A Geometric Theorem for Wireless Network Design Optimization

Massimo Franceschetti *Matthew Cook*
Jehoshua Bruck

California Institute of Technology
Mail Code 136-93
Pasadena, CA 91125
Email: {massimo,cook,bruck}@paradise.caltech.edu

***Abstract*—Consider an infinite square grid G. How many discs of given radius r, centered at the vertices of G, are required, in the worst case, to completely cover an arbitrary disc of radius r placed on the plane? We show that this number is an integer in the set $\{3, 4, 5, 6\}$ whose value depends on the ratio of r to the grid spacing.**

This result can be applied at the very early design stage of a wireless cellular network to determine, under the recent International Telecommunication Union (ITU) proposal for a traffic load model, and under the assumption that each client is able to communicate if it is within a certain range from a base station, conditions for which a grid network design is cost effective, for any expected traffic demand.

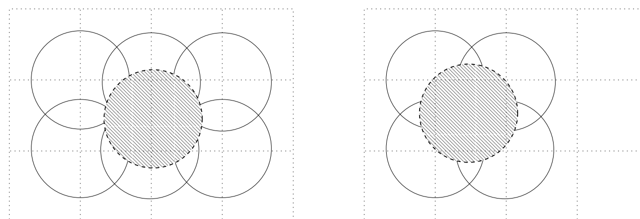

Fig. 1. **The disc game.** *Player one's shaded disc requires six discs placed at the vertices of the grid to be fully covered, while player two's disc requires only four. In this case, player one beats player two 5 to 7.*

I. Introduction

A. The disc game

Consider the following two-players game: each player draws a disc of radius r on a square grid and then tries to completely cover the disc drawn by his opponent by new discs of the same radii r, but centered only at grid points. The objective of the game is to end up drawing less discs than the opposite player. What is a strategy that guarantees at least a tie with any opponent? A winning combination example for player one is depicted in Fig. 1. Player one's (shaded) disc requires six discs placed at the vertices of the grid to be fully covered, while player two's disc requires only four grid discs. Hence, in this case, player one beats player two 5 to 7.

A general version of this problem can be phrased in terms of the following fundamental question of combinatorial geometry: how many discs of given radius r, centered at the vertices of an infinite square grid, are required, in the worst case, to completely cover an arbitrary disc of radius r placed on the plane, and how is this worst case achieved?

The answer to this question depends on the ratio of the radius r to the grid spacing L, as it is illustrated in Fig. 2. On the left hand side of the figure six solid line discs, centered at grid vertices, are required to cover a dashed line disc placed half way between two adjacent grid vertices. If the ratio of r to the grid spacing L is increased (right hand side of the figure), then fewer discs centered at grid vertices are required to cover the same dashed line disc. In Section III we present a geometric theorem that shows that the number of discs of radius r centered at the vertices of a square grid that are necessary and sufficient to completely cover an arbitrary disc of radius r placed on the plane is an integer $\mathcal{N} \in \{3, 4, 5, 6\}$, whose value depends on the ratio r/L. The necessary condition of the theorem is proved constructively, by showing a disc placement that requires the maximum number of $\mathcal{N}$ grid discs to be covered, thus providing the optimal strategy to play the game.

This work was partially supported by the Lee Center for Advanced Networking at Caltech.

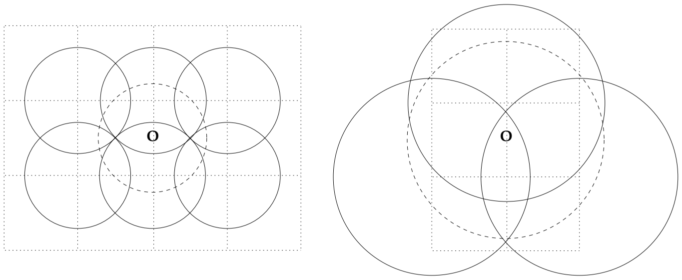

Fig. 2. **Scaling the picture.** *On the left hand side of the figure, the dashed line disc requires six discs centered at lattice vertices to be fully covered. On the right hand side of the figure, it requires only three discs.*

B. Network design optimization

How can we apply this result to the design of radio cellular networks? Let us consider a scenario in which a telecommunication company plans the deployment of a new network in a city. One key problem is to decide where to position base stations, according to a distribution of demand points, to achieve optimum quality of service, with minimum costs. At the early design stage interference effects are neglected, simplistic propagation models are assumed, and the problem is to suggest initial design strategies. We assume that, by paying a local tax, the company can install base stations at the city traffic lights. In this way, all base stations are placed at some vertices of a square grid, corresponding to the street intersections, in a way that each demand point is within a given distance from a base station. Following this design, the company estimates a cost of deployment of, say, 500 monetary units (mu) per base station. Alternatively, the company can place the stations optimally, without constraining them to be at the street intersections. This means identifying a minimum cardinality set of locations of the base stations that cover all the demand points. This strategy can potentially lead to a smaller number of installations, but its estimated cost of deployment is, for instance, 1700 mu per base station, due to higher manufacturing and installation costs, and because the company may need to pay, or to give a discount, to the owner of the building, or land, where it intends to place a base station. Hence, the company faces the following dilemma: is it better to choose the grid design, that may lead to a larger number of less expensive base stations, or is it better to choose locations optimally, potentially using less, but more expensive base stations?

The solution to this puzzle depends on the ratio of the communication range r of a base station to the city block length L. Consider the limiting case in which all demand points lie inside a circle of radius r, thus requiring a single non-grid base station of cost C_{ng}. Alternatively, we can serve all the points with a number $\mathcal{N}$ of grid base station that depends on the ratio r/L, and that is provided by our theorem. The corresponding cost is in this case $\mathcal{N} \cdot C_g$, where C_g is the cost of one grid base station. By applying our geometric result, we are therefore in a position to solve the puzzle. Accordingly, we will show that the following holds for any distribution of the demand points:

- For $C_{ng} > 6C_g$, the network grid design is cost effective if $r/L \geq \sqrt{2}/2$.
- For $C_{ng} > 5C_g$, the network grid design is cost effective if $r/L \geq \sqrt{10}/4$.
- For $C_{ng} > 4C_g$, the network grid design is cost effective if $r/L \geq 1$.
- For $C_{ng} > 3C_g$, the network grid design is cost effective if $r/L \geq 5\sqrt{2}/4$.

In all remaining cases a non-grid network design can be cost effective for some distribution of demand points. By referring to the cost values given in the example we have: $1700/500 = 3.4 > 3$, hence the company should always choose a grid design if the ratio of the communication range of the base stations to the city block length is greater or equal to $\frac{5\sqrt{2}}{4}$.

Note that we are comparing the best covering of the demand points by grid discs and the best covering of the demand points by arbitrarily located discs. In practice one may use a polynomial time approximation algorithm to solve any of these two problems sub-optimally (see for example the approximation algorithms in [1] [2] [6]). In this case the comparison can easily be done applying our theorem in conjunction to the appropriate approximation factors of the algorithms used.

C. Model Assumptions

We now discuss the assumptions that we make in our model. The described scenario relies on the concept of identifying demand nodes, on the concept of using existing traffic light poles as potential transmitting locations, and on the assumption that a demand node can communicate if it is within a given distance from a base station.

The concept of demand nodes was introduced by Tutschku and Tran-Gia [11] [12], and the International Telecommunication Union [7] has recently proposed its standardization. According to their definition, a demand node represents the center of an area with a certain traffic demand and each node stands for the same portion of traffic load. Hence, different traffic patterns correspond to different node distributions: highly populated business districts typically lead to dense distributions of demand nodes, while suburban and rural areas lead to sparser distributions. More details on the identification of demand nodes can be found in the survey of Tran-Gia, Leibnitz, and Tutschku [10].

The idea of using existing traffic light poles to place base stations in a city has been exploited by several telecommunication companies in the recent years, both in U.S. and Europe, to build microcellular networks with higher capacities than traditional cellular systems [3].

Finally, one could argue that constraining each demand point to be within a given distance from a base station corresponds to assuming base station transmitters to have rotational symmetric range, that is not an accurate physical representation of what is often in practice an anisotropic and time-varying communication range, due to shadowing and fading effects. However, we argue that a circle that bounds the maximal range can be used as a first order approximation at the early design stage of the network, as hexagonal cell shapes are universally adopted to approximate circular radiation patterns in the design and analysis of cellular systems [9], and as circular radiation patterns are assumed in the calculation of the throughput capacity of ad-hoc wireless networks [4] [5] [8]. We also point out that our assumption of having a city formed by regularly spaced blocks better applies to some U.S. urban and suburban areas than to older, and more irregular, European cities.

There is no doubt that our postulated model can be improved, relaxing some, or all previous assumptions. This, however, does not lead to a simple analytical evaluation, as it is provided in this paper, of the range of parameters that suggest a grid or non-grid design. Our contribution is in determining, at the early stage of the design, if there is an indication of convenience of a grid design, that can be later validated by more accurate numerical solvers.

II. A THEOREM IN GEOMETRY.

Theorem 1: Consider a square lattice where the distance between two neighboring lattice vertices is L. Call a disc of fixed radius r, centered at a lattice vertex, a grid disc. The number $\mathcal{N}$ of grid discs that are necessary and sufficient to cover any disc of radius r placed on the plane, is given by:

- CASE 1. For $r/L < \frac{\sqrt{2}}{2}$, $\mathcal{N}$ does not exist.
- CASE 2. For $\frac{\sqrt{2}}{2} \leq r/L < \frac{\sqrt{10}}{4}$, $\mathcal{N} = 6$.
- CASE 3. For $\frac{\sqrt{10}}{4} \leq r/L < 1$, $\mathcal{N} = 5$.
- CASE 4. For $1 \leq r/L < \frac{5\sqrt{2}}{4}$, $\mathcal{N} = 4$.
- CASE 5. For $r/L \geq \frac{5\sqrt{2}}{4}$, $\mathcal{N} = 3$.

The rest of the paper is devoted to the proof of Theorem 1. Conclusions and future work are discussed in Section IV. The proofs below make use of simple geometric arguments, but are by no means trivial. They involve finding a disc that requires the prescribed number of grid discs to be fully covered, and then finding different configurations of grid discs that are sufficient to cover any disc on the plane. The number of these configurations that we need to find increases with r/L.

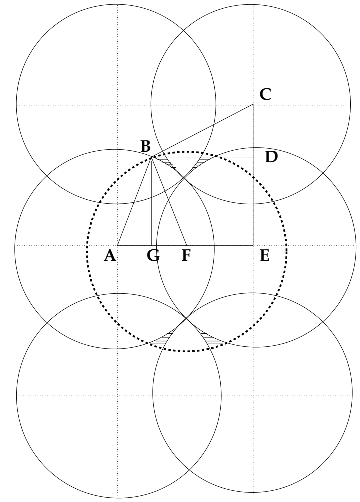

Fig. 3. **CASE 2, necessary condition.** *The distance between the two lattice vertices A and E is 1. Therefore, $\overline{AF} = \overline{FE} = \frac{1}{2}$. Since $\overline{AB} = \overline{BF} = r$, $\overline{AG} = \overline{GF} = \frac{1}{4}$.*

III. NOTHING BUT PROOFS

To simplify the notation, in the following we fix $L = 1$.

A. Proof of CASE 1.

For $r < \frac{\sqrt{2}}{2}$ the grid discs do not cover the plane compactly, since they do not cover the centers of the lattice squares. It follows that any number of grid discs is not sufficient to cover any disc that covers the center of a lattice square. □

B. Proof of CASE 2.

The necessary condition is proven by showing that there exists a disc that requires six grid discs to be covered. The sufficient condition is proven using a tiling argument: we first show that there exists a triangle ABC, such that any disc centered inside triangle ABC is covered by six grid discs, and then we show that, by symmetry, we can tile the entire plane using triangles that have this property.

1) Necessary Condition: Consider $r \geq \frac{\sqrt{2}}{2}$ and a disc centered halfway between two neighboring lattice vertices. Such disc is the dashed disc depicted in Figure 3. We have that the six (solid) grid discs depicted in Figure 3 are necessary to completely cover the dashed disc if $\overline{BC} > r$, i.e., when the four shaded areas in Figure 3

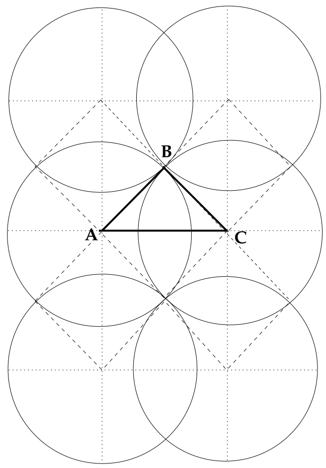

Fig. 4. **CASE 2, sufficient condition.** *Any disc centered inside triangle ABC is covered by the six grid discs.*

are not null. By repeatedly applying the Pythagorean theorem, we have:

$$\overline{BC} = \sqrt{\left(1 - \sqrt{r^2 - \frac{1}{16}}\right)^2 + \frac{9}{16}}\ ; \tag{1}$$

imposing $\overline{BC} > r$, we obtain $r < \frac{\sqrt{10}}{4}$.

2) Sufficient Condition: Call $\mathcal{A}$ the area covered by the six grid discs in Figure 4. Any point inside triangle ABC has distance greater than r from the border of area $\mathcal{A}$. Therefore, a disc can be centered inside triangle ABC and be covered by the six grid discs depicted in Figure 4. By symmetry, we can tile the plane with triangles inside which discs can be centered and covered by six grid discs. □

C. Proof of CASE 3.

1) Necessary Condition: Consider $r \geq \frac{\sqrt{10}}{4}$ and a disc centered at a distance ϵ to the left from halfway between two neighboring lattice vertices (dashed disc in figure 5). By the same reasoning of CASE 2, we have that the five (solid) grid discs depicted in Figure 5 are necessary to completely cover the dashed disc, if $\overline{BC} > r$. By repeatedly applying the Pythagorean theorem, we have in this case:

$$\overline{BC} = \sqrt{\left[1 - \sqrt{r^2 - \left(\frac{1}{4} - \frac{\epsilon}{2}\right)^2}\right]^2 + \left(\frac{3}{4} + \frac{\epsilon}{2}\right)^2}\ ; \tag{2}$$

imposing $\overline{BC} > r$ we obtain:

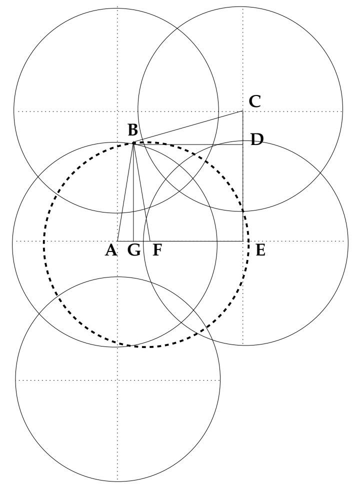

Fig. 5. **CASE 3, necessary condition.** *Point F is the center of the dashed disc and is shifted by ϵ to the left from halfway between the lattice vertices A and E. Therefore, we have: $\overline{AF} = \frac{1}{2} - \epsilon$, $\overline{FE} = \frac{1}{2} + \epsilon$.*

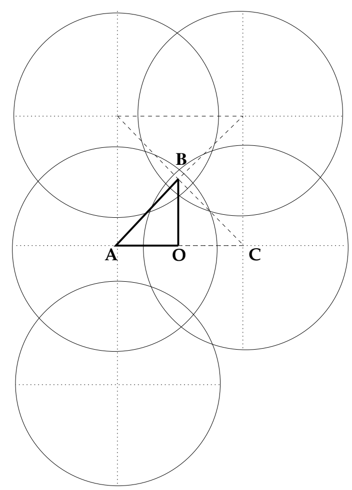

Fig. 6. **CASE 3, sufficient condition.** *Point O is halfway between lattice vertices A and C. Any disc centered inside triangle AOB is covered by the five grid discs.*

$$\sqrt{\frac{\epsilon^2 + \epsilon}{2} + \frac{5}{8}} > r\ . \tag{3}$$

By symmetry we can restrict the ϵ range: $0 < \epsilon < \frac{1}{2}$ and for any $r < 1$ inequality (3) is verified.

2) Sufficient Condition: Call $\mathcal{A}$ the area covered by the five grid discs in Figure 6. Any point inside triangle AOB has distance greater than r from the border of area $\mathcal{A}$. Therefore, a disc can be centered inside triangle AOB and be covered by the five grid discs depicted in Figure 6. By symmetry, we can tile the plane with triangles inside which discs can be centered and covered by five grid discs. □

D. Proof of CASE 4.

1) Necessary Condition: Consider $r \geq 1$ and a disc placed at the center of a lattice square. If we place the grid discs as depicted in the right section of Figure 7, four grid discs are always necessary to cover the dashed disc placed at the center of a lattice square, because:

$$\overline{EF} = \overline{DF} > \overline{OF} = r\,; \tag{4}$$

the same holds whenever two grid discs are centered at neighboring lattice vertices or at lattice vertices on the same diagonal of a lattice square. If we examine the remaining possible placement of grid discs, depicted in the left section of Figure 7, we have that four grid discs are necessary only if $\overline{BC} = \overline{CD} > r$. By the Pythagorean theorem, we have in this case:

$$\overline{BC} = \sqrt{\left(r - \frac{\sqrt{2}}{2}\right)^2 + 2} \tag{5}$$

imposing $\overline{BC} > r$ we obtain $r < 5\frac{\sqrt{2}}{4}$.

2) Sufficient Condition: Consider the left section of Figure 8. The four grid discs cover any disc centered inside the shaded area $ABCE$. This area is defined by the triangle ABC and by the circle centered at point P. This circle is the locus of the centers of the discs to be covered that touch point P. Any disc centered inside the remaining area ACE is not covered by the four grid discs. Consider now the right section of Figure 8, the four (solid) grid discs cover, in this case, any disc centered inside the shaded area $CDEF$. This area is defined by triangle BCD and by the circle centered at point Q. This circle is the locus of the centers of the discs to be covered that touch point Q. Such circle passes by point E, therefore, adjoining the two shaded areas $ABCE$ and $CDEF$, we fully cover the area of triangle BCD. Any disc centered inside triangle BCD is covered by four grid discs: the four depicted in the left section or the four depicted in the right section of Figure 8. By symmetry, the same holds for triangle ABD and we can tile the plane with triangles inside which discs can be centered and covered by four grid discs. □

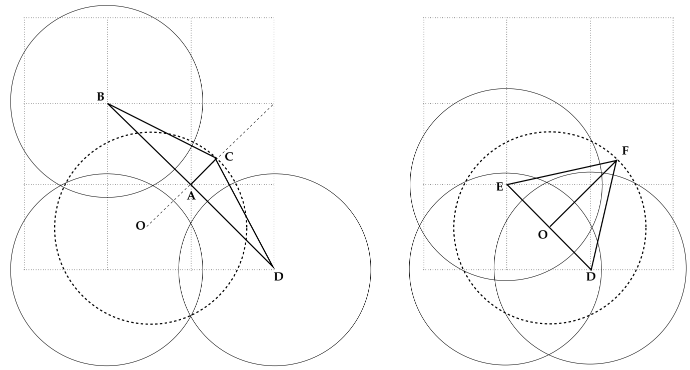

Fig. 7. **CASE 4, necessary condition.** *Point O is at the center of a lattice square. In the left section of the figure, the dashed disc centered at point O requires one more grid discs to be covered, if $\overline{BC} > r$. In the right section of the figure, the dashed disc centered at point O always requires one more grid disc to be covered.*

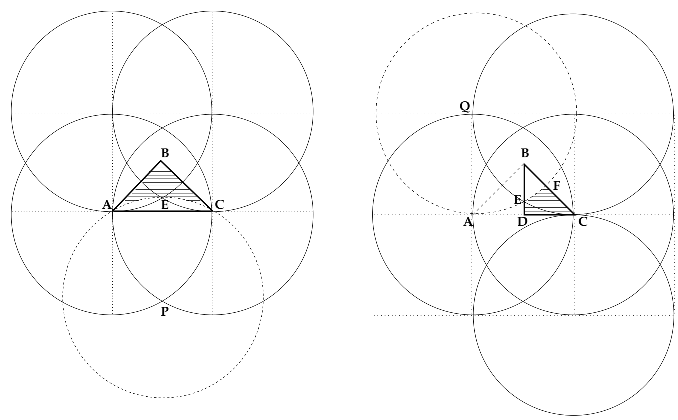

Fig. 8. **CASE 4, sufficient condition.** *Four grid discs cover any disc centered inside the shaded areas. Intersecting the two shaded areas, we obtain triangle BCD.*

E. Proof of CASE 5

1) Necessary Condition: By symmetry, any two discs with the same radius, centered far apart by an arbitrary $\epsilon > 0$, cover less than half of each other's perimeter, therefore, any grid disc must cover less than half of the perimeter of any other disc not centered at a lattice point. It follows that any two grid discs must cover less than the entire perimeter of any other disc not centered at a lattice point. Hence, any disc not centered at a lattice point requires at least three grid discs to be covered.

2) Sufficient Condition: It is enough to prove this condition when r is minimum, therefore, we carry out calculations fixing $r = \frac{5\sqrt{2}}{4}$. We consider three different placements of three grid discs on the lattice and show that they are enough to cover any disc arbitrarily placed on the plane. Consider the upper left section of Figure 9. Taking point O as the origin of the coordinate system, the three grid discs are placed at points: $(-1, 0), (-1, 2), (1, 0)$. The coordinates of point P are calculated by applying the Pythagorean theorem to triangle AOP obtaining: $P = \left(0, -\frac{\sqrt{34}}{4}\right)$. Therefore, the locus of the centers of the discs to be covered that touch point P, given by the circle

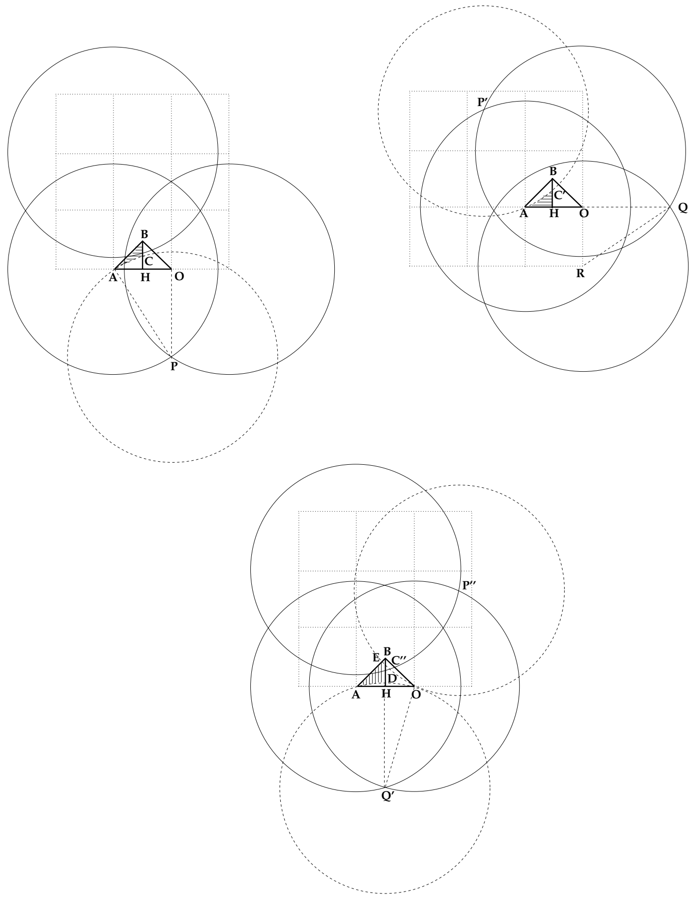

Fig. 9. **CASE 5, sufficient condition.** *Three grid discs cover any disc centered inside the shaded areas. Intersecting the three shaded areas, we obtain triangle ABH.*

centered at point P, is defined by the equation:

$$x^2 + \left(y + \frac{\sqrt{34}}{4}\right)^2 = r^2\,; \qquad (6)$$

this circle passes by point A and intersects segment $\overline{BH}$ at point $C = \left(-\frac{1}{2}, \frac{\sqrt{46}-\sqrt{34}}{4}\right)$. It is easy to see that any disc centered inside the shaded area ABC is covered by the three grid discs centered at points: $(-1,0), (-1,2), (1,0)$. We now consider the three grid discs depicted in the upper right section of Figure 9, centered at points: $(-1,0), (0,1), (0,-1)$. First, let us focus on point Q. Its coordinates are calculated by applying the Pythagorean theorem to triangle OQR, obtaining: $Q = \left(\frac{\sqrt{34}}{4}, 0\right)$. Since $\overline{HQ} = \overline{HO} + \overline{OQ} = \frac{1}{2} + \frac{\sqrt{34}}{4} > r$, the locus of the centers of the discs to be covered that touch point Q, given by the circle centered at point Q, does not intersect triangle ABH. Now, let us focus on point P'. The coordinates of point P' are calculated by intersecting the two grid circles:

$$\begin{cases} (x+1)^2 + y^2 &= r^2 \\ x^2 + (y-1)^2 &= r^2\,, \end{cases} \qquad (7)$$

obtaining: $P' = \left(\frac{-2-\sqrt{21}}{4}, \frac{2+\sqrt{21}}{4}\right)$. Therefore, the locus of the centers of the discs to be covered that touch point P', given by the circle centered at point P', is defined by the equation:

$$\left(x + \frac{2+\sqrt{21}}{4}\right)^2 + \left(y - \frac{2+\sqrt{21}}{4}\right)^2 = r^2\,; \qquad (8)$$

this circle passes by point A and intersects segment $\overline{BH}$ at point $C' = \left(-\frac{1}{2}, \frac{2+\sqrt{21}-\sqrt{29}}{4}\right)$. It is easy to see that any disc centered in the shaded area $AC'H$ is covered by the three grid discs centered at points: $(-1,0), (0,1), (0,-1)$.

Intersecting the two circles centered at points P and P', defined by equations (6) and (8) respectively, we obtain two points:

$$\begin{aligned} (x_0, y_0) &= (-1, 0) \\ (x_1, y_1) &= \left(\frac{2-\sqrt{21}}{4}, \frac{2+\sqrt{21}-\sqrt{34}}{4}\right) \end{aligned}$$

depicted in the left section of Figure 10. By the coordinates of these points, (x_1, y_1) is placed inside triangle ABH. Hence, any disc centered inside the shaded area given by the intersection of the two discs depicted in the left section of Figure 10 is covered by neither the three grid discs depicted in the upper left section, nor by the three grid discs depicted in the upper right section of Figure 9.

In order to cover such a disc, we consider the three grid discs depicted in the lower section of Figure 9, placed at points: $(-1,0), (-1,2), (0,0)$. In this case, the coordinates of point Q' are calculated by applying the Pythagorean theorem to triangle $Q'OH$, obtaining: $Q' = \left(-\frac{1}{2}, -\sqrt{\frac{23}{8}}\right)$. Therefore, the locus of the centers of the discs to be covered that touch point Q', given by the circle centered at Q', is defined by the equation:

$$\left(x + \frac{1}{2}\right)^2 + \left(y + \sqrt{\frac{23}{8}}\right)^2 = r^2\,; \qquad (9)$$

this circle passes by points A and O and intersects segment $\overline{BH}$ at point $D = \left(-\frac{1}{2}, \frac{5\sqrt{2}-\sqrt{46}}{4}\right)$. The coordinates of point P'' are calculated by intersecting the two grid circles:

$$\begin{cases} (x+1)^2 + (y-2)^2 &= r^2 \\ x^2 + y^2 = r^2\,, \end{cases} \qquad (10)$$

obtaining: $P'' = \left(\frac{\sqrt{6}-1}{2}, \frac{\sqrt{6}+4}{4}\right)$. Therefore, the locus of the centers of the discs to be covered that touch point P'', given by the circle centered in P'', is defined by the equation:

$$\left(x - \frac{\sqrt{6}-1}{2}\right)^2 + \left(y - \frac{\sqrt{6}+4}{4}\right)^2 = r^2 \, ; \qquad (11)$$

this circle passes by point O and intersects segment $\overline{BH}$ at point $C'' = \left(-\frac{1}{2}, \frac{4+\sqrt{6}-\sqrt{26}}{4}\right)$. It is easy to see that any disc centered inside the shaded area $AEC''D$ is covered by the three grid discs centered at points: $(-1, 0), (-1, 2), (0, 0)$. In order to check that this placement of grid discs covers any disc centered inside the shaded area given by the intersection of the two discs depicted in the left section of Figure 10, we intersect the two circles centered at points P' and Q' in Figure 9, defined by equations (8) and (9) respectively, obtaining two points:

$$\begin{aligned} (x_0, y_0) &= (-1, 0) \\ (x_2, y_2) &= \left(-\frac{\sqrt{21}}{4}, \frac{2+\sqrt{21}-\sqrt{46}}{4}\right) \end{aligned}$$

depicted in the right section of Figure 10. Given the coordinates of these points, since (x_2, y_2) is placed outside the area of triangle ABH, we conclude that the three grid discs placed at points: $(-1, 0), (-1, 2), (0, 0)$, cover any disc centered inside the shaded area given by the intersection of the two discs depicted in the left section of Figure 10. It follows that intersecting the three shaded areas of Figure 9: ABC,$AC'H$,$AEC''D$, we cover triangle ABH completely. Hence, any disc centered inside triangle ABH is covered by three grid discs, placed in one of the three configurations depicted in Figure 9. By symmetry, the same holds for triangle OBH and we can tile the plane with triangles inside which discs can be centered and covered by three grid discs. □

IV. Conclusions

We presented a basic theorem in combinatorial geometry that can be applied in the design of a wireless cellular network. Using the concept of demand nodes to model the expected network traffic, this theorem can be useful to give an indication of a cost effective choice at a very early design stage of the network planning, for any expected traffic load.

In the future, we plan to generalize the theorem to other lattice structures. We also plan to study a related problem also inspired by wireless communication networks. That is the problem of placing a minimum number of discs of given radius r on the plane, in a way that they cover a set of given points, and that the resulting graph, which has the centers of the discs as vertices and vertices joined by an edge if the corresponding discs intersect, is connected.

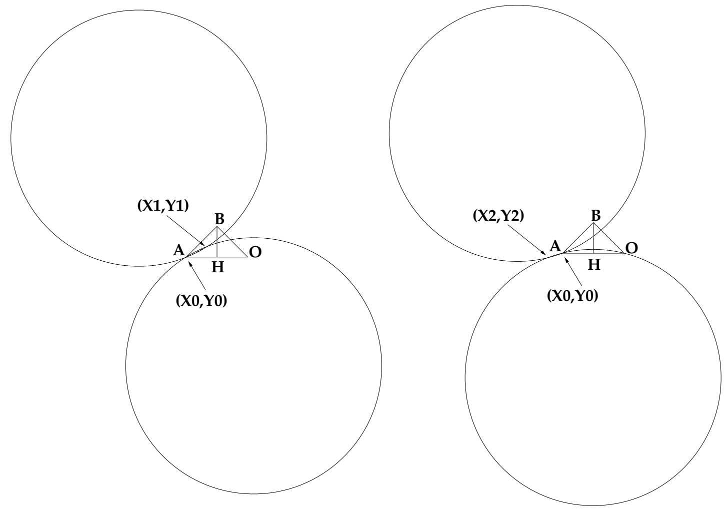

Fig. 10. **CASE 5, sufficient condition.** *Point (x_1, y_1) is inside triangle ABH, point (x_2, y_2) is outside triangle ABH.*

References

[1] C. Glaber, S. Reith, and H. Vollmer. "The Complexity of Base Station Positioning in Cellular Networks." *Workshop on Approximation and Randomized Algorithms in Communication Networks (ARACNE), 2000.*

[2] T. Gonzalez. "Covering a Set of Points in Multidimensional Space." *Information Processing Letters, 40(4), pp.181-188, 1991.*

[3] L. Greenstein, N. Amitay, T. Chu, L.J. Cimini Jr., G.J. Foschini, M.J. Gans, A.J. Rustako Jr., R.A. Valenzuela, and G. Vannucci. "Microcells in Personal Communication Systems." *IEEE Communications magazine, December, 76-88, 1992.*

[4] M. Grossglauser and D. Tse. "Mobility Increases the Capacity of Ad-Hoc Wireless Networks." *Proceedings of the International Conference in Information and Communication (INFOCOM), 2001.*

[5] P. Gupta and P.R. Kumar. "The Capacity of Wireless Networks." *IEEE Transactions on Information Theory, 46(2), pp.388-404, 2000.*

[6] D.S. Hochbaum and W. Maass. "Approximation Schemes for Covering and Packing Problems in Image Processing and VLSI." *Journal of the ACM, 32(1), pp.130-136, 1985.*

[7] ITU-T Focus Group on Traffic Engineering for Personal Communication. E750-Series of Recommendations on Traffic Engineering Aspects of Networks Supporting Mobile and UPT Services." *Technical Reference of the International Telecommunication Union, Geneva, 1999.*

[8] S. R. Kulkarni, and P. Viswanath. "A deterministic approach to throughput scaling in wireless networks." *Proceedings of the International Symposium on Information Theory (ISIT), 351, July 2002.*

[9] T. Rappaport. "Wireless communications, principles and practice." *Prentice Hall, 1996.*

[10] P. Tran-Gia, K. Leibnitz, and K. Tutscku. "Teletraffic issues in mobile communication network planning." *Telecommunication Systems, 15, 3-20,2000.*

[11] K. Tutschku. "Demand based radio network planning of cellular communication systems." *Proceedings of the IEEE Symposium on Information and Communication (INFOCOM), San Francisco, USA, 1998*.

[12] K. Tutschku, and P. Tran-Gia. "Spatial traffic estimation and characterization for mobile communication network design." *IEEE Journal of Selected Areas in Communications, 16(5), 804-811,1998.*