaries, and with the Reynolds number of the flow. Several equations for C are used in hydraulics. One of the most common equations is the Manning eq.

$$C = \frac{1.49\ r^{1/6}}{n} \tag{1.13}$$

where n is the Manning roughness coefficient. Eq. 1.13 is valid only for the ft-sec system of units i.e. for velocity in fps and hydraulic radius in ft. For other systems of units the constant 1.49 must be changed for instance for the meter-second system the value of the constant becomes unity. Values of n are tabulated in text books on hydraulics (e.g. see p 230 ref. 1.1). These values apply to clearwater flows in rigid channels. As outlined elsewhere the Manning coefficients for alluvial channels are variable.

Another very useful equation for C is

$$C = \sqrt{\frac{8g}{f}} \tag{1.14}$$

where g is the acceleration of gravity and f is the Darcy-Weisbach friction factor commonly used in pipe flow analysis. The factor f is given on a graph as a function of Reynolds number and the roughness of the boundary (for example see p 182, ref. 1.1). Values of f in such charts, like the usual tabulated values of n, are for clear water flows in rigid channels.

The Chezy equation with the Manning equation for the coefficient is

$$V = \frac{1.49}{n}\ r^{2/3}\ S^{1/2} \tag{1.15}$$

where V is expressed in fps and r is in ft. The Chezy equation with equation 1.14 for C is

$$V = \sqrt{\frac{8g}{f}}\ \sqrt{rS}\ . \tag{1.16}$$

Equation 1.16 will apply for any consistent set of units. For example if V is in fps then r must be expressed in ft and g in ft per sec per sec.

1.8 Equations for Two-Dimensional Flow. The hydraulic radius of a rectangular channel is given by (fig. 1.3)

$$r = \frac{A}{p} = \frac{bd}{b + 2d} \tag{1.17}$$

Fig. 1.3 Section through rectangular channel

When d is small compared with b, b + 2d will differ from b by a small percentage and r will be but little different from d. Under these circumstances r can be taken as equal to d without introducing significant errors. When b is infinitely long we can say r = d. Infinitely wide channels are said to be two-dimensional and for such cases the velocity, depth and other quantities do not change with distance across the stream and one need consider only a strip of flow one foot wide since all strips are the same.

Substituting d for r in equations 1.15 and 1.16 gives the equations for flcw in two-dimensional channels.

$$V = \frac{1.49}{n} d^{2/3} S^{1/2} \tag{1.18}$$

$$V = \sqrt{\frac{8g}{f}} \sqrt{dS} \tag{1.19}$$

VELOCITY DISTRIBUTION

1.9 Shear Stress. Figure 1.4 represents a short reach of a flow with cuts made at the end sections of the reach. One imagines that the water supporting the ends of the reach is removed

and that the forces that were exerted on the remaining water by that removed have been duplicated exactly by forces F_1 and F_2. Assuming uniform flow the depth d and velocity V are uniform throughout the reach and there is no acceleration of the flow. Under these conditions the relation between forces acting on the prism of water parallel to the flow is that their vector sum is zero. The forces acting

Fig. 1.4 Forces on water prism in channel

parallel to the flow are F_1, F_2, the component of the weight acting down the slope which is $\gamma AL \sin\theta$, and the bed friction force exerted by the bed on the water. The bed friction force is given by $\tau_o pL$ where τ_o is the average shear stress in lbs per sq ft exerted on the water by the bed. Summing up all the forces gives

$$F_1 + \gamma AL \sin\theta - F_2 - \tau_o pL = 0.$$

Since F_1 and F_2 are numerically equal one gets,

$$\tau_o = \frac{\gamma AL}{pL} \sin\theta = \gamma r \sin\theta.$$

Since the angle θ is usually very small one can write $\sin\theta = \tan\theta = S$ where S is the slope of the channel. The expression then becomes,

$$\tau_o = \gamma rS \tag{1.20}$$

For two dimensional flow the equation for τ_o becomes

$$\tau_o = \gamma dS \tag{1.21}$$

1.10 Velocity Distribution. The velocity of flow of a fluid past a solid boundary is zero at the boundary and increases with distance from it. This is also true of flow in a channel. The equation for the

velocity in a two-dimensional channel as a function of distance y vertically up from the bed is,

$$v = V + \frac{1}{k}\sqrt{gdS}\,(1 + 2.3 \log_{10} \frac{y}{d}) \qquad (1.22)$$

where v is the velocity at distance y from the bed, V is the mean velocity over the depth, k is the von Karman constant which has a value of 0.4 for clear water and the other symbols are as defined previously. Equation 1.22 will plot a straight line on a graph of v against $\log \frac{y}{d}$. Such a graph is shown in fig. 1.5a where two velocity profiles have been plotted for flows 0.295 ft deep. One of the flows is with clear water and the other is with a suspended load of 0.1 mm sand with a mean concentration of 15.8 grams per liter. It is seen that the measured values fit very closely the straight lines and are of the form of equation 1.22. For comparison the same data have been plotted on rectangular coordinates in Fig. 1.5b.

The quantity $\sqrt{gdS}$ in eq. 1.22 is the so-called shear velocity used extensively in fluid mechanics and will be denoted here by u_*. It is related to τ_o by

$$u_* = \sqrt{\frac{\tau_o}{\rho}} = \sqrt{gdS}\,.$$

Introducing u_* into eq. 1.19 gives

$$\frac{V}{u_*} = \sqrt{\frac{8}{f}}\,.$$

Due to misnumbering of pages, this page was omitted.

Fig. 1.5. Linear and semi-logarithmic graphs of velocity profiles in a flow 0.195 ft deep and 33.5 in wide with clear water and with a heavy suspended load of 0.1 mm sand.

NON-UNIFORM FLOW IN OPEN CHANNELS

1.11 Definition. Non-uniform steady flow varies in depth and mean velocity as one moves up and downstream but the discharge is constant and conditions at any position in the flow do not change with time. Non-uniform flow occurs in a stream when the width and/or slope changes causing the mean velocity, depth, etc., to change from one station to another.

1.12 Characteristics of Non-Uniform Flow in Open Channels. The shapes, cross sections and slopes of natural streams vary along their length and give rise to non-uniform flow. Therefore, in analyzing flows in natural streams the methods of non-uniform flow must be used. The specific energy diagram, an example of which is given in fig. 1.2, shows clearly the relationships between depth and velocity head that may occur in non-uniform flow. As the depth decreases the velocity head increases until for low depths in shooting flows the velocity head is many times the depth and most of the specific energy is kinetic energy. The ratio of velocity head to depth is expressed conveniently in terms of the Froude number.

$$F^2 = \frac{V^2}{gd} = 2\,\frac{V^2/2g}{d}.$$

Since the maximum possible rise of the water surface is equal to $V^2/2g$ it is seen that F^2 gives a direct measure of this height of disturbance relative to the depth.

Non-uniform flow is treated analytically with eq. 1.1 and 1.3. It will be noted from fig. 1.1 that $z_1 - z_2 = SL$ and $h_f = S_f L$ where S is the channel slope and S_f is the slope of the line joining the points which have elevations above the datum equal to the total head h. This line is known as the energy grade line and S_f is known as the slope of the energy grade line or the energy slope. S_f may be greater or less than the channel slope S. The two slopes are equal when the flow is uniform. The flow depth for uniform flow is called the normal depth. A useful relation for non-uniform flow is

$$\tau_o = \gamma d S_f . \qquad (1.23)$$

Since in uniform flow $S_f = S$ eq. 1.23 can be considered as a general relation that applies to steady flow regardless of whether it is uniform or non-uniform. The subject of non-uniform flow is treated in most hydraulics books, for examples see ref. 1.1 pp 252-274.

References:

1.1 Daugherty, R. L., and Ingersoll, A. C. Fluid Mechanics, McGraw Hill, 1954

1.2 Rouse, H., editor, Engineering Hydraulics, John Wiley and Sons, 1950.

Problems:-

1.1 In the logarithmic equation for velocity distribution (eq. 1.22) (a) find the values of y/d and d-y/d at which the local velocity v, is equal to the mean velocity V. (b) show that the average of the velocities at points which are 0.2d and 0.8d down from the surface is equal to the velocity at a point which is 0.6d down from the surface.

1.2 The following data were obtained in a determination of the effect of plant growth on the roughness in a trapezoidal concrete-lined irrigation ditch.

Bottom width	= 3.0 ft
Side slopes	1:1
Discharge (steady)	= 24 cfs
Slope of channel bottom	= 0.0040

	Sta. 91+00	Sta. 94+73
Water surface elevation, ft	875.13	973.52
Bottom elevation	873.56	

Find the value of the Manning roughness coefficient "n" and the average shear stress (τ_o) on the boundary.

CHAPTER 2 - FLOW IN ALLUVIAL CHANNELS

2.1. Introduction. In the last century, man's knowledge of the mechanics of rigid-boundary open channels has improved until it is now possible to design and analyze such channels quite successfully. In the case of channels whose boundaries are composed of erodible material which is carried in significant quantities by the flowing water, the status of our knowledge is not satisfactory. Such streams are called alluvial streams. This chapter will be concerned with the nomenclature connected with alluvial streams, the pecularities of such streams which make them difficult to treat analytically, and a brief discussion of their mechanics. Most of the topics introduced in this chapter will be treated in more detail in later chapters.

2.2. Bed Forms and Their Formation.

(a) Dunes. Consider a flow with a given depth. At very low velocities the flow will be unable to move the individual sand grains and the bed form (whatever it might be) will remain unchanged. In this case the sand bed acts as a rigid boundary. As the velocity is increased a critical value will be reached at which the individual sand grains are moved by the flowing water. This critical value of velocity or bed shear stress at which the particles of bed material start to move depends on the depth of flow, the properties of the bed material and fluid, and configuration of the channel. (This topic will be discussed further in lecture no. 4.) When the velocity is above this critical value but lower than another value which will be discussed presently, irregular, haphazardly located, sharp crested features which move downstream are formed. These features are called dunes. In appearance they are not unlike dunes formed by wind-driven sand. An example of a typical dune pattern is shown in fig. 2-1. A dune pattern changes continuously and hence the bed must be considered as a deformable boundary. At low velocities, the water surface above dunes is very smooth. At higher velocities, so-called "boils" are frequently observed. These appear as highly turbulent, sediment laden areas and usually occur just downstream from dune crests. The exact mechanism involved in their

occurrence has not been explained.

Some writers make a distinction between small, somewhat regular and orderly arranged dunes which form at lower velocities and the larger, more randomly shaped and located dunes which accompany higher velocities. These writers refer to the former as ripples and the latter as dunes. There appears to be little rational reason for this distinction since the mechanism by which they are formed and sustained is apparently the same in both cases, and their effects are qualitatively the same and cannot be logically distinguished quantitatively.

Although several explanations of the mechanism of dune formation have been presented, none of them is entirely satisfactory and the process by which a given flow seeks out and sustains a unique dune pattern cannot be regarded as known. The difficulties stem from several sources. First, the basic physical principles of the interaction between the fluid and bed material, and hence the mechanism of particle entrainment and transport are not fully understood and cannot be described mathematically. Secondly, it is now impossible to describe a dune pattern quantitatively except in a gross, statistical way and consequently one cannot describe the boundary conditions in a mathematical analysis of the problem. The third difficulty is that dune formation involves scour and deposition in a certain pattern and the physics of the sediment transport, either on a small, local scale or the gross transport for a flow, is not understood well enough to be described mathematically.

Despite these difficulties, several conceptual models have been proposed to explain the formation of dunes and their effects. Anderson (ref. 2.1) assumed that dunes result from the action of a periodic variation of velocity. By assuming that the initial form of this velocity variation is the same as that caused by surface water waves, he was able to derive a relation between the depth and velocity of flow, and the dune wave length which appears to be qualitatively correct. However, this approach cannot explain the existence of dunes in a flow where there is no free surface such as in closed pipes or the atmosphere.

von Karman (ref. 2.2) has offered an explanation which assumes

that turbulence is instrumental in forming the dunes. By making certain assumptions about the relation between the magnitudes of the fluid velocity at the surface of the bed, the vertical turbulent velocity, and the fall velocity of the particles, he arrived at some qualitative functional relations between the characteristics of the dunes and the flow parameters. These relations have never been developed further.

Liu (ref. 2.3) concluded that dunes are the result of an instability between the fluid and the sand bed similar to the Helmholtz instability which occurs at the interface of fluids with different densities and velocities. The obvious dissimilarity between a sand bed and a dense fluid, and the differences between their reactions to an applied pressure distribution make this analogy rather tenuous.

Albertson et al (ref. 2.4) extended Liu's idea and presented a graph based on dimensional analysis and experiment on which they delineated the flow conditions for which different bed forms could be expected. This graph has not proved to be too reliable.

In 1956, Bagnold (ref. 2.5) hypothesized that dunes are the result of another type of instability. Bagnold's idea was that the component along the bed of the weight of the fluid (bed shear) is supported by the stress due to the deformation rate of the fluid near the bed (viscosity of the fluid) and a stress which results from the intergranular bombardment of the bed material by the material which is in transport. When this intergranular stress exceeds the stress which the material in the bed can support by friction between successive layers of grains in the bed, the bed deforms into dunes. The form drag of the dunes then supports the stress in excess of that which the sand grains can support. There is currently considerable controversy regarding the validity of this idea and the bulk of experimental data neither proves nor disproves it.

Many other explanations have been offered for this intriguing phenomenon but each has some major deficiency. The present state of our knowledge of the formation and sustenance of dunes is eminently unsatisfactory.

(b) Flat bed. As the velocity is increased, a value is reached at which

Fig. 2.1. Side view of a typical dune configuration in a laboratory flume. Flow is from left to right.

Fig. 2.2 Side view of a typical flat bed in a laboratory flume. Ripples in foreground are due to effects of flume walls. Flow is from left to right.

Fig. 2.3. Top view of a sand wave in a 33.5-inch wide laboratory flume. Flow is from left to right.

Fig. 2.4. Side view of antidunes in a laboratory flume. The wave at the right has just broken, and the center wave is at incipient breaking. Flow is from left to right.

the dunes disappear and the bed becomes flat. Usually a very smooth water surface accompanies the flat bed configuration. A flow with a flat bed is shown in fig. 2.2. The reason for the bed assuming this configuration in preference to a dune pattern cannot be given until the mechanics of dune formation is understood.

At velocities in the vicinity of the transition from dunes to flat bed, sand waves (also called sand bars) may occur. These are reaches of flat bed, several feet long, which occur between reaches of dune covered bed. Over the flat bed, the flow velocity and sediment transport rate are larger and the mean water depth is lower than over the dune covered reaches. Such sand waves are reported by some laboratory investigators and may or may not be a laboratory curiosity. Fig. 2.3 shows a plan view of a sand wave in a flume which is 33.5 inches wide.

(c) Antidunes. As the velocity is further increased and the Froude number, F, based on depth (see eq. 1.11) approaches unity, the water surface becomes somewhat unstable and even small disturbances can give rise to stationary waves of significant amplitude. Under the wave troughs, the flow velocity and sediment transport rate are larger than under the wave crests. This differential transport of material from under the wave troughs to under the crests causes the formation of bed features called antidunes which are somewhat sinusoidal in form. A typical antidune pattern is shown in fig. 2.4. Antidunes occur in trains of three or four to twenty or more waves and usually move upstream, if at all. The antidunes and accompanying surface waves increase in amplitude and the surface waves often break in a manner similar to the breaking of ocean waves. It has been shown by Kennedy (ref. 2.6) that the wave length of antidunes is given by

$$\lambda = \frac{2\pi V^2}{g} \qquad (2.1)$$

where λ (lambda) is the wave length (distance between adjacent crests or troughs). Kennedy also found that the waves break when their height (vertical distance from bottom of trough to top of crest) is one-seventh the wave length.

2.3. Effect of Bed Forms on Roughness*

Each of the different bed forms discussed in the preceeding section and illustrated in figs. 2.1 to 2.4 obviously has a different roughness**. For a flat bed without sediment movement, the only roughness is that of the sand grains and the friction factor can be estimated from a pipe friction diagram if the effective size of the sand as roughness is known. In flow over dunes, the fluid encounters two types of resistance, that due to the roughness of the sand grains and the form drag of the dunes. Further, the presence of the dunes alters the magnitude of the grain resistance since the magnitude of the shear on the bed varies from point to point on the deformed bed. The roughness of a dune bed is much greater than that of a flat bed without movement, and the friction factor of the accompanying flow is also much larger.

A flat bed with sediment movement presents still another situation. In this case the roughness elements on the bed, the sand grains, are moving and cannot be expected to offer the same resistance to flow as fixed elements, and indeed they do not. The friction factor of flow over a flat bed with movement is somewhat lower than that of flow over a flat bed without movement. The presence of antidunes does not appreciably change the magnitude of the effective roughness of the bed from that of a flat bed with sediment motion. However, if the waves break, the friction factor of the flow will be increased due to the energy dissipation in wave breaking.

It must be borne in mind that, contrary to many widely held beliefs and widely accepted texts, the roughness of an alluvial channel

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* A more complete discussion of the roughness of alluvial streams will be presented in lecture no. 8. The preliminary remarks here are intended only for background for discussion of the mechanics of alluvial streams and the difficulties involved in their analysis.

** It is important to bear in mind the distinction between roughness and friction factor. The roughness pertains to the character of the flow boundary and is a property of the size, shape and spacing of the protuberances on the bed. The friction factor, such as that of Darcy-Weisbach (eq. 1.16) relates the rate of energy dissipation (slope) of a flow to the roughness, fluid properties, flow velocity and channel geometry.

and the friction factor for flow in such a channel are not constants, but can vary widely depending on the form of the bed. The bed form is determined by the sand characteristics, depth and velocity of flow, and water temperature. Experiments have shown that the friction factor can vary by a factor of six or more.

Sediment in suspension has a small but still significant effect on the friction factor of a given flow. Indeed there is no reason to expect a mixture of sediment and water to behave the same as just water. Vanoni and Nomicos (ref. 2.7) have shown that by interfering with the turbulence pattern in the fluid, suspended sediment reduces the friction factor by as much as 28 percent from that for the same flow without suspended sediment.

2.4. Description of Transportation and Definition of Load*

The total sediment discharge of a stream is defined as the average quantity of sediment passing a section of the stream per unit time. The material which is being transported is referred to as the load of the stream.

It is convenient to distinguish several classes of load. First, a distinction is made on the basis of the mode of transport. Some particles move along the bed or very close to it by rolling, sliding, and making short excursions into the fluid which do not carry the material more than a few diameters above the bed. This material is called the bed load. Other particles are carried by the fluid at some distance from the bed. These particles are supported by the upward components of the turbulence velocities and are swept along at roughly the forward velocity of the fluid. Material carried in this manner is said to be transported as suspended load, and is called the suspended load. The distinction between bed load and suspended load is quite nebulous and material which is bed load at one time can be suspended load an instant later since there is a continuous interchange of material between the two modes. However, there are certain advantages in making this distinction and treating the two modes individually.

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* See Chapters 5 and 6 for further discussion of the theory of transportation and its application.

The second distinction, which is based on the relative particle size, classifies the load into bed material load and wash load. The bed material load is composed of particle sizes found in appreciable quantities in the actively moving part of the bed. The wash load is composed of the finer particles which are found only in relatively small quantities in the bed. It is carried almost entirely in suspension and its magnitude depends primarily on the amount of fine material available to the stream and is not determined by the depth and velocity.

The final distinction is based on the technique used in measuring the sediment discharge. Due to the physical size of the samplers used to measure the concentration, it is impossible to measure the bed load and the suspended load very near the bed. The product of the concentration determined from such measurements and the water discharge is called the measured sediment discharge of the stream. The difference between the total and the measured sediment discharge is called the unmeasured sediment discharge.

In flow over a flat bed the material on and near the bed is swept along rather uniformly and there is relatively little variation in the local sediment transport rate. When dunes are present, material is carried up the upstream faces of the dunes to the crest. Some of the material is then carried forward to the upstream slope of the next dune where it continues its movement and the rest drops into the sheltered area on the lee side of the dune. This differential transport from the upstream side to the lee side accounts for the downstream movement of the dune.

A similar explanation accounts for the upstream movement of antidunes. In flow over antidunes the transport capacity on the adverse slope of the upstream face of the dune is less than on the downstream face. Therefore deposition occurs on the upstream face and erosion occurs on the downstream face and the profile moves upstream.

2.5. Statement of the Problem of Hydraulics of Alluvial Channels

With the background of the preceding sections it is possible to briefly discuss the mechanics of alluvial channels and the difficulties encountered in the design and analysis of such channels. Consider a

natural or man-made alluvial channel which is in equilibrium with its bed (i.e., there is no net scour or deposition of bed material). It will be assumed for the purpose of this discussion that the slope of the stream does not change during changes in discharges such as occur during the passage of a flood. Whether or not this is strictly true is a moot question.

Fed into each reach of the stream is a certain water discharge and a sediment load which the stream must transport. Since the channel has no control over the magnitude of these quantities, they may be considered as the independent variables. By changing its bed configuration (roughness) the stream can adjust the depth and velocity of flow and the factors (whatever they may be) which govern the sediment transport capacity of the flow in such a way that the imposed water and sediment load can be accomodated. Thus a stream that might have a dune bed with a very large roughness at a low flow will change its dune configuration to reduce the roughness as the water and sediment load increase. Finally the bed will become flat to offer the minimum resistance to the flow. Also, as the load increases, the suspended sediment acts to further decrease the friction factor of the flow by affecting the intensity and distribution of the turbulence as was discussed in section 2.3. Thus, since it can change its roughness, the fluctuation in depth for a given change in discharge will be less for an alluvial stream than for a channel with fixed roughness. Considering the large range of discharge found in natural streams and the limited permissible variations in depth before damage results, it is fortunate indeed that alluvial streams can change their roughness.

These same factors that are the boon of natural rivers are the bane of the sedimentation engineer. His colleagues who deal with fixed-roughness open channels which carry clear water have only one independent variable, discharge, and one dependent variable, depth. Further, they have a great deal of reliable information available to determine the roughness corresponding to the channel surface and a selection of several fairly reliable flow equations (e.g., Manning's) to relate the discharge depth and roughness. Alluvial streams have to transport the

sediment load imposed on the stream with the available water discharge. To assure that solutions exist for all combinations of discharge and load, the depth must be related to both of these quantities. In alluvial streams, this relation is accomplished through the intermediary of variable roughness. The roughness adjusts so that the depth and velocity give the required discharge and the factors that govern the capacity of the flow for sediment are such that the imposed load is carried by the available discharge. Thus, to achieve a solution, one relation in addition to the flow equation is needed: a relation between roughness, discharge and load. The available techniques for solving problems of flow in alluvial channels effect this relation by various ways, none of which is entirely reliable or can be expressed in a simple formula.

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CHAPTER 3 - PROPERTIES OF SEDIMENT

Only the properties of sediment pertinent to sediment transport will be discussed in this section. Many other properties of sediments and soils of interest to geologists, soil physicists, and foundations engineers will not be considered.

3.1 Particle Size and Shape. Grain size is the most important property affecting transportability of sediment. At one extreme of the spectrum of sizes are large boulders which are rolled only by violent mountain streams, and at the other, fine clay, which once suspended, requires days to settle. Hydraulic engineers use the usual classification of sizes into boulders and cobbles (64 - 4000 mm), gravel (2 - 64 mm), sand (0.062 - 2 mm), silt (4 - 62 microns), and clay (0.24 - 4 microns). It should be noted that terms such as wash load, suspended load, bed load, etc., refer to method of transport in any particular situation and do not correspond to any fixed size graduations.

Since natural grains are not spherical, the "diameter" or grain size depends on how it is determined. Common definitions are as follows:

(a) Sedimentation diameter - diameter of sphere of same density with same settling velocity in same fluid at same temperature.

(b) Sieve diameter - opening of square mesh sieve which will just pass the particle.

(c) Nominal diameter - diameter of sphere of equal volume.

(d) Triaxial dimensions. - a, b, c.

For clay, silt, and fine sand it is practical to determine the sedimentation diameter by settling analysis methods such as 1) pipette, 2) bottom withdrawal tube, 3) hydrometer, and 4) visual accumulation tube. For sand and gravel sieve analysis is convenient, and for gravel and boulders direct measurements can be made by calipers or by volumetric displacement.

Although particle shape affects sediment transport there is at present no direct, practical, quantitative way to assess particle shape and its effects, except on fall velocity. (See ref. 3.3e and fig. 3.1-A).

3.2 Density and Petrographic Analysis. River sand is naturally composed of a variety of minerals, quartz and feldspar being predominant. The mean specific gravity varies little, usually being in the range 2.6 to 2.7. Heavy minerals (such as magnetite) often become segregated from lighter ones, forming bands along crests of dunes for example, because they are less easily transported.

3.3 Settling Velocity. The settling or fall velocity of a grain is the rate at which it falls in a still fluid. The fluid drag on the particle equals its submerged weight.

When particles are carried as suspended load they are continually settling back toward the bed, with the turbulence tending to diffuse them upward into the flow, thereby counteracting the settling. Thus the fall velocity is the most pertinent characteristic of fine sediment (sand, silt and clay), and furthermore it reflects the combined influence of particle size, shape, and density, and fluid density and viscosity.

In a turbulent fluid, such as in a stream, the mean fall velocity can be expected to be nearly the same as in a quiescent fluid. As a rough first approximation one may visualize the particle settling at a uniform rate relative to the surrounding fluid, which is sometimes going up, sometimes down owing to turbulence. The problem is complicated by acceleration affects, and cannot be solved rigorously.

(a) Spheres. For spheres, the fall velocity can readily be determined from the drag-coefficient-Reynolds-number curve (see ref. 3.1, p.122 or 3.2, p. 304). Curves of fall velocity versus diameter of quartz spheres (spec. grav. = 2.65), settling in water, are especially useful in finding the sedimentation diameter for observed values of the fall velocity. (See fig. 3.1).

(b) Stokes Law for settling of small spheres is

$$w = \frac{1}{18} \left(\frac{s_s}{s_f} - 1\right) \frac{gd^2}{\nu} \qquad (3.1)$$

where w = settling velocity

s_s = specific gravity of particle

s_f = specific gravity of fluid (= 1 for water)

g = gravitational acceleration

Fig. 3.1 Terminal velocity of fall for quartz spheres in air and water.

Fig. 3.1-a Relation of sieve diameter and fall diameter for naturally worn quartz particles for various shape factors (S.F.). (From ref. 3.3e, p. 37).

Notes: 1. Fall diameter is sedimentation diameter for fall in water at 24°C only.
2. Shape factor S.F. is defined in terms of the triaxial dimensions as:

$$S.F. = c/\sqrt{ab}$$

where
a = longest axis
b = intermediate axis
c = shortest of the three mutually perpendicular axes of the particle.

d = diameter of sphere

ν = kinematic viscosity.

It applies only when the sphere is small enough so that

$$\frac{wd}{\nu} < 0.1.$$

For quartz spheres in water, the upper limit of Stokes Law is approximately 0.05 mm, or roughly the dividing size between silt and sand.

(c) Sedimentation Diameter. A recent find is that the sedimentation diameter for a natural sand grain is not unique for a given particle, but depends on the fluid characteristics as well (ref. 3.3e). For a sphere the sedimentation diameter is the same (namely the actual diameter) whether the fall test is made in water, air, oil, or a fluid of any density or viscosity. However, for irregular particles the shape appears to have a different relative effect on the settling velocity at different Reynolds numbers ($\frac{wd}{\nu}$). Thus in ref. 3.3e a new, but related definition is introduced; namely, the standard fall diameter which is the sedimentation diameter when the fall test is made in water of 24°C.

It should be remembered, however, that the determination of fall velocity by direct observation is a valuable aim in itself. How one converts to an equivalent diameter is not as important as consistency of procedure.

No straightforward way has been devised for adjusting fall velocity w of a natural grain observed at one temperature to w at another temperature. It can be done by ref. 3.3e, but not in a direct or highly reliable way, because of the emphasis on diameters in that report. Ref. 3.3e also gives information relating sieve diameter to fall diameters and fall velocity for grains of various shape factors.

(d) Hindered settling is the retardance of the settling of a single particle due to the presence of neighboring particles. For example, using ref. 3.5, it can be shown that the standard hydrometer test for particle size analysis using 50 grams/liter of clay causes particle settling velocities to be 30% less than for unhindered settling, and computed sedimentation diameters to be 16% too small.

Hindrance also occurs when a particle settles in a confined column. Even when the column diameter is 100 times the sphere diameter, a 2.5% reduction in fall velocity may be observed (ref. 3.1, p.779).

(e) Flocculation of fine sediments (mainly clays) often greatly increases settling rates, because many clay particles may become agglomerated and settle as larger units, namely flocs. Rate of flocculation depends not only on turbulence levels and particle concentrations, but also on the physical chemistry of the particles and the chemical composition of suspending fluid.

In making tests of settling velocity of fine sediments, attention should be given to whether laboratory conditions of flocculation are at all similar to field conditions. If a deflocculating agent is used in the laboratory, the true individual particle settling velocity may be obtained; but the sediment may never settle at such a slow rate under field conditions (ref. 3.4).

3.4 Frequency Distributions. All natural sediments have a distribution of sizes and settling velocities, as they are never perfectly uniform. The fluvial behavior of sediment may be expected to depend on the spread of sizes, as well as on the mean size. Generally, research has dealt primarily with mean size, with little attention to characteristics of the distribution.

Further complication arises in natural streams because there are obvious variations in grain size from place to place, and between bed and banks. Rivers are constantly sorting their sediment loads, and hence obtaining representative samples of river bed material is difficult.

(a) Definition of cumulative frequency distribution. The distribution of sizes (or settling velocities) may be represented graphically by plotting "percent by weight finer" vs. "grain size". Such a graph (see fig. 3.2) is technically known as a cumulative frequency distribution. Although there are many mathematical formulations available for fitting frequency distributions, only the "normal" or "Gaussian" distribution, which is the most useful, will be discussed here. Table 3.1 gives pertinent data on this distribution.

Fig. 3.2 Example of cumulative frequency distribution of sieve diameter on semi-logarithmic graph paper. (See also Fig. 3.5) (Kizu River, Japan).

Table 3.1

Values of the Cumulative Normal

Frequency Distribution Function

$\frac{x}{\sigma}$	$A = \int_0^{x/\sigma} f(x)\,dx$	$A = \int_0^{x/\sigma} f(x)\,dx$	$\frac{x}{\sigma}$
0.1	3.98	25.00	0.6745
0.2	7.93	40.00	1.2817
0.3	11.79	45.00	1.645
0.4	15.54	49.00	2.327
0.5	19.15	49.90	3.090
0.6	22.57	49.99	3.722
0.7	25.80	49.999	4.265
0.8	28.81	49.9999	4.7529
0.9	31.59	50.00	∞
1.0	34.13		
1.1	36.43		
1.2	38.49		
1.3	40.32		
1.4	41.92		
1.5	43.32		
1.6	44.52		
1.7	45.54		
1.8	46.41		
1.9	47.13		
2.0	47.73		
2.1	48.21		
2.2	48.61		
2.3	48.93		
2.4	49.18		
2.5	49.38		
2.6	49.53		
2.7	49.65		
2.8	49.74		
2.9	49.81		
3.0	49.865		
3.1	49.903		
3.2	49.93129		
3.3	49.95166		
3.4	49.96631		
3.5	49.97674		
3.6	49.98409		
3.7	49.98922		
3.8	49.99277		
3.9	49.99519		
4.0	49.99683		
∞	50.00		

Equation: $f(x) = \frac{1}{\sigma\sqrt{2\pi}} e^{-x^2/2\sigma^2}$

where $f(x)$ = probability density

X = random variable

M = mean = $\frac{\Sigma X}{n}$

$x = X - M$ = random variable adjusted to zero mean

σ = standard deviation = $\sqrt{\frac{\Sigma x^2}{n}}$

Note: $\int_{-\frac{x}{\sigma}}^{0} f(x)\,dx = \int_0^{x/\sigma} f(x)\,dx$

So $\int_{-\infty}^{\infty} f(x)\,dx = 1$

Space does not permit treatment of the subject of mathematical statistics (see ref. 3.6 and 3.7 for elementary treatment); only some practical procedures for application can be covered in the following sections.

(b) Arithmetic probability paper for normal distribution. In fig. 3.2 the curve has an S-shape on semi-logarithmic graph paper with the ends being asymptotic to the 0% and 100% lines. Because of the spread of grain sizes the logarithmic scale is almost always used for grain size. One might also consider the abscissa to be an arithmetic scale of logarithms of grain size. For simplicity the following discussion is based at first on the assumption that the abscissa scale is arithmetic, and for this purpose we shall use as an example the distribution of annual rainfall at Springfield, Mass. (fig. 3.3).

The normal distribution also plots as an S-shape curve on a graph like fig. 3.3, but one cannot readily determine whether the S-curve plotted through a set of data points is an S-curve representing a normal distribution. Thus it is useful to devise a distorted type of graph paper on which the cumulative normal distribution appears as a straight line; such graph paper is called arithmetic probability paper, a sample of which is shown in fig. 3.4 showing same rainfall data as fig. 3.3. Note that 0 and 100% do not even appear on this graph.

Now when data points plotted on probability paper can be fitted with a straight line, we have automatically fitted a normal distribution. The intercept and slope of the line correspond to the mean (D_m) and standard deviation, σ, of the distribution. The mean is the value read off the graph at 50% and the standard deviation σ is

$$\sigma = D_{84.1} - D_{50} = D_{50} - D_{15.9} \qquad (3.2)$$

where $D_{84.1}$ represents the size for which 84.1% is finer, and so on. One of the great advantages of probability paper (rarely mentioned in textbooks) is that one can determine D_m and σ graphically without resorting to lengthy calculations.

The standard deviation is a measure of the spread of a distribution: the larger σ, the greater the range of values. For a normal

Fig. 3.3 Example of cumulative frequency distribution on arithmetic graph paper (See also Fig. 3.4). (Annual rainfall at Springfield, Mass., 1848 - 1938).

Fig. 3.4 Example of cumulative frequency distribution on arithmetic probability paper (same data as Fig. 3.3). (Annual rainfall at Springfield, Mass. 1848-1938).

distribution, note the following values:

Range	Percent within indicated range
Mean $\pm\ \sigma$	68.2
Mean $\pm\ 2\sigma$	95.4
Mean $\pm\ 3\sigma$	99.73

The distribution is symmetric about the mean. For other multiples of σ, the percentage can be read from probability paper or table 3.1.

(c) Logarithmic probability paper. For this type of paper the probability scale is just like arithmetic probability paper, but the scale for the variable is simply changed from arithmetic to logarithmic. Natural sands often plot as nearly a straight line on this kind of paper, with deviations only at the extreme ends (last few percent). (See fig. 3.5 for example, showing same sieve analysis as fig. 3.2). A distribution which can be represented by a straight line on log probability paper is called a log normal distribution.

For this case D_{50} now becomes the geometric mean D_{mg}. In place of σ, we have the geometric standard deviation σ_g given by

$$\sigma_g = \frac{D_{84.1}}{D_{50}} = \frac{D_{50}}{D_{15.9}} \tag{3.3}$$

Note that division now replaces subtraction in eq. 3.2. Furthermore for the logarithmic normal distribution the ranges are given as follows:

Range	Percent within indicated range
$\frac{D_{mg}}{\sigma_g}$ to $D_{mg}\,\sigma_g$	68.2
$\frac{D_{mg}}{\sigma_g^2}$ to $D_{mg}\,\sigma_g^2$	95.4
$\frac{D_{mg}}{\sigma_g^3}$ to $D_{mg}\,\sigma_g^3$	99.73

In cases where the plotted curve is not exactly straight on log

Fig. 3.5 Example of cumulative distribution of sieve diameter on logarithmic probability paper (same sample as Fig. 3.2). (Kizu River, Japan).

probability paper, Otto (ref. 3.8) suggests connecting the points $D_{15.9}$ and $D_{84.1}$ with a straight -line segment, and computing D_{mg} and σ_g from this straight-line segment.

It is important to remember that whether or not the plotted curve is a straight line on probability paper, these two kinds of paper are still very useful for presenting size or fall velocity distribution data.

Table 3.2

Sediment Samples from Natural River Beds, Beaches and Dunes

River Beds	Place	Geometric Mean Sieve Diameter	Geometric Std. Deviation
		D_{mg} (mm)	σ_g
Missouri R.	Omaha, Neb.	0.20	1.20
Chao Phya R.	5 mi. u/s Singburi, Thailand	0.28	1.28
Colorado River	Topock Gorge, Calif.	0.20	1.34
Colorado River	Taylor's Ferry, Calif.	0.32	1.44
Virgin River	Weeping Rock, Zion Nat'l Park, Utah	0.17	1.40
Virgin River	St. George, Utah	0.18	1.39
Umqua R. (South Fork)	0.5 mi. S. of Roseburg, Oregon	0.14	1.45
Little Colorado R.	Cameron, Ariz.	0.16	1.47
West Goose Creek	Nr. Oxford, Miss.	0.29	1.50
Dry Creek	A.R.S. Sta. 10, Nr. Holly Springs, Miss.	0.38	1.54
Niobrara R.	Near Cody, Neb.	0.28	1.60
Tiber River	Rome, Italy	0.42	1.69
Saco River	Nr. highway U.S. 302, nr. Fryeburg, Maine	0.46	1.71
Salinas River	8 mi. N. of Bradley, Calif.	0.64	1.71

(table continued)

Table 3.2 (cont'd)

Rivers	Place	Geometric Mean Sieve Diameter	Geometric Std. Deviation
Santa Ana R.	4 mi. N. of Corona, Calif.	0.70	1.73
Mountain Creek	Nr. Greenville, S. C.	0.86	1.80
Mojave River	Victorville, Calif.	1.10 1.16	1.98 2.15 analysis of dune sa 4/2/62
Kizu River	At confluence with Uji and Katsura Rivers near Yawata, Japan	1.14	2.12
Loire River	Nr. Tours, France	1.06	2.38
Ping River	Nr. highway bridge, Lampun, Thailand	0.94	2.40
Kiowa Creek	Nr. Elbert, Colo.	1.00	2.73
Beaches			
Santa Barbara, Calif.		.16	1.24
Oahu Island, Hawaii; North tip		.56	1.35
Oahu Island, Hawaii; Waikiki Beach		.38	1.53
Desert Dunes			
Kelso Dunes	Nr. Kelso, Calif.	.26	1.17

(d) Typical values of σ_g. Some selected values of D_g and σ_g are listed in table 3.2, in order of increasing σ_g. Some of the values were computed from data in the literature, and others are based on sieve analyses of samples collected by the authors.

It is interesting to note that smaller values of σ_g appear to be associated with the smaller D_g values, often for the larger rivers. For the coarser material (larger D_g) the spread of sizes is relatively greater (larger σ_g).

3.5 Measurement of Size Distribution.

(a) Sieve analysis is good down to 0.074 mm (200 mesh), fair to 0.044 mm (325 mesh). Sieve sizes (openings) are made in a geometric series with every sieve being $\sqrt[4]{2}$ larger in size than the preceding. Taking every other sieve gives a $\sqrt{2}$ series, while taking every fourth gives a ratio of 2 between adjacent sieve sizes. When the sand is fairly uniform (i.e. σ_g is fairly small) the $\sqrt[4]{2}$ series should be used. Although the basic principle of sieving is self-evident, there are a number of points of procedure which should be observed for first class work (the research engineer needs better data than the aggregate tester!). A recent C.I.T. memorandum on the subject is reproduced as Appendix 3-A.

(b) Settling analysis for finer particles. For medium and fine sand, the visual accumulation tube recently developed by the Interagency Committee on Sedimentation (ref. 3.3d) is useful for rapid analysis, although less accurate than sieve analyses. It has the advantage of giving fall diameter, which is more closely related to behavior in water than sieve diameter.

For silts and clays the principal acceptable methods are pipette, bottom withdrawal tube, and hydrometer. See ref. 3.3 a, b, c for a thorough description of these and other methods. A brief comparative evaluation of these methods is given in table 3.3.

Table 3.3

ADVANTAGES AND DISADVANTAGES OF VARIOUS METHODS OF SETTLING ANALYSIS OF FINE SEDIMENT

I. Pipette method.

Advantages

1. Most accurate method (results within a few percent).
2. Data yield distribution curve directly.
3. Simple equipment.
4. With multiple-depth sampling flocculation can be studied.
5. Can give accurate results for concentrations as low as 1 gr/l. (1000 ppm).

Disadvantages

1. Pipette withdrawals are very small, and are samples of a sample, thus increasing sampling error.
2. Pipette does not withdraw fluid from exactly the depth of tip of pipette, but from a region around the tip.
3. Volume of suspension decreases due to successive small withdrawals.
4. Drying and weighing pipette samples requires very careful laboratory technique, because weights may be extremely small (a few milligrams).

II. Bottom withdrawal tube method.

Advantages

1. Less sampling error, because all settled material is collected and weighed.
2. Suitable for very small concentrations (down to 100 ppm).
3. Good accuracy has been established (although not as good as pipette).

Disadvantages

1. Curve of raw data must be differentiated to yield distribution curve (increases errors especially for smaller sizes).
2. Sediment sometimes sticks to sides of bottom hopper and will not completely flush out.
3. Volume of suspension (and height of column) changes requiring corrections in results.

III. Hydrometer method.

Advantages

1. Quickest and easiest method; no samples to dry and weigh.
2. Simple equipment
3. Data yield distribution curve directly.

Disadvantages

1. Must use fairly concentrated suspension (5%) in order to measure accurately the increment in density due to suspended particles.

(table continued)

Table 3.3 (cont.)

Hydrometer method (cont.)

Disadvantages

2. Accuracy only fair; test more suited to soil mechanics than to sediment transport studies.
3. Immersing and removing hydrometer disturbs settling column.
4. Hydrometer bulb extends over a finite depth and is assumed to register density at its center of buoyancy.
5. As hydrometer reading changes, the depth to center of buoyancy changes (thus a variable height of settling column is analyzed).
6. Must determine specific gravity of particles (as a separate test).
7. Hindered settling effect is appreciable at high concentrations required.

IV. All the various decantation methods are unacceptable because it is impossible to achieve an upflow of water without turbulent fluctuations, and without variation in velocity across the flow cross section.

Summary:

1. In order of accuracy the pipette method is considered best, the bottom withdrawal tube next, and hydrometer last. Decantation is unacceptable.
2. The pipette method is used as a standard to calibrate other methods, but is tedious.
3. The bottom withdrawal tube is favorable for sediment transport studies; although less accurate, it is also less tedious, and can be used for very dilute suspensions, such as suspended load samples.
4. The hydrometer method is unsuitable for sediment transport studies, because very few liquid samples are dense enough for direct analysis. (Thickening a suspension by removing water may radically change settling characteristics due to flocculation.)

References:

3.1 Rouse, H. Ed., Engineering Hydraulics, John Wiley and Sons New York, 1950, Chapter XII, "Sediment Transportation" by Brown, C. B. pp 769-857.

3.2 Daugherty, R. D. and Ingersoll, A. C., Fluid Mechanics with Engineering Applications, McGraw-Hill, New York, 1954.

3.3 Sub-Committee on Sedimentation, U. S. Interagency Committee on Water Resources, Measurement and Analysis of Sediment Load in Streams, Reports as follows:

3.3a Report No. 4, "Methods of Analyzing Sediment Samples"

3.3b Report No. 7, "A Study of New Methods for Size Analysis of Suspended-Sediment Samples".

3.3c Report No. 10, "Accuracy of Sediment Size Analysis Made by Bottom Withdrawal Tube Method".

3.3d Report No. 11, "Development and Calibration of Visual Accumulation Tube".

3.3e Report No. 12, "Some Fundamentals of Particle Size Analysis".

3.4 McLaughlin, R. T., "Settling Properties of Suspensions", Jour. of Hy. Div., ASCE, Vol. 85, No. HY 12, Dec. 1959, pp 9-41.

3.5 McNown, J. S., and Lin, P. N., "Sediment Concentration and Fall Velocity", Proc. 2nd Midwest Conf. on Fluid Mechanics, Ohio State Univ., 1952, p. 401-411.

3.6 Johnstone, D. and Cross, W. P., Elements of Applied Hydrology, Ronald Press, New York, 1949, Chapter 10.

3.7 Hoel, P. G., Introduction to Mathematical Statistics, second edition, John Wiley and Sons, New York.

3.8 Otto, Geo. H., "A Modified Logarithmic Probability Graph for the Interpretation of Mechanical Analysis of Sediments". Jour. of Sedimentary Petrology. Vol. 9, No. 2, 1939, pp. 62-76.

3.9 Kennedy, J.F. and Koh, R.C.Y., "On the Fall Velocity of Spheres and Sand Grains", paper in preparation, California Inst. of Technology, 1960.

3.10 Cadle, R. D., Particle Size Determination, Interscience Publishers, New York 1955.

3.11 Lane, E.W., Ch., "Report of Sub-Committee on Sediment Terminology", Trans. A.G.U., Vol. 28, No. 6, Dec. 1947, p. 936-938.

APPENDIX 3-A

Memorandum on Procedures for Sieve Analyses of Natural Sands

The purpose of this memorandum is to recommend procedures for the use of sieves to determine the particle-size distribution of natural sands. The recommendations are based on a brief literature survey and experience at the Sedimentation Laboratory at the California Institute of Technology.* The principal items of procedure which will be discussed are sieving time and sample size. To obtain reproducible results with a set of testing sieves, it is necessary to follow standardized procedures.

I. Principles of Sieving

There are three conditions necessary for the passage of a particle through a sieve: (1) that the particle have a cross section small enough that it can pass through the sieve opening when properly oriented; (2) that it be properly oriented; and (3) that the particle have access to an opening. If the first condition is met, the probability that a given particle will pass a sieve depends on the number of chances it has to meet the second and third conditions and this, in turn, is determined by the duration of sieving, the number of openings available, and on the interference a particle suffers from other particles.

If the particle is much smaller than the opening, it will pass easily, of course, once it gains access to an opening. However, two factors can reduce the probability of access, namely (1) overloading, i.e., so much material on the sieve that the particle never gets shaken down near an opening, and (2) clogging, which may occur when there is a large proportion of the sample just a little too large to pass the screen. The effect of clogging can be reduced by reducing the size of the sample.

If the particle is only slightly smaller than the opening, then orientation becomes important, and (with the exception of plate-shaped

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* Appendix to Progress Report (Nov. 24, 1957 to May 24, 1958) on Contract No. 12-14-100-995 (41) for Agricultural Research Service, U. S. Dept. of Agriculture, by R.H. Taylor, Jr., and N.H. Brooks, Sedimentation Laboratory, California Institute of Technology, Pasadena, California.

particles) can be considered to be random. Thus the probability of the grain passing through is increased by longer exposure - i.e., by sieving for a longer time. However, there seems to be no end-point, perhaps due to small variations in the size of openings in the sieve. Furthermore, if the material which has passed a sieve is re-run on the same sieve for the same amount of time, not all the grains will pass the second time. Thus, an end-point must be arbitrarily defined. One of the principal points of discussion in the literature is the definition of this end-point and the means for attaining it, and recognizing when it has been attained. Because of the presence of some oversized openings in every sieve, the accuracy does not necessarily increase with time of sieving, because an oversize particle has more time to find an oversized hole.

It is the general consensus that natural sands and gravels are sufficiently uniform and regular in shape that special precautions need not be taken for shape variations.

If a sample is very large, it is doubtful whether any amount of sieving will produce dependable or reproducible separation because of the lack of opportunity for undersized particles to gain access to the sieve, and because of the increased amount of clogging, as mentioned above. If the sample size is very small, then a disproportionate degree of care must be used in order to maintain a given level of accuracy. Ideally, it has been stated, the amount of material remaining on each sieve should be one layer of particles, but on the finer screens this amount may not be sufficient for accuracy. Using a larger amount represents a compromise among the above considerations and requires the definition of an end-point. Specific suggestions will be discussed below.

II. Discussion and Recommendations

A. Variations in Sieve Openings. The size of openings in testing sieves is naturally subject to some variation, being larger for the fine-mesh screens. The tolerances set by the U.S. Bureau of Standards for the U.S. Sieve Series (Fine Series) are reproduced in Table I. Even though a testing sieve should be considered a precision instrument,

Table I

U.S. Sieve Series (Fine Series)

Width of opening (microns)	Screen no.	Permissible Variation (%) Average opening (±)	Maximum opening* (+)	Wire diameter (mm)
5660	3.5	3	10	1.28 -1.90
4760	4	3	10	1.14 -1.68
4000	5	3	10	1.00 -1.47
3360	6	3	10	0.87 -1.32
2830	7	3	10	0.80 -1.20
2380	8	3	10	0.74 -1.10
2000	10	3	10	0.68 -1.00
1680	12	3	10	0.62 -0.90
1410	14	3	10	0.56 -0.80
1190	16	3	10	0.50 -0.70
1000	18	5	15	0.43 -0.62
840	20	5	15	0.38 -0.55
710	25	5	15	0.33 -0.48
590	30	5	15	0.29 -0.42
500	35	5	15	0.26 -0.37
420	40	5	25	0.23 -0.33
350	45	5	25	0.20 -0.29
297	50	5	25	0.170-0.253
250	60	5	25	0.149-0.220
210	70	5	25	0.130-0.187
177	80	6	40	0.114-0.154
149	100	6	40	0.096-0.125
125	120	6	40	0.079-0.103
105	140	6	40	0.063-0.087
88	170	6	40	0.054-0.073
74	200	7	60	0.045-0.061
62	230	7	90	0.039-0.052
53	270	7	90	0.035-0.046
44	325	7	90	0.031-0.040
37	400	7	90	0.023-0.035

* For sieves from the 1000 micron (No. 18) to the 37 micron (No. 400) size, inclusive , not more than 5% of the openings shall exceed the nominal opening by more than one-half of the permissible variations in maximum opening.

Note: The specifications above were set by the U.S. Bureau of Standards. (Table reproduced from reference 2, p. 180.)

still the average opening of a particular 200-mesh sieve, for example, may vary $\pm$ 7% from the nominal size (0.074 mm), or range from 0.069 to 0.079 mm. The maximum opening for the 200-mesh sieve may be 60% larger than the nominal size or 0.118 mm.

Keeping in mind these permissible variations, it is clear that it is necessary to use the same set of sieves in order to get reproducible results. In a laboratory where more than one of each sieve is in use, the individual sieves should have suitable identification marks.

Where high accuracy is required, sieves may be sent to the Bureau of Standards for calibration. Weber and Moran[4] state that direct measurement of a sample of sieve openings or wire diameters is the only satisfactory method of calibration. They also present an empirical equation for correcting the results of sieve analysis for the effect of the variation in the size of openings in a particular sieve. As indicated by Table I, the variations become larger for the finer sieves.

B. Sample Size and Preparation. The American Society for Testing Materials[1] requires, for material 95% of which is smaller than 2.0 mm (10 mesh), that the sample be approximately 100 grams. However, their purpose is to specify a standard test for aggregates, where the desired degree of accuracy is not as high as in research on sediment transportation. In particular, for well-sorted fine sands, 100 grams could lead to considerable overloading.

Shergood[3] has studied the problem of sieve loading by a number of detailed tests. When using a series of sieves with ratio of openings of 2, he recommended maximum allowable residues, as shown in Table II. When he had larger residues he found that the sieves retained significant numbers of particles which should have passed through.

However, when a $\sqrt{2}$ series is being used (i.e., twice the number of sieves), he states that the allowable residues should be halved because the percentage of near-mesh size particles is doubled. By extending his line of reasoning one step further, one deduces that for a $\sqrt[4]{2}$ series, the allowable residues should be quartered, as indicated

in Table II. The intermediate sieves in the table can be interpolated, taking care to interpolate in the proper column depending on the closeness of spacing of sieve openings used.

Table II

Recommended Maximum Residues on Individual 8-Inch Diameter Sieves (after Shergood[3])

British Sieve No.	Sieve Opening mm	Equiv. U.S. Sieve No.	Maximum Residue (grams)		
			2-Series	$\sqrt{2}$-Series	$\sqrt[4]{2}$-Series*
7	2.411	8	150	75	38
14	1.204	16	100	50	25
25	0.599	30	70	35	18
52	0.295	50	50	25	12
100	0.152	100	35	18	9
200	0.076	200	25	12	6

* Extension based on Shergood's reasoning.

Shergood's reasoning apparently overlooks particle interference, for he implies that the allowable residue depends only on the number of "near-mesh" particles without regard to the total number of particles. Thus, the values obtained by extension (last column in Table II) appear more reasonable than the values given by him for the 2-series. A loading of 25 grams on a 200-mesh sieve is too much under any circumstances in the opinion of the writers.

As a general rule Shergood suggests that for river sands, the sample should be limited to between 100 and 150 grams for coarse sand, and between 40 and 60 grams for fine sands. He admits, however, that under special circumstances (such as fine well-sorted sands) samples of this size may be too large.

The writers recommend for medium or fine sands that the samples be limited to about 25 to 50 grams for 8-inch sieves. For fine well-sorted sands the size of sample should be reduced to less than 10 to 20

grams. (For 3-inch diameter sieves these values should, of course, be reduced by the factor 9/64.) These small weights are necessary to conform to Table II because, for very well-sorted sands, the largest residue in a $\sqrt[4]{2}$ series may be as much as 1/3 to 1/2 of the total sample weight. Furthermore, for sieves finer than the 200-mesh the allowable residues may be expected to decrease still further.

There is no disadvantage to using samples much smaller than the limits suggested above, except for the increased care required in collecting material from the sieves, weighing it, and keeping the sieves clean so that particles which become stuck in the sieves during one analysis are not jarred loose later. In the Sedimentation Laboratory a full sieve analysis on a sample of only one gram of fine sand was recently performed on an 8-inch set of sieves without any unusual difficulty.

ASTM also wisely recommends that "the selection of samples of an exact predetermined weight shall not be attempted". Rather, one should obtain the approximate weight desired by successive sample splitting only. This procedure still allows considerable leeway; for example, a sample of about 40 grams can be reduced to 30 grams by splitting twice and keeping all <u>except</u> the 1/4 portion (10 grams) normally considered the "yield" of the splitting procedure.

C. <u>Duration of Sieving</u>. ASTM requires that sieving be continued until less than one percent of the residue on any sieve passes in one minute. Weber and Moran[4] state that enough time must be allowed for the fractions retained on the several sieves to approach constancy, but that a protracted time allows additional opportunity for the closely sized particles to seek out openings slightly larger than average in each sieve. They suggest that the period after which the greatest sieve fraction begins to lose a constant amount for each succeeding minute is the optimum period on which to standardize, and that this should be approximately 3 to 5 minutes. Shergood concludes that 9 minutes is adequate.

A sieving time of 10 minutes is recommended as being adequate, but not excessive, when a mechanical shaking and tapping machine is used, such as the W.S. Taylor Company's Ro-Tap Testing Sieve Shaker. This has been the standard practice in the Sedimentation Laboratory.

Shergood also quite correctly points out that inaccuracies are controlled much more effectively by reducing sample size than by increasing sieving duration.

III. Summary of Recommendations

The principal recommendations for making sieve analyses of natural sands are as follows:

1. For medium or fine sands (less than 0.5 mm) use a sample size less than 25 to 50 grams for 8-inch diameter sieves, and 3 to 7 grams for 3-inch diameter sieves; for fine well-sorted sands, the sample size should be reduced to about 10 to 20 grams (or 1.5 to 3 grams for 3-inch sieves).
2. Smaller samples are permissible but require greater care in collecting and weighing sieve fractions.
3. Larger samples are not recommended because of the possibilities of overloading of individual sieves (especially for well-sorted sands), and excessive clogging.
4. Samples should not be made exactly equal to some predetermined weight, but should be obtained by successive subdivision of a larger sample by means of a sample splitter.
5. Ten minute sieving time in a mechanical shaking machine is recommended.
6. For very accurate work, sieves should be sent to the Bureau of Standards for calibration.
7. For reproducible results, the same individual sieves should be used. Individual sieves should be marked to distinguish them from other sieves of same nominal opening.

References:

1. ASTM Standards 1955, Part 3, ASTM Designation C 136-46, "Sieve Analysis of Fine and Coarse Aggregates", American Society for Testing Materials, Philadelphia.

2. Cadle, R. D., "Particle Size Determination", Interscience Publishers, Inc., New York, 1955 (Ch. VI).

3. Shergood, F. A., "The Effect of Sieve Loading on the Results of Sieve Analysis of Natural Sands", J. of the Society of Chemical Industry (London), 65, 245, (1946).

4. Weber, M. Jr., and R. F. Moran, Industrial Engineering Chemistry, Analytical Edition, 10, 180 (1938).

Problems:-

3.1 A sieve analysis of the sand currently in the large flume in the Sedimentation Laboratory gave the following data:

Tyler Mesh (per in.)	Sieve Opening (mm.)	Grams Retained	Percent Retained
32	0.495	0.85	2.47
35	0.417	1.56	4.53
42	0.351	3.88	11.27
48	0.295	3.82	11.10
60	0.246	5.35	15.54
65	0.208	5.69	16.53
80	0.175	4.31	12.52
100	0.147	5.06	14.70
115	0.124	2.37	6.89
150	0.104	1.16	3.37
170	0.088	0.21	0.61
200	0.074	0.12	0.35
Pan		0.04	0.12
			100.00 %

a) Plot the above analysis on logarithmic probability paper.

b) Fit a log normal distribution (i.e. a straight line) by eye and determine the geometric mean D_{mg} and geometric standard deviation σ_g.

c) Can the distribution be represented as an ordinary normal distribution (straight line on arithmetic probability paper)?

CHAPTER 4 - INITIATION OF SEDIMENT MOTION

4.1 Introduction. In connection with certain problems which arise in the sedimentation field the engineer is often interested in knowing what flow conditions cause incipient motion of the particles of bed material. For example if one is designing an earth canal to transport clear water, the flow conditions must be such that no motion of the bed material occurs or the channel will scour. Also, some formulas for the transport of bed load require certain parameters (usually bed shear stress or velocity) of the flow at which motion is incipient. This chapter will present the theory and available techniques for predicting incipient motion, discuss the effects of the bed form, and present an example of analysis of earth canals to determine if the bed material will be moved.

4.2 Theory; White's Analysis. Probably the two most significant and widely known analyses of initiation of motion are those of C. M. White and A. Shields. These analyses will be discussed separately.

White (4.1) sought to determine the factors which govern the initiation of motion by equating the moments which tend to move the grains to those which resist movement. Fig. 4.1 shows the forces acting on a typical grain which is in the top layer of grains on a flat bed. The forces parallel to the bed which tend to rotate the particle forward are the result of the bed shear stress which was discussed in section 1.9. If each surface grain resists the applied bed shear of an element of bed area which is proportional to the square of the

Fig. 4.1 Forces acting on a sediment particle which is in the top layer of grains of a flat bed.

diameter of the particle then the force parallel to the bed, F_h, will be

$$F_h = \tau_o C_1 d_s^2 \tag{4.1}$$

where

τ_o = bed shear stress

d_s = particle diameter

C_1 = constant which relates d_s^2 to area over which particle resists the applied bed shear stress.

If the distance above the point of rotation to the point of action of F_h is proportional to the diameter, then the overturning moment M_o will be

$$M_o = \tau_o C_1 C_2 d_s^3 \tag{4.2}$$

where C_2 is the constant ratio of the particle diameter to the moment arm of F_h.

The resisting moment is due to the submerged weight of the particle. If this weight is proportional to the cube of the diameter, $C_3(\gamma_s - \gamma_f)d_s^3$, and if its moment arm is proportional to the diameter, $C_4 d$, then the resisting moment is

$$M_r = C_4 C_3 (\gamma_s - \gamma_f) d_s^4 \tag{4.3}$$

where γ_s and γ_f are respectively the unit weights of the particle material and the fluid. The particle will start to move when τ_o is such that $M_r = M_o$. This critical value of τ_o is called the critical shear stress and denoted by τ_c. Thus, equating eq. 4.2 and 4.3 and combining the various constants into a single constant C, the result is

$$\tau_c = C (\gamma_s - \gamma_f) d_s \tag{4.4}$$

where τ_c is in lb per sq ft, d_s is in mm and C is a dimensionless constant. Thus the critical shear stress is proportional to the particle diameter. The factor C depends on the density and shape of the particles, the properties of the fluid, and the arrangement of the

surface particles. Values of $C(\gamma_s - \gamma_f)$ for quartz sand in water reported by various authors range from 0.013 to 0.040 if τ_c is in lbs per square foot and γ_s and γ are in lbs per cu ft. It will be seen from Shields' analysis in the next section that eq. 4.4 is valid only for a certain range of flow conditions.

4.3 Theory; Shields' Analysis. Shields (ref. 4.2) believed that it was not possible to express analytically the forces acting on a typical sediment particle. He avoided any attempt at rationality by making certain gross assumptions and then confirmed and supplemented the analysis experimentally. In the interest of brevity, Shields' relation will be derived here by means of dimensional analysis* rather than from the more involved physical reasoning which he used.

In the case of motion of bed particles the important quantities are shear stress, τ_o, the difference in density between the sediment and the fluid $(\rho_s - \rho_f)$, the diameter of the particles, d_s, the kinematic viscosity, ν, and gravity, g. These five quantities can be grouped into two dimensionless quantities,

$$\frac{d_s \sqrt{\frac{\tau_o}{\rho_f}}}{\nu} = \frac{d_s u_*}{\nu} \tag{4.5}$$

and

$$\frac{\tau_o}{d_s(\rho_s - \rho_f)\, g} = \frac{\tau_o}{d_s \gamma\, (s_s - 1)} \tag{4.6}$$

where γ is the unit weight of water and s_s is the specific gravity of the bed particles. The last quantity pertains only to particles in water. This is as far as dimensional analysis takes us and experiments must be made to determine the relation between these two

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* Dimensional analysis is a technique which seeks significant relations among the variable involved in a problem by studying the units of the variables and combining the variables in different dimensionless forms. The three basic units are force, length and time. For example, if we knew that force F, mass M, and acceleration A were related, but not how, dimensional analysis would lead us to examine the quantity F/MA, since this quantity has no units (dimensions). From experimental evidence we conclude that F/MA = 1.

quantities. Shields made experiments to evaluate this relation and the results are shown in fig. 4.2. At points on the line the shear stress

Fig. 4.2 Results of experiments by Shields to determine the conditions for incipient motion.

is at the critical value, i.e., $\tau_o = \tau_c$. At points above the line, the particles will be moved, while at points below the line the flow will not move the particles. Fig. 4.2 is valid only for flat beds. Shields' work is widely accepted and apparently quite reliable.

Fig. 4.2 shows that relations of the type given in eq. 4.4 are valid only when $\frac{d_s u_*}{\nu}$ is greater than about 60, since only beyond this point is the critical shear stress proportional to the particle diameter.

$$\frac{\tau_c}{d_s \gamma (s_s - 1)} = \text{constant} = 0.06 . \qquad (4.7)$$

Taking γ = 62.4 lb per cut ft and s_s = 2.65, the value of $C(\gamma_s - \gamma_f)$ in

eq. 4.4 is $C(\gamma_s - \gamma_f) = 0.020$ if τ_c is in lb per sq ft, d_s is in mm and γ and γ_s are in lbs per cu ft.

4.4 Probable Effect of Bed Form on Results. The preceding analyses pertain only to flat beds for which the slope is so small that the component of the weight of the particle parallel to the bed is negligible. In practical applications this situation is encountered infrequently. If the flow has ever been great enough to move the bed particles, dunes will have formed on the bed and will have remained when the flow was reduced. Thus when the flow is again increased to the point where motion is incipient the analyses for flat beds will not be applicable.

To date no experiments or analyses have been conducted to determine the conditions for incipient motion on a bed which is not flat. Such an analysis is much more complex than that for a flat bed for two reasons. First, the bed shear is not uniformly distributed. For example, in the case of dunes, the shear stress is greater on the upstream face of the dune where the flow is accelerating and the velocity at the level of the sand grains is relatively large. On the downstream side of the dune which is sheltered, the shear stress will be less than the computed average value. The second major complication is that the component of the weight of the particle parallel to the bed can no longer be neglected since the local slope of the bed can be quite large due to presence of the bed features. On the upstream face this factor tends to resist motion while on the downstream face the particle weight aids motion.

Since the two factors described above have opposite effects on the motivating forces on the particles, it is not certain whether incipient motion on a bed which is not flat will occur at larger or smaller values of shear stress than on a flat bed. There is some evidence however which indicates that higher values of bed shear stress are required in the case of the dune-covered bed.

4.5 Example Problem. To illustrate the use of Shields' diagram (fig. 4.2) and the type of problem in which the concept of critical shear stress arises, an example problem will be worked out in this section.

Given: A channel has been designed to convey clear water to a fish hatchery. The design calls for a flow

depth of 2.0 ft and a velocity of 2.0 ft per sec. The channel is very wide so the side effects can be neglected and the hydraulic radius taken as equal to the depth. It is to be constructed in sand which has a median diameter d_s = 0.915 mm (0.003 ft). For a flat bed in this material the Darcy friction factor (see section 1.7, eq. 1.14) is estimated to be $f = 0.0185$ and the required channel slope is $S = 0.000145$. The water temperature is 20°C and the kinematic viscosity is $\nu = 1.08 \times 10^{-5}$ ft^2 per sec. The specific gravity of the sand is 2.65. Clear water is admitted to the channel and it is imperative that no material be scoured from the bed since any sand in the hatchery will result in instantaneous death of all the fish.

Required: Determine if the existing design is adequate to deliver clear water to the hatchery. Use Shields' diagram (fig. 4.2).

Analysis: The first step is to evaluate the Shields parameters given in eq. 4.5 and 4.6. These require the shear velocity u_* and the bed shear stress τ_o. The shear velocity is given in section 1.10 as

$$\frac{V}{u_*} = \sqrt{\frac{8}{f}}\,.$$

Therefore

$$u_* = 2.0\sqrt{\frac{0.0185}{8}} = 0.0962 \text{ ft/sec.}$$

The parameter on the horizontal scale is then

$$\frac{d_s u_*}{\nu} = \frac{(0.003)\,(0.0962)}{1.08 \times 10^{-5}} = 19.5.$$

The shear stress τ_o is given in eq. 1.21 as

$$\tau_o = \gamma d S. \qquad (1.21)$$

For this problem

$$\tau_o = (62.4)(2.0)(0.000145) = 0.0181 \text{ lb/ft}^2$$

and the parameter on the vertical scale is

$$\frac{\tau_o}{\gamma(s_s - 1)\ d_s} = \frac{0.0181}{(62.4)(2.65-1)(0.003)} = 0.0587.$$

Using these values in fig. 4.2, the resultant point falls above the line so scour will occur in the channel.

Conclusion: If the fish are to be saved, the channel must be redesigned.

References:

4.1 White, C.M., "Equilibrium of Grains on Bed of a Stream", Proc. of Royal Soc. of London, vol. 174 A, 1940, pp. 322-334.

4.2 Shields, A., "Application of Similarity Principles and Turbulence Research to Bed-Load Movement", Tr. by W.P. Ott and J. C. van Uchelen, Soil Conservation Service Cooperative Laboratory, California Institute of Technology, Pasadena, California.

Problem:-

4.1 Using the data obtained in the laboratory experiment on initiation of motion, compute Shields' parameters (eqs. 4.5 and 4.6). Plot the parameters on Shields' diagram (fig. 4.2) to determine how accurately the fitted curve predicts the conditions for initiation of motion in this case. Use $\gamma = 62.4\ \mathrm{lb/ft^3}$, $S_s = 2.68$, and $d_s = 0.23$ mm $= 0.00075$ ft. Values of ν are given in ref. 1.1, page 15. The flume is 2.79 ft wide.

CHAPTER 5 - THEORY OF SEDIMENT TRANSPORTATION

5.1. Introduction. The sediment transported by a stream can be divided into (a) bed load and (b) suspended load. The bed load moves on or near the bed and the suspended load is carried in the fluid away from the bed. There is no sharp division between these two components of the load. The two terms are used for convenience in discussing transportation but actually have no other use. The sum of the bed load and suspended load is called the total load.

The load of a stream can also be divided into two parts according to grain size. The fine fraction is called the wash load and consists of fine grains found only in very small quantities in the bed. The other part of the load is called the bed material load and is made up of grain sizes found in appreciable quantities in the bed.

The rate of transportation of sediment by a stream is called sediment discharge and the discharge is considered in parts like the load. For example, one can refer to bed load discharge, suspended load discharge, bed material discharge, etc. The sediment discharge of a stream is usually given in lbs per sec or tons per day.

The early studies of transportation dealt with sands where the sediment discharges were low and most of the load was bed load. The main objective of these studies was to develop relations for total sediment discharge. Many such relations appear in the literature. These are mostly empirical in nature because little progress has been made in analyzing bed movement.

Since about 1935 much progress has been made in the mechanics of suspension and this phase of the problem is relatively well understood. However, before a complete transport theory can be developed, further progress must be made with the problem of movement near the bed.

5.2. Mechanism of Sediment Suspension. Sediment is lifted off of the bed of a stream and carried up into the body of the flow by the vertical components of the turbulence velocities. In fig. 5.1 v'_y is the instantaneous value of the vertical turbulence velocity and v is the mean velocity at a point a distance y above the bed. The velocity v'_y

Fig. 5.1. Upward transport of sediment by turbulence fluctuations.

measured at the small rectangle with sides dx and dz is sometimes up and sometimes down and also varies in magnitude. Since there is no net flow in the vertical direction the mean value of v_y' is zero or in other words in a given interval of time the upward and downward flow will be equal in volume. It is known from experience that the concentration of sediment in a stream decreases with distance up from the bed. Therefore the upward flow through the area dxdz will tend to be richer in sediment than the downward flow and there will be a net upward transfer of sediment despite the fact that there is no net flow of water. The rate of upward transfer of sediment by the diffusion process just described is given in text books (ref. 1.2, p. 95) in terms of a diffusion coefficient. The differential equation for the steady state distribution of the concentration of suspended sediment is obtained by equating the rate of upward flow of sediment due to turbulence diffusion to the downward flow rate due to settling under the force of gravity (see ref. 1.2, p. 97).

5.3. <u>Distribution of Suspended Load.</u> Eq. 5.1 gives the relative distribution of sediment concentration in a two dimensional steady uniform flow.

$$\frac{c}{c_a} = \left[\frac{d-y}{y} \; \frac{a}{d-a} \right]^z . \qquad (5.1)$$

$$z = \frac{w}{k\sqrt{\tau_o/\rho}} = \frac{w}{k\,u_*}. \qquad (5.2)$$

In these equations c is the concentration at distance y up from the bed, c_a is the concentration at some reference level $y = a$, w is the settling velocity of the sediment grains in the stream, k is the von Karman constant with a value of 0.4 for clear flows, τ_o is the bed shear stress in lbs per sq ft as given by eq. 1.21 and ρ is the density of the water in slugs per cu ft (1.94 slugs per cu ft). The concentrations c and c_a can be measured in any units, for instance, grams per liter or parts per million. The eqs. 5.1 and 5.2 apply to that fraction of the sediment with settling velocity w. The settling velocity should be determined under concentrations and other conditions that exist in the stream but it is often taken as the velocity of a grain in a clear unbounded fluid.

Fig. 5.2 is a graph of the relative concentration of suspended sediment according to eq. 5.1 for several values of z and with the reference level $a = .05d$. The ratio $(y-a)/(d-a)$ is the fraction of the distance from the reference level to the water surface. It will be seen that the concentration tends to become uniform for low values of z and that for high z values the suspended load is concentrated near the bed. Since z is proportional to the settling velocity w in a given flow, i.e., a given k and u_*, the coarsest material will have the highest z value. Also the coarse material will be concentrated near the bed and the fine materials will tend to be more nearly uniformly distributed in the flow. A given sediment with a given settling velocity will yield a low value of z in a stream with high bed shear stress τ_o and a high value of z for low τ_o. Since $\tau_o = \gamma d s$, large shear stress and low z are associated with deep and steep streams which will also have high velocities and transport capacities. Thus it can be seen that the term u_* in the denominator of the formula for z is a measure of the transporting capacity of a stream. The settling velocity w is a measure of the effort necessary to transport or suspend a grain. Since z is proportional to the ratio of w to u_* it can be considered as a measure of the

Fig. 5.2. Distribution of suspended load in a flow according to eq. 5.1.

effort which a stream exerts to transport the sediment, i.e., a small value of z means that the stream needs to exert only a small part of the available effort to transport the sediment.

5.4. Validity of Distribution Equation. Eq. 5.1 will plot a straight line on a logarithmic graph of concentration c against d-y/y. The slope of this line will be z logarithmic cycles on the c axis per cycle on the axis of d-y/y. The value of z can be determined by plotting measured concentration of a given size fraction at several depths on such a logarithmic graph. An example of several graphs for measurements in a flume are shown in fig. 5.3 which illustrates that the form of eq. 5.1 is correct. In fig. 5.4 are shown measured concentration plotted against y-a/d-a along with curves which show values calculated by eq. 5.1 with z values determined graphically as outlined above. The data on the streams on which the concentration measurements of fig. 5.4 were made are shown in table 5.1. It will be seen that fig. 5.4

Table 5.1. Data for Curves of Figure 5.4.

Exponent z	Flow depth d-ft	Slope ft per ft	Size of suspended sand mm	Conc. at reference level c_a gm/lit	Site of measurement
0.16	9.9	.000125	.044 to .062	0.134	Missouri R. at Omaha 10-18-51
0.32	0.590	.00125	.10	6.95	Laboratory
0.43	7.7	.000121	.062 to .074	0.240	Missouri R. at Omaha 10-17-51
0.56	0.295	.00125	.10	3.20	Laboratory
0.81	0.295	.00125	.10	17.0	Laboratory
1.12	7.7	.000121	.149 to .210	2.53	Missouri R. at Omaha 10-17-51
1.93	0.600	.00125	.208 to .295	0.56	Laboratory

Fig. 5.3. Measured concentration of suspended load at center of flume 10.5 in wide.

Fig. 5.4. Vertical distribution of relative concentration of suspended sediment over a wide range of conditions of flow and sediment size.

represents a wide range of streams and that the curves fit the data very well.

To calculate the z value one must first calculate the settling velocity w, the shear velocity u_* and the von Karman constant k. Values of z calculated for laboratory streams using values of w for which the effect of concentration has been estimated and measured k values have agreed within a few percent of z values determined by the graphical procedure outline above.

Values of k have been observed to decrease from 0.4 for clear water to as little as 0.2 for high concentration of suspended load (ref. 5.1). Fig. 1.5 shows velocity profiles for a clear flow with $k = 0.4$ and a heavily laden flow with $k = 0.2$. The value of k can be calculated from graphs of velocity versus $\log y$ or $\log y/d$ as shown in fig. 5.1. From eq. 1.22 it can be seen that the slope N of the straight line in fig. 1.5a is

$$N = \frac{\log \frac{y_2}{y_1}}{v_2 - v_1} = \frac{k}{2.3\sqrt{g\,d\,S}} .$$

N can be determined from the semi-log graph like fig. 5.1a and k then calculated using known values of d and S.

The problem of determining z for a stream where no measurements are available is not solved satisfactorily. No good way is available of determining w even if the concentration of suspended sediment were known and this concentration cannot be estimated very closely. The value of k also depends on the concentration. Einstein and Chien (ref. 5.2) have plotted observed values of k against the parameter

$$\sum \frac{c_i\, w_i}{V\, S_f} \; \frac{\gamma_s - \gamma}{\gamma}$$

where c_i is the concentration of grains with settling velocity w_i, V is the mean flow velocity, S_f is the friction slope and γ_s and γ are respectively the specific weight of the sediment and water. The Σ denotes summation for all values of c_i and w_i. This graph can be used to estimate k after one has estimated the concentration.

In view of the imperfect state of theories of sediment suspension the prediction of the distribution of sediment in a flow must continue to involve the exercise of judgement and the result can only be considered an estimate.

5.5. Application of Distribution Equation. The exponent z in eq. 5.1 can be used directly to judge qualitatively if appreciable quantities of sediment are being carried in suspension. As can be seen from fig. 5.2 when z exceeds about 2 or 3 most of the sediment is moving near the bed. For such cases one would expect the stream to appear clear when viewed from the bank. On the other hand if z was in the neighborhood of 0.1 one would expect to see sediment on the surface of the stream. The knowledge of z values will immediately enable one to visualize the stream and the kind of load it will carry.

The suspended load discharge of a stream can be calculated by summing up the discharge of all the filaments of the flow from the bed to the surface of the stream. A filament of flow one foot wide and a small distance dy in height will discharge a volume of water per sec equal to $v\,dy$ where v is the mean velocity of the filament. If the concentration of suspended sediment at the level y of the filament is c lbs per cu ft then the sediment discharged by the filament per sec is $c\,v\,dy$. To sum up the sediment discharge of all filaments one integrates $c\,v\,dy$ or

$$\int_a^d c\,v\,dy. \tag{5.3}$$

Eq. 5.1 for c and eq. 1.22 for v can be substituted into eq. 5.3 to yield an involved relation for the suspended load discharge in terms of c_a the reference concentration. Since there is no straight forward relation for c_a, the integral cannot be evaluated precisely. Einstein has evaluated c_a using a bed load equation in his paper on the bed load function (ref. 5.3). Since river sediments are well graded they must be broken up into several size fractions and the integration of eq. 5.3 performed for each size fraction.

5.6. Bed Load. There are no reliable theories for bed-load discharge and all expressions available are based almost entirely on experimental

results. The interaction between flow and the grains on the bed and the formation of dunes makes up a very complicated system that has yet to yield to analysis. Solution of the problem of the behavior of the bed must progress before transport relations can be put on a firm foundation. Until such progress is made, calculation of the bed load discharge as well as the total discharge must rely on empirical relations.

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5.2 Einstein, H. A., and Chien, Ning, "Effect of Heavy Sediment Concentration near the Bed on the Velocity and Sediment Distribution", MRD Sediment Series No. 8, Missouri River Division Corps of Engineers, Omaha, Nebraska and Univ. of California, Berkeley, Aug. 1955.

5.3 Einstein, H. A., "The Bed-load Function for Sediment Transportation in Open Channel Flows", U.S. Dept. of Agriculture, Soil Conservation Service Tech. Bull. No. 1026, Sept. 1950.

Problem:-

5.1 The table below gives velocity distribution and suspended sand distribution for the fraction of the material passing the .105 mm sieve and retained on the .074 mm sieve at vertical C-3 on the Missouri River at Omaha on 4 Nov. 1952. On this day the slope of the stream was .000120, the depth was 7.8 ft, the width was 800 ft, the water temperature was 7°C, and the flow was approximately uniform.

y distance up from bottom ft	v velocity ft/sec	c concentration (in size fraction 0.074-0.105 mm) gr/l
0.7	4.30	0.411
0.9	4.50	0.380
1.2	4.64	0.305
1.4	4.77	0.299
1.7	4.83	0.277
2.2	5.12	0.238
2.7	5.30	0.217
2.9	5.40	-
3.2	5.42	0.196
3.4	5.42	-
3.7	5.50	0.184
4.2	5.60	-
4.8	5.60	0.148
5.8	5.70	0.130
6.8	5.95	-
7.8 (surface)	-	-

a) Plot the velocity profile on semi-logarithmic graph paper (v vs. log y) and concentration profile on log-log paper (c vs. $\frac{d-y}{y}$). Draw straight lines giving the best fit to the plotted points.

b) Compute from data given and your graphs the following quantities:

u_* = shear velocity
V = mean velocity (at vertical C-3)
k = von Karman constant
f = Darcy friction factor
z = exponent of suspended load equation from graph
z = w/ku_*.

c) Compute the rate of transport of this particular size fraction of sand in pounds per second per foot by graphical integration of the product cv.

CHAPTER 6 - SEDIMENT DISCHARGE FORMULAS

6.1. Introduction. The most important practical objective of research in sedimentation is to obtain an expression for the sediment discharge in terms of hydraulic parameters and sediment properties. With such a relation it would be possible to predict the amount of degradation, aggradation or bank erosion to be expected in a stream, and it would be possible also to plan work to limit these occurrences to tolerable amounts or to eliminate them if undesirable. Because sedimentation mechanics is only imperfectly developed, sediment discharge relations are largely empirical in nature and do not give precise results.

The older formulas are very often referred to as bed load formulas apparently because the bed load was probably the largest part of the load in the experiments on which the formulas were based. Newer formulas which account explicitly for the suspended load are sometimes called total load or total sediment discharge formulas. Actually all of the transport formulas are total load formulas but each one applies to a limited set of conditions depending on the data upon which it is based.

In the following paragraphs several of the better known formulas will be presented and compared with a view to evaluating them for use in engineering work.

6.2. Sediment Discharge Formulas. The following formulas are given below:

(a) Meyer Peter (ref. 6.1)	(eq. 6.1)
(b) Schoklitsch (ref. 6.2)	(eq. 6.2)
(c) Duboys (ref. 1.2, p. 794)	(eq. 6.3)
(d) Shields (ref. 1.2, p. 795)	(eq. 6.4)
(e) Einstein-Brown (ref. 1.2, p. 797)	(eq. 6.5)
(f) Meyer Peter-Muller (ref. 6.1)	(eq. 6.6)
(g) Laursen (ref. 6.3)	(eq. 6.7)
(h) Einstein Bed Load Function (ref. 5.3)	(eq. 6.8)

In the following formulas g_s is the bed material discharge per ft of width of stream. The wash load is not given by the formulas. As can be seen the equations apply to two-dimensional flow.

(a) Meyer Peter Formula (ref. 6.1).

$$g_s^{2/3} = 39.25\, q^{2/3} S - 9.95\, d_m. \qquad (6.1)$$

$$d_m = \frac{1}{100} \sum_i p_i d_{si}. \qquad (6.1a)$$

g_s = sediment discharge in lbs per sec per ft of channel width.

q = water discharge in cfs per ft of channel width.

d_m = mean size of sediment in ft according to eq. 6.1a.

p_i = weight percent of bed sediment with mean size d_{si} where d_{si} is in ft.

$\sum_i$ denotes summation over all size fractions of bed sediment.

To determine d_m, a mechanical analysis of a representative sample of bed material must first be made. The bed material is then divided into a convenient number of size fractions and the mean size d_{si} and weight percent p_{si} of each fraction determined. The sum of the products $p_i d_{si}$ is then obtained and divided by 100 as in eq. 6.1a. The Meyer Peter formula as written in eq. 6.1 is valid only for the foot-lbs-sec system of units.

(b) Schoklitsch formula (ref. 6.2).

$$g_s = \sum_i \frac{p_i}{100} \frac{86.7}{\sqrt{d''_{si}}} S^{3/2} (q - q_{oi}). \qquad (6.2)$$

$$q_{oi} = .00532 \frac{d''_{si}}{S^{4/3}} \qquad (6.2a)$$

S = slope of channel in ft per ft.

d''_{si} = mean size of a size fraction of the bed sediment in inches.

p_i = weight percent of bed sediment with mean size d''_{si}.

q_{oi} = the value of q at which sediment motion begins in cfs per ft.

$\sum_i$ denotes summation over all size fractions.

(c) Duboys formula (ref. 1.2, p. 794).

$$g_s = \psi \tau_o (\tau_o - \tau_c). \tag{6.3}$$

ψ = coefficient depending on mean size of bed sediment, ft^3 per lb per sec.

$\tau_o = \gamma d S$ = bed shear stress in lbs per sq ft.

τ_c = critical bed shear stress in lbs per sq ft.

ψ and τ_c are given in fig. 6.1 as functions of the mean size of bed sediment. (after Straub)

γ = specific weight of water lbs per cu ft.

d = water depth.

S = slope of channel.

(d) Shields formula (ref. 1.2, p. 795).

$$g_s = 10 q S \frac{\tau_o - \tau_c}{(s_s - 1)^2 d_s}. \tag{6.4}$$

s_s = specific gravity of sediment.

d_s = mean size of bed sediment.

τ_c = critical bed shear stress given in graph, fig. 4.2,

$$\frac{u_* d_s}{\nu} \text{ versus } \frac{\tau_c}{\gamma(s_s - 1) d_s}.$$

ρ = mass density of water.

$u_* = \sqrt{\tau_o/\rho}$

ν = kinematic viscosity of water.

γ = specific weight of water.

Since the Shields formula is dimensionally homogeneous, any consistent set of units can be used in it.

(e) Einstein-Brown formula (ref. 1.2, p. 797).

$$\phi = f\left(\frac{1}{\psi}\right). \tag{6.5}$$

$$\phi = \frac{g_s}{\gamma\sqrt{g(s_s - 1)}\, F\, d_s^{3/2}} . \tag{6.5a}$$

$$\psi = \frac{d_s\,(s_s - 1)}{dS} . \tag{6.5b}$$

$$F = \sqrt{\frac{2}{3} + \frac{36\nu^2}{g\, d_s^3\,(s_s - 1)}} - \sqrt{\frac{36\nu^2}{g\, d_s^3\,(s_s - 1)}} . \tag{6.5c}$$

The function f is given in fig. 6.2. Other symbols are as defined previously. Any system of units can be used with the above formula.

(f) Meyer Peter-Muller formula (ref. 6.1).

$$\gamma\left(\frac{k_s}{k_r}\right)^{3/2} dS = .047\,(\gamma_s - \gamma)\, d_m + 0.25\left(\frac{\gamma}{g}\right)^{1/3}\left(1 - \frac{\gamma_w}{\gamma_s}\right)^{2/3} g_s^{2/3} . \tag{6.6 *}$$

$$\frac{k_s}{k_r} = \sqrt{\frac{f}{8}}\,\frac{V}{\sqrt{g\, dS}} . \tag{6.6a}$$

f = function of Reynolds number and relative roughness ϵ/D given in chart form, for example, see fig. 8.8, p. 182, ref. 1.1.

$$R = \frac{4\,V\,d}{\nu} = \text{Reynolds number.} \tag{1.6}$$

$$\frac{\epsilon}{D} = \frac{d_{90}}{4d} = \text{relative roughness.} \tag{6.6b}$$

d_{90} = size of sediment in bed for which 90 percent by weight is finer, ft.

Other symbols are as defined for eqs. 6.1 and 6.4. Any system of units may be used in eq. 6.6.

(g) Laursen formula (ref. 6.3).

$$\bar{c} = \sum_i p_i \left(\frac{d_{si}}{d}\right)^{7/6} \left(\frac{\tau'_{oi}}{\tau_{ci}} - 1\right) f\left(\frac{u_*}{w_i}\right). \tag{6.7}$$

*Eq. 6.6 like other formulas is for two-dimensional flow. For methods of correcting for effects of banks and reducing channel flow to equivalent two-dimensional flow see ref. 6.1.

$$\tau'_{oi} = \frac{V^2 d_{si}^{1/3}}{30\, d^{1/3}} . \qquad (6.7a)$$

$$g_s = \frac{1}{100} \gamma q \bar{c}. \qquad (6.7b)$$

$\bar{c}$ = mean sediment discharge concentration in percent by weight.

w_i = settling velocity of particle with size d_{si} in fps.

d_{si} = mean size of the grains in a size fraction of the bed material in ft.

τ'_{oi} = bed shear stress due to grain roughness of a grain of size d_{si} in lbs per sq ft.

$\tau_{ci} = 4\, d_{si}$ = critical bed shear stress for particle of size d_{si} ft.

$u_* = \sqrt{g d S}$ = total shear velocity in fps.

V = mean flow velocity in fps.

$f(\frac{u_*}{w_i})$ = graphical function given in fig. 6.3.

$\sum_i$ indicates summation of values for each size fraction.

(h) Einstein bed load function (ref. 5.3). Space does not permit the presentation and explanation of all of the equations and charts needed to make calculations by this method. For this information the reader is referred to the original publications (ref. 6.5 and 5.3).

6.3. Flow Formulas. A sediment discharge formula is used in design work to give a relation between the water discharge q and sediment discharge g_s. For the Meyer Peter and Schoklitsch formulas, eqs. 6.1 and 6.2, this result can be obtained in a straight forward manner. Once the slope S and mechanical analysis of the bed material have been determined, values of q are assumed and g_s is calculated. A curve of g_s versus q can then be prepared. Such a curve will be called a sediment discharge rating curve.

All of the other formulas presented contain terms such as the shear stress, depth and mean velocity. In order to express g_s in terms of q in such equations, a relation between q and depth is needed. The

Fig. 6.1. Graph of coefficient ψ and critical shear stress, τ_c for Duboys eq. 6.3.

Fig. 6.2. Graph of function $\phi = f(1/\psi)$ for Einstein-Brown eq. 6.5.

Fig. 6.3. Graph of functions $f(u_*/w)$ for Laursen formula, eq. 6.7.

flow equation, i.e., the Chezy equation gives such a relation, however, in order to use it, one must know the friction factor. As already stated the friction factor of alluvial streams varies and its precise determination is not possible. There are two methods of analyzing flow in alluvial channels which give indirectly the friction factor (refs. 6.5, 6.6). The first of these presented by Einstein and Barbarossa (ref. 6.5) is based on river measurements while the other is based mainly on laboratory data. In the calculations by formulas presented in the next section the discharge q was determined by the Einstein-Barbarossa method. This gave a relation between q and the depth which could be presented graphically as a flow rating curve.

The Einstein-Barbarossa method of developing a rating curve is based on the idea that the bed friction of an alluvial stream can be divided into two parts, one due to the sand grain roughness and the other to the dunes or bed irregularities. The total bed friction results in a bed shear stress τ_o which according to the above idea is made up of a part τ_o' due to the sand grain roughness and τ_o'' due to the bed irregularities.

Because of space limitations, the relations of the Einstein-Barbarossa method cannot be presented. The reader is referred to the original publication (ref. 6.5) for complete information on the method.

References:

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6.6 Liu, Ksin-Kuan and Hwang, Shoi-Yan, "Discharge Formulas for Straight Alluvial Channels", J. of Hyd. Div., A.S.C.E., Nov. 1959, pp. 65-97.

Problem:-

6.1 Calculate the sediment discharge per unit width for the Colorado River at Taylor's Ferry for a depth of 10 ft and flow rate of 40 cfs per ft using the following 7 equations,

(a) Meyer Peter (eq. 6.1)

(b) Schoklitsch (eq. 6.2)

(c) Duboys (eq. 6.3)

(d) Shields (eq. 6.4)

(e) Einstein-Brown (eq. 6.5)

(f) Meyer Peter-Muller (eq. 6.6)

(g) Laursen (eq. 6.7)

The slope and data on sediment are given in tables 7.1, 7.2 and 7.3. (The class will be divided into 7 groups of 4 to 5 men each and each group shall solve the problem for 2 equations.)

CHAPTER 7 - COMPARISON OF SEDIMENT DISCHARGE FORMULAS

7.1 Basis for Comparison. As indicated previously, sediment discharge formulas are based almost entirely on laboratory data. Since very little is known about how transport relations will change, if at all, as the size of streams increases there is no basis for judging how well formulas based on data from small laboratory streams will apply to large streams such as rivers. Further uncertainties are introduced because natural streams differ from laboratory flows in that their channels are irregular in cross-section and alignment. The ideal method of evaluating formulas is to compare actual measured sediment discharges of rivers with the values given by formulas. Unfortunately, very few good total sediment discharge measurements on rivers are available due to the difficulty and expense of making them.

Measurements made at the following sites were used in comparing the formulas:

1. Colorado River at Taylor's Ferry (ref. 7.1)
2. Niobrara River near Cody, Nebraska (ref. 7.2)
3. Mountain Creek near Greenville, South Carolina (ref. 7.3)
4. West Goose Creek near Oxford, Mississippi (ref. 7.3).

The total bed material load of the Colorado River was based on suspended load samples and calculations by the modified Einstein method developed by Colby and Hembree (ref. 7.2). This method uses measured velocities, depths and suspended load samples in modifications of the equations presented by Einstein in his bed load function (ref. 5.3). It has the advantage that actual measured values for some important quantities are used in the equations instead of calculated values.

In the other three streams, 2, 3 and 4, total load, including bed load was measured directly.

Values of the main properties and dimensions of the four streams are listed in table 7.1. The streams vary in size from the Colorado with depth up to 12 ft and widths of about 350 ft to west Goose Creek with depth of a few inches and a width of 13 ft. The bed material was medium sand except for Mountain Creek which has coarse sand.

Table 7.1

Properties of Streams at Sites of Sediment Discharge Measurements

Data	Stream			
	Colorado R.	Niobrara R.	Mountain Cr.	W. Goose Cr.
depth range - - - - d-ft	4-12	0.7-1.3	.16-.45	.12-.25
range in q - - - - cfs/ft	8-35	1.7-5	.2-1.1	0.2-0.6
width - - - - - - ft	350	110	14	13
slope - - - - - - ft/ft	.000217	.00129	.00157	.00305
temperature - - - ^{o}F	60	60	78	68
Geom. mean sed. size, mm	.320	.283	.86	.287
Geom. std. deviation, σ_g	1.44	1.6	1.8	1.5
d_{35} - - - - - - - - mm	.287	.233	0.68	.245
d_{50} - - - - - - - - mm	.330	.277	0.86	.287
d_{65} - - - - - - - - mm	.378	.335	1.08	.330
d_{90} - - - - - - - - mm	- -	.530	1.88	.470
mean size - - - d_m-mm	.396	.342	1.02	.308

Mountain Creek had the coarsest bed material

The sieve analyses of the bed sediments for the streams are shown in table 7.2. Table 7.3 shows the size fractions into which the sediments were divided in the calculations with formulas of Schoklitsch, Laursen and the Einstein bed load function. This table also give the settling velocities calculated according to charts of ref. 6.4 with a shape factor of 0.7.

The observed sediment discharges used in comparing formulas exclude the wash load and include only bed material load. The wash load for the Niobrara River consisted of particles finer than .125 mm. The wash loads for the other streams were not given but are presumed to have been small.

Fig. 7.1. Flow rating curve for Colorado River at Taylor's Ferry calculated by Einstein-Barbarossa method compared with measurements.

Fig. 7.2. Flow rating curve for Mountain Creek calculated by Einstein-Barbarossa method, compared with measurements.

Fig. 7.3. Flow rating curve for West Goose Creek calculated by Einstein-Barbarossa method, compared with measurements.

Fig. 7.4. Sediment rating curves for Colorado River at Taylor's Ferry according to several formulas, compared with measurements.

Fig. 7.5. Sediment rating curves for Niobrary River near Cody, Nebraska according to several formulas, compared with measurements.

Fig. 7.6. Sediment rating curves for Mountain Creek near Greenville, South Carolina according to several formulas, compared with measurements.

Fig. 7.7. Sediment rating curves for West Goose Creek near Oxford, Mississippi according to several formulas, compared with measurements.

The observed data for the four streams showed fluctuations in quantities as surface slope, water temperature and grain size of bed sediment. In making the calculations mean values of these quantities were used. The following notes indicate how some of the important quantities were selected.

Colorado River at Taylor's Ferry. The measured surface slopes (ref. 7. 1) varied from .000147 to .000333. In the calculations the arithmetic average of all measured slopes or .000217 was used. The water temperature varied between 48 and 81°F. A temperature of 60°F was used in the calculations.

Niobrara River. All sediment discharge data in ref. 7.2 were used. The temperature of the water varied between 33 and 86°F. In the calculations a temperature of 60°F was used. To obtain q_s the total sediment discharge was divided by 135 ft, a mean width for the stream obtained for Fig. 24, ref. 7.2. A slope of .00129 was used in the calculations. The mean water surface slope from data of Table 24, p. 26 of ref. 7.2 is .00121.

Mountain Creek. The surface slope of Mountain Creek varied between .00136 and .00175 during the measurements. Only those data were used for which the slope fell within the range .00155 to .00160. A value of .00157 for the slope was used in the calculations. The water temperature for the measurements used varied between 15 and 25.5°C. A temperature of 78°F (25.5°C) was used in the calculations. A width of 14.22 ft was assumed to obtain q and g_s.

West Goose Creek. T he measured water surface slope of West Goose Creek varied between .00248 and .00315. Only those data were used in the sediment rating curve for which the slope ranged between .00291 and .00315. A value of .00305 was used for the slope in the calculations. A width of 12.87 ft and a temperature of 68°F were used in the calculations.

Table 7-2

Sieve Analysis of Bed Material of Streams

Sieve opening mm	Colorado % finer	Niobrara % finer	Mt. Cr. % finer	W. Goose Cr. % finer
.062	0.22	0.5		0.42
.074			.07	
.125	1.33	4.2	.33	1.7
.175				8.8
.246			1.7	34.0
.250	21.4	40.0		
.351			6.2	70.0
.495			19.0	93.1
.500	88.7	89.0		
.701			37.3	99.0
.991			60.5	99.9
1.00	98.0	96.5		
1.40			79.4	
1.98			90.4	
2.00	99.0	98.0		
3.96			99.3	
4.00	99.5	99.0		

7.2 Results of Calculations. Figs. 7.1, 7.2 and 7.3 show flow rating curves for the Colorado River, Mountain Creek and West Goose Creek respectively.* The solid line shows the calculated values according to the Einstein-Barbarossa method and the dots indicate measured values. The curve for the Colorado River agrees well with the measurements. The curves for the other two streams, figs. 7.2 and 7.3, indicate too large a depth for a given discharge. The large discrepancy between the curves and the actual data will no doubt introduce errors into the sediment discharge calculations. However in order to compare all formulas on the basis of calculations only the calculated flow rating curves were used.

The sediment discharge rating curves according to the formulas and the measurements are plotted on figs. 7.4 to 7.7. It is immediately apparent that there is a wide scatter of the results for the different formulas. However the measurements fall within the calculated curves

*The flow rating curve for the Niobrara River was omitted.

Table 7.3

Size Fractions of Bed Sediment used in Calculations

Stream	% by wt. p_i	mean size, d_{si}		settling vel. w	
		mm	ft	cm/sec	fps
Colorado River	20.8	.177	.000580	1.9	.063
T = 60°F	69.6	.354	.00116	4.8	.158
	9.6	.707	.00232	9.6	.314
Niobrara River	3.9	.088	.000288	.77	.0253
T = 60°F	37.3	.177	.000580	1.9	.063
	51.0	.354	.00116	4.8	.158
	7.8	.707	.00232	9.6	.314
Mountain Creek	17.7	.349	.00114	5.3	.173
T = 78°F	42.6	.700	.00229	10.3	.338
	31.0	1.40	.00459	10.7	.417
	8.7	2.80	.00918	19.5	.640
West Goose Creek	32.9	.176	.00577	2.1	.069
T = 68°F	60.2	.349	.00114	4.9	.161
	6.9	.700	.00229	9.8	.322

in the four cases presented. It also seems pertinent to note that there is scatter in the measurements themselves which frequently may vary as much as 50 to 100 percent from the mean.

In observing the graphs it is seen that some of the formulas give consistently steeper rating curves than others. For instance, the curves for the Einstein bed load function, Laursen and Meyer Peter-Muller equations are usually steeper than the others. The latter three equations seem to give slopes that are reasonably close to those of curves through the data, although the curves do not always agree with the data.

The sediment rating curves on logarithmic graphs can be approximated by straight lines with the equation

$$g_s = B\, q^m \qquad (7.1)$$

where m is the slope of the line on the logarithmic sediment discharge rating curves and B is given by the position of the line on the chart. The mean concentration c_m of sediment in the flow is given by

$$c_m = \frac{g_s}{q} = B\, q^{m-1}. \qquad (7.2)$$

c_m is called sediment discharge concentration. If $m = 1$ the concentration c_m will be independent of flow rate q and if m exceeds unity c_m will increase as q increases. Experience has shown, and it is reasonable to expect, that the concentration increases with q. Therefore one can expect that m should be greater than unity, i.e. that the sediment discharge rating curves on logarithmic graphs should have steeper slopes than unity. Experience indicates that the slopes should be between 2 and 3 (ref. 7.4). Curves fitted to the data points shown in figs. 7.4 to 7.7 have slopes, m, in this range.

It will be noted from the graphs that the rating curves from the formulas of Meyer-Peter, Schoklitsch, Duboys, Shields and Einstein-Brown rise too slowly especially for high discharges. The fact that the above 5 formulas give realistic slopes at low sediment discharges and slopes that are too flat at high discharges is compatible with the idea that these are "bed load" formulas intended to apply only when the load was mostly bed load. Two of the streams, the Colorado and Niobrara were known to have appreciable suspended load and hence one would not expect "bed load" formulas to apply to them. As a further check the value of the exponent z can be used to judge the distribution between bed and suspended load. Values of z for the four streams are shown in table 7.4. These values were calculated for the geometric mean sediment size and for maximum and minimum flows. Mountain Creek is the only stream with values in excess of 2.5 and which strictly speaking is a "bed load" stream. Because of this one would expect the so-called "bed load" formulas to fit the Mountain Creek measurements better than those of the other streams. However this is not the case.

Table 7.4

Values of Exponent z for Streams, Based on Mean Sediment Size and a Value for von Karman k of 0.4

Stream	Temp °F	q - cfs/ft		$z = w/0.4u_*$	
		min	max	max	min
Colorado River	60	6.0	60	2.2	1.4
Niobrara River	60	1.0	6.0	1.6	1.0
Mountain Creek	78	0.2	3.0	6.4	4.0
West Goose Creek	68	0.2	1.0	2.5	1.8

7.3 Recommendations. From the limited evidence presented here it is not possible to recommend strongly any formula or formulas. It is clear that the statement by the Fontana Sedimentation Conference in 1954 (ref. 7.5), that, in applying formulas, errors of 100 percent are to be expected applies to the present cases. Based on the evidence presented, the following statements are made with the idea that they will serve as a guide in the use of formulas until better information is available.

1. Values of sediment discharge calculated by formulas are to be considered as estimates only since the errors involved may be 100 percent or more.
2. When possible, it is desirable to use more than one formula to calculate sediment discharge. By comparing the results from several formulas one gets a rough idea of their reliability.
3. Calculated sediment discharge rating curves which have slopes on logarithmic graphs which are in the neighborhood of unity are too flat. These curves should not be used especially in cases where one is interested in the difference between, or the ratio of, sediment discharges for different flows in the same channel. In such cases the use of sediment discharge rating curves with slopes of between 2 and 3 on logarithmic graphs should give more reliable results.

References:

7.1 Interim Report, Total Sediment Transport Program Lower Colorado River, U. S. Bureau of Reclamation, Jan. 1955

7.2 Colby, B. R. and Hembree, C.H., "Computation of Total Sediment Discharge Niobrara River near Cody, Nebraska," U. S. Geological Survey Water Supply Paper 1357, 1955.

7.3 Einstein, Hans Albert, "Bed-Load Transportation in Mountain Creek," U.S. Dept. of Agriculture, Soil Conservation Service Rep. SCS-TP-55, August 1944.

7.4 Leopold, Luna B. and Maddock, Thomas Jr., "The Hydraulic Geometry of Stream Channels and Some Physiographic Implications", U.S. Geological Survey Prof. Paper 252.

7.5 Notes on the Fontana Conference, Oct. 11, 1954.

CHAPTER 8 - ROUGHNESS OF ALLUVIAL CHANNELS

8.1. Need for Knowledge of Roughness of Alluvial Channels. The problem of clear flow in fixed boundary channels has been studied extensively and the friction factor of such flows can be predicted quite accurately. For such channels the roughness of the boundaries is constant and can be determined once for all time for each material. The fluid properties vary only with temperature and in a systematic way which can be determined quite accurately.

In the case of streams which transport their bed material, and which have movable, deformable beds, the problem of determining the resistance to flow is much more complicated. Since the form of the boundary is related to the depth and velocity of flow, the properties of the fluid and bed material, and the geometry of the channel, the roughness of the channel is not constant but can take on an infinite number of values. Furthermore, there is no reason to suspect that a mixture of water and sediment will have the same properties and behavior as clear water, and indeed it does not. The effects of the suspended sediment in this respect are not understood well enough to be expressed quantitatively. Since the depth and velocity of flow, channel roughness, sediment concentration in the flow, and in some cases the channel geometry (if there is significant scour or deposition of bed material) are all interrelated, it is not possible to make experiments to study each variable separately by holding all variables constant except the one under consideration. In the language of mathematics, the problem is inherently nonlinear.

All of these difficulties do not lessen the need for knowledge of the roughness of alluvial channels since no design or analysis of any water conveyance channel can procede very far without such knowledge. To illustrate the types of problems which require knowledge of roughness, a hypothetical problem will be discussed.

Suppose that you are an engineer responsible for the flood protection of a city on a large river. You are notified by the river agencies upstream that a flood with a maximum discharge of Q is going to pass

your city. You must determine to what height the levees must be raised to protect the city. Assuming the width b is large enough that the depth can be taken as equal to the hydraulic radius, Manning's equation (eq. 1.15) may be written as

$$Q = (b\,d)\,\frac{1.49}{n}\,d^{2/3}\,S^{1/2}.$$

This equation cannot be solved for d until n is known. As will be seen in section 8.3, this roughness can vary by a factor of four or more. Without some knowledge of n for the river, or some analytical technique to predict it, the depth of the river at the anticipated discharge cannot be predicted. This same problem arises in designing channel sections in alluvial material.

8.2. Example of Variable Roughness; Laboratory Data. Fig. 8.1 shows the results of a set of laboratory experiments which were performed in a laboratory flume 10-1/2 inches wide and 40 ft long. Each point represents a different flow condition. The depth is the same for all points (d = 0.241 ft) but the velocity is different for each. The bed form accompanying each flow is identified by the shading of the point as explained in the legend. The sand has a median diameter of 0.152 mm. The slope of the flume is variable so that uniform flow could be achieved for each velocity.

The experiments were performed by selecting a velocity and then adjusting the flume slope until uniform flow was achieved. The slopes of the flume and energy gradient were then identical. This slope for each flow is shown in the top graph. The second and third graphs show the bed shear velocity U_{*_b} and the bed friction factor f_b. These quantitites are the same as the shear velocity and friction factor defined and discussed in section 1.9 and 1.7 except that a mathematical procedure has been applied to the values f and u_* computed from the measured depth, velocity and slope to eliminate the effects of the smooth walls of the flume. Thus U_{*_b} and f_b are the values which would exist if the same flow were in an infinitely wide channel. The data shown in fig. 8.1 are valid only for the stated depth, sand, and flume and are not generally

Fig. 8.1. Variation of slope, s, bed friction factor, f_b, and bed shear velocity, u_{*b}, for laboratory experiments in a 10.5-inch wide flume.

applicable. These experiments have been discussed in detail by Vanoni and Brooks (ref. 8.1).

The bed friction factor is seen to decrease with increasing velocity in the dune and sand wave regimes and achieve a nearly constant value in the flat bed regime. Thus it appears that at this depth the bed configuration with the greatest roughness is the dune pattern which occurs at low velocity. As the velocity is increased the dune pattern changes to reduce the roughness and friction factor. Finally at higher velocities the bed becomes flat and the bed roughness is the irreducible minimum.

As the velocity increases the concentration of sediment in the fluid also increases (data not shown here) and this also reduces the friction factor as was discussed in section 2.2. However, this effect is small compared to the change in roughness due to the change in bed form.

The important point to note is that for flow over a flat bed, the friction factor is only about one-eighth that for flow over the roughest dune bed. Thus the channel roughness is <u>not</u> constant as implied or stated in many texts.

This change in friction factor has a marked effect on the relation between the velocity and slope. Flow formulas such as Manning's (eq. 1.15) and the Darcy-Weisbach (eq. 1.16) indicate that for each depth and slope there is one and only one velocity if the friction factor is constant. However for the flows in the laboratory flume under discussion, the graph of slope versus velocity indicates that for some slopes, two and possibly more velocities were possible. For example, at a slope of 0.0024, flows with velocities of 0.91 ft/sec and 1.38 ft/sec were studied. The fitted curve indicates that a flow with a velocity of 2.14 ft/sec might have also been possible with the same slope. The sediment discharges for the two flows studied at this slope were 0.59 lb/min and 3.8 lb/min for the low velocity and high velocity flow, respectively. This multiplicity of velocity with respect to slope is caused by the decrease of the friction factor which accompanies increasing velocity which in turn is brought about primarily by changes in the bed configuration. The details of the processes involved in this

change in friction factor are not understood well enough to be predicted or described analytically. In fact, the phenomenon has received attention in the literature only in the last few years.

8.3. Example of Variable Roughness; Field Data. There is ample evidence to prove that the variable roughness phenomenon discussed in section 8.2 is not just a laboratory curiosity but exists in natural streams also. Fig. 8.2 (taken from ref. 8.1) shows data from the Rio Grande River near Bernalillo, New Mexico taken in 1952. The data were taken during the passage of the spring flood and extends from April 25 to July 24. Section F is located approximately 1-1/2 miles downstream from section A-2 and the river is much wider at this location. The slope,friction factor, and D_{65} (65% of the bed material is finer than this value) are shown for each point. The river is very wide so the hydraulic radius can be taken as equal to the depth. The sediment discharge (data not given here) generally increased with the unit discharge q but was not uniquely related to it. The sediment discharge was generally higher for a given value of q on the rising stage than on the falling stage.

From the data given in fig. 8.2 it is seen that the friction factor at section A-2 varied by a factor of over three and at section F by a factor of six. These variations are much too large to be accounted for by the relatively small changes in the size of the bed material which occurred. Fig. 8.2 shows the same trend of friction factor as the laboratory data discussed in section 8.2. The friction factor generally decreases with increasing unit discharge (velocity) and increases with decreasing unit discharge. However, as shown in table 8.1 the friction factor is not uniquely related to the unit discharge but apparently depends also on the sediment discharge.

Thus slight differences in unit discharge can be accompanied by relatively large changes in the friction factor, the smaller friction factor accompanying the larger sediment discharges. These observations lend strength to the conjecture put forth in section 2.5 that the roughness of the channel adjusts so that the imposed discharge and sediment load may be accomodated.

The variability of roughness and its non-uniqueness with respect

Fig. 8.2. Variation of hydraulic radius, slope, friction factor, and bed material size with changing discharge for Rio Grande River at Bernalillo, New Mexico, April-July 1952.

Table 8.1

Data for the Rio Grande River at Bernalillo, New Mexico

Section	Date	Unit Discharge q cfs/ft	Hydraulic Radius r=d ft	Total Discharge Q cfs	Velocity U ft/sec	Friction Factor f	Suspended Load Discharge tons/day
A-2	April 25	10.4	2.49	2820	4.14	0.036	25,300
	June 26	10.4	2.75	2800	3.74	0.040	7,070
F	April 25	4.41	1.18	2820	3.72	0.018	21,600
	July 24	3.49	1.58	2030	2.20	0.078	10,560

to discharge is probably responsible for discontinuous rating curves observed on many rivers. On these rivers the discharge cannot be determined from the stage since over a certain range of stage there are different stages which will correspond to the same discharge at different times. The data from the Rio Grande given in table 8.1 indicate that it might have such a rating curve. At station A-2 for nearly identical discharges the depths on different days were 2.49 ft and 2.75 ft. Bruce Colby (ref. 8.2) has described in detail his observations and measurement of such streams. He too attributes the discontinuity to roughness changes.

8.4. Estimating Channel Roughness. The roughness of an alluvial channel (or the friction factor of the flow) may be determined by a procedure proposed by Einstein and Barbarossa (ref. 8.3) once the discharge, bed material properties, channel geometry, and slope are known. Data from natural streams were used to determine the relationship between the derived dimensionless quantities. However, the derivation of some of the quantities which enter into the analysis is rather involved and the physical significance of these quantities is somewhat obscure. By this procedure it is possible to construct a depth-discharge curve (rating curve) for a stream when the quantities mentioned above are known. For some streams the predicted depth-discharge relations are fairly accurate, but in other instances, the procedure will yield a relation which is not nearly accurate enough for

engineering application. The big difficulty is that there is no way to determine beforehand if the procedure will yield the required accuracy. The results of calculations using this procedure should be viewed with this in mind. (See Fig. 7.1, 7.2 and 7.3.)

Because methods of calculations are imperfect the engineer is forced to rely on his experience and judgment or on empirical approaches such as regime theory (see Chapter 10) in estimating the roughness of a stream for which no measured values are available. However if such data are available for the stream for conditions comparable to those for which the friction factor is desired, this data should be used in preference to values computed from the Einstein-Barbarossa method.

References:

8.1. Vanoni, Vito A., and Brooks, Norman H., "Laboratory Studies of the Roughness and Suspended Load of Alluvial Streams", Sedimentation Laboratory, California Institute of Technology, Pasadena, Calif., Report No. E-68, December 1957.

8.2. Colby, Bruce R., "Discontinuous Rating Curves for Pigeon Roost and Cuffawa Creeks in Northern Mississippi", U. S. Dept. of Agriculture, Agricultural Research Service Publication ARS 4-136, April 1960.

8.3. Einstein, Hans A., and Barbarossa, Nicolas L., "River Channel Roughness", Trans. A.S.C.E., Vol. 117, 1952. pp 1121-1146.

8.4. Liu, Hsin-Kuon and Hwang, Shoi-Yan, "Discharge Formulas for Straight Alluvial Channels", J. of Hyd. Div., A.S.C.E., Nov. 1959, pp 65-97.

Problem:-

8.1 Calculate Mannings n (eq. 1.15) and the Darcy-Weisbach f (eq. 1.16) from the laboratory data obtained for flows with the following bed configurations:

1) Flat bed with incipient sediment motion.
2) Dune covered bed.
3) Flat bed with high sediment transport rate.

CHAPTER 9 - PRINCIPLES OF CHANNEL STABILITY

9.1. Introduction. A brief description of general ideas on alluvial channel stability will be presented in this chapter. No attempt is made to provide all working tools because they are available in detail in other publications. (See reference list at end of chapter, especially 9.1, 9.2 and 9.3.) Discussion of regime theory and similar empirical relations is deferred to Chapter 10.

9.2. Definition of Stable Channel. A stable channel in alluvial material is one in which scour of banks and changes in alignment do not occur, and deposition or scour at the bed, if it occurs, is not objectionable. This definition allows some changes in bed during short periods of time, provided that over a period of years there is general equilibrium of the bed.

9.3. Basic Requirements.

(a) Overall. A stable channel must convey both the water and sediment discharges which are delivered into it. In the long run, rivers are self-adjusting in this regard, as the general shape of a valley landscape is generally such that the water and sediment transport are roughly in balance in each channel. Any gross unbalance would be quickly reflected in aggradation or degradation. A stream in this kind of balance is called a graded stream (ref. 9.8).

(b) Local. Overall balance between water flow and sediment load is necessary, but not sufficient, as there may be local scour at one point and deposition at another. On a meandering natural stream the outside bank of a stream is eroded by locally high velocities near the bank, at the same time deposition on bars occurs on the inside bank. Even graded streams may thus be unstable in this regard and require artificial bank protection (revetments or groins). Canals must be laid out with suitable width-depth ratios and side-slopes to avoid serious bank erosion.

9.4. Stream Variables for Overall Stability. First consider the problem of overall balance without regard to local erosion which will be discussed in a later section.

For canals or streams transporting very little or no sediment (except wash load), the computed shear stress on the bed should be less than the critical shear for the bed material (see Chapter 4). This so-called analysis by "limiting tractive force" (ref. 9.3) is preferred to analysis by "limiting velocity" (ref. 9.9) because the limiting velocity varies considerably with the size of the channel. Scour of the banks is a special problem considered in sections 9.8 and 9.9 below.

As shown in fig. 9.1, Lane (ref. 9.3) suggests values of limiting tractive force (i.e. bed shear stress) which are considerably greater than critical shear values as computed by Shields or White for sand and fine gravel. For example for 1.0-mm bed material in a canal carrying clear water, Lane suggests $\tau_{max} = 190$ gr/m^2, whereas Shields' curve yields $\tau_c = 57$ gr/m^2. In the writer's opinion this discrepancy is due to presence of bed features such as ripples and dunes at near-critical conditions in contrast to flat beds used in Shields' and White's experiments. Thus for design Lane's values are probably more reasonable than White's or Shields'. However, it should be noted that Lane obtained his values, not by direct observations, but by calculating backward from various published recommendations for limiting velocity.

Fig. 9.1 from Lane also shows a variety of recommendations from different sources for different conditions. The diversity of curves gives an indication of the large margin of error possible in analysis of stable channels.

When the bed-material load is appreciable the problem becomes more complicated. The hydraulic characteristics of the channel must provide a transport capacity commensurate with the sediment load supplied to the stream. But since we have seen in Chapter 8 that channel roughness is intimately related to transport of sediment, the hydraulic characteristics cannot be taken as fixed or known quantities which may be directly applied in sediment transport calculations. There is what might be called feedback, because it is the sediment-carrying requirement that may fix the stream hydraulics for given discharge instead of vice versa. At the present time, this whole problem is not thoroughly understood, and the designer may find himself in as much of a quandary

Fig. 9.1 Recommended limiting tractive forces (or shear stresses) for canals (after Lane, ref. 9.3, p. 1253).

as the researcher!

To understand the relationships it is helpful to consider even the simplest possible case: a straight channel to be built in uniform non-cohesive alluvium. The major variables are as follows:

S = slope

d = mean depth (assume ≈ hydr. radius as in wide channel)

w = width of channel

V = mean velocity

Q = discharge

Q_s = sediment discharge

f = friction factor

D = mean sediment size.

Neglected are variables such as the shape of cross section, water temperature and viscosity, geometric standard deviation of sediment, shape factor and fall velocity of sediment, wash-load concentration, flow-duration curve, hydrograph characteristics, stream curvature, etc. Although obviously pertinent, there is no way of handling them analytically by present methods. Nevertheless, the idealized problem is still instructive.

There are eight variables above satisfying five equations or functional relationships as follows:

Continuity:

$$Q = V w d \tag{9.1}$$

Flow equation, defining f or n:

$$V = \sqrt{\frac{8}{f}}\ \sqrt{g d S} = \frac{1.49}{n}\ d^{2/3}\ S^{1/2} \tag{9.2}$$

Equation for f or n:

$$f \text{ or } n = \text{function of other variables} \tag{9.3}$$

(such as in Einstein-Barbarossa analysis, Ch. 8)

Sediment transport equation:

$$Q_s = \text{function of hydraulic variables} \tag{9.4}$$

(such as bed-load formulas in Chapters 6 and 7)

Relation of width to depth:

$$\frac{w}{d} = \text{function of other variables} \tag{9.5}$$

For the last equation, there is no general theory (although a later section will discuss scour of banks); one must rely on observations summarized empirically as in regime theory, or by Leopold and Maddock (see Chapter 10). Sediment transport equations are almost universally for two-dimensional flow (i.e. per unit of width).

9.5. Solution of Equations for Overall Stability. With the five relations, eqs. 9.1 to 9.5, only three variables may be considered independent, and five must be dependent. The designer in his analysis can specify three variables such as D, Q and S; or perhaps D, Q and Q_s. In this way we arrive at the paradox of which variables are really independent; basically probably none are. For instance, geologically speaking grain size is certainly dependent on watershed characteristics, including hydraulic ones.

The problem of whether a variable is independent or dependent is not just an academic question, because for some choices of independent variables (such as D, d, S) there is more than one solution, or perhaps none at all for the remaining variables. This fact is indicated by laboratory results as well as field data, but in transport and roughness equations it is usually obscured in the "generous" scatter of points (refs. 9.12, 9.13). It is likely, for example, that the sediment-rating curve and flow-rating curves are not unique for some streams with rapidly changing channel roughness; in these cases the "curves" may be "bands" of some considerable width, with scatter of points not being due to observational errors but to real variations. For further discussion see Chapter 8.

A concrete suggestion for the designer is to seek from the various

equations, by trial-and-error if necessary, combinations of variables which seem consistent and reasonable considering the whole range of natural flows. Several different approaches should be tried and compared. It is necessary to depend heavily on experience, and in case computed results are at variance with field experiences, the equations should probably be considered less reliable than the field observations.

Unfortunately, at the present state of knowledge of mechanics of transport, channel stability problems involving overall transport relations cannot be solved with high confidence without confirmation by various empirical formulas based on observations of existing streams and canals (regime theory, etc., described in Chapter 10).

9.6. Continuity Principle for Sediment. Departures from overall stability may be considered as storage or depletion of sediment in a given reach, i.e., non-uniform flow of sediment in the channel. If the reach has an equilibrium sediment discharge of Q_{se} and the actual sediment discharge entering the reach is Q_s, the stream will try to adjust Q_s to Q_{se}. Both the actual rate of transport Q_s and the equilibrium rate Q_{se} will change, but they will tend to approach each other.

Thus if $Q_s < Q_{se}$, the stream is "starved" and degradation occurs to supply the material required for an increase in Q_s. At the same time, however, the bed may become coarser or even armored by depletion of finer fractions, and the roughness and the depth will probably increase; all these events will tend to reduce Q_{se} in the direction of Q_s. This occurs for example downstream from a dam which cuts off the normal flow of sediment.

Conversely, when $Q_s > Q_{se}$, sediment is stored in the reach, causing aggradation, as upstream from a reservoir where Q_{se} is decreased in the backwater zone.

There are many examples of river engineering problems caused by non-uniform sediment transport. They may be roughly categorized as follows:

$Q_s > Q_{se}$ (Deposition)

1. Aggradation upstream from a reservoir.
2. Sedimentation in reservoirs and lakes.
3. Tributary channel bringing heavy sediment load to main channel, causing local aggradation.
4. Canyon streams discharging on alluvial fans, causing widespread deposition (as in Los Angeles area).
5. Desilting works at water intakes returning all sediment load to main channel in which flow of water is depleted (e.g. Imperial Dam).
6. River regulation eliminates floods which formerly periodically cleared channel of accumulated sediment and vegetation (e.g. Colorado River at Needles).

$Q_s < Q_{se}$ (Scour)

1. Degradation downstream from dams.
2. Canals in too fine material carrying clear water.
3. Realigned channels, with increased slope.

9.7. Bank Stability. Attention will now be directed to problems of local stability. These are situations where local erosion may occur, although the total transport in the channel is in general equilibrium. Material may be eroded from unstable banks but deposited on the bed, thus effecting a change in cross section or alignment.

Channel banks may be damaged basically in two ways: (1) direct removal of material at the surface by scour; and (2) internal shear failures resulting in sudden caving, sliding, or sloughing of large bodies of earth. Such shear failures may be caused by surface erosion at the toe of slope, general bed degradation, excessive saturation of bank at low water, slope angle being too steep, or earthquake.

When examining damaged banks in the field it is necessary to make the distinction between scour and shear failure, for the treatment is different. If scour is the problem then protective measures such as groins, revetments or vegetation cover are needed to keep flow with scouring velocity safely away from the bank material (ref. 9.4). On the other hand, if sliding is the problem, the embankment slope should be

reduced, or an intermediate berm might be provided to increase stability; or the soil can be compacted to improve shear strength; or drainage can be improved to reduce seepage pressures following rapid drawdown (i.e. revetments should be pervious). Methods of soil mechanics, not reviewed here, are utilized for evaulating stability of slopes against internal slips.

The discussion which follows is concerned only with the surface scour aspect of bank stability. Of necessity it is also limited to cohesionless materials, as only empirical information is available on scour of cohesive banks (see Chapter 10).

9.8. Critical Shear Stress on Sloping Banks. When particles are resting on a slope, it takes less shear to dislodge them, inasmuch as there is a downhill component of the weight of the grain. In fact when the slope of a sand or gravel pile reaches θ, the angle of repose, the gravity force alone on each particle is sufficient to send it tumbling down the slope. Clearly then to provide substantial particle resistance to the flowing water, the side slope ϕ must be less than the angle of repose.

E. W. Lane (refs. 9.1, 9.3) has presented an ingenious analysis of critical shear on side slopes, using the factor K, which is defined for a given grain size as

$$K = \frac{\tau_c \text{ on side slope}}{\tau_c \text{ on flat bed}} .$$

By analysis of forces on the particles, Lane and others (refs. 9.1, 9.3) show that

$$K = \cos\phi \sqrt{1 - \frac{\tan^2\phi}{\tan^2\theta}}$$

where ϕ = angle of side slope

and θ = angle of repose.

This equation is shown graphically in fig. 9.2. Note that $K \to 0$ as $\phi \to \theta$, and $K \to 1$ as $\phi \to 0$, as expected.

Experimentally determined values of the angle of repose θ for various materials are given in fig. 9.4 from Simons (ref. 9.1). θ is seen to be larger for more angular material and θ always increases

gradually with grain size.

Hence for a given material, the designer estimates θ, selects side slope, reads K from fig. 9.2; then he computes τ_c for a flat bed (see Chapter 4 or fig. 9.1). Finally the product $K\tau_c$, the critical shear for the side slope, is compared with actual shear on the side walls, to see if any grains will move. But the shear on the side walls under given flow conditions is not readily known and must be estimated as explained in the next section.

9.9 Shear Distribution in Channels. There is no really satisfactory solution to the problem of determining the variation of shear stress around the boundary of an open channel. The only analysis known to the writers is by Olsen and Florey (ref. 9.15, also ref. 9.3, p. 1241-3) using a differential equation which is simple enough to be solvable, but not proven to represent the given physical problem. In the absence of a more reliable procedure these results (see fig. 9.3) may be used, as the possible errors are not so much as in other steps of design of stable channels. This figure shows, for example, that as a trapezoidal canal becomes wide, the bottom shear in the center where it is a maximum approaches 1.00 γdS (as it should for a two-dimensional channel); and for the maximum shear on the sides, the curves show 0.75 to 0.80 γdS for wide canals (where γ = unit weight of water, d = depth, and S = slope).

With the aids in sections 9.8 and 9.9, one can also design a loose rock revetment, as well as banks in non-cohesive natural material.

9.10 Curves. For a curving channel, there is more danger of scouring the banks because the thread of the current sweeps close to the outside bank of a curve. Lane has given some empirical estimates on reduction of permissible tractive force to protect curves which are reproduced below in table 9.1.

Nevertheless, it is desirable to make canals and realigned river channels with slight curvature to concentrate the main current of the river.* While special bank protection may be needed on the downstream part of outside curves, the revetment requirements on

* Examination of aerial photographs of the natural river will provide clues to the maximum radius of curvature at which the stream maintains a well defined channel.

Fig. 9.2 Relation K to ϕ, angle of side slope, and θ, angle of repose of cohesionless bank material. (K is the ratio of critical shear on a side slope to what it would be for the same material on the bed.) (after Lane, ref. 9.3, p. 1245).

Fig. 9.3 Maximum shear stresses on sides and bottom of a channel (after Lane, ref. 9.3, p. 1241, based on solution of approximate idealized equations). Note: In this figure w = unit weight; y = depth; S_o = energy slope; b = width.

Fig. 9.4 Angle of repose of non-cohesive materials (after Simons, ref. 9.1, p. 25).

Table 9.1

Comparison of Permissible Tractive Forces in Sinuous Canals with Permissible Values in Straight Canals

Degree of sinuousity	Relative limiting tractive force	Corresponding Relative limiting velocity
Straight canals - - - - - -	1.00	1.00
Slightly sinuous canals - -	0.90	0.95
Moderately sinuous canals	0.75	0.87
Very sinuous canals - - -	0.60	0.78

all the rest of the bank may be completely eliminated, as is often found on the Missouri River.

9.11 Local Scour Due to Structures. Man-made structures may cause localized scour which should be distinguished from general degradation due to interruption of the sediment supply.

(a) Contractions such as bridge openings. At bridges the flow velocity may be increased due to reduced cross-section, causing local scour. Especially around bridge piers scour may be pronounced due to acceleration of the flow past the nose of the pier, and turbulence in wake of pier (see ref. 9.14 and 9.16).

(b) Energy dissipators. Every spillway and drop structure must be equipped with an adequate stilling basin within which to dissipate the energy before the turbulent flow is released onto an alluvial bed. The design of such structures to avoid scour can be successfully accomplished with hydraulic models or generalized laboratory studies (for example, ref. 9.11). There is extensive literature on dissipators, and no attempt is made to cover the subject herein.

Nevertheless, an apparently well-designed stilling basin may still fail by being undermined in the process of general channel degradation wherever the supply of sediment in the stream is reduced. Thus stilling basins must be built low enough to withstand any anticipated degradation.

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Problem:-

9.1 A revetment of loose crushed rock is to be built on the outside of a bend on a meandering stream (consider "moderately sinuous"). The following data are known for design flow:

Q = discharge = 300 cfs

w = average width 40 ft

d = mean depth = 2 ft

n = Manning roughness = 0.025

Using lane's analysis select a side slope and mean size of revetment rock.

CHAPTER 10 - REGIME OF CHANNELS

10.1. Introduction. The preceding chapters have presented and discussed analytic and semi-analytic methods for determining the various quantities needed to design or analyze an alluvial channel. With the possible exception of the equation for predicting the distribution of suspended load (eq. 5.1), relations have either been inadequate or unreliable for general application and have provided the engineer who must design or analyze alluvial channels with few usable tools. Another approach to the problem is to put aside the analytic approach, adopt the philosophy that nothing succeeds like success, and then study what constitutes success in alluvial channels. To this end, numerous studies have been conducted of man-made and natural alluvial channels which have given satisfactory performance. These studies have attempted to find how discharge, velocity, depth, channel shape, water and sediment properties, and slope are related in these successful channels and to express the relations in a form which can be conveniently used for design of other channels. These techniques which use this approach are referred to as regime theories, although, strictly speaking, there is practically no theory involved. The results have been presented graphically and by equations, but all are derived from plotting data from the studied channels in different forms and deducing the necessary relations among the different variables.

Usually three relations are presented: (1) a flow formula of some type which gives the required slope; (2) a formula for channel depth; (3) a formula for channel width. These three quantities are presented as functions of the discharge and channel material.

Regime theory had its origin in India among the early engineers who were charged with designing and operating the extensive canal systems built by the British. In its early stages it was quite unreliable since little data was available to base it on and the data analysis was not complete. As more data have been accumulated and more thoroughly analyzed, regime theory has become more popular and is now used in different forms quite extensively. It is reliable to the extent that the

canal being designed or analyzed is similar in composition and operation to the canals upon which the regime relations being used are based.

In this chapter, three of the better regime theories will be presented. The mechanics of channel adjustment to regime will also be discussed in a qualitative way and a specific example will be presented to show how a river reacts when its regime is upset.

10.2. Definition of Regime. The term "regime" does not admit to concise definition since it is a general concept. Professor Blench (ref. 10.1), a proponent of regime theory, defines the term by analogy. According to Blench, to declare that a river has acquired a regime is comparable in a general sense to stating that a territory has acquired a climate. The vagaries of weather from day to day are great and have not yielded to exact analysis. Nevertheless, a regime of weather such as a climate, is recognizable, and reasonably definite laws controlling it have been discovered.

A regime-type river is defined as one that has formed a major part of every cross section from material that has been transported or could be transported by the river at some stage of flow. A river is said to be "in regime" in a reach if its mean measurable behavior during a certain time interval does not differ significantly from its mean measurable behavior during comparable times before or after the given interval. In short, a river "in regime" is one in which the different variables have achieved a relation to each other such that no objectionable scour or deposition occurs, and the shape, alignment and slope of the channel remain constant.

Most regime methods give three equations, a flow formula and two equations which relate channel width and depth to the discharge and channel characteristics. Thus the channel slope, width, and depth are treated as the dependent variables.

10.3. Blench's Regime Theory. Professor Blench has probably devoted more time and energy to the study and improvement of regime theory than any other person. In his recent book (ref. 10.1) he recommends the following equations for the design of canals with a small bed load:

$$b = \sqrt{\frac{F_b Q}{F_s}} \tag{10.1}$$

$$d = \sqrt[3]{\frac{F_s Q}{F_b^2}} \tag{10.2}$$

$$S = \frac{F_b^{5/6} F_s^{1/12} \nu^{1/4}}{3.63 g Q^{1/6}} \tag{10.3}$$

In these equations,

b = mean channel width; i.e., width which multiplied by depth gives area.

d = mean depth of flow measured from the bed of the channel.

$F_b = V^2/d$ = bed factor.

$F_s = V^3/b$ = side factor.

The other quantities are as previously defined. Blench recommended side factors of 0.1, 0.2 and 0.3 for loams of very slight, medium, and high cohesiveness, respectively. For the bed factor he recommended

$$F_{bo} = 1.9\sqrt{d_s} \tag{10.4}$$

where the zero subscript denotes that this value of F_b is applicable for cases of small bed load discharge and d_s is the median diameter of the bed material in mm. This value of F_b applies only if the bed material is in the sand range.

For cases in which the bed load discharge concentration is greater than 20 ppm, Blench recommended the following equations:

$$b = \sqrt{\frac{F_b Q}{F_s}} \tag{10.1}$$

$$d = \sqrt[3]{\frac{F_s Q}{F_b^2}} \tag{10.2}$$

$$S = \frac{F_b^{5/6} F_s^{1/12} \nu^{1/4}}{3.63\, Q^{1/6} g(1 + C/2330)} \qquad (10.5)$$

where C is the bed load concentration in ppm. In these equations, the following values of bed factor are to be used*:

(i) If the velocity is subcritical (see section 1.5)

$$F_b = F_{bo}\,(1 + 0.012\,C) \qquad (10.6)$$

where F_{bo} is defined in eq. 10.4.

(ii) If the velocity is supercritical

$$F_b = 32.2 + 0.006\,(C - C_c) \qquad (10.7)$$

where C_c is the bed load concentration in ppm at critical velocity. The same values of F_s are applicable to both high and low bed load discharge canals.

These equations are derived mostly from data on canals in India, most of which have cohesive banks and consequently are not strictly applicable to canals which have sand banks. For this reason these equations, and similar regime equations based on Indian data, have not been used extensively outside the area where they were developed.

Example**:

The Ft. Morgan I canal west of Ft. Morgan, Colorado, has the following channel and operating characteristics:

$Q = 146$ cfs.

$d_s = 0.318$ mm.

Temperature = 77° F

$\nu = 0.97 \times 10^{-5}$ ft/sec

$C = 200$ ppm.

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* Blench erroneously assumed that dunes occur at velocities which are less than critical, flat bed at higher velocities. This is the basis he used for distinguishing between the bed factors for the two flow conditions.

** Data taken from ref. 10.2.

Bank material: highly cohesive.

Flow is subcritical

Since C is greater than 20 ppm the large bed load equations must be used. From eq. 10.3,

$$F_{bo} = 1.9\sqrt{0.318}$$
$$= 1.07$$

Using this value in eq. 10.6 yields the bed factor

$$F_b = 1.07\,(1 + 2.4) = 3.64.$$

For highly cohesive banks,

$$F_s = 0.3.$$

Eqs. 10.1, 10.2 and 10.3 then yield $d = 3.22$ ft, $b = 42.1$, and $S = 0.00051$, respectively. The actual values for the canal are $d = 3.51$ ft, $b = 30.6$ ft and $S = 0.000135$. In this case, the Blench equations yield a channel which is too wide and over estimate the roughness.

10.4. Simons' and Albertson's Regime Method. Recently Simons and Albertson (ref. 10.2) have presented a set of graphs which can be used for design or analysis of alluvial channels. Their approach appears superior to any that have been presented previously for several reasons. First, it is based on more data than previous regime methods, and these data represent a wide range of types (i.e. cohesive and non-cohesive) of bed and bank material. In presenting their recommended relations, the different types of bed and banks are distinguished so it is possible to select regime relations applicable to the channel in question. Second, they give more information on the shape of the channel. Values of the bank slope, mean channel width, and top channel width may be derived from their relations. Finally, they present three different methods for estimating the slope and thus give an idea of the possible range of slope to be considered.

The graphical relations used in this method are presented and

discussed in appendix 10A.

10.5. Leopold and Maddock's Regime Analysis of Natural Streams. Natural streams would be expected to have a different regime than irrigation canals because of the wide variation of discharge and load they experience. Leopold and Maddock (ref. 10.3) analyzed data from several rivers and found that for most rivers it was possible to relate the top width w, mean depth d, mean velocity V, and suspended load L to discharge Q in the following meaningful relations:

$$w = aQ^b \tag{10.8}$$

$$d = cQ^f \tag{10.9}$$

$$V = kQ^m \tag{10.10}$$

$$L = pQ^j \ * \tag{10.11}$$

Substituting eqs. 10.8, 10.9 and 10.10 into the continuity equation, $Q = wdV$, yields

$$Q = ack\,Q^{b+f+m}$$

from which we conclude

$$ack = 1 \tag{10.12}$$

and

$$b + f + m = 1 \tag{10.13}$$

Leopold and Maddock studied the rivers from two points of view. First, they considered the variation of w, d, V, L with Q at given sections of rivers. They found that the variations could be expressed quite well by equations of exponential form (eqs. 10.8 through 10.11), and while the coefficients a, c and k were different for different streams, the exponents b, f and m were fairly constant for all the streams studied. For the twenty river cross sections studied, they obtained the following average values:

$$b = 0.26$$

$$f = 0.40$$

$$m = 0.34.$$

* Actually values of L are the measured loads including the measured wash load.

The exponent j was not found to be so constant, varying from 2.0 to 3.0.

They then studied the variations of channel geometry and suspended load along each stream. The discharge of a river generally increases in a downstream direction due to the inflow of tributary streams. In order to compare different sections of a river, it is necessary to select in a special way the discharges for which the comparisons are made. Leopold and Maddock assumed that the most rational basis for analysis was to select discharges at each section which were equaled or exceeded the same percent of the time. Stated another way, the comparison was made for discharges of the same frequency. For a given river, the constants in eqs. 10.8 through 10.11 will depend on the frequency of the discharges selected. They further assumed that the mean annual discharges have about the same frequency at all stations. Comparing the width, depth, velocity and suspended load of different stations along the study rivers at the mean annual discharge, they found that the fit of the measured data to the exponential relations was quite good for each stream and the exponents were again quite constant. The average values they obtained were about $b = 0.5$, $f = 0.4$ and $m = 0.1$*. Due to insufficient data on suspended load, they were unable to obtain a significant number of values of the exponent j to estimate its value or determine if it is constant. From the limited data available and some qualitative physical arguments, they concluded that the suspended load increases in a downstream direction less rapidly than the discharge and therefore j is less than unity.

Eqs. 10.8 through 10.11 describe a set of curves for each station on a river and for each discharge of a different frequency along a river. These curves are called the hydraulic geometry of the stream. Using the hydraulic geometry of the streams which they studied and some other measured quantities (such as slope) from these streams, Leopold and Maddock reached the following conclusions about natural streams:

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* Note that if the Blench regime eqs. 10.1 and 10.2 were written in exponential form, the values of b and f would be $b = 0.5$ and $f = 0.33$. Thus the regime equations specifying channel geometry are similar to Leopold and Maddock's relations governing variations along a river. By manipulating the equations, it can be shown that the exponent in the velocity equation, m, is not the same as that given by Blench's flow equation.

1) The mean velocity of a stream increases in the downstream direction. This is apparent from eq. 10.10 since m is greater than zero.

2) The concentration of suspended sediment decreases in the downstream direction along a stream. This is due to the increase of suspended load in the downstream direction being less than the increase in discharge.

3) A wide river with a particular velocity carries a smaller suspended load than a narrow river having the same velocity and discharge.

4) For a given width and a given discharge, an increase in suspended load requires an increase in velocity and a decrease in depth.

5)* At constant discharge, an increased velocity at constant width is associated with an increase of suspended load and bed load.

6)* At constant velocity and discharge an increase in width is associated with a decrease of suspended load and an increase of bed load.

7) The roughness of a stream is nearly constant along the stream for discharges of the same frequency.

8) In rivers whose beds are composed of sand, significance of changes in slope (such as might occur during the passage of a flood) are very small compared to the significance of changes in roughness which take place.

Their analysis also led them to the conclusion that changes in channel geometry and velocity and the relation between them are due to variations in the discharge <u>and</u> the sediment load and hence these latter quantities are the independent variables for a stream.

10.6. <u>Example of Disturbed Regime</u>. It was Leopold and Maddock's thesis that the relation between discharge and the other variables is

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* Conclusions 5 and 6 are based primarily on Gilbert's flume data (ref. 10.4), but appear to be valid for natural streams also.

brought about by a long term adjustment of the channel geometry and slope to the discharge and load that the stream must carry. Man's span on earth has been too short on the geologic time scale to obtain quantitative information on the development and formation of rivers. However, we do have information on how rivers react when their discharge and/or load is altered and its regime is upset. Such an example will be discussed here.

After the closure of Hoover Dam, the resulting lake acted as a gigantic settling tank and essentially clear water was released from the reservoir. Thus the stream was deprived of its load and its regime was upset. The progressive changes which occurred in the river channel at Yuma, Arizona, which is about 350 miles downstream from Hoover Dam are shown in fig. 10.1. The annual discharge and sediment load are also shown.

From fig. 10.1 it is seen that the annual suspended load decreased from an average of nearly 150 million tons to less than 25 million tons. The water discharge fluctuated depending on the releases from the dam and the inflow of the tributaries below the dam. In the period shown there was no large consistent sustained decrease in the discharge following closure of the dam. The decrease in load caused the channel to degrade markedly and the depth to increase. Despite a decrease in width, the velocity decreased considerably. Since the slope was essentially unchanged and the depth increased, the reduced velocity reflects a change in the roughness which was observed to increase by a factor of about three. It is doubtful that this increase was caused directly by the increased size of the bed material. More likely it was due to changes in the bed form and the decrease in the suspended load. Thus with the dam closure and reduction of load, the river altered its regime to accomodate about the same discharge, but a much smaller load.

10.7. River Channel Patterns. Natural streams are classified as braided, meandering, or straight according to their geometry in plan view, as when viewed from an airplane. Each of these patterns is part of the regime of the stream for the conditions under which it exists. The

different channel patterns and the factors associated with each will be briefly discussed here. For a more complete treatment the reader is referred to ref. 10.5.

A straight channel is a very uncommon occurrence in nature. Indeed, reaches which are straight for distances greater than ten times the channel width are rare. Even in reaches of rivers which appear straight the cross section is by no means uniform and the thalweg (line of maximum depth) wanders back and forth across the stream. Another characteristic of straight (also meandering) natural channels is the occurrence of short portions of shallow cross section (riffles) which separate longer portions in which the cross section is significantly deeper (pools). Thus a straight channel implies neither a uniform cross section nor a straight thalweg.

In fig. 10.2, Cottonwood Creek above station 1500 has a typical meander pattern. Pools and riffles which are always found in straight channels are also characteristic of meandering channels and thus the straight pattern can be considered as a special case of the meander pattern in which the amplitudes of the meander loops are very small. Riffles are commonly found at the points of inflection (reversal of curvature) in meanders while pools are found at the points of greatest curvature (bends). This is not always the case, as can be seen in fig. 10.2. Leopold and Wolman (ref. 10.5) found that the distance between riffles in a straight channel equals the straight line distance between successive points of inflection in the wave pattern of a meandering river of the same width. They also found that the wave length of the meanders (which is twice the distance between riffles) is related to the bank-full width w by

$$\lambda = 6.5\, w^{1.1} \tag{10.14}$$

where λ is the meander wave length where λ and w are in feet.

No satisfactory explanation has been given for either the how or why of meander formation. Leopold and Maddock's findings indicate that it is related to the formation of riffles. Taylor (ref. 10.6) has shown that the analytic models which have been proposed to explain meandering are insufficient.

The braided pattern is characterized by channel division around alluvial islands. In fig. 10.2, Cottonwood Creek in the vicinity of station 2000 has a typical braided pattern. The growth of the islands apparently begins with the deposition of a central bar which is composed of the coarser fraction of the bed material. As the bar moves downstream it becomes larger forcing the water into the flanking channels which deepen and grow laterally. Eventually the deepening channels lower the water surface until an island emerges and becomes stabilized by vegetation. Thus the braiding of streams is related to the problem of bar deposition which is not understood except in a very general qualitative way. Braided reaches of a channel are usually steeper (see fig. 10.2) wider, and shallower than undivided reaches carrying the same flow.

Fig. 10.1 Progressive changes of Colorado River at Yuma, Arizona resulting from construction of engineering works upstream. (From Leopold and Maddock, U.S.G.S. Prof. Paper 252, Fig. 28).

Fig. 10.2 Plan and profile of Cottonwood Creek near Daniel, Wyo. In this reach the river changes its pattern from meandering to braided. (From Leopold and Wolman, U.S.G.S. Prof. Paper 282-B, Fig. 47).

References:

10.1 Blench, T., Regime Behaviour of Canals and Rivers, Butterworths Scientific Publications, London, 1957.

10.2 Simons, Daryl B., and Albertson, Maurice L., "Uniform Water Conveyance Channels in Alluvial Material", Proc. Am. Soc. of Civ. Engrs., Paper 2484, Vol. 86, No. HY 5, May, 1960. pp 33-71.

10.3 Leopold, Luna B., and Maddock Thomas, Jr., "The Hydraulic Geometry of Stream Channels and some Physiographic Implications", U.S. Geological Survey Prof. Paper 252, 1953.

10.4 Gilbert, Grove Karl, "The Transportation of Debris by Running Water", U. S. Geological Survey, Prof. Paper No. 86, 1914.

10.5 Leopold, Luna B., and Wolman, M. G., "River Channel Patterns: Braided, Meandering, and Straight", U. S. Geological Survey Prof. Paper 282B, 1957.

10.6 Taylor, R. Hugh, "Periodicity and Channel Pattern Determinants in Alluvial Channels", unpublished report, California Institute of Technology, 1960.

Problem:-

10.1 The Ft. Morgan Canal west of Ft. Morgan, Colorado has the following channel and operating characteristics:

$Q = 146$ cfs

$d_s = 0.318$ mm

Temperature $= 77^{\circ}F$

$= 0.97 \times 10^{-5}$ ft^2/sec

Banks are cohesive.

Using the curves of Simons and Albertson (ref. 10.2) determine the channel depth, width, top width, and the slope. Use each of their three different approaches for determining the slopes to be considered. Compare the results obtained by this method with those predicted by the Blench regime equations, and with the actual channel properties given in section 10.2.

Appendix 10-A

Notes on the Simons-Albertson Regime Relations*

10A.1. Introduction.

The results of thoroughgoing study of the available techniques for designing channels in alluvial materials, and of the data upon which these techniques are based have recently been published by D. B. Simons and M. L. Albertson. These authors have developed regime-type graphical relations between the channel and flow parameters which can be used to design alluvial channels. Their approach appears superior to other regime techniques that have been presented because it is based on more data, reflects more thorough analysis of the data, differentiates between different bed and bank materials, and gives three methods for estimating slope to give some concept of the range of slope to be considered. These graphs and their use are presented and briefly discussed in this paper.

10A.2. Data analyzed.

The data upon which the recommended design relations are based were taken from straight, stable reaches of canals in which no objectionable scour or deposition had been observed. The groups of data are summarized in table 1. The mean size of the bed material varied from about 0.1 mm to over 7.5 mm.

10A.3. Design of channels.

Before treating the details of canal design, two prime requisites should be mentioned. First, the Froude number should be less than 0.3 to prevent wave erosion of the banks. Second, the sediment discharge concentration should be less than 500 ppm since the high velocities required to transport greater concentration make stability difficult to obtain.

The Simons-Albertson relations for designing stable channels are

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* The information and figures contained herein are taken from "Uniform Water Conveyance Channels in Alluvial Material" by Daryl B. Simons and Maurice L. Albertson, Proc. of the A.S.C.E., J. of the Hyd. Div., vol. 86, no. HY 5, May 1960, pp. 33-71.

Table 1

Summary of Groups of Canal Data Used in Simons-Albertson Regime Analysis

Location of Canals	No. of Reaches Studied	Discharge cfs min	Discharge cfs max	Slope x 10^3 min	Slope x 10^3 max	Avg. Sed. Disch. Conc. ppm
San Luis Valley, Colorado	15	17	1500	0.79	9.7	- -
Punjab, India	42	5	9000	0.12	0.34	238
Sind, India	28	311	9057	0.059	0.100	156 to 3590
Imperial Valley, California	4	- -	- -	- -	- -	2500 to 8000
Misc. Data	24	43	1039	0.058	0.387	- -

shown in figs. 1 through 9. Channel design proceeds in the following steps.

a) For the design discharge select from fig. 1 the wetted perimeter appropriate to the bed and bank materials. Note that curves B and E are both for canals with a sand bed and cohesive banks, but curve E is based on data from the Imperial Valley Canals which carry very large sediment loads;therefore, curve E should be used only for canals which must carry comparable loads.

b) Fig. 2 can be used to estimate the mean channel width, and the top channel width can be estimated from fig. 3. These values will give an estimate of the side slope to be used.

c) For the discharge, select from fig. 4 the hydraulic radius for the bed and bank materials of the channel. The channel area, A, can now be computed,

$$A = PR$$

and the mean velocity, V, determined,

$$V = Q/A.$$

d) From fig. 5, the average bed depth can now be estimated. This is the average depth above the <u>bed</u> (not the banks) of the channel.

The detailed dimensions of the channel can not be determined. Usually a trapezoidal channel shape is used. The values of hydraulic radius and wetted perimeter determined from fig. 1 and fig. 4 must be retained. The values of bed depth, average width, and top width can be adjusted as required to maintain these values of hydraulic radius and wetted perimeter. If the bank material is non-cohesive, the side slope must not exceed the values of angle of repose given in fig. 6. It is good practice to use a value of side slope which is 5° to 10° less than the indicated angle of repose.

e) Using the calculated velocity, determine R^2S from fig. 7 for the appropriate bed and bank materials. Since R is known, S can be computed.

f) Using the values of velocity and width determined previously, determine the value of V^2/gDS from fig. 8 for the appropriate bed and bank materials. The slope can then be computed .

g) For mean size of the bed material and the bed and bank materials of the channel being designed, determine the critical tractive force from fig. 9. From the critical tractive force, the slope can be computed.

h) The designer must now invoke his engineering judgement, guided by the slopes determined in steps e, f and g to arrive at the design slope.

Fig. 1 Variation of wetted perimeter with discharge for regime channels.

Fig. 2 Variation of average width with wetted perimeter for regime channels.

Fig. 3 Variation of top width with average width for regime channels.

Fig. 4 Variation of hydraulic radius with discharge for regime channels.

Fig. 5 Variation of average bed depth with hydraulic radius for regime channels.

Fig. 6 Angle of repose for non-cohesive materials.

Fig. 7 Variation of mean velocity with R^2S for regime channels. (Lacey type slope relation.)

Fig. 8 Variation of $\frac{V^2}{gDS}$ with $\frac{VW}{\nu}$ for regime channels. (Blench-King type slope relation.)

Fig. 9 Variation of tractive force with mean size of bed material for regime channel. (Tractive-force type slope relation.)