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NONSTATIONARY RESPONSE OF STRUCTURES AND ITS APPLICATION TO EARTHQUAKE ENGINEERING

by

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Abstract

This thesis presents a simplified state-variable method to solve for the nonstationary response of linear MDOF systems subjected to a modulated stationary excitation in both time and frequency domains. The resulting covariance matrix and evolutionary spectral density matrix of the response may be expressed as a product of a constant system matrix and a time-dependent matrix, the latter can be explicitly evaluated for most envelopes currently prevailing in engineering. The stationary correlation matrix of the response may be found by taking the limit of the covariance response when a unit step envelope is used. The reliability analysis can then be performed based on the first two moments of the response obtained.

The method presented facilitates obtaining explicit solutions for general linear MDOF systems and is flexible enough to be applied to different stochastic models of excitation such as the stationary models, modulated stationary models, filtered stationary models, and filtered modulated stationary models and their stochastic equivalents including the random pulse train model, filtered shot noise, and some ARMA models in earthquake engineering. This approach may also be readily incorporated into finite element codes for random vibration analysis of linear structures.

A set of explicit solutions for the response of simple linear structures subjected to modulated white noise earthquake models with four different envelopes are presented as illustration. In addition, the method has been applied to three selected topics of interest in earthquake engineering, namely, nonstationary analysis of primary-secondary systems with classical or nonclassical dampings, soil layer response and related structural reliability analysis, and the effect of the vertical components on seismic performance of structures. For all the three cases, explicit solutions are obtained, dynamic characteristics of structures are investigated, and some suggestions are given for aseismic design of structures.

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Chapter 1

Introduction

Many environmental loads applied to structures are random in nature, such as earthquake loads or wind loads applied to buildings, oceanwave loads to offshore platforms, aerodynamic pressure on aircraft, etc. Random vibration theory is employed to study the dynamic behavior of structures under such random loads. In random analysis, the load is often modeled as a stochastic process and the desired response is given in the form of statistical quantities, such as the probability density function, the mean and covariance of the response, etc. The reader who is interested in the fundamental theory of random vibration is refered to Stratonovich (1963), Crandall and Mark (1967), and Lin (1967). Extensive reviews on the recent development of random vibration may be found in Crandall and Zhu (1983), and Spanos and Lutes (1986).

Whenever possible, the first objective of dynamic analysis of structures subjected to a random excitation is to find the joint probability density functions of the response since these functions completely describes the stochastic properties of the response. Any statistical quantity of the response may be evaluated in terms of an integral of the probability density function. This approach is applicable only if the process considered is exactly, or approximately, a Markov process and, therefore, the forward and backward Kolmogorov equations can be used to determine the probability density function. Unfortunately, it is very difficult to solve for the probability density function in general cases, especially in nonstationary analysis. Therefore, the main focus of random analysis is often placed on obtaining the first two moments of the response, or their equivalent, such as the mean and covariance response or the first two cumulants of the response.

The first two moments of the response are of great importance. If the structure is linear and the excitation is Gaussian, the response will be Gaussian and these two quantities will completely determine the probability density of the response. In the case that the excitation is a non-Gaussian process, even though the first two moments do not completely characterize the probability structure of the response process, they still provide important information about the process. For instance, an upper-bound estimate of the probability of a random process $x(t)$ may be obtained from a Chebyshev type inequality as

$$\begin{aligned} &Prob(|x(t) - \mu_x(t)| \geq \epsilon \quad \text{for some t in a} \leq \text{t} \leq \text{b}) \\ &\qquad \leq \frac{1}{2\epsilon^2}[\sigma_x^2(a) + \sigma_x^2(b)] + \frac{1}{\epsilon^2}\int_a^b \sigma_x(t)\sigma_{\dot{x}}(t)dt \end{aligned} \qquad (1-1)$$

where $\mu(t), \sigma_x(t), \sigma_{\dot{x}}(t)$ are the transient mean and standard deviation of the process and its derivative process; and a, b, and ϵ are arbitrary constants satisfying $0 \leq a < b$ and $\epsilon > 0$. It is noted that only the first two moments are needed in this estimation.

The importance of the first two moments is also justified by their engineering application. The properties of random environmental loads in real engineering problems are often given in the form of their first two moments or equivalents. The information can only be used to solve for the first two moments of the structural response, but is not adequate to determine the moments of higher order or the probability density, unless further information is given or additional assumptions are made. One of the important applications of random vibration theory is in structural reliability analysis. In most of the methods available for reliability analysis, only information on the first two moments is needed.

Most of the early works on random vibration were confined to stationary response analysis. However, load processes encountered in many engineering problems can exhibit strongly nonstationary features. For instance, it has been recognized that earthquake ground motion usually has build-up, stationary, and tail stages (Amin, Ts'ao, and Ang, 1968). Barnoski and Maurer (1973) showed that for an excitation with an exponentially damped cosine correlation, the nonstationary response of a simple linear system may exceed its stationary value by a factor in excess of two. Therefore, it is important to determine the nonstationary stochastic properties of the response of structures under nonstationary excitation, especially for realistic reliability estimation.

Though the problem of determining the second moment response of a linear structure subjected to nonstationary excitation has been well-formulated (Caughey, 1963; Lin, 1967), algebraic difficulty has frequently been reported in solving the problem. Continuing effort has been devoted to developing accurate and efficient approaches to solve for the second moment response of linear systems and then to use the results so obtained to investigate dynamic behavior of structures under nonstationary random excitation. Various methods have been proposed, depending on the type of structure, the modeling of the random load, and the stochastic quantities of interest.

The existing methods to solve for the second moment response may be classified into two categories: time domain approaches and frequency domain approaches. The former include the impulsive response approach used by Bogdanoff, Goldberg, and Bernard (1961), Caughey and Stumpf (1961), Bucciarelli and Kuo (1970), and Iwan and Hou (1988); the incremental load approach or Heaviside response approach used by Madsen and Krenk (1982), Krenk et al. (1983), Di Paolr (1985), Langley (1986), Muscolino (1988), Borino et al. (1988), and Igusa (1989a, 1989b); and the Kolmogorov-Fokker-Plank (KFP) equation approach used by Lin (1964). The latter include the double Fourier transform approach employed by Holman and Hart (1971, 1972, 1974); Page's instantaneous power spectrum method used by Liu (1972); Bendat's instantaneous power spectrum method used by Corotis and Vanmarcke (1975); and Priestley's evolutionary power spectrum method used by Hammond (1968), Barnoski and Maurer (1969, 1973), Corotis and Marshall (1977), Ahmadi (1986), Borino et al. (1988), and Shihab and Preumont (1989). For the second moment analysis of MDOF systems, the modal superposition approach has been used by many investigators including Hommand (1968), Hart (1970), Masri (1978, 1979), Madsen and Krenk (1982), and To (1984, 1986, 1987). Alternatively, the state-variable approach has been employed by Gasparini (1979), Gasparini and DebChaudhury (1980), DebChaudhury and Gasparini (1982), DebChaudhury and Gazis (1988), To (1987), and Yang, Sarkani, and Long (1988). The Lyapunov direct method is often used to find numerical solutions for the second moment response and different approximation techniques have been introduced to simplify the problem including those by Roberts (1971), Hasselman (1971), Holman and Hart (1974), Sun and Kareem (1989), and Bucher (1988). A selected review of these methods is given in Appendix I.

It may be concluded from the review that even though many different formulations for the analysis of the second moment response of linear MDOF systems have been developed in both the time and frequency domains, explicit solutions are still difficult to obtain. This is mainly due to the algebraic difficulties caused by nonstationarity of the loads and the size of the system. Considering the importance of explicit solutions in both theoretical and practical studies of the dynamic behavior of structures, a new method may be needed to overcome these difficulties and facilitate obtaining the explicit solution for the covariance response. This is precisely the objective of this thesis.

In this thesis, a new more precise and efficient method is developed to find the nonstationary covariance response and the evolutionary power spectral density of the response of general MDOF linear structures subjected to modulated white noise and the method is applied to some problems of interest in earthquake engineering. These problems may be studied by other methods, but unless otherwise specified, the results presented herein are the explicit solutions. The problem of acquiring and processing the necessary data to characterize the input process will not be discussed here. It is assumed throughout the thesis that this information is always available. Since most of the current random ground motion models may be associated with the modulated white noise model, as seen from Appendix II, it is expected that the method will offer a unified way to solve for the nonstationary response of MDOF systems under general excitations.

Chapter 2 provides the fundamental formulation of the method both in the time and frequency domains. As an illustration, explicit solutions for the nonstationary response of simple structures subjected to a modulated stationary white noise are presented. The envelope functions used are the step function, rectangular function, exponential function and the product of a polynomial and exponential function, which prevail in earthquake engineering. Some results may be found in individual references, but here the results are complete and the procedure is unified and much simpler as compared to other methods.

Chapter 3 shows an application of the method to MDOF systems. Exact solutions are found for the covariance response and the mean square value of the energy envelope response of two-degree-of-freedom primary-secondary systems with classical and nonclassical damping. Interaction effects between the primary and secondary systems are also included. The work is an extention of Smith's research (1985).

Chapter 4 addresses the application of the method to linear continuous systems. A soil layer response analysis under random earthquake input at the bedrock and the corresponding structural reliability analysis is chosen as an example. The results are presented for both absolute ground acceleration and structural response. The work may be thought of as a continuation of Lin's work (1987) where the concept of evolutionary Kanai-Tajimi models has been proposed.

To show the potential of the method, Chapter 5 presents an application to the dynamic analysis of simple structures subjected to correlated and uncorrelated external and parametric excitations. Due to existence of the parametric excitation, the problem becomes nonlinear. As an earthquake engineering application, a simple structure subjected to combined vertical and horizontal earthquake loads is studied. The primary difference from the original work done by Lin and Shih (1980) is that an explicit solution is presented instead of a numerical solution, and a general solution for the response for correlated external and parametric excitations is also included. The work is also motivated by results presented by Benaroya and Rehak (1989a, 1989b) where only the stationary solution was given.

Finally, Chapter 6 gives a brief summary of this study and discusses the application of the present method to some other problems.

Chapter 2

A Simplified State-Variable Method for Nonstationary Response of Linear MDOF Systems

2.1 Introduction

As mentioned in Chapter 1, environmental loads, such as earthquake ground motion, are often modeled as a random process. In general, nonstationary random analysis is required to account for the nonstationarity of such environmental excitations. Since the probability density function of the nonstationary response is generally difficult to obtain, the main focus is often placed on the first two response moments. If the excitation is Gaussian, these two quantities will completely determine the probability structure of the response. Even in the case that the excitation is a non-Gaussian process, they still give important probability information about the response based on the well-known Chebyshev's inequality.

The analysis of the nonstationary response of linear MDOF structures subjected to random excitation has been well-formulated by several authors including Caughey (1963) and Lin (1967), and different techniques are available to find the second moment response. However, explicit solutions have been obtained in only a very few cases. It is frequently reported that the mathematical expressions for the statistics of the response are cumbersome to manipulate. Therefore, numerical schemes or approximation techniques are often employed in the solution of nonstationary problems.

In this chapter, a new approach for determining the nonstationary response of linear MDOF systems subjected to modulated stationary excitation is developed based on complex modal analysis. The approach is referred to as a simplified state-variable method. Using this approach, the final solution for the covariance matrix of the response of a linear MDOF system, $\mathrm{Q}(t)$, can be expressed in a very compact analytical form as

$$\mathbf{Q}(t) = \mathbf{L}\mathbf{B}(t)\mathbf{L}^T \tag{2-1}$$

where the constant matrix $\mathbf{L}$ depends only on the system parameters, and each entry of the time-dependent matrix $\mathbf{B}(t)$ is a certain easily evaluated standard integration of the product of exponential fuctions, triangular functions as well as the envelope function chosen, which may be easily evaluated. As a consequence, an explicit solution becomes much easier to obtain. The potential usefulness of the approach can be seen from the various applications in this and following chapters.

In the following sections, the basic formulation of the method is first presented in the time domain and then the general expressions for stationary and nonstationary covariance responses are obtained. An alternative version in the frequency domain is also given to find the evolutionary spectral density of the response. Finally, the extension of the method to different loading cases is discussed and some conclusions are drawn.

2.2 Time Domain Formulation

Consider a general linear multiple-degree-of-freedom (MDOF) system subjected to a nonstationary random excitation represented as a modulated stationary load. The equation of motion can be written as

$$\begin{gathered} \mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \\ \mathbf{x}(0) = \mathbf{x}_0 \qquad \dot{\mathbf{x}}(0) = \mathbf{v}_0 \end{gathered} \tag{2-2}$$

where $\mathbf{M}, \mathbf{C}, \mathbf{K}$ are, as usual, the mass, damping, and stiffness matrices respectively, $\mathbf{x}(t)$ is the N-dimensional response vector, and $\mathbf{x}_0$ and $\mathbf{v}_0$ are the initial displacement and velocity vectors. The load vector $\mathbf{f}(t)$ is assumed to be of the form

$$\mathbf{f}(t) = f_0(t)\mathbf{r} \tag{2-3}$$

where $\mathbf{r}$ is a constant vector and $f_0(t)$ is a modulated stationary process, namely,

$$f_0(t) = \eta(t)n(t) \tag{2-4}$$

in which $\eta(t)$ is a deterministic envelope function used to account for the nonstationarity of the excitation, and $n(t)$ is a stationary process whose stochastic properties are specified by its covariance function $R_n(\tau)$ or its power spectral density $S_n(\omega)$. $R_n(\tau)$ and $S_n(\omega)$ are related to each other by the well-known Wiener-Khintchine relationship

$$\begin{aligned} S_n(\omega) &= \frac{1}{2\pi}\int_{-\infty}^{\infty} R_n(\tau)e^{-i\omega\tau}d\tau \\ R_n(\tau) &= \int_{-\infty}^{\infty} S_n(\omega)e^{i\omega\tau}d\omega \end{aligned} \qquad (2-5)$$

Note that Eq. (2-4) includes most stochastic models prevailing in earthquake engineering and a discussion on the extension of the method to different load cases will be given later in this chapter.

Recasting Eq. (2-2) into the $2N$-space form yields

$$\begin{aligned} \frac{d}{dt}\mathbf{Y}(t) &= \mathbf{A}\mathbf{Y}(t) + \mathbf{F}(t) \\ \mathbf{Y}(0) &= \mathbf{Y}_0 \end{aligned} \qquad (2-6)$$

where

$$\begin{aligned} \mathbf{Y} &= \begin{pmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{pmatrix} \qquad \mathbf{Y}_0 = \begin{pmatrix} \mathbf{x}_0 \\ \mathbf{v}_0 \end{pmatrix} \\ \mathbf{A} &= \begin{pmatrix} 0 & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{pmatrix} \\ \mathbf{F} &= \begin{pmatrix} 0 \\ \mathbf{M}^{-1}\mathbf{f}(t) \end{pmatrix} = \mathbf{F}_0 f_0(t) \end{aligned} \qquad (2-7)$$

in which $\mathbf{I}$ denotes a N x N unit matrix. The focus will be placed on Eq. (2-6) instead of (2-2) since (2-6) is more flexible in dealing with both classical and nonclassical damping.

2.2.1 Solution for the Response of MDOF Systems Under Modulated Stationary Excitation

Assuming that the $2N$ eigenvalues, $\lambda^{(i)}$, and eigenvectors, $\mathbf{v}^{(i)}$, of the system matrix $\mathbf{A}$ in Eq. (2-6) can be obtained, and noticing that for a sufficiently lightly damped system they appear in N complex conjugate pairs,

$$\begin{aligned} \lambda^{(2k-1)} &= \bar{\lambda}^{(2k)} \\ \mathbf{v}^{(2k-1)} &= \bar{\mathbf{v}}^{(2k)} \qquad k = 1, \ldots, N \end{aligned} \qquad (2-8)$$

where $\bar{(\cdot)}$ denotes the complex conjugate operation, the fundamental matrix of the system $\mathbf{\Phi}(t)$ can be constructed as

$$\mathbf{\Phi}(t) = \mathbf{U}\mathbf{\Lambda}(t)\mathbf{U}^{-1} \tag{2-9}$$

where

$$\mathbf{U} = (\mathbf{u}_1, \mathbf{u}_2, \ldots, \mathbf{u}_{2k-1}, \mathbf{u}_{2k}, \ldots, \mathbf{u}_{2N-1}, \mathbf{u}_{2N})$$

$$\mathbf{\Lambda}(t) = \begin{pmatrix} \mathbf{\Lambda}_{11}(t) & 0 & \ldots & 0 \\ 0 & \mathbf{\Lambda}_{22}(t) & \ldots & 0 \\ \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & \ldots & \mathbf{\Lambda}_{NN}(t) \end{pmatrix}$$

and the column components of $\mathbf{U}$ and the submatrices of $\mathbf{\Lambda}(t)$ are found as

$$\begin{gathered} \mathbf{u}_{2k-1} = Re(\mathbf{v}^{(2k-1)}) \qquad \mathbf{u}_{2k} = Im(\mathbf{v}^{(2k-1)}) \\ \mathbf{\Lambda}_k(t) = e^{-\varsigma_k \omega_k t} \begin{pmatrix} \cos \omega_{dk} t & \sin \omega_{dk} t \\ -\sin \omega_{dk} t & \cos \omega_{dk} t \end{pmatrix} \\ k = 1, \ldots, N \end{gathered} \tag{2-10}$$

where

$$\begin{aligned} -\varsigma_k \omega_k &= Re(\lambda_k) \\ \omega_{dk} &= \omega_k \sqrt{1 - \varsigma_k^2} = Im(\lambda_k) \\ k &= 1, \ldots, N \end{aligned} \tag{2-11}$$

$Re(\cdot)$ and $Im(\cdot)$ denote the real and imaginary parts of the argument respectively. The parameters ς_k, ω_k, and $\omega_{dk}, k = 1, \ldots, N$ represent respectively the damping ratio, natural frequency, and damped natural frequency of the kth mode of the system. It is assumed that all the parameters are positive for physically realistic structures, though the method can be extended to a general case.

Define

$$\mathbf{D} = \mathbf{U}^{-1}\mathbf{F}_0 \tag{2-12}$$

The general solution of (2-6) can then be expressed as

$$\begin{aligned} \mathbf{Y}(t) &= \mathbf{\Phi}(t)\mathbf{Y}_0 + \int_0^t \mathbf{\Phi}(t-\tau)\mathbf{F}(\tau)d\tau \\ &= \mathbf{\Phi}(t)\mathbf{Y}_0 + \mathbf{L}\int_0^t \mathbf{P}(t-\tau)\eta(\tau)n(\tau)d\tau \end{aligned} \tag{2-13}$$

where

$$\mathbf{P}(t) = \begin{pmatrix} \mathbf{P}_1(t) \\ \mathbf{P}_2(t) \\ \cdots\cdot \\ \mathbf{P}_N(t) \end{pmatrix}$$

$$\mathbf{L} = \mathbf{U}\mathbf{V} \qquad (2-14)$$

$$\mathbf{V} = diag(\mathbf{V}_1, \mathbf{V}_2, \ldots, \mathbf{V}_N)$$

and the submatrices are defined as

$$\mathbf{P}_k(t) = e^{-\varsigma_k \omega_k t} \begin{pmatrix} \cos \omega_{dk} t \\ \sin \omega_{dk} t \end{pmatrix}$$

$$\mathbf{V}_k = \begin{pmatrix} D_{2k-1} & D_{2k} \\ D_{2k} & -D_{2k-1} \end{pmatrix} \qquad k = 1, \ldots, N \qquad (2-15)$$

The first two moments of the random response can now be easily obtained from equation (2-13) and, in turn, the probability density function of the response may be found if the excitation is assumed to be Gaussian.

2.2.1.1 Mean Value of the Response

Taking mathematical expectation on both sides of Eq. (2-13) yields the mean value of the response expressed as

$$\begin{aligned} E[\mathbf{Y}(t)] &= \mathbf{\Phi}(t)E[\mathbf{Y}_0] + E[\mathbf{L}\int_0^t \mathbf{P}(t-\tau)\eta(\tau)n(\tau)d\tau] \\ &= \mathbf{\Phi}(t)E[\mathbf{Y}_0] + \mathbf{L}\int_0^t \mathbf{P}(t-\tau)\eta(\tau)E[n(\tau)]d\tau \end{aligned} \qquad (2-16)$$

where $E[\mathbf{Y}_0]$ is determined from the mean values of the initial displacement vector $\mathbf{x}_0$ and the initial velocity vector $\mathbf{v}_0$ and $E[n(t)]$ is the mean of the stationary process.

Without loss of generality, the mean of the stationary process may be assumed to be zero. Therefore, the above expression is reduced to

$$E[\mathbf{Y}(t)] = \mathbf{\Phi}(t)E[\mathbf{Y}_0] \qquad (2-17)$$

If the mean of the initial condition is also zero, which is the case for most engineering applications, the mean of the response remains zero for all time. Zero mean of the response is assumed hereinafter unless further notification is given.

2.2.1.2 Covariance Response

The covariance matrix of the response may be found as follows

$$\begin{aligned}\mathbf{Q}(t_1,t_2) &= E\left[(\mathbf{Y}(t_1) - E[\mathbf{Y}(t_1)])(\mathbf{Y}(t_2) - E[\mathbf{Y}(t_2)])^T\right] \\ &= \mathbf{L}E[\int_0^{t_1}\int_0^{t_2} \mathbf{P}(t_1-\tau_1)\mathbf{P}^T(t_2-\tau_2)\eta(\tau_1)n(\tau_1)\eta(\tau_2)n(\tau_2)d\tau_1 d\tau_2]\mathbf{L}^T \\ &= \mathbf{L}\int_0^{t_1}\int_0^{t_2} \mathbf{P}(t_1-\tau_1)\mathbf{P}^T(t_2-\tau_2)\eta(\tau_1)\eta(\tau_2)E[n(\tau_1)n(\tau_2)]d\tau_1 d\tau_2 \mathbf{L}^T\end{aligned} \qquad (2-18)$$

The final solution may be written as

$$\mathbf{Q}(t_1,t_2) = \mathbf{L}\mathbf{B}(t_1,t_2)\mathbf{L}^T \qquad (2-19)$$

where the constant matrix $\mathbf{L}$ is given by Eq. (2-14) and the transient matrix $\mathbf{B}(t_1,t_2)$ is defined as

$$\mathbf{B}(t_1,t_2) = \int_0^{t_1}\int_0^{t_2} \eta(\tau_1)\eta(\tau_2)R_n(\tau_1-\tau_2)\mathbf{P}(t_1-\tau_1)\mathbf{P}^T(t_2-\tau_2)d\tau_1 d\tau_2 \qquad (2-20)$$

Note that the covariance response is the same as the mean square response if the mean of the response is zero.

In contrast to other methods, the final solution is well-separated into a constant part and a transient part as shown in Eq. (2-19). The constant matrix $\mathbf{L}$ depends only on the system parameters, i.e., mass matrix, damping matrix, and stiffness matrix and, therefore, it remains the same no matter how the load changes. The time-variant matrix $\mathbf{B}(t_1,t_2)$ can be evaluated from the time integration of some products of exponential functions, triangular functions, and the envelope functions chosen, and the integrations have the same form for all the components of $\mathbf{B}(t)$. Thus an explicit solution can be easily obtained provided the explicit representations of these integrals can be found.

The evaluation of the transient part generally involves double integrals. However, the procedure may be simplified in certain special cases. For instance, if $R_n(t_1-t_2)$ is separable, i.e., if

$$R_n(t_1-t_2) = R_1(t_1)R_2(t_2), \qquad (2-21)$$

the transient part may be expressed as

$$\begin{aligned}\mathbf{B}(t_1,t_2) &= \mathbf{B}_1(t_1)\mathbf{B}_2^T(t_2)\\ \mathbf{B}_1(t_1) &= \int_0^{t_1} \eta(\tau_1)R_1(\tau_1)\mathbf{P}(t_1-\tau_1)d\tau_1 \\ \mathbf{B}_2(t_2) &= \int_0^{t_2} \eta(\tau_2)R_2(\tau_2)\mathbf{P}(t_2-\tau_2)d\tau_2\end{aligned} \tag{2-22}$$

If the power spectral density of the stationary process, $S_n(\omega)$, is given, the concept of evolutionary spectral density may facilitate the evaluation, as seen in section 2.3. If the stationary process is a white noise, the double integral may be reduced to a single integral, which will be shown in section 2.2.2.

2.2.1.3 Probability Density Function

If the stationary process is assumed to be Gaussian, the response as the output of a linear system is also Gaussian. Therefore, the probability density function of the response can be determined based on the first two moments of the response as follows

$$p(\mathbf{Y}(t)) = \frac{1}{(2\pi)^{N/2}|\mathbf{Q}(t)|^{1/2}} e^{-\frac{1}{2}(\mathbf{Y}(t)-E[\mathbf{Y}(t)])^T\mathbf{Q}^{-1}(t)(\mathbf{Y}(t)-E[\mathbf{Y}(t)])} \tag{2-23}$$

Note that the probability density function depends on time.

2.2.2 Covariance Response of MDOF Structures Under a Modulated White Noise

Let $n(t)$ be a stationary white noise process with properties:

$$\begin{aligned}E[n(t)] &= 0\\ E[n(t_1)n(t_2)] &= S_0\delta(t_1-t_2)\end{aligned} \tag{2-24}$$

in which S_0 is the intensity of the white noise. Then, Eq. (2-20) may be reduced to a single integral. The solution is given by Eq. (2-19)

$$\mathbf{Q}(t_1,t_2) = \mathbf{L}\mathbf{B}(t_1,t_2)\mathbf{L}^T$$

with

$$\mathbf{B}(t_1,t_2) = S_0\int_0^{min(t_1,t_2)} \mathbf{P}(t_1-\tau)\mathbf{P}^T(t_2-\tau)\eta^2(\tau)d\tau \tag{2-25}$$

2.2.2.1 Stationary Solution

As a special case, the stationary correlation of the response $\mathbf{R}(\tau)$ can be obtained by using a unit step envelope function. Letting $t_2 = t_1 + \tau$ and letting t_1 approach ∞ in (2-19) yields

$$\mathbf{R}(\tau) = \mathbf{L}\bar{\mathbf{B}}(\tau)\mathbf{L}^T \qquad (2-26)$$

where

$$\bar{\mathbf{B}}(\tau) = S_0 \int_0^{+\infty} \mathbf{P}(s)\mathbf{P}^T(s+\tau)ds \qquad (2-27)$$

In detail, the submatrices of $\bar{\mathbf{B}}(\tau)$ are given by

$$\bar{\mathbf{B}}_{mn}(\tau) = S_0 \int_0^{\infty} \mathbf{P}_m(s)\mathbf{P}_n^T(s+\tau)ds \qquad (2-28)$$

$$m, n = 1, \ldots, N$$

where

$$\mathbf{P}_m(\tau) = e^{-\varsigma_m \omega_m \tau} \begin{pmatrix} \cos \omega_{dm}\tau \\ \sin \omega_{dm}\tau \end{pmatrix} \qquad (2-29)$$

An explicit expression for the components $\bar{\mathbf{B}}_{mn}(\tau)$ can be found to be

$$\bar{\mathbf{B}}_{mn}(\tau) = S_0 e^{-\varsigma_n \omega_n \tau} \begin{pmatrix} c_{mn}^{11} & c_{mn}^{12} \\ c_{mn}^{21} & c_{mn}^{22} \end{pmatrix} \begin{pmatrix} \cos \omega_{dn}\tau & \sin \omega_{dn}\tau \\ -\sin \omega_{dn}\tau & \cos \omega_{dn}\tau \end{pmatrix} \qquad (2-30)$$

where the constants $c_{mn}^{ij}, i, j = 1, 2, m, n = 1, \ldots, N$ are defined by

$$c_{mn}^{11} = \frac{(\varsigma_m\omega_m + \varsigma_n\omega_n)(\omega_m^2 + \omega_n^2 + 2\varsigma_m\varsigma_n\omega_m\omega_n)}{(\omega_m^2 + \omega_n^2 + 2\varsigma_m\varsigma_n\omega_m\omega_n)^2 - 4\omega_{dm}^2\omega_{dn}^2}$$

$$c_{mn}^{21} = \frac{\omega_{dn}(\omega_m^2 + \omega_n^2 + 2\varsigma_m\varsigma_n\omega_m\omega_n) - 2\omega_{dm}^2\omega_{dn}}{(\omega_m^2 + \omega_n^2 + 2\varsigma_m\varsigma_n\omega_m\omega_n)^2 - 4\omega_{dm}^2\omega_{dn}^2} \qquad (2-31)$$

$$c_{mn}^{12} = \frac{\omega_{dm}(\omega_m^2 + \omega_n^2 + 2\varsigma_m\varsigma_n\omega_m\omega_n) - 2\omega_{dm}\omega_{dn}^2}{(\omega_m^2 + \omega_n^2 + 2\varsigma_m\varsigma_n\omega_m\omega_n)^2 - 4\omega_{dm}^2\omega_{dn}^2}$$

$$c_{mn}^{22} = \frac{2\omega_{dm}\omega_{dn}(\varsigma_m\omega_m + \varsigma_n\omega_n)}{(\omega_m^2 + \omega_n^2 + 2\varsigma_m\varsigma_n\omega_m\omega_n)^2 - 4\omega_{dm}^2\omega_{dn}^2}$$

Eqs. (2-26), (2-30) and (2-31) complete the stationary solution for the correlation matrix.

The stationary mean square response $\mathbf{R}(0)$ can be obtained by setting $\tau = 0$ in (2-26). That is

$$\mathbf{R}(0) = \mathbf{L}\bar{\mathbf{B}}(0)\mathbf{L}^T \qquad (2-32)$$

The submatrices $\bar{\mathbf{B}}_{mn}(0), m, n = 1, \ldots, N$ are

$$\bar{\mathbf{B}}_{mn}(0) = S_0 \begin{pmatrix} c_{mn}^{11} & c_{mn}^{12} \\ c_{mn}^{21} & c_{mn}^{22} \end{pmatrix} \qquad (2-33)$$

where $c_{mn}^{ij}, i, j = 1, 2, m, n = 1, \ldots, N$ are defined in Eq. (2-31). Note that in the stationary case the probability density (2-23) becomes time-independent.

Alternatively, $\mathbf{R}(0)$ may be found directly from Eq. (2-25) by choosing $\eta(\tau)$ as the unit step function, and letting $t_1 = t_2 = t$ and t approach ∞. As result,

$$\mathbf{R}(0) = \mathbf{L}\mathbf{B}(\infty)\mathbf{L}^T \qquad (2-34)$$

2.2.2.2 Nonstationary Solution

The nonstationary solution of MDOF systems subjected to modulated white noise is given by Eq. (2-25). In many cases, an alternative form of the solution is more convenient based on the following argument. If the system is subjected to a modulated white noise excitation, the covariance of the responses at two different times $t_1 \neq t_2$, $\mathbf{Q}(t_1, t_2)$, may be evaluated in terms of that for $t_1 = t_2$, i.e., $\mathbf{Q}(t_1, t_1)$. Assume, without loss of generality, that $t_1 < t_2$, then, the relationship can be expressed by

$$\mathbf{Q}(t_1, t_2) = \mathbf{Q}(t_1, t_1)\mathbf{\Phi}^T(t_2 - t_1) \qquad (2-35)$$

where $\mathbf{\Phi}(t)$ is the fundamental matrix solution of the system.

Letting $t_1 = t_2 = t$ in Eqs. (2-19) and (2-25), one obtains

$$\mathbf{Q}(t) = \mathbf{L}\mathbf{B}(t)\mathbf{L}^T \qquad (2-36)$$

where

$$\mathbf{B}(t) = S_0 \int_0^t \mathbf{P}(t-\tau)\mathbf{P}^T(t-\tau)\eta^2(\tau)d\tau \qquad (2-37)$$

Eqs. (2-35)-(2-37) provide an alternative version of the nonstationary solution which places emphasis on the mean square response. Once the mean square response is found, the covariance matrix $\mathbf{Q}(t_1, t_2)$ can be evaluated without any difficulty. Therefore, many results are presented only for the mean square response throughout the thesis.

2.2.2.3 Evaluation of the Transient Matrix $\mathbf{B}(t)$

It is observed that an explicit solution for the nonstationary response covariance is possible whenever the time-varying matrix $\mathbf{B}(t)$ can be explicitly evaluated, which is the case for a class of envelopes of the form

$$\eta(t) = \begin{cases} At^{\gamma} e^{-\alpha t}, & \text{if } t \geq 0; \\ 0, & \text{otherwise.} \end{cases} \qquad (2-38)$$

where A is positive, and α and γ are nonnegative. The normalizing factor A is chosen such that the maximum of the envelope is unity. Note that most envelope functions commonly used in earthquake engineering are special cases of the above form, including the step fuction used by Caughey and Stumpf (1961), the boxcar type function used by Barnoski and Maurer (1969), the exponential function used by Shinozuka and Sato (1967), the product of polynomial and exponential function used by R.N. Iyengar and K.T. Iyengar (1969), Saragoni and Hart (1974), and the piecewise expression used by Jennings, Housner, and Tsai (1968), etc.

The evaluation of $\mathbf{B}(t)$ involves the calculation of a general integral of the form

$$\mu(n, \alpha, \beta, a, b) = \int_0^t s^n e^{(\alpha + i\beta)s} e^{(a+ib)(t-s)} ds \qquad (2-39)$$

where i is the imaginary unit and n ia a nonnegative integer. An explicit expression for the integral may be found as

$$\mu(n, \alpha, \beta, a, b) = e^{\alpha t} \sum_{k=1}^{n+1} C_k(t) e^{i(\beta t - k\theta)} - e^{at} C_{n+1}(0) e^{i(bt-(n+1)\theta)} \qquad (2-40)$$

By separating the real and imaginary parts of μ,

$$\begin{aligned} Re[\mu] &= e^{\alpha t} \sum_{k=1}^{n+1} C_k(t) \cos(\beta t - k\theta) - e^{at} C_{n+1}(0) \cos(bt - (n+1)\theta) \\ Im[\mu] &= e^{\alpha t} \sum_{k=1}^{n+1} C_k(t) \sin(\beta t - k\theta) - e^{at} C_{n+1}(0) \sin(bt - (n+1)\theta) \end{aligned} \qquad (2-41)$$

where $C_k(t), k = 1, \ldots, n+1$ are defined as

$$C_k(t) = \frac{(-1)^{k-1}}{\rho^k} \frac{d^{k-1}}{dt^{k-1}}(t^n) \tag{2-42}$$

in which ρ and θ can be determined from the relationship

$$\rho e^{i\theta} = (\alpha - a) + i(\beta - b) \tag{2-43}$$

Note that

$$\begin{aligned} C_k(0) &= 0, \qquad k = 1, \ldots, n \\ C_{n+1} &= \frac{(-1)^n n!}{\rho^{n+1}} \end{aligned} \tag{2-44}$$

Using the integral (2-39), the submatrices of $\mathbf{B}(t)$ can be expressed as

$$\mathbf{B}_{mn}(t) = S_0 \int_0^t \mathbf{P}_m(t-\tau)\mathbf{P}_n^T(t-\tau)\eta^2(\tau)d\tau \tag{2-45}$$

$$m, n = 1, \ldots, N$$

and their components can be found as follows:

$$\begin{aligned} B_{mn}^{(11)} &= \frac{S_0 A^2}{2}\Big(Re[\mu(n, -2\alpha, 0, -(\varsigma_m\omega_m + \varsigma_n\omega_n), \omega_{dm} + \omega_{dn})] \\ &\qquad + Re[\mu(n, -2\alpha, 0, -(\varsigma_m\omega_m + \varsigma_n\omega_n), \omega_{dm} - \omega_{dn})]\Big) \\ B_{mn}^{(12)} &= \frac{S_0 A^2}{2}\Big(Im[\mu(n, -2\alpha, 0, -(\varsigma_m\omega_m + \varsigma_n\omega_n), \omega_{dm} + \omega_{dn})] \\ &\qquad - Im[\mu(n, -2\alpha, 0, -(\varsigma_m\omega_m + \varsigma_n\omega_n), \omega_{dm} - \omega_{dn})]\Big) \\ B_{mn}^{(21)} &= \frac{S_0 A^2}{2}\Big(Im[\mu(n, -2\alpha, 0, -(\varsigma_m\omega_m + \varsigma_n\omega_n), \omega_{dm} + \omega_{dn})] \\ &\qquad + Im[\mu(n, -2\alpha, 0, -(\varsigma_m\omega_m + \varsigma_n\omega_n), \omega_{dm} - \omega_{dn})]\Big) \\ B_{mn}^{(22)} &= \frac{S_0 A^2}{2}\Big(Re[\mu(n, -2\alpha, 0, -(\varsigma_m\omega_m + \varsigma_n\omega_n), \omega_{dm} - \omega_{dn})] \\ &\qquad - Re[\mu(n, -2\alpha, 0, -(\varsigma_m\omega_m + \varsigma_n\omega_n), \omega_{dm} + \omega_{dn})]\Big) \end{aligned} \tag{2-46}$$

or, in detail,

$$B_{mn}^{11}=\frac{S_0A^2}{2}\Bigg(e^{-2\alpha t}\sum_{k=1}^{n+1}(-1)^{k-1}\frac{d^{k-1}}{dt^{k-1}}(t^n)[\frac{\cos k\theta_1}{\rho_1^k}+\frac{\cos k\theta_2}{\rho_2^k}]-e^{-(\varsigma_m\omega_m+\varsigma_n\omega_n)t}\cdot$$
$$(-1)^n n!\left[\frac{\cos[(\omega_{dm}+\omega_{dn})t-(n+1)\theta_1]}{\rho_1^{n+1}}+\frac{\cos[(\omega_{dm}-\omega_{dn})t-(n+1)\theta_2]}{\rho_2^{n+1}}\right]\Bigg)$$

$$B_{mn}^{12}=\frac{S_0A^2}{2}\Bigg(e^{-2\alpha t}\sum_{k=1}^{n+1}(-1)^{k-1}\frac{d^{k-1}}{dt^{k-1}}(t^n)\left[\frac{\sin k\theta_2}{\rho_2^k}-\frac{\sin k\theta_1}{\rho_1^k}\right]-e^{-(\varsigma_m\omega_m+\varsigma_n\omega_n)t}\cdot$$
$$(-1)^n n!\left[\frac{\sin[(\omega_{dm}+\omega_{dn})t-(n+1)\theta_1]}{\rho_1^{n+1}}-\frac{\sin[(\omega_{dm}-\omega_{dn})t-(n+1)\theta_2]}{\rho_2^{n+1}}\right]\Bigg)$$

$$B_{mn}^{21}=\frac{S_0A^2}{2}\Bigg(e^{-2\alpha t}\sum_{k=1}^{n+1}(-1)^{k-1}\frac{d^{k-1}}{dt^{k-1}}(t^n)\left[-\frac{\sin k\theta_2}{\rho_2^k}-\frac{\sin k\theta_1}{\rho_1}\right]-e^{-(\varsigma_m\omega_m+\varsigma_n\omega_n)t}\cdot$$
$$(-1)^n n!\left[\frac{\sin[(\omega_{dm}+\omega_{dn})t-(n+1)\theta_1]}{\rho_1^{n+1}}+\frac{\sin[(\omega_{dm}-\omega_{dn})t-(n+1)\theta_2]}{\rho_2^{n+1}}\right]\Bigg)$$

$$B_{mn}^{22}=\frac{S_0A^2}{2}\Bigg(e^{-2\alpha t}\sum_{k=1}^{n+1}(-1)^{k-1}\frac{d^{k-1}}{dt^{k-1}}(t^n)\left[\frac{\cos k\theta_1}{\rho_1^k}-\frac{\cos k\theta_2}{\rho_2^k}\right]-e^{-(\varsigma_m\omega_m+\varsigma_n\omega_n)t}\cdot$$
$$(-1)^n n!\left[-\frac{\cos[(\omega_{dm}+\omega_{dn})t-(n+1)\theta_1]}{\rho_1^{n+1}}+\frac{\cos[(\omega_{dm}-\omega_{dn})t-(n+1)\theta_2]}{\rho_2^{n+1}}\right]\Bigg) \tag{2-47}$$

where θ_1, ρ_1 and θ_2, ρ_2 are determined from the following relationship:

$$\begin{aligned}\rho_1 e^{i\theta_1} &= (\varsigma_m\omega_m+\varsigma_n\omega_n-2\alpha)-i(\omega_{dm}+\omega_{dn})\\ \rho_2 e^{i\theta_2} &= (\varsigma_m\omega_m+\varsigma_n\omega_n-2\alpha)-i(\omega_{dm}-\omega_{dn})\end{aligned} \tag{2-48}$$

in which i is the imaginary unit.

2.3 Evolutionary Spectral Analysis

This section is devoted to frequency domain analysis. As pointed out in Appendix I, there is no unique definition of the spectral density for a nonstationary process. The evolutionary spectral density defined by Priestley (1965) is herein employed due to its physical interpretation and mathematical convenience.

2.3.1 Background

As is well known, every stationary random process has a spectral representation of the form

$$x(t) = \int_{-\infty}^{+\infty} e^{i\omega t} dZ(\omega) \qquad (2-49)$$

where $Z(\omega)$ is another random process with orthogonal increments, namely

$$E[dZ(\omega_1)dZ^*(\omega_2)] = G_x(\omega_1)\delta(\omega_1 - \omega_2)d\omega_1 \qquad (2-50)$$

where $G_x(\omega)$ is the spectral density of $x(t)$ and $\delta(\cdot)$ is the Delta function. The Wiener-Khintchine relationship exists between the power spectral density function $G_x(\omega)$ and the autocorrelation function $R_x(\tau)$ of the stationary process $x(t)$, as shown in Eq. (2-5). The physical interpretation of ω as a frequency is justified from the following expression

$$R_x(0) = E[x^2(t)] = \int_{-\infty}^{\infty} G_x(\omega)d\omega \qquad (2-51)$$

The concept has been extended to the study of nonstationary processes by Priestley (1965, 1967). The spectral representation of a class of nonstationary processes $x(t)$ may be found as

$$x(t) = \int_{-\infty}^{+\infty} A(t,\omega)e^{i\omega t} dZ(\omega) \qquad (2-52)$$

in which $Z(\omega)$ is an orthogonal increment random process and $A(t,\omega)$ is a deterministic function of t and ω. If $A(t,\omega)$ varies slowly with time, then $A(t,\omega)e^{i\omega t}$ retains physical significance as an amplitude modulated harmonic function. The correlation function can be obtained as

$$E[x(t_1)x(t_2)] = \int_{-\infty}^{+\infty} A(t_1,\omega)A^*(t_2,\omega)d\omega \qquad (2-53)$$

and the mean square value of $x(t)$ is given by

$$E[x^2(t)] = \int_{-\infty}^{+\infty} |A(t,\omega)|^2 d\omega \qquad (2-54)$$

where $|A(t,\omega)|^2$ is often referred to as the *Evolutionary Spectral Density* of the nonstationary process $x(t)$. This quantity provides a local energy distribution in the neighborhood of the instant t which has the same physical interpretation as the power spectral density function of a stationary process.

A nonstationary random process is called an evolutionary process if it has an evolutionary spectral representation of the form of Eq. (2-52). Obviously, a stationary process is a special case with $A(t,\omega) = 1$ in Eq. (2-52). Another special case is the uniformly modulated process with $A(t,\omega)$ depending only on t. Note that not every nonstationary process has such a representation. The concept of an evolutionary process can be formally extended to a vector process, namely, a random vector process $\mathbf{x}(t)$ is called an evolutionary vector process if it has a spectral representation of the vector version of Eq. (2-52).

2.3.2 Evolutionary Spectral Density of the Response

Assume a linear MDOF system is subjected to a modulated stationary process. The nonstationary covariance matrix of the response can be found as Eq. (2-18). That is,

$$\mathbf{Q}(t_1,t_2) = \mathbf{L}\int_0^{t_1}\int_0^{t_2}\mathbf{P}(t_1-\tau_1)\mathbf{P}^T(t_2-\tau_2)\eta(\tau_1)\eta(\tau_2)E[n(\tau_1)n(\tau_2)]d\tau_1 d\tau_2\mathbf{L}^T$$

Using the Wiener-Khintchine relationship yields

$$\begin{aligned}\mathbf{Q}(t_1,t_2) = \int_{-\infty}^{\infty} S_n(\omega)\Big(\mathbf{L}\int_0^{t_1}\int_0^{t_2}&\mathbf{P}(t_1-\tau_1)\mathbf{P}^T(t_2-\tau_2) \\ &\eta(\tau_1)\eta(\tau_2)e^{i\omega(\tau_2-\tau_1)}d\tau_1 d\tau_2\mathbf{L}^T\Big)d\omega\end{aligned} \qquad (2-55)$$

2.3.2.1 Evolutionary Spectral Representation of the Response

Eq. (2-55) may be reduced to

$$\mathbf{Q}(t_1,t_2) = \int_{-\infty}^{\infty} S_n(\omega)\mathbf{A}(t_1,\omega)\mathbf{A}^{*T}(t_2,\omega)d\omega \qquad (2-56)$$

by introducing

$$\mathbf{A}(t,\omega) = \mathbf{L}\int_0^{t}\mathbf{P}(t-\tau)\eta(\tau)e^{-i\omega\tau}d\tau \qquad (2-57)$$

where matrix $\mathbf{L}$ and vector $\mathbf{P}(t)$ are defined as before and $\eta(t)$ is the envelope function.

Therefore, the nonstationary response of any linear MDOF system subjected to a modulated white noise is an evolutionary vector process. The response has the evolutionary spectral representation

$$\mathbf{Y}(t) = \int_{-\infty}^{+\infty} \mathbf{A}(t,\omega) dZ(\omega) \qquad (2-58)$$

where $Z(\omega)$ has the properties

$$\begin{aligned} E[dZ(\omega)] &= 0 \\ E[dZ(\omega_1)dZ^*(\omega_2)] &= S_n(\omega_1)\delta(\omega_1 - \omega_2)d\omega_1 \end{aligned} \qquad (2-59)$$

and $\mathbf{A}(t,\omega)$ is defined by Eq. (2-57). For the special case that $n(t)$ is white noise process, $S_n(\omega)$ is simply a constant. That is

$$S_n(\omega) = \frac{S_0}{2\pi} \qquad (2-60)$$

where the constant S_0 is the intensity of the white noise.

2.3.2.2 Evolutionary Spectral Density of the Response

The evolutionary spectral density matrix can be expressed as

$$\mathbf{S}(t,\omega) = S_n(\omega)\mathbf{A}(t,\omega)\mathbf{A}^{*T}(t,\omega) \qquad (2-61)$$

and the second moment response is then expressed as

$$E[\mathbf{Y}(t)\mathbf{Y}^T(t)] = \int_{-\infty}^{\infty} \mathbf{S}(t,\omega)d\omega \qquad (2-62)$$

Note that the definition of $\mathbf{A}(t,\omega)$ may not be unique in the sense that a difference of a factor of $e^{i\omega t}$ is allowable, but the resulting $\mathbf{S}(t,\omega)$ is still uniquely determined.

Sometimes the one-sided spectral density is preferred. For this case,

$$\begin{aligned} \mathbf{G}(t,\omega) &= 2\mathbf{S}(t,\omega) \\ \mathbf{Q}(t) &= \int_0^{\infty} \mathbf{G}(t,\omega)d\omega \end{aligned} \qquad (2-63)$$

where $\mathbf{G}(t,\omega)$ is the one-sided spectral density matrix.

2.3.2.3 Evaluation of the Evolutionary Spectral Density Matrix

Explicit solutions for the evolutionary spectral density matrix can be obtained if $\mathbf{A}(t,\omega)$ can be exactly evaluated, which is the case for the envelope function described by Eq. (2-38). Define

$$\mathbf{w}(t,\omega) = \int_0^t \mathbf{P}(t-\tau)\eta(\tau)e^{-i\omega\tau}d\tau \qquad (2-64)$$

Then, its subvectors $\mathbf{w}_m(t,\omega), m = 1,\ldots,N$ can be expressed as

$$\begin{aligned}\mathbf{w}_m(t,\omega) &= \begin{pmatrix} w_m^{(1)}(t,\omega) \\ w_m^{(2)}(t,\omega)\end{pmatrix} \\ &= \int_0^t e^{-\varsigma_m\omega_m(t-\tau)}\begin{pmatrix}\cos\omega_{dm}(t-\tau) \\ \sin\omega_{dm}(t-\tau)\end{pmatrix} e^{-i\omega\tau}\eta(\tau)d\tau\end{aligned} \qquad (2-65)$$

Using the definition (2-39) yields

$$\begin{aligned} w_m^{(1)}(t,\omega) &= \frac{A}{2}\Big(\mu(\gamma,-\alpha,-\omega,-\varsigma_m\omega_m,\omega_{dm}) + \mu(\gamma,-\alpha,-\omega,-\varsigma_m\omega_m,-\omega_{dm})\Big) \\ w_m^{(2)}(t,\omega) &= \frac{A}{2i}\Big(\mu(\gamma,-\alpha,-\omega,-\varsigma_m\omega_m,\omega_{dm}) - \mu(\gamma,-\alpha,-\omega,-\varsigma_m\omega_m,-\omega_{dm})\Big)\end{aligned} \qquad (2-66)$$

or, explicitly,

$$\begin{aligned} w_m^{(1)}(t,\omega) &= \frac{A}{2}\Big(e^{-\alpha t}\sum_{k=1}^{n+1}(-1)^{k-1}\frac{d^{k-1}}{dt^{k-1}}(t^n)[\frac{e^{-i(\omega t+k\theta)}}{\rho^k} + \frac{e^{-i(\omega t+k\bar{\theta})}}{\bar{\rho}^k}] \\ &\quad - e^{-\varsigma_m\omega_m t}(-1)^n n![\frac{e^{i(\omega_{dm}t-(n+1)\theta)}}{\rho^{n+1}} + \frac{e^{-i(\omega_{dm}t+(n+1)\bar{\theta})}}{\bar{\rho}^{n+1}}]\Big) \\ w_m^{(2)}(t,\omega) &= \frac{A}{2i}\Big(e^{-\alpha t}\sum_{k=1}^{n+1}(-1)^{k-1}\frac{d^{k-1}}{dt^{k-1}}(t^n)[\frac{e^{-i(\omega t+k\theta)}}{\rho^k} - \frac{e^{-i(\omega t+k\bar{\theta})}}{\bar{\rho}^k}] \\ &\quad - e^{-\varsigma_m\omega_m t}(-1)^n n![\frac{e^{i(\omega_{dm}t-(n+1)\theta)}}{\rho^{n+1}} - \frac{e^{-i(\omega_{dm}t+(n+1)\bar{\theta})}}{\bar{\rho}^{n+1}}]\Big)\end{aligned} \qquad (2-67)$$

where $\rho_m, \theta_m, \bar{\rho}_m$, and $\bar{\theta}_m \quad m = 1.\ldots,N$ are determined from the relationships

$$\begin{aligned}\rho_m e^{i\theta_m} &= (\varsigma_m\omega_m - \alpha) - i(\omega+\omega_{dm}) \\ \bar{\rho}_m e^{i\bar{\theta}_m} &= (\varsigma_m\omega_m - \alpha) - i(\omega-\omega_{dm})\end{aligned} \qquad (2-68)$$

From the definition of vector $\mathbf{A}(t,\omega)$, it follows that

$$\mathbf{A}(t,\omega) = \mathbf{Lw}(t,\omega) \qquad (2-69)$$

and, in turn, the evolutionary spectral density matrix can be found from Eq. (2-61)

$$\mathbf{S}(t,\omega) = S_n(\omega)\mathbf{A}(t,\omega)\mathbf{A}^{*T}(t,\omega)$$

or, directly expressed as

$$\mathbf{S}(t,\omega) = S_n(t,\omega)\mathbf{Lw}(t,\omega)\mathbf{w}^{*T}(t,\omega)\mathbf{L}^T \qquad (2-70)$$

Note that the off-diagonal terms of $\mathbf{G}(t,\omega)$ are generally complex, but the integration of the imaginary parts over the entire frequency domain is zero, which is justified by Eq. (2-62).

2.4 Nonstationary Response of Simple Systems Subjected to a Modulated White Noise

As an illustration of the proposed method, the results for the nonstationary covariance matrix response and the evolutionary spectral density are given for a simple single-degree-of-freedom system with natural frequency, ω_n, and critical damping ratio, ς, subjected to modulated white noise with different envelopes including the unit step envelope, rectangular envelope, Saragoni-Hart envelope, and Shinozuka-Sato envelope prevailing in earthquake engineering. All the results presented herein are explicit solutions.

For this simple system, the system matrix L can be found as

$$\mathbf{L} = \begin{pmatrix} 0 & \dfrac{1}{\omega_d} \\ 1 & -\dfrac{\varsigma\omega_n}{\omega_d} \end{pmatrix} \qquad (2-71)$$

The nonstationary covariance matrix is expressed as

$$\begin{aligned} \mathbf{Q}(t) &= \mathbf{LB}(t)\mathbf{L}^T \\ &= \sum_{k=1}^{3} \mathbf{B}_k J_k(t) \end{aligned} \qquad (2-72)$$

where

$$\mathbf{B}_1 = \frac{1}{2}\begin{pmatrix} -\dfrac{1}{\omega_d^2} & \dfrac{\varsigma\omega_n}{\omega_d^2} \\ \dfrac{\varsigma\omega_n}{\omega_d^2} & 1-\dfrac{\varsigma^2\omega_n^2}{\omega_d^2} \end{pmatrix}$$

$$\mathbf{B}_2 = \frac{1}{2}\begin{pmatrix} 0 & \dfrac{1}{\omega_d} \\ \dfrac{1}{\omega_d} & -2\dfrac{\varsigma\omega_n}{\omega_d} \end{pmatrix} \tag{2-73}$$

$$\mathbf{B}_3 = \frac{1}{2}\begin{pmatrix} \dfrac{1}{\omega_d^2} & -\dfrac{\varsigma\omega_n}{\omega_d^2} \\ -\dfrac{\varsigma\omega_n}{\omega_d^2} & \dfrac{\omega_n^2}{\omega_d^2} \end{pmatrix}$$

and

$$J_1(t) = S_0\int_0^t \eta^2(t-\tau)e^{-2\varsigma\omega_n\tau}\cos 2\omega_d\tau\, d\tau$$

$$J_2(t) = S_0\int_0^t \eta^2(t-\tau)e^{-2\varsigma\omega_n\tau}\sin 2\omega_d\tau\, d\tau \tag{2-74}$$

$$J_3(t) = S_0\int_0^t \eta^2(t-\tau)e^{-2\varsigma\omega_n\tau}\, d\tau$$

where constant S_0 is the intensity of the white noise.

The evolutionary spectral density can be written as

$$\mathbf{G}(t,\omega) = \frac{S_0}{\pi}\mathbf{L}\mathbf{w}(t,\omega)\mathbf{w}^{*T}(t,\omega)\mathbf{L}^T \tag{2-75}$$

Note that for all four envelopes considered herein the system matrix $\mathbf{L}$ remains the same and only $\mathbf{J}_k(t), k=1,2,3$ and $\mathbf{w}_k(t,\omega), k=1,2$ need to be changed. The results are presented as follows:

2.4.1 Unit Step Envelope

The step function has been widely used as an envelope by many researchers to study the transient and the asymptotically stationary behavior of structures subjected to a suddenly applied stationary load. The step function is defined as

$$\eta^{(1)}(t) = \begin{cases} 1; & \text{if } t \geq 0 \\ 0. & \text{otherwise} \end{cases} \tag{2-76}$$

which is a special case of (2-38) with $\gamma = 0$ and $\alpha = 0$.

The stationary correlation matrix of the response can be obtained by using Eqs. (2-26)-(2-31). That is

$$\mathbf{R}(\tau) = \mathbf{L}\bar{\mathbf{B}}(\tau)\mathbf{L}^T \tag{2-77}$$

where $\mathbf{L}$ is defined in (2-71) and $\bar{\mathbf{B}}(\tau)$ is given by

$$\bar{\mathbf{B}}(\tau) = e^{-\varsigma\omega_n\tau}\begin{pmatrix} c_{11} & c_{12} \\ c_{21} & c_{22} \end{pmatrix}\begin{pmatrix} \cos\omega_d\tau & \sin\omega_d\tau \\ -\sin\omega_d\tau & \cos\omega_d\tau \end{pmatrix} \tag{2-78}$$

and the constants $c_{ij}, i, j = 1, 2$ are found as

$$\begin{aligned} c_{11} &= \frac{1+\varsigma^2}{4\varsigma\omega_n} & c_{12} &= \frac{\omega_d}{4\omega_n^2} \\ c_{21} &= \frac{\omega_d}{4\omega_n^2} & c_{22} &= \frac{\omega_d^2}{4\varsigma\omega^3} \end{aligned} \tag{2-79}$$

Substituting Eqs. (2-78) and (2-79) into (2-77) yields

$$\mathbf{R}(\tau) = e^{-\varsigma\omega_n\tau}\begin{pmatrix} \frac{1}{4\omega_n^2\omega_d}\sin\omega_d\tau + \frac{1}{4\varsigma\omega_n^3}\cos\omega_d\tau & -\frac{1}{4\varsigma\omega_n\omega_d}\sin\omega_d\tau \\ \frac{1}{4\varsigma\omega_n\omega_d}\sin\omega_d\tau & \frac{1}{4\varsigma\omega_n}\cos\omega_d\tau - \frac{1}{4\omega_d}\sin\omega_d\tau \end{pmatrix} \tag{2-80}$$

Note that $\mathbf{R}(\tau)$ vanishes as the time lag τ approaches ∞, which implies that the responses at time t_1 and t_2 become uncorrelated for a sufficiently large difference in t_1 and t_2. It is observed that $R_{12}(\tau) = R_{21}(-\tau)$. All the components of $\mathbf{R}(\tau)$ are exponentially decaying harmonic functions of the time leg τ with a period of $\frac{2\pi}{\omega_d}$.

The second moment response is given by

$$\mathbf{R}(0) = \begin{pmatrix} \frac{1}{4\varsigma\omega_n^3} & 0 \\ 0 & \frac{1}{4\varsigma\omega_n} \end{pmatrix} \tag{2-81}$$

which agrees with the results from previous studies.

Numerical results for the correlation matrix for three different values of damping ratio, namely, $\varsigma = 0.05, 0.1, 0.2$, are shown in Fig. 2.1. It is confirmed that all the components of the correlation matrix are exponentially decaying harmonic functions. Two autocorrelation functions start from their maxima and the cross-correlation function starts from zero. All the curves vanish as the time lag τ approaches ∞.

The nonstationary covariance matrix can be obtained in terms of Eqs. (2-72) - (2-74). Particularly, $\mathbf{J}_k(t)$ can be found to be

$$\begin{aligned} J_1^{(1)}(t) &= e^{-2\varsigma\omega_n t}\left(-\frac{\varsigma}{2\omega_n}\cos 2\omega_d t + \frac{\omega_d}{2\omega_n^2}\sin 2\omega_d t\right) + \frac{\varsigma}{2\omega_n} \\ J_2^{(1)}(t) &= e^{-2\varsigma\omega_n t}\left(-\frac{\varsigma}{2\omega_n}\sin 2\omega_d t - \frac{\omega_d}{2\omega_n^2}\cos 2\omega_d t\right) + \frac{\omega_d}{2\omega_n^2} \\ J_3^{(1)}(t) &= \frac{1}{2\varsigma\omega_n}(1 - e^{-2\varsigma\omega_n t}) \end{aligned} \qquad (2-82)$$

where constant S_0 is the intensity of the white noise.

Substituting Eqs. (2-73) and (2-82) into (2-72) yields

$$\begin{aligned} Q_{11}^{(1)}(t) &= S_0\left(\frac{1}{4\varsigma\omega_n^3} - \frac{e^{-2\varsigma\omega_n t}}{4\varsigma\omega_n\omega_d^2}\left(1 + \frac{\varsigma\omega_d}{\omega_n}\sin 2\omega_d t - \varsigma^2\cos 2\omega_d t\right)\right) \\ Q_{12}^{(1)}(t) &= Q_{21}(t) = \frac{S_0}{4\omega_d^2}e^{-2\varsigma\omega_n t}(1 - \cos 2\omega_d t) \\ Q_{22}^{(1)}(t) &= S_0\left(\frac{1}{4\varsigma\omega_n} + \frac{e^{-2\varsigma\omega_n t}}{4\varsigma\omega_n\omega_d^2}\left(-\omega_n^2 + \varsigma\omega_n\omega_d\sin 2\omega_d t + \varsigma^2\omega_n^2\cos 2\omega_d t\right)\right) \end{aligned} \qquad (2-83)$$

A comparison shows that the expression for $Q_{11}^{(1)}(t)$ is identical to Caughey and Stumpf's result (1961) by setting $S_0 = 2D$. Here, in addition, the results for $Q_{12}^{(1)}$ and $Q_{22}^{(1)}$ are also obtained in closed form. It is interesting to note that $Q_{12}(t)$ is an exponentially decaying harmonic function of t with a period of $\frac{\pi}{\omega_d}$.

The results for the nonstationary case are illustrated in Fig. 2.2 for four damping ratios of $\varsigma = 0.005, 0.02$, 0.05 and 0.1. T is the natural period of the system. As $t \to \infty$, the responses approach their stationary values, namely,

$$\begin{aligned} Q_{11}^{(1)} &= \frac{S_0}{4\varsigma\omega_n^3} \\ Q_{12}^{(1)} &= Q_{21} = 0 \\ Q_{22}^{(1)} &= \frac{S_0}{4\varsigma\omega_n} \end{aligned} \qquad (2-84)$$

In the case of $\varsigma = 0$, the above solution must be revised. By direct integration, or by taking the limit as $\varsigma \to 0$ in Eq. (2-83), one obtains

$$
\begin{aligned}
Q_{11}^{(1)}(t) &= \frac{S_0}{\omega_n^2}\left(\frac{t}{2} - \frac{\sin 2\omega_d t}{4\omega_n}\right) \\
Q_{12}^{(1)}(t) &= \frac{S_0}{4\omega_n^2}(1 - \cos 2\omega_d t) \\
Q_{22}^{(1)}(t) &= S_0\left(\frac{t}{2} + \frac{\sin 2\omega_d t}{4\omega_n}\right)
\end{aligned}
\tag{2-85}
$$

As t approaches ∞, the solutions become unbounded.

In order to obtain the evolutionary spectral density, the term $\mathbf{w}^{(1)}(t,\omega)$ needs be evaluated. Its components can be expressed as

$$
\begin{aligned}
w_k^{(1)}(t,\omega) = &\Big((a_k \cos\omega t + b_k \sin\omega t) + e^{-\varsigma\omega_n t}(c_k \cos\omega_d t + d_k \sin\omega_d t)\Big) \\
&+ i\Big((\alpha_k \cos\omega t + \beta_k \sin\omega t) + e^{-\varsigma\omega_n}(\gamma_k \cos\omega_d t + \delta_k \sin\omega_d t)\Big)
\end{aligned}
\tag{2-86}
$$

$$k = 1, 2$$

where the constants are

$$
\begin{aligned}
a_1 &= -\beta_1 = -c_1 = -d_2 = \varsigma\omega_n(\omega^2 + \omega_n^2)|H(i\omega)|^2 \\
b_1 &= \alpha_1 = -\gamma_1 = -\delta_2 = \omega\Big(\omega^2 - \omega_n(1 - 2\varsigma^2)\Big)|H(i\omega)|^2 \\
d_1 &= a_2 = -c_2 = -\beta_2 = \omega_d(\omega_n^2 - \omega^2)|H(i\omega)|^2 \\
\delta_1 &= b_2 = \alpha_2 = -\gamma_2 = 2\varsigma\omega\omega_n\omega_d|H(i\omega)|^2
\end{aligned}
\tag{2-87}
$$

in which

$$|H(i\omega)|^2 = \frac{1}{(\omega_n^2 - \omega^2)^2 + 4\varsigma^2\omega_n^2\omega^2} \tag{2-88}$$

The evolutionary spectral density is then expressed as

$$\mathbf{G}^{(1)}(t,\omega) = \frac{S_0}{\pi}\mathbf{L}\mathbf{w}^{(1)}(t,w)\mathbf{w}^{(1)*T}(t,\omega)\mathbf{L}^T \tag{2-89}$$

In particular, the evolutionary spectral density of the displacement can be found as

$$
\begin{aligned}
G_{11}^{(1)}(t,\omega) = &\frac{S_0}{\pi}|H(i\omega)|^2\Big(1 + e^{-2\varsigma\omega_n t}[1 + a(t) + \omega^2 b(t)] \\
&- e^{-\varsigma\omega_n t}[c(t)\cos\omega t + \omega d(t)\sin\omega t]\Big)
\end{aligned}
\tag{2-90}
$$

where

$$
\begin{aligned}
a(t) &= \frac{\varsigma\omega_n}{\omega_d}\sin 2\omega_d t + \frac{\varsigma^2\omega_n^2 - \omega_d^2}{\omega_d^2}\sin^2\omega_d t \\
b(t) &= \frac{1}{\omega_d^2}\sin^2\omega_d t \\
c(t) &= 2\cos\omega_d t + \frac{2\varsigma\omega_n}{\omega_d}\sin\omega_d t \\
d(t) &= \frac{2}{\omega_d}\sin\omega_d t
\end{aligned}
\qquad (2-91)
$$

The result is similar to that obtained by Corotis and Vanmarke (1975) except that certain errors exist in their presentation.

Some numerical results are shown in Fig. 2.3 for five different times, i.e., $t =$ 2.5, 5.0, 10.0, 20.0, and 40.0 sec. The results show the change in the amplitude and frequency content of the spectral densities of the response. It is observed that these curves approach their stationary counterparts for a sufficiently large time.

2.4.2 Boxcar Envelope

The boxcar envelope function has been introduced to account for the effect of finite duration of excitation. The function is defined as

$$
\eta^{(2)}(t) = \begin{cases} 1, & \text{if } 0 \le t \le T_d; \\ 0, & \text{otherwise.} \end{cases} \qquad (2-92)
$$

where T_d denotes the duration of the excitation. Note that the boxcar envelope can be thought of as a superposition of two step functions starting from different times with opposite signs, namely,

$$
\eta^{(2)}(t) = \eta^{(1)}(t) - \eta^{(1)}(t - T_d) \qquad (2-93)
$$

which leads to the following piecewise solution:

$$
\mathbf{Q}^{(2)}(t) = \begin{cases} 0, & \text{if } t < 0; \\ \mathbf{Q}^{(1)}(t), & \text{if } 0 \le t \le T_d; \\ \mathbf{Q}^{(1)}(t) - \mathbf{Q}^{(1)}(t - T_d), & \text{if } T_d < t. \end{cases} \qquad (2-94)
$$

where $Q^{(1)}(t)$ denotes the nonstationary solution for the step envelope.

The result is shown in Fig. 2.4 for the four damping ratios, $\varsigma = 0.005, 0.02, 0.05$ and 0.1. As anticipated, the mean square response gradually increases toward the stationary value before $t = T_d$ and then decreases toward zero afterwards.

Similarly, the one-sided evolutionary spectral density matrix can be found as

$$\mathbf{G}^{(2)}(t,\omega) = \frac{S_0}{\pi}\mathbf{L}\mathbf{w}^{(2)}(t,\omega)\mathbf{w}^{(2)*T}(t,\omega)\mathbf{L}^T \tag{2-95}$$

where

$$\mathbf{w}^{(2)} = \begin{cases} 0, & \text{if } 0 < t; \\ \mathbf{w}^{(1)}(t,\omega), & \text{if } 0 \le t \le T_d; \\ \mathbf{w}^{(1)}(t,\omega) - e^{-i\omega T_d}\mathbf{w}^{(1)}(t-T_d,\omega), & \text{if } T_d < t. \end{cases} \tag{2-96}$$

Note that it is $\mathbf{w}^{(2)}(t,\omega)$ instead of $\mathbf{G}^{(2)}(t,\omega)$ itself that obeys the above superposition principle. The evolutionary spectral density for four different times is shown in Fig. 2.5.

2.4.3 Saragoni-Hart Type Envelope

In ground motion simulation, Saragoni and Hart used a product of a power function and a exponential function as an envelope to account for different stages of ground motion, such as build-up, stationarity, and decay. The envelope is exactly of the form of Eq. (2-38). However, $\gamma = 1$ is assumed here for simplicity. That is

$$\eta^{(3)}(t) = \begin{cases} Ate^{-\alpha t}, & \text{if } t \ge 0; \\ 0, & \text{otherwise.} \end{cases} \tag{2-97}$$

where $A = \alpha e$ is assumed to normalize the envelope such that its peak value is one and α is assumed to be positive. Note that if γ is not an integer, an infinite series may be expected.

The nonstationary covariance matrix can be generally expressed by

$$\mathbf{Q}^{(3)}(t) = \sum_{k=1}^{3} \mathbf{B}_k(t) J_k^{(3)}(t) \tag{2-98}$$

where $B_k(t), k = 1,2,3$ are defined as in (2-73) and $J_k^{(3)}(t) k = 1,2,3$ are found as

$$\begin{aligned} J_1^{(3)}(t) &= A^2 S_0 \int_0^t \tau^2 e^{-2\alpha\tau} e^{-2\varsigma\omega(t-\tau)} \cos 2\omega_d(t-\tau) d\tau \\ &= A^2 S_0 \Bigg(e^{-2\alpha t}(\frac{t^2}{\rho}\cos\theta - 2\frac{t}{\rho^2}\cos 2\theta + \frac{2}{\rho^3}\cos 3\theta) \\ &\quad - e^{-2\varsigma\omega_n t}\frac{2}{\rho^3}\cos(2\omega_d t - 3\theta)\Bigg) \end{aligned}$$

$$J_2^{(3)}(t) = A^2 S_0 \int_0^t \tau^2 e^{-2\alpha\tau} e^{-2\varsigma\omega(t-\tau)} \sin 2\omega_d(t-\tau) d\tau$$

$$= -A^2 S_0 \Big(e^{-2\alpha t}(\frac{t^2}{\rho}\sin\theta - 2\frac{t}{\rho^2}\sin 2\theta + \frac{2}{\rho^3}\sin 3\theta) \qquad (2-99)$$

$$+ e^{-2\varsigma\omega_n t}\frac{2}{\rho^3}\sin(2\omega_d t - 3\theta)\Big)$$

$$J_3^{(3)}(t) = A^2 S_0 \int_0^t \tau^2 e^{-2\alpha\tau} e^{-2\varsigma\omega(t-\tau)} d\tau$$

$$= A^2 S_0 \Big(e^{-2\alpha t}(\frac{t^2}{2(\varsigma\omega_n - \alpha)} - \frac{t}{2(\varsigma\omega_n - \alpha)^2} + \frac{1}{4(\varsigma\omega_n - \alpha)^3})$$

$$- \frac{1}{4(\varsigma\omega_n - \alpha)^3} e^{-2\varsigma\omega_n t}\Big)$$

in which ρ, θ are determined from the relationship

$$\rho e^{i\theta} = (2\varsigma\omega_n - 2\alpha) - i(2\omega_d) \qquad (2-100)$$

Note that in the above $\varsigma\omega_n - \alpha \neq 0$ is assumed, otherwise $J_3^{(3)}(t)$ should be replaced by

$$J_3^{(3)}(t) = A^2 S_0 \frac{t^3}{3} e^{-2\varsigma\omega_n t} \qquad (2-101)$$

Eqs. (2-98)-(2-101) complete the solution for covariance response. Some numerical results are presented in Fig. 2.6.

The one-sided evolutionary spectral density can be determined fron

$$\bar{\mathbf{G}}^{(3)}(t,\omega) = \frac{S_0}{\pi} L \mathbf{w}^{(3)}(t,\omega) \mathbf{w}^{(3)*T} L^T \qquad (2-102)$$

where two components of $\mathbf{w}^{(3)}$ are expressed as

$$w_k^{(3)} = \frac{A}{2}\Big(e^{-\alpha t}[(a_k t + b_k)\cos\omega t + (c_k t + d_k)\sin\omega t]$$

$$+ e^{-\varsigma\omega_n t}(g_k \cos\omega_d t + f_k \sin\omega_d t)\Big)$$

$$+ i\frac{A}{2}\Big(e^{-\alpha t}[(\sigma_k t + \theta_k)\cos\omega t + (\gamma_k t + \delta_k)\sin\omega t] \qquad (2-103)$$

$$+ e^{-\varsigma\omega_n t}(\lambda_k \cos\omega_d t + \mu_k \sin\omega_d t)\Big)$$

$$k = 1, 2$$

and where the constants can be found as

$$a_1 = -\gamma_1 = (\varsigma\omega_n - \alpha)(\frac{1}{\rho^2} + \frac{1}{\bar{\rho}^2})$$

$$b_1 = -g_1 = -f_2 = -\delta_1 = -\left(\frac{(\varsigma\omega_n - \alpha)^2 - (\omega + \omega_d)^2}{\rho^4} + \frac{(\varsigma\omega_n - \alpha)^2 - (\omega - \omega_d)^2}{\bar{\rho}^4}\right)$$

$$c_1 = \sigma_1 = \frac{\omega + \omega_d}{\rho^2} + \frac{\omega - \omega_d}{\bar{\rho}^2}$$

$$d_1 = \theta_1 = -\lambda_1 = -\mu_2 = -2(\varsigma\omega_n - \alpha)\left(\frac{\omega + \omega_d}{\rho^4} + \frac{\omega - \omega_d}{\bar{\rho}^4}\right)$$

$$f_1 = b_2 = -g_2 = -\delta_2 = -2(\varsigma\omega_n - \alpha)\left(\frac{\omega + \omega_d}{\rho^4} - \frac{\omega - \omega_d}{\bar{\rho}^4}\right)$$

$$\mu_1 = d_2 = \theta_2 = -\lambda_2 = \frac{(\varsigma\omega_n - \alpha)^2 - (\omega + \omega_d)^2}{\rho^4} - \frac{(\varsigma\omega_n - \alpha)^2 - (\omega - \omega_d)^2}{\bar{\rho}^4}$$

$$a_2 = -\gamma_2 = -\frac{\omega + \omega_d}{\rho^2} + \frac{\omega - \omega_d}{\bar{\rho}^2}$$

$$c_2 = \sigma_2 = -(\varsigma\omega_n - \alpha)(\frac{1}{\rho^2} - \frac{1}{\bar{\rho}^2})$$

$$(2-104)$$

with ρ^2 and $\bar{\rho}^2$ defined as

$$\begin{aligned} \rho^2 &= (\varsigma\omega_n - \alpha)^2 + (\omega + \omega_d)^2 \\ \bar{\rho}^2 &= (\varsigma\omega_n - \alpha)^2 + (\omega - \omega_d)^2 \end{aligned} \qquad (2-105)$$

Eqs. (2-102)-(2-105) complete the solution for the evolutionary spectral density matrix. Some numerical results are shown in Fig. 2.7.

2.4.4 Shinozuka-Sato Envelope

Shinozuka and Sato (1967) proposed the following envelope

$$\eta^{(4)}(t) = \begin{cases} A(e^{-\alpha t} - e^{-\beta t}), & \text{if } t \leq 0 \\ 0 & \text{otherwise.} \end{cases} \qquad (2-106)$$

for earthquake ground motion. The parameters A, α, and β are chosen such that

$$\begin{aligned} &\beta > \alpha \geq 0 \\ &A = \frac{\beta}{\beta - \alpha}(\frac{\alpha}{\beta})^{\frac{\alpha}{\alpha-\beta}} \end{aligned} \qquad (2-107)$$

The envelope takes the nonstationarity of earthquake ground motion into account without loss of simplicity, therefore, it is preferred by some investigators.

Note that $\eta^{(4)}$ consists of two terms both of which are the special cases of equation (2-38) with $\gamma = 0$ and different exponentials, therefore, the principle of superposition can be imposed. Substituting Eq. (2-106) into (2-72)-(2-74) yields the nonstationary covariance matrix as

$$Q^{(4)}(t) = \sum_{k=1}^{3} \mathbf{B}_k J_k^{(4)}(t) \qquad (2-108)$$

where $\mathbf{B}_k, k = 1,2,3$ are defined as before and $J_k^{(4)}(t), k = 1,2,3$ are evaluated by

$$J_1^{(4)}(t) = A^2 S_0 \int_0^t (e^{-2\alpha t} - 2e^{-(\alpha+\beta)t} + e^{-2\beta t}) e^{-2\varsigma\omega_n(t-\tau)} \cos 2\omega_d(t-\tau)d\tau$$

$$J_2^{(4)}(t) = A^2 S_0 \int_0^t (e^{-2\alpha t} - 2e^{-(\alpha+\beta)t} + e^{-2\beta t}) e^{-2\varsigma\omega_n(t-\tau)} \sin 2\omega_d(t-\tau)d\tau$$

$$J_3^{(4)}(t) = A^2 S_0 \int_0^t (e^{-2\alpha t} - 2e^{-(\alpha+\beta)t} + e^{-2\beta t}) e^{-2\varsigma\omega_n(t-\tau)} d\tau$$

$$(2-109)$$

or, explicitly,

$$J_k^{(4)} = A_k e^{-2\alpha t} + B_k e^{-2\beta t} + C_k e^{-(\alpha+\beta)t} + e^{-2\varsigma\omega_n t}(D_k \cos 2\omega_d t + E_k \sin 2\omega_d t)$$

$$k = 1,2 \qquad (2-110)$$

in which the constants A_k, B_k, C_k, D_k and E_k $\quad k = 1,2$ are given by

$$A_1 = \frac{2\varsigma\omega_n - 2\alpha}{(2\varsigma\omega_n - 2\alpha)^2 + (2\omega_d)^2}$$

$$B_1 = \frac{2\varsigma\omega_n - 2\beta}{(2\varsigma\omega_n - 2\beta)^2 + (2\omega_d)^2}$$

$$C_1 = -2\frac{2\varsigma\omega_n - \alpha - \beta}{(2\varsigma\omega_n - \alpha - \beta)^2 + (2\omega_d)^2}$$

$$A_2 = \frac{2\omega_d}{(2\varsigma\omega_n - 2\alpha)^2 + (2\omega_d)^2}$$

$$B_2 = \frac{2\omega_d}{(2\varsigma\omega_n - 2\beta)^2 + (2\omega_d)^2}$$

$$C_2 = -\frac{4\omega_d}{(2\varsigma\omega_n - \alpha - \beta)^2 + (2\omega_d)^2}$$

$$D_1 = E_2 = -(A_1 + B_1 + C_1)$$
$$E_1 = -D_2 = A_2 + B_2 + C_2 \qquad (2-111)$$

and $J_3^{(4)}(t)$ is given by

$$J_3^{(4)} = A_3 e^{-2\alpha t} + B_3 e^{-2\beta t} + C_3 e^{-(\alpha+\beta)t} + F_3 e^{-2\varsigma\omega_n t} \qquad (2-112)$$

in which

$$A_3 = \frac{1}{2\varsigma\omega_n - 2\alpha}$$
$$B_3 = \frac{1}{2\varsigma\omega_n - 2\beta}$$
$$C_3 = -\frac{2}{2\varsigma\omega_n - \alpha - \beta} \qquad (2-113)$$
$$F_3 = -(A_3 + B_3 + C_3)$$

Numerical results are shown in Fig. 2.8. It is observed that the response approaches zero for a sufficiently large time and the decay rate depends on the minimum of $2\alpha, 2\beta$, and $2\varsigma\omega_n$. Note that the solution needs to be revised if any denominator in Eq. (2-113) becomes zero.

The one-sided evolutionary spectral density matrix of the response is found to be

$$\mathbf{G}^{(4)}(t,\omega) = \frac{S_0}{\pi}\mathbf{L}\mathbf{w}^{(4)}(t,\omega)\mathbf{w}^{(4)*T}(t,\omega)\mathbf{L}^T \qquad (2-114)$$

where

$$\begin{aligned} w_k^{(4)} = {} & \frac{A}{2}\Big(e^{-\alpha t}(a_k\cos\omega t + b_k\sin\omega t) - e^{-\beta t}(c_k\cos\omega t + d_k\sin\omega t) \\ & + e^{-\varsigma\omega_n t}(g_k\cos\omega_d t + f_k\sin\omega_d t)\Big) \\ & + i\frac{A}{2}\Big(e^{-\alpha t}(l_k\cos\omega t + m_k\sin\omega t) - e^{-\beta t}(n_k\cos\omega t + u_k\sin\omega t) \\ & + e^{-\varsigma\omega_n t}(v_k\cos\omega_d t + s_k\sin\omega t)\Big) \end{aligned} \qquad (2-115)$$

$$k = 1, 2$$

where the constants are evaluated as

$$a_1 = -m_1 = (\varsigma\omega_n - \alpha)(\frac{1}{\rho_\alpha^2} + \frac{1}{\bar{\rho}_\alpha^2})$$

$$b_1 = l_1 = \frac{\omega + \omega_d}{\rho_\alpha^2} + \frac{\omega - \omega_d}{\bar{\rho}_\alpha^2}$$

$$c_1 = -u_1 = (\varsigma\omega_n - \beta)(\frac{1}{\rho_\beta^2} + \frac{1}{\bar{\rho}_\beta^2})$$

$$d_1 = n_1 = \frac{\omega + \omega_d}{\rho_\beta^2} + \frac{\omega - \omega_d}{\bar{\rho}_\beta^2}$$

$$\begin{aligned} g_1 &= -a_1 + c_1 \\ f_1 &= (\omega + \omega_d)\frac{(1}{\rho_\alpha^2} - \frac{1}{\rho_\beta^2}) - (\omega - \omega_d)(\frac{1}{\bar{\rho}_\alpha^2} - \frac{1}{\bar{\rho}_\beta^2}) \\ v_1 &= -b_1 + d_1 \\ s_1 &= -(\varsigma\omega_n - \alpha)(\frac{1}{\rho_\alpha^2} - \frac{1}{\bar{\rho}_\alpha^2}) + (\varsigma\omega_n - \beta)(\frac{1}{\rho_\beta^2} - \frac{1}{\bar{\rho}_\beta^2}) \end{aligned} \qquad (2-116)$$

and

$$a_2 = -m_2 = \frac{\omega + \omega_d}{\rho_\alpha^2} - \frac{\omega - \omega_d}{\bar{\rho}_\alpha^2}$$

$$b_2 = l_2 = -(\varsigma\omega_n - \alpha)(\frac{1}{\rho_\alpha^2} - \frac{1}{\bar{\rho}_\alpha^2})$$

$$c_2 = -u_2 = \frac{\omega + \omega_d}{\rho_\beta^2} - \frac{\omega - \omega_d}{\bar{\rho}_\beta^2}$$

$$d_2 = n_2 = -(\varsigma\omega_n - \beta)(\frac{1}{\rho_\beta^2} - \frac{1}{\bar{\rho}_\beta^2})$$

$$\begin{aligned} g_2 &= -a_2 + c_2 \\ f_2 &= -(\varsigma\omega_n - \alpha)(\frac{1}{\rho_\alpha^2} + \frac{1}{\bar{\rho}_\alpha^2}) + (\varsigma\omega_n - \beta)(\frac{1}{\rho_\beta^2} + \frac{1}{\bar{\rho}_\beta^2}) \\ v_2 &= -b_2 + d_2 \\ s_2 &= -(\omega + \omega_d)(\frac{1}{\rho_\alpha^2} - \frac{1}{\rho_\beta^2}) - (\omega - \omega_d)(\frac{1}{\bar{\rho}_\alpha^2} - \frac{1}{\bar{\rho}_\beta^2}) \end{aligned} \qquad (2-117)$$

Some numerical results are shown in Fig. 2.9.

2.5 Application to Other Stochastic Load Models

A complete formulation for the nonstationary response of linear MDOF systems subjected to a modulated stationary excitation has been presented in the previous sections. The approach can be extented to many other stochastic excitation models, such as those mentioned in the review in Appendix II.

If the load is modeled as a stationary process, the stationary response may be obtained by taking the limit of the nonstationary solution with a unit step envelope as the time becomes sufficiently large, as described in the section 2.2.2.1.

If a random impulse train model is employed in the analysis, the formulation still applies by first defining the equivalent modulated white noise process with a proper envelope. The solution is valid due to the equivalence between these two models up to the second moment, as shown in Appendix II.

In order to describe the nonstationary characteristics of the excitation both in magnitude and frequency content, the filtered modulated stationary model is introduced. Since the filter may be treated as an substructure attached to the original system, the analysis of an MDOF system under the filtered modulated stationary model including the filtered random pulse train model, may be replaced by the analysis of an augmented MDOF system subjected to the modulated stationary model, to which the simplified state-variable method presented can apply.

If the ARMA model is used and a corresponding linear differential equation can be defined, this method may also be employed to find the nonstationary response of MDOF systems subjected to the ARMA model based on the same argument as the above.

Furthermore, the method may be extended to the case of multiple stochastic load processes, such as those applied to long structures. In the corresponding solution for the nonstationary covariance of the response, the single-term expression will be replaced by a summation of similar terms. A discussion about detailed results is beyond the scope of this thesis.

2.6 Conclusions

A simplified state-variable method is presented to solve for the nonstationary covariance matrix and the evolutionary spectral density matrix of the response of MDOF structures subjected to modulated stationary excitation. In contrast to other methods, the final solution can be expressed as a product of a constant matrix $\mathbf{L}$ which is determined by the system parameters only, and a time-dependent matrix $\mathbf{B}(t)$, whose components are the products of exponential functions, triangular functions, and the envelope function chosen. An explicit solution can be found whenever these integrals can be explicitly evaluated, which is the case for many engineering applications. The method applies for systems with classical or nonclassical damping subjected to many different stochastic load models.

In contrast to Gasparini's approach (1979), this method formulates the problem directly in terms of physical state variables, i.e., displacements and velocities. As a result, this method is a one-step procedure to obtain the covariance response and the evaluation of the transient part of the solution becomes much easier. Furthermore, the method is flexible enough to be applied to a variety of systems including those with nonclassical damping.

Compared with the formulation given by Yang, Sarkani, and Long (1988) which is basically a frequency domain approach where a numerical technique such as FFT is generally needed there to find the mean square response, both methods stem from the canonical modal analysis and can be applied to the analysis of nonclassically damped systems. However, the present method formulates the problem in both the time and frequency domains and, again, the method is an one-step procedure which does not need to explicitly solve the problem on the modal basis first. As a consequence, the method proposed herein provides a complete set of solutions including the stationary correlation matrix, the nonstationary covariance matrix and the evolutionary spectral density matrix, which may be explicitly evaluated for a large class of envelopes.

The method is particularly suitable for the seismic analysis of structures since earthquake loads are often modeled as a modulated stationary process. Explicit solutions can be easily obtained by this method for a class of envelopes which include most of the envelopes frequently used in earthquake engineering as special cases. The method is sufficiently flexible to be applied to different stochastic earthquake ground motion models, including white noise, filtered white noise, modulated white noise, filtered modulated white noise, and their equivalent.

As an illustration, a complete set of results for the stationary correlation matrix, the nonstationary covariance matrix, and the evolutionary spectral density matrix of the response of a simple linear system subjected to a modulated white noise excitation are presented for certain chosen envelopes. The results obtained show the efficiency and the completeness of the method even in this simple case. More applications in earthquake engineering will be presented in the following chapters.

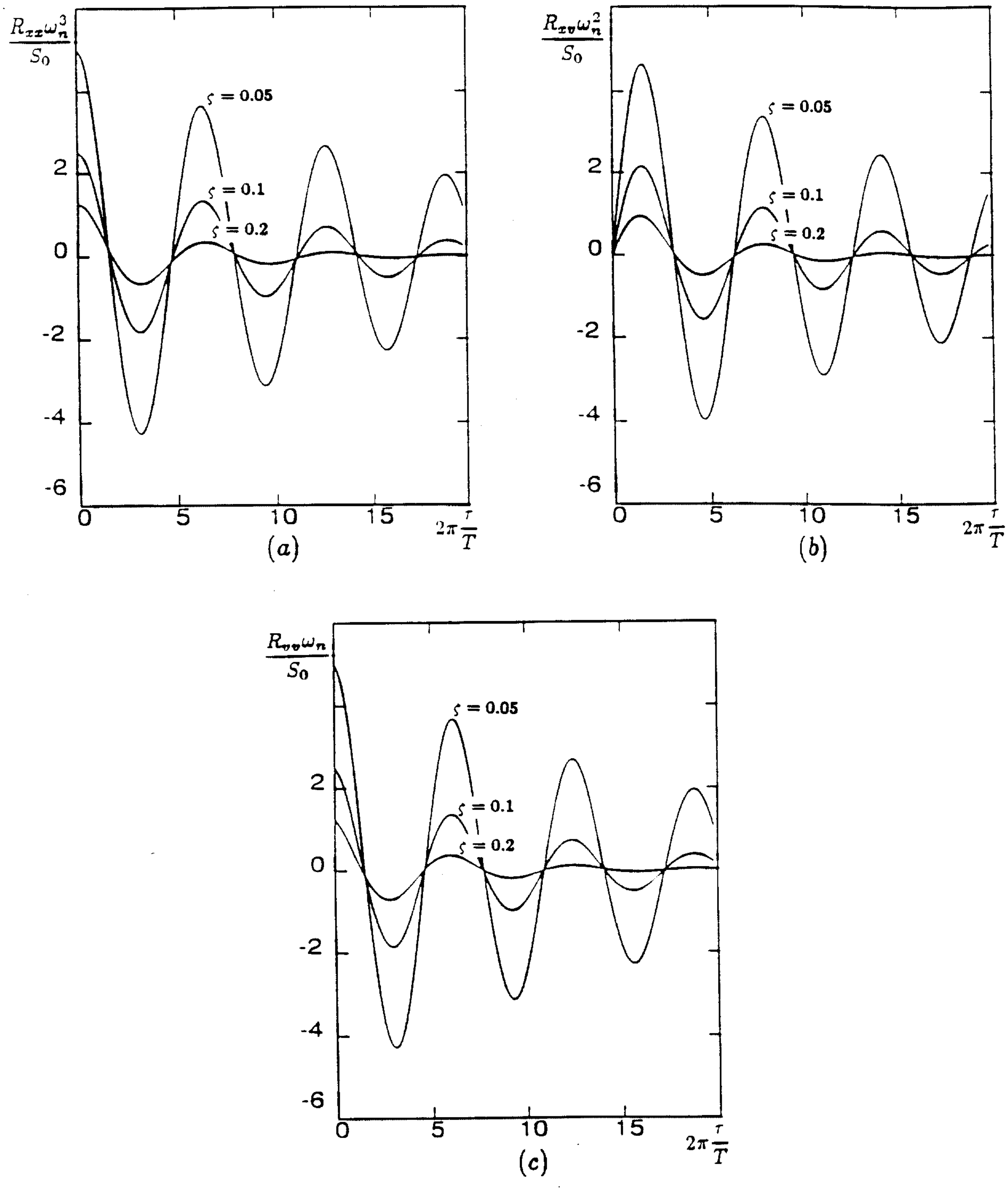


Figure 2.1. Nondimensional stationary correlation of the response of linear simple systems with damping ratio $\varsigma = 0.05, 0.1$, and 0.2 subjected to a stationary white noise.

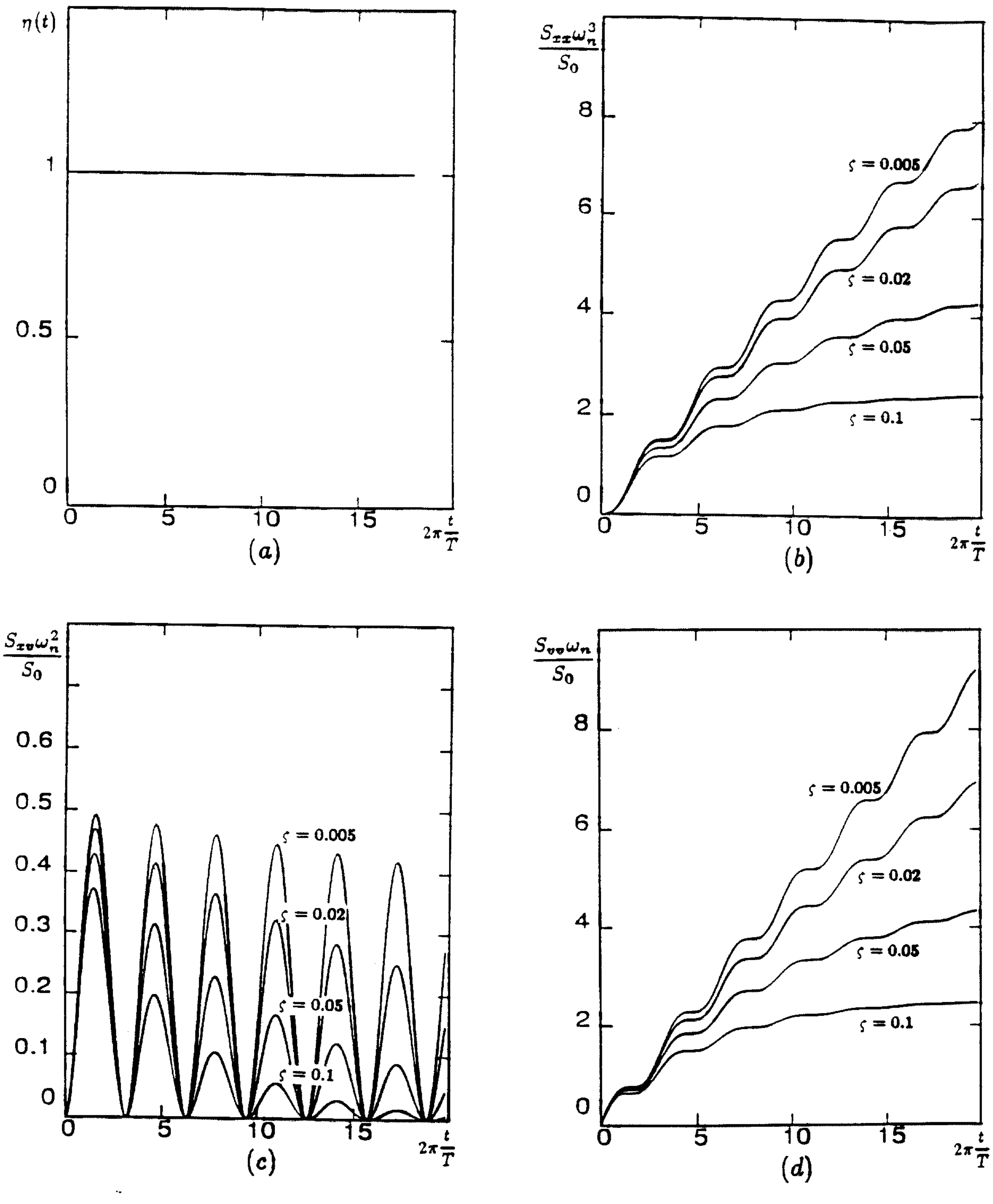


Figure 2.2. Nonstationary covariance of the response of linear simple systems with damping ratio $\varsigma = 0.005, 0.02, 0.05$, and 0.1 subjected to a modulated white noise. Case 1: Unit step envelope.

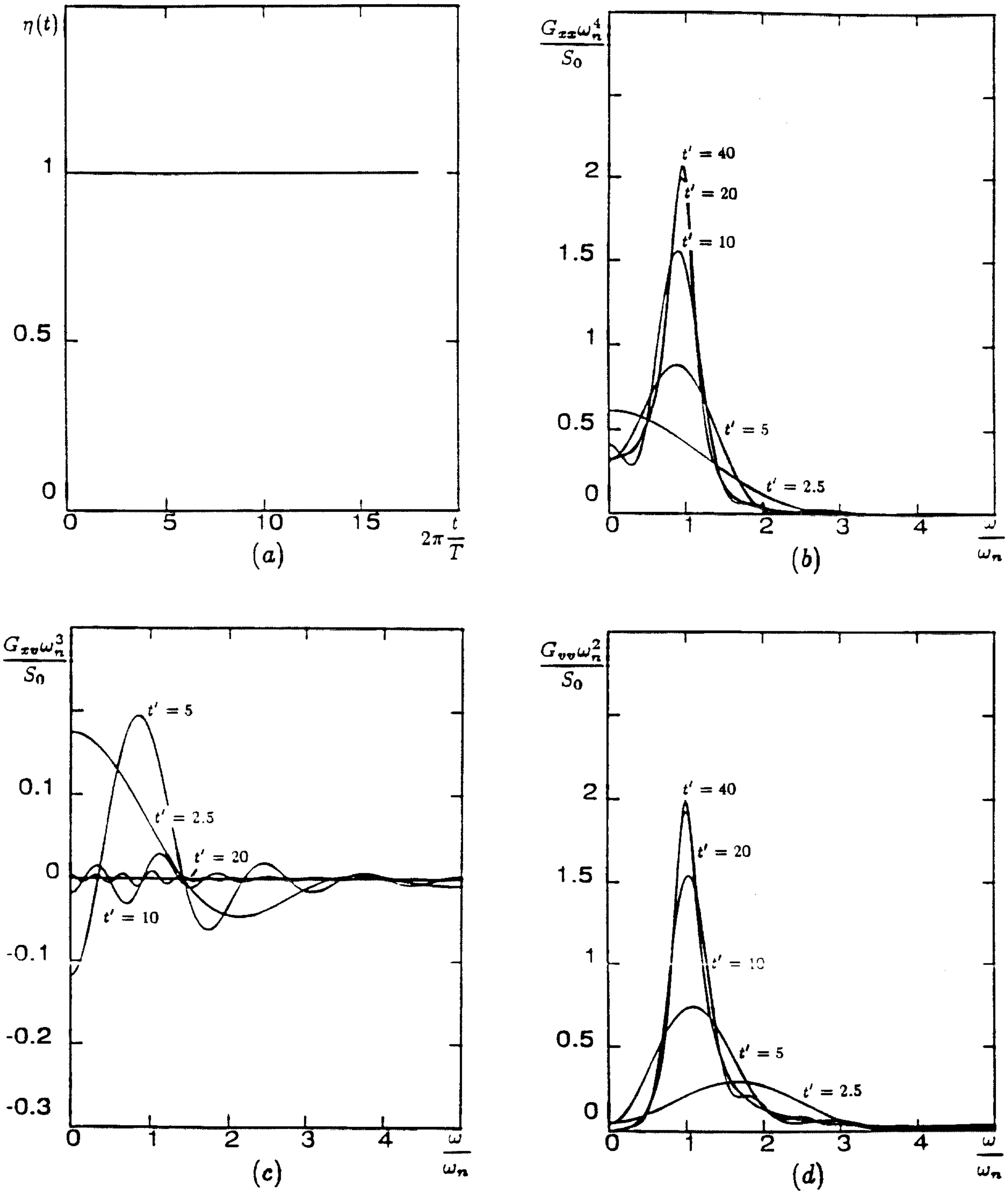


Figure 2.3. Evolutionary spectral densities of the response of linear simple systems with damping ratio $\varsigma = 0.2$ subjected to a modulated white noise. $t' = 2\pi\frac{t}{T}$. Case 1: Unit step envelope.

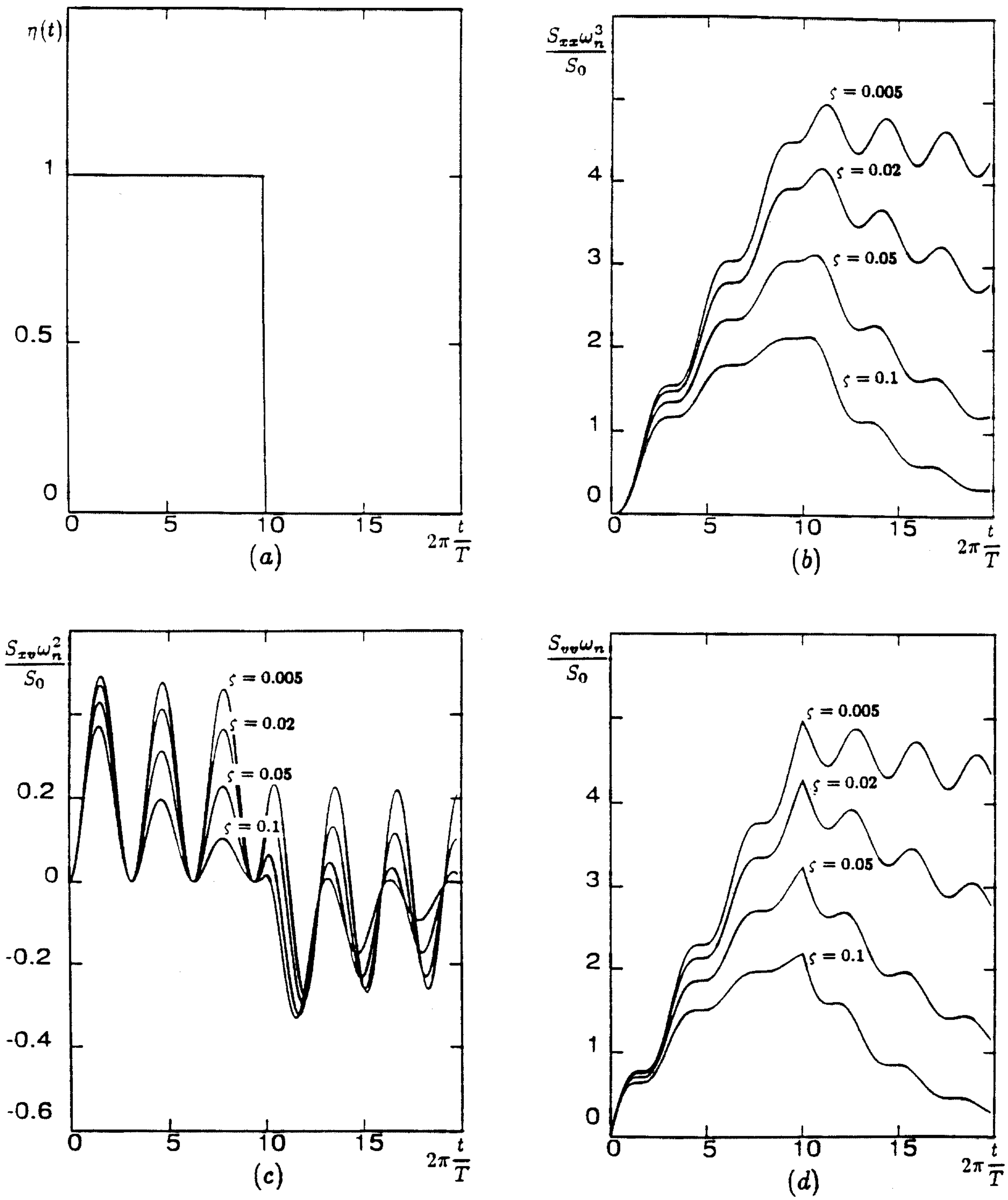


Figure 2.4. Nonstationary covariance of the response of linear simple systems with damping ratio $\varsigma = 0.005, 0.02, 0.05$, and 0.1 subjected to a modulated white noise. Case 2: Rectangular envelope with $T_d\omega_n = 10$.

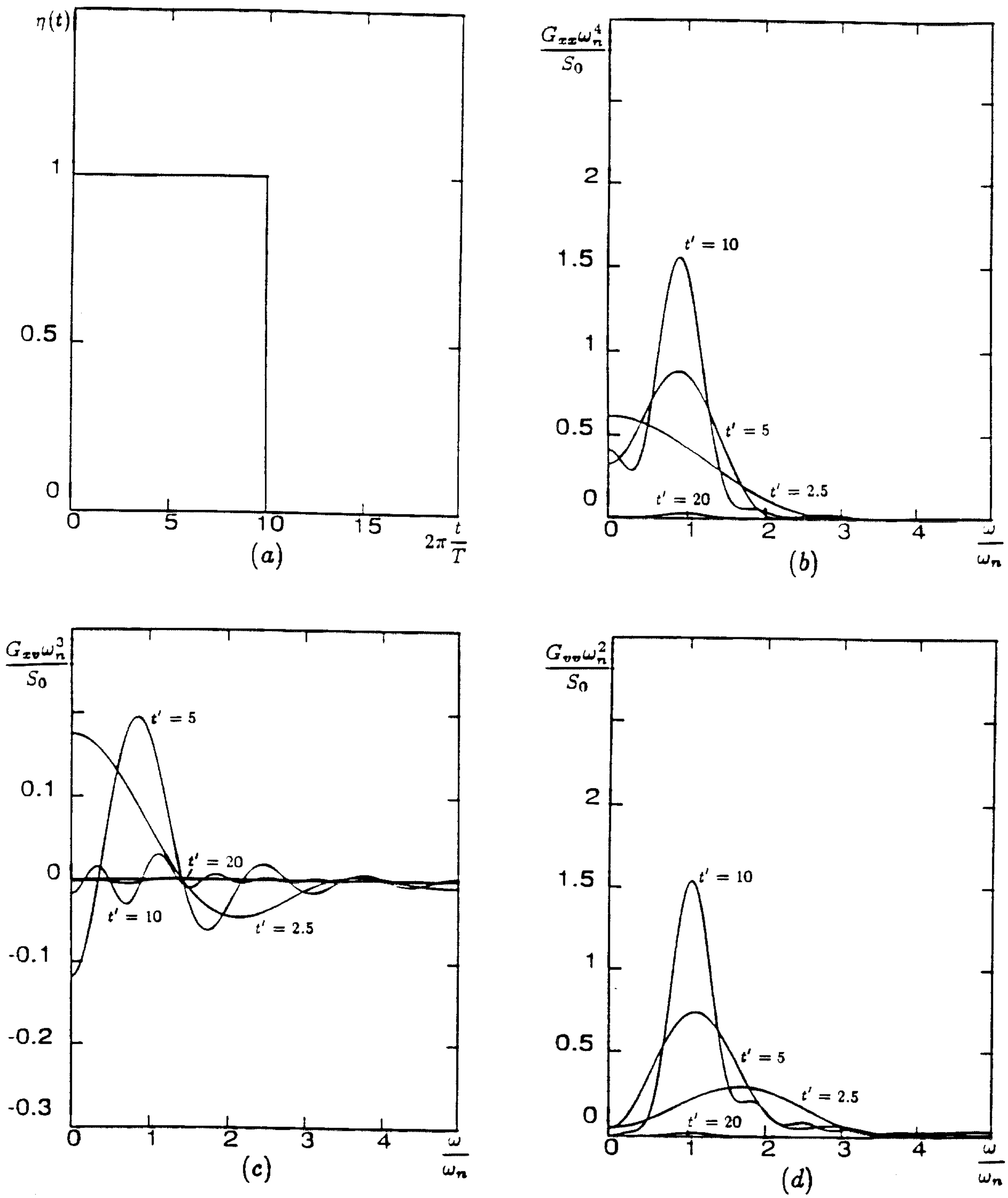


Figure 2.5. Evolutionary spectral densities of the response of linear simple systems with damping ratio $\varsigma = 0.2$ subjected to a modulated white noise. $t' = 2\pi\frac{t}{T}$. Case 2: Rectangular envelope with $T_d\omega_n = 10$.

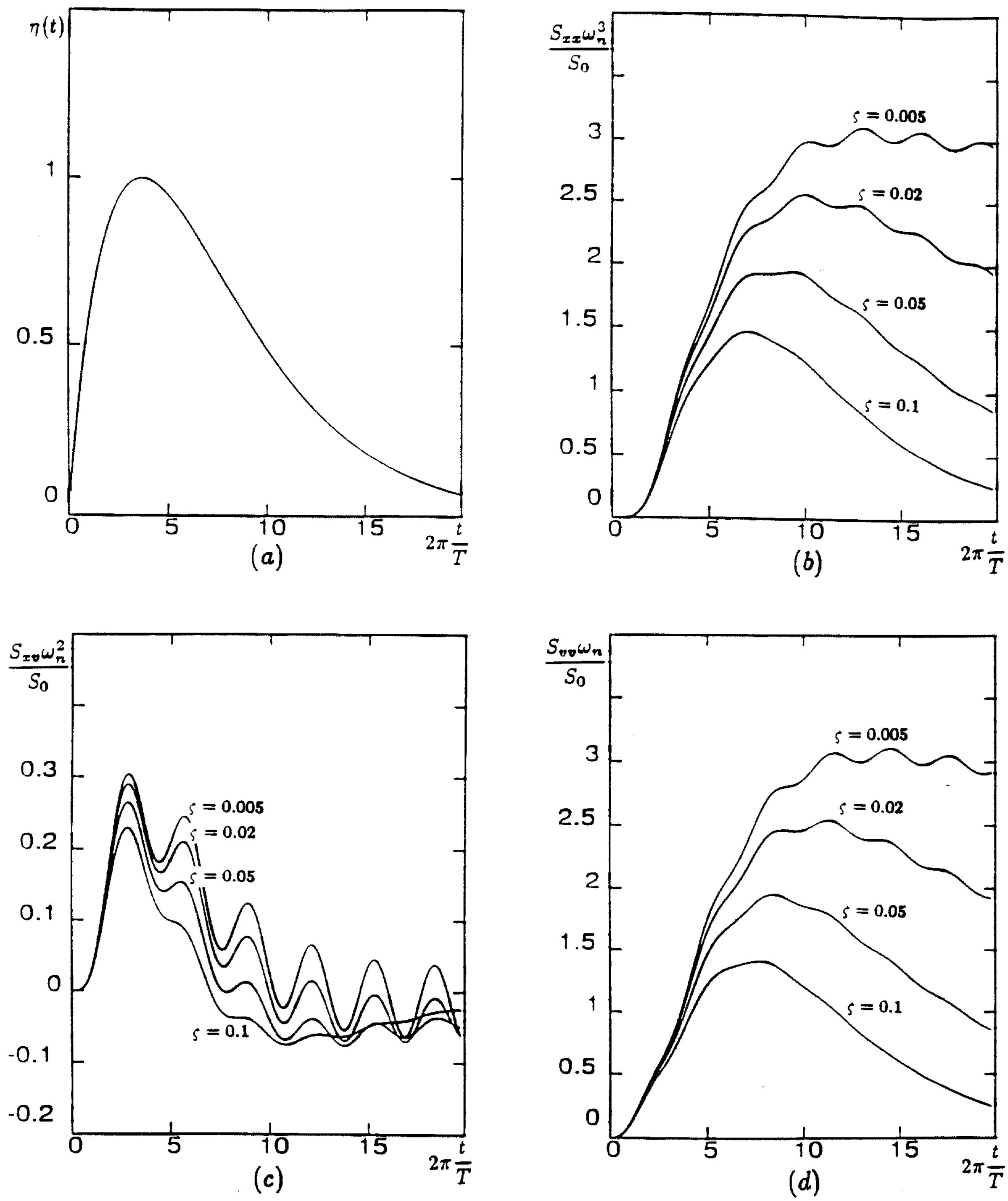


Figure 2.6. Nonstationary covariance of the response of linear simple systems with damping ratio $\varsigma = 0.005, 0.02, 0.05$, and 0.1 subjected to a modulated white noise. Case 3: Saragoni–Hart envelope with $A = 0.7398$ and $\frac{\alpha}{\omega_n} = 0.2718$.

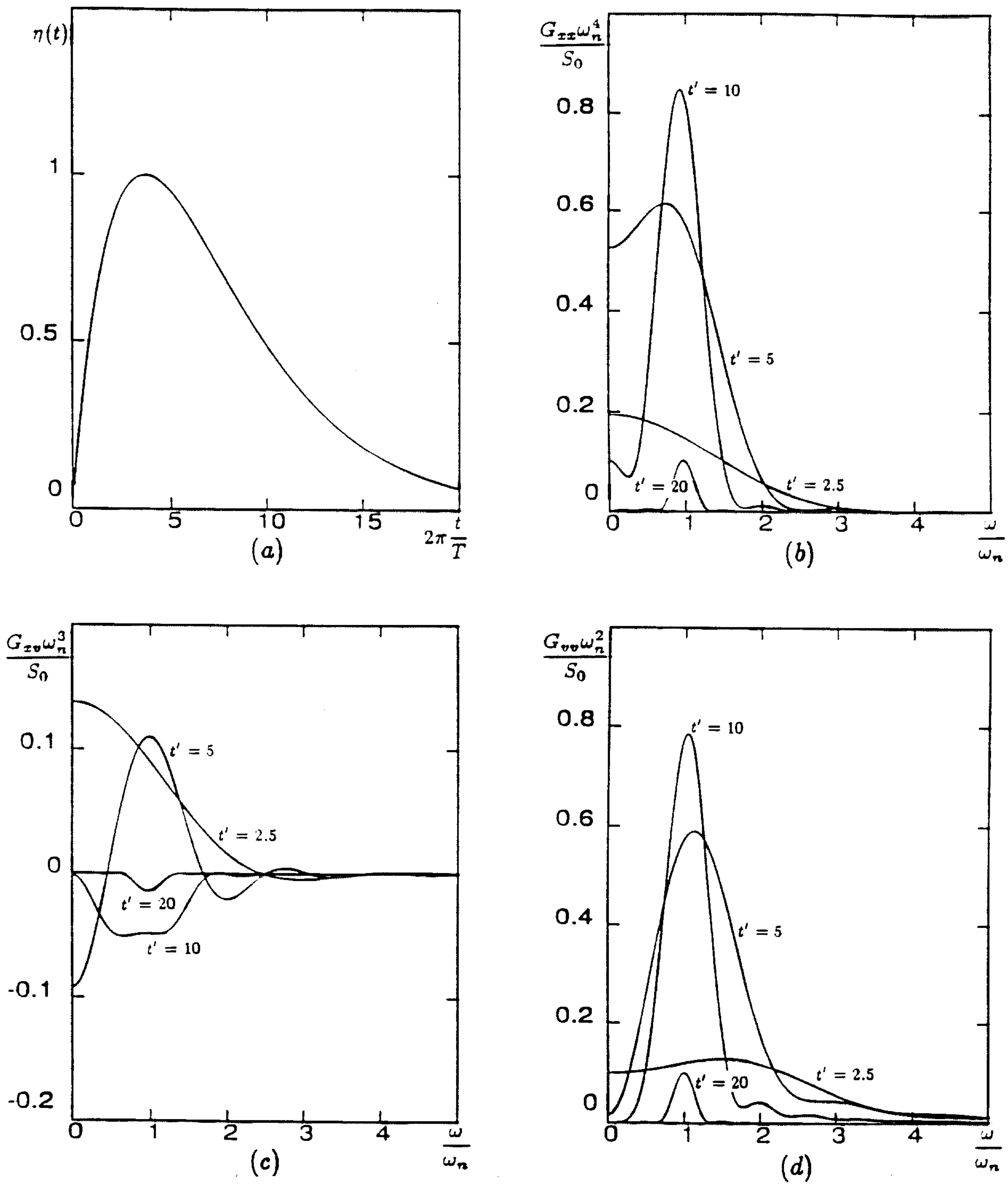


Figure 2.7. Evolutionary spectral densities of the response of linear simple systems with damping ratio $\varsigma = 0.2$ subjected to a modulated white noise. $t' = 2\pi\frac{t}{T}$. Case 3: Saragoni–Hart envelope with $A = 0.7398$ and $\frac{\alpha}{\omega_n} = 0.2718$.

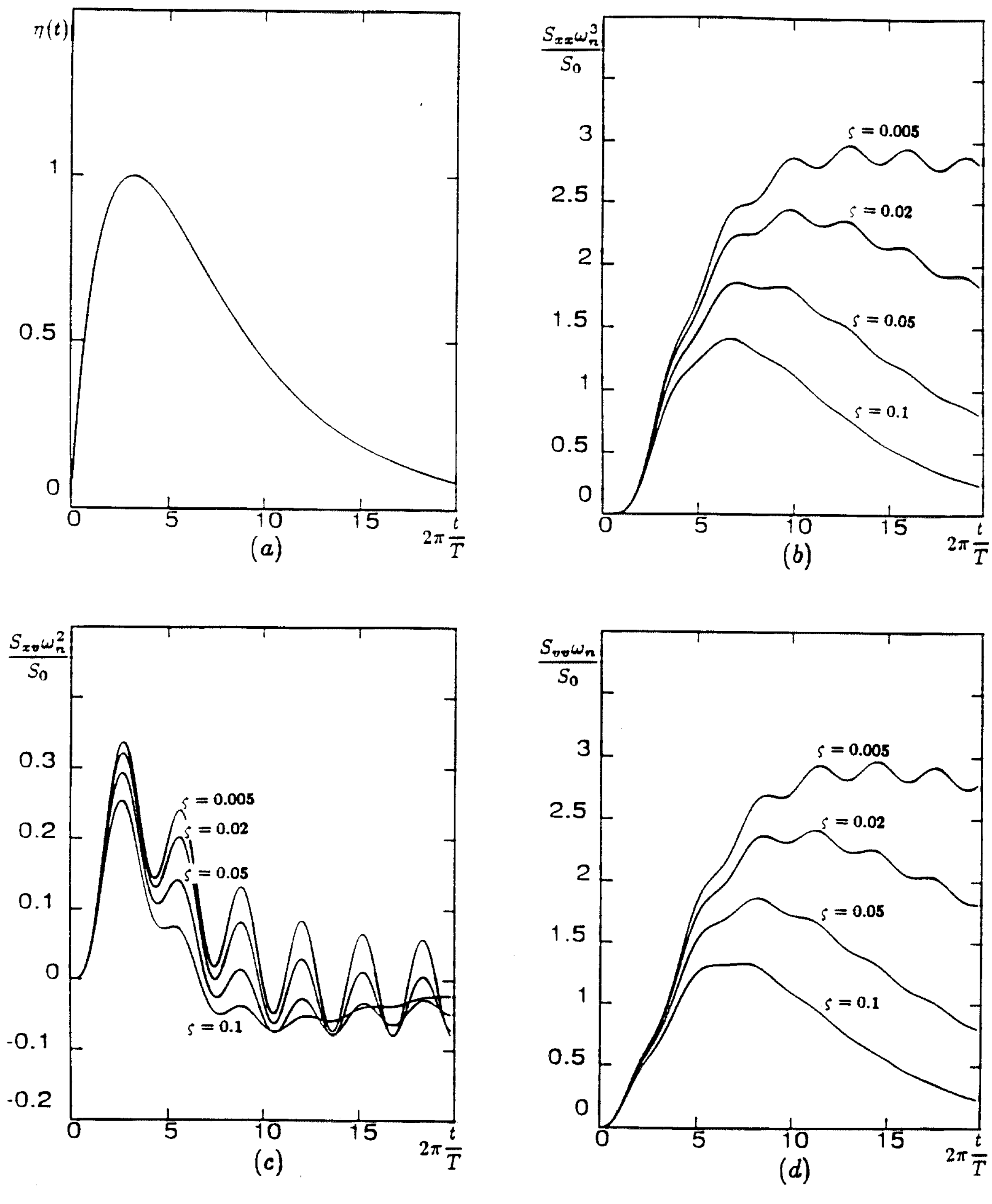


Figure 2.8. Nonstationary covariance of the response of linear simple systems with damping ratio $\varsigma = 0.005, 0.02, 0.05$, and 0.1 subjected to a modulated white noise. Case 4: Shinozuka–Sato envelope with $A = 2.598$, $\frac{\alpha}{\omega_n} = 0.1732$, and $\frac{\beta}{\omega_n} = 0.5196$.

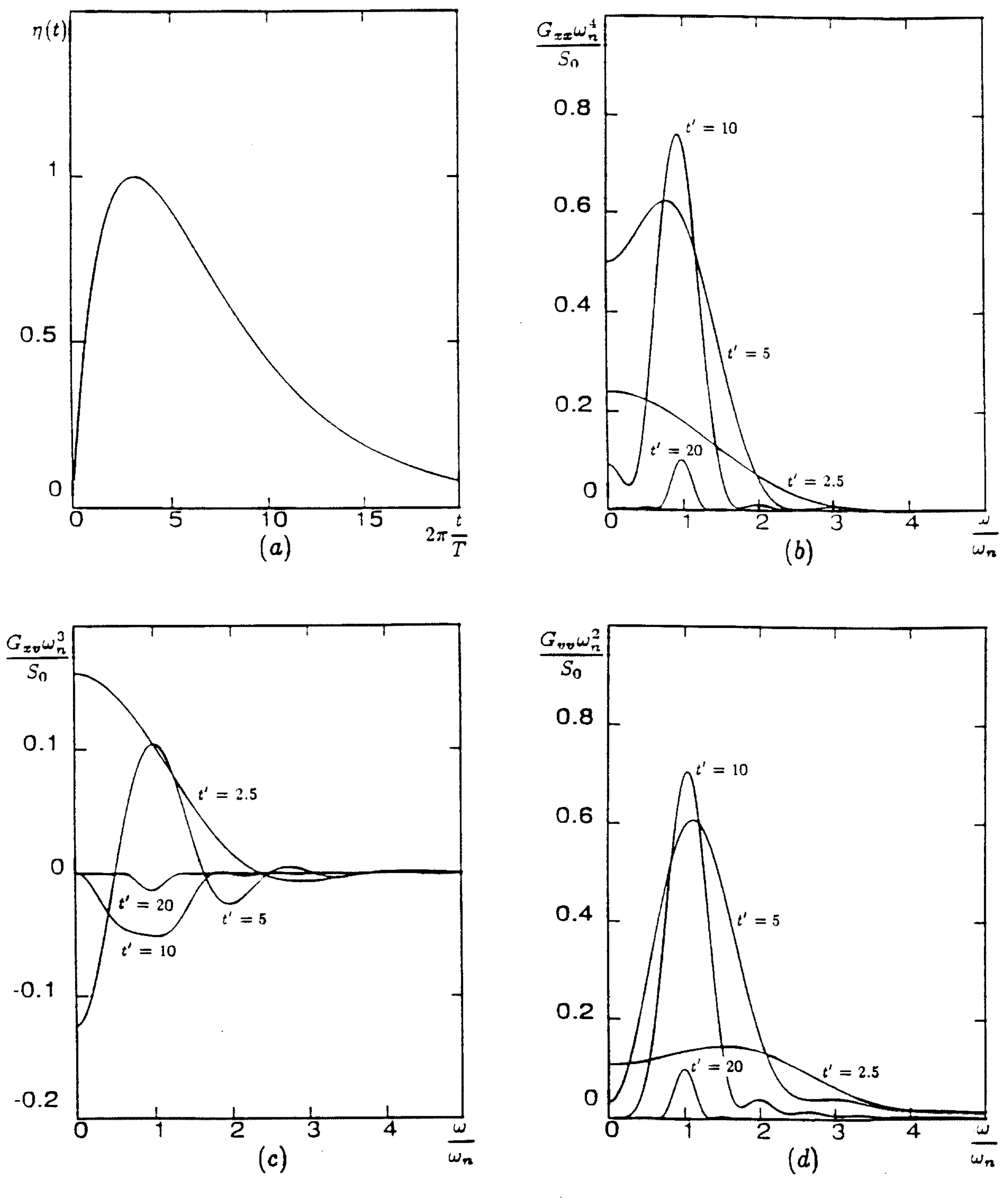


Figure 2.9. Evolutionary spectral densities of the response of linear simple systems with damping ratio $\varsigma = 0.2$ subjected to a modulated white noise. $t' = 2\pi\frac{t}{T}$. Case 4: Shinozuka–Sato envelope with $A = 2.598$, $\frac{\alpha}{\omega_n} = 0.1732$, and $\frac{\beta}{\omega_n} = 0.5196$.

Chapter 3

Nonstationary Response of Primary-Secondary Systems

3.1 Introduction

Over the last two decades, increasing attention has been given to the dynamic performance of secondary systems. This concern has been motivated by the design and analysis of equipment and piping networks in critically important structures, such as nuclear power plants, or refining facilities. A secondary system is characterized by having an effective mass which is small in comparison with its supporting structure, referred to as the primary system. As a consequence of the large difference in mass, certain computational difficulties may be found in a standard dynamic analysis and new features are also exhibited in the dynamic behaviour of the response of both the primary and secondary systems. Numerous investigations have been devoted to these two aspects and a detailed review may be found in Chen and Soong (1988).

A conventional method for the analysis and design of secondary systems is adopted by the ASME boiler and pressure vessel code (1981). This method neglects the interaction between the primary and secondary systems. Therefore, the primary system can be analyzed first in the absence of the secondary system and the response(s) at the supporting point(s) so obtained can then be used as the input to the secondary system. Considerable error would be generated by this method when interaction becomes important, which necessitates a complete dynamic analysis. However, the large difference in mass might result in ill-conditioned matrices in a complete analysis and computational instability might make the analysis fail. Many studies have been conducted to overcome the numerical difficulties, and various efficient approximate techniques have been developed to find an accurate solution for the dynamic response and the floor spectra. The concept of floor spectra is useful in the aseismic design of secondary systems.

Attention has also been directed towards studying the effects of some inherent dynamic characteristics such as tuning and detuning, attachment configuration, nonclassical damping, interaction, etc., on the behavior of combined primary-secondary systems. These effects change as the ratio of the mass of the secondary system to that of the primary system changes. The investigation gives certain guidelines to the dynamic analysis and design of secondary systems. For instance, some criteria have been developed regarding whether the interaction can be neglected without significantly changing the final results.

Though considerable progress has been made in the seismic analysis of primary-secondary systems, refered to as P-S systems hereafter, relatively few results are available for their stochastic response, especially for systems subjected to a nonstationary excitation. Chakravorty and Vanmarcke (1973) obtained the mean square relative displacement of an SDOF secondary system attatched to an SDOF primary system subjected to a suddenly applied white noise. Singh (1975) simplified the computation of the mean square response of an SDOF secondary system attached to an MDOF primary system. Igusa and Der Kiureghian (1983,1985) used the peturbation method to simplify expressions for the stationary response of an MDOF secondary system attached arbitrarily to an MDOF primary system. Iwan and Smith (1987) presented approximate expressions for the envelope response of an SDOF-SDOF P-S system subjected to a general nonstationary load where an assumption of narrow-band response and broad-band excitation is appropriate. Yang et al. (1988) studied the effect of nonclassical damping on the nonstationary mean square response of the secondary system.

In this chapter, as an application to MDOF systems, the simplified state-variable method proposed in Chapter 2 is employed to obtain exact solutions for the covariance matrix response, mean square energy envelope response and evolutionary spectral density of the response of P-S systems subjected to a nonstationary modulated white noise excitation. The concept of an evolutionary floor spectral density is introduced and this quantity is used to calculate the nonstationary response of the secondary system. The effect of interaction, tuning and detuning, and nonclassical damping are discussed. In addition, the effect of the nonstationarity of excitation on the response is also investigated. Finally, some remarks are made concerning the aseismic design and analysis of secondary systems.

3.2 Formulation

For the sake of simplicity, a simple combined primary-secondary system is discussed, as shown in Fig. 3.1. The secondary system is modeled as an SDOF system with mass m_s, stiffness k_s and damping c_s attached to an SDOF primary system of mass m_p, stiffness k_p, and damping c_p. The discussion may be extended to the case of MDOF secondary systems and/or MDOF primary systems without difficulty.

Let $x_s(t)$ denote the displacement of the secondary system relative to the primary system, $x_p(t)$ the displacement of the primary system relative to the ground, and $G(t)$ the input ground acceleration. The governing differential equations of the combined system are

$$\begin{aligned} m_s(\ddot{x}_s + \ddot{x}_p + G) + c_s\dot{x}_s + k_s x_s &= 0 \\ m_p(\ddot{x}_p + G) + c_p\dot{x}_p + k_p x_p - c_s\dot{x}_s - k_s x_s &= 0 \end{aligned} \qquad (3-1)$$

where the first equation is the equation of motion for the secondary system and the second is for the primary system. In addition, zero initial conditions are assumed hereafter. As a convention in this chapter, the subscripts p, s are used for the quantities associated with the primary system and the secondary system respectively.

Introduce the following parmeters

$$\begin{aligned} &\epsilon = \frac{m_s}{m_p} \\ &\omega_s^2 = \frac{k_s}{m_s} \qquad 2\zeta_s\omega_s = \frac{c_s}{m_s} \\ &\omega_p^2 = \frac{k_p}{m_p} \qquad 2\zeta_p\omega_p = \frac{c_p}{m_p} \end{aligned} \qquad (3-2)$$

Then, rewriting Eq. (3-1) in a matrix form yields

$$\mathbf{M}\ddot{\mathbf{Y}}(t) + \mathbf{C}\dot{\mathbf{Y}}(t) + \mathbf{K}\mathbf{Y}(t) = G(t)\mathbf{F}_0 \qquad (3-3)$$

where

$$\mathbf{Y}(t) = \begin{pmatrix} x_s(t) \\ x_p(t) \end{pmatrix} \qquad \mathbf{F}_0 = \begin{pmatrix} 0 \\ -1 \end{pmatrix}$$
$$\mathbf{M} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$
$$\mathbf{C} = \begin{pmatrix} (1+\epsilon)2\varsigma_s\omega_s & -2\varsigma_p\omega_p \\ -\epsilon 2\varsigma_s\omega_s & 2\varsigma_p\omega_p \end{pmatrix} \tag{3-4}$$
$$\mathbf{K} = \begin{pmatrix} (1+\epsilon)\omega_s^2 & -\omega_p^2 \\ -\epsilon\omega_s^2 & \omega_p^2 \end{pmatrix}$$

$G(t)$ is assumed be a modulated white noise expressed by

$$G(t) = \eta(t)n(t) \tag{3-5}$$

where the envelope $\eta(t)$ and the stationary white noise process $n(t)$ are defined as in (2-38) and (2-24).

Using the state-variable formulation, Eq. (3-3) becomes

$$\dot{\mathbf{Z}}(t) = \mathbf{A}\mathbf{Z}(t) + \mathbf{F} \tag{3-6}$$

where

$$\mathbf{Z}(t) = \begin{pmatrix} \mathbf{Y}(t) \\ \dot{\mathbf{Y}}(t) \end{pmatrix} \qquad \mathbf{F} = G(t)\begin{pmatrix} \mathbf{O} \\ \mathbf{F}_0 \end{pmatrix}$$
$$\mathbf{A} = \begin{pmatrix} \mathbf{O} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{pmatrix} \tag{3-7}$$

in which $\mathbf{I}$ is a 2 x 2 unit matrix. In the following discussion, Eqs. (3-3) and (3-6) will be alternatevely used whenever it is more convenient.

3.2.1 Covariance Response

Assuming the eigenvalues and eigenvectors of Eq. (3-6) are λ_k and $\mathbf{u}_k$, $k = 1, 2, 3, 4$ respectively, the general solution can be expressed as

$$\mathbf{Z}(t) = \mathbf{L}\int_0^t \mathbf{P}(t-\tau)G(\tau)d\tau \tag{3-8}$$

where $\mathbf{P}(t)$ is the same as defined in (2-14)-(2-15) and the system matrix $\mathbf{L}$ can be obtained in terms of the eigensolutions $\mathbf{u}_k, \lambda_k, k = 1, 2, 3, 4$, as described in Chapter 2.

The mean value of the response $\mathbf{Z}(t)$ remains zero at any moment since the mean of the initial condition is assumed be zero with probability 1. The nonstationary covariance matrix response $\mathbf{Q}(t)$ is given by

$$\mathbf{Q}(t) = S_0\mathbf{L}\int_0^t \eta^2(\tau)\mathbf{P}(t-\tau)\mathbf{P}^T(t-\tau)d\tau\mathbf{L}^T \tag{3-9}$$

where S_0 measures the intensity of the white noise. Without loss of generality, the covariance matrix response describing the correlation of the responses at these two different moments t_1 and t_2 $(t_1 < t_2)$ may be written as

$$\mathbf{Q}(t_1,t_2)) = \mathbf{Q}(t_1)\mathbf{\Phi}^T(t_1-t_2) \tag{3-10}$$

The eigensolutions $\mathbf{u}_k$ and $\lambda_k, k = 1,2,3,4$, can be obtained in terms of the solutions $\mathbf{v}_k$ and $\mu_k, k = 1,2,3,4$, of the following generalized eigenvalue problem corresponding to Eq. (3-3)

$$(\mu^2\mathbf{M} + \mu\mathbf{C} + \mathbf{K})\mathbf{v} = \mathbf{O} \tag{3-11}$$

by the relationship

$$\begin{aligned} \lambda_k &= \mu_k \\ \mathbf{u}_k &= \begin{pmatrix} \mathbf{v}_k \\ \mu_k\mathbf{v}_k \end{pmatrix} \qquad k = 1,2,3,4 \end{aligned} \tag{3-12}$$

Substituting Eq. (3-4) into (3-11) yields

$$\begin{vmatrix} \mu^2 + 2\varsigma_s\omega_s(1+\epsilon)\mu + (1+\epsilon)\omega_s^2 & -2\varsigma_p\omega_p\mu - \omega_p^2 \\ -\epsilon(2\varsigma_s\omega_s\mu + \omega_s^2) & \mu^2 + 2\varsigma_p\omega_p\mu + \omega_p^2 \end{vmatrix} = 0 \tag{3-13}$$

where $|*|$ denotes the determinent of corresponding matrix. Expanding Eq. (3-13) yields

$$\mu^4 + a_1\mu^3 + a_2\mu^2 + a_3\mu + a_4 = 0 \tag{3-14}$$

where the coefficients a_k are as follows.

$$\begin{aligned} a_1 &= 2\varsigma_p\omega_p + 2(1+\epsilon)\varsigma_s\omega_s \\ a_2 &= \omega_p^2 + (1+\epsilon)\omega_s^2 + 4\varsigma_s\varsigma_p\omega_s\omega_p \\ a_3 &= 2\varsigma_s\omega_s\omega_p^2 + 2\varsigma_p\omega_p\omega_s^2 \\ a_4 &= \omega_s^2\omega_p^2 \end{aligned} \tag{3-15}$$

Eq. (3-14) can generally be solved by solving two reduced quadratic equations, as shown later in this chapter. For some special cases, such as the case where there is no interaction and the case where classical damping is assumed, the solution of Eq. (3-14) can be easily obtained. For simplicity, distinct values of μ_k are assumed hereafter, but the approach can be extended to the degenerate case where multiple roots of μ exist.

After solving for μ_k, the eigenvectors $\mathbf{v}_k$ can be expressed as

$$\mathbf{v}_k = \begin{pmatrix} 1 \\ r_k \end{pmatrix} \tag{3-16}$$

where

$$r_k = \frac{\mu_k^2 + 2\varsigma_s\omega_s(1+\epsilon)\mu_k + (1+\epsilon)\omega_s^2}{2\varsigma_p\omega_p\mu_k + \omega_p^2} \tag{3-17}$$

The results for μ_k and $\mathbf{v}_k$ can be used to construct the system matrix $\mathbf{L}$.

3.2.2 Evolutionary Spectral Density

The evolutionary spectral density matrix of the response can be represented by

$$\mathbf{S}(t,\omega) = \frac{S_0}{2\pi}\mathbf{L}\mathbf{w}(t,\omega)\mathbf{w}^{*T}(t,\omega)\mathbf{L}^T \tag{3-18}$$

where $\mathbf{w}(t,\omega)$ is defined in Eq. (2-64). If one-sided spectral density $\mathbf{G}(t,\omega)$ is desired,

$$\mathbf{G}(t,\omega) = 2\mathbf{S}(t,\omega) \tag{3-19}$$

and the nonstationary covariance response can be found from

$$\mathbf{Q}(t) = \int_0^\infty \mathbf{G}(t,\omega)d\omega \tag{3-20}$$

3.2.3 Mean Square Energy Envelope Response

In some engineering applications, the statistics of the envelope response are more interesting. Different envelope processes have been defined and here an energy-based definition originating from Crandall (1963) is employed. According to this definition, the envelope process $A(t)$ of a random process $x(t)$ is given by the relationship

$$\frac{1}{2}A^2 = \frac{1}{2}\frac{\dot{x}^2}{\omega^2} + \frac{1}{2}x^2 \tag{3-21}$$

The right-hand side of Eq. (3-21) is the sum of the kinetic and potential energies per unit mass. Therefore, $A(t)$ would be the displacement if the total energy were converted entirely to potential energy. It can be shown that a closed form representation for the probability density function of the energy envelope process $A(t)$ can be obtained if the excitation is assumed to be Gaussian.

Recall the definition of (3-21) and assume the joint probability density function of the response to be of the form

$$p(x,\dot{x}) = \frac{1}{2\pi\sigma_x\sigma_{\dot{x}}\sqrt{1-\rho^2}} e^{-\frac{1}{2(1-\rho^2)}\left(\frac{x^2}{\sigma_x^2} - 2\rho\frac{x}{\sigma_x}\frac{\dot{x}}{\sigma_{\dot{x}}} + \frac{\dot{x}^2}{\sigma_{\dot{x}}^2}\right)} \tag{3-22}$$

where σ_x and $\sigma_{\dot{x}}$ are the standard deviations of the displacement and velocity respectively, and ρ is the correlation coefficient. Then, the probability function of the energy envelope can be expressed by

$$\begin{aligned} F(a) &= Prob(|A| \le a) \\ &= Prob\left(\frac{\dot{x}^2}{\omega^2} + x^2 \le a^2\right) \end{aligned} \tag{3-23}$$

and the probability density can be written as

$$\begin{aligned} p(a) &= \frac{dF(a)}{da} \\ &= \frac{a}{\sigma_x^2} e^{-\frac{a^2}{2\sigma_x^2}} S(a) \end{aligned} \tag{3-24}$$

Introduce the variable, α, defined as

$$\alpha = \frac{\omega\sigma_x}{\sigma_{\dot{x}}} \tag{3-25}$$

then, $S(a)$ in Eq. (3-24) can be expressed as

$$\begin{aligned} S(a) &= \frac{\alpha}{\sqrt{1-\rho^2}} e^{-\frac{\rho^2}{2(1-\rho^2)}\frac{a^2}{\sigma_x^2}} \\ &\frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} e^{\frac{a^2}{2(1-\rho^2)\sigma_x^2}\left[(1-\alpha^2)\cos^2\theta + \rho\alpha\sin 2\theta\right]} d\theta \end{aligned} \tag{3-26}$$

This can be evaluated based on the covariance matrix obtained. Numerical integration is generally needed in the nonstationary case. However, for the stationary case, $\rho = 0$ and $\omega\sigma_x = \sigma_{\dot{x}}$, which results in $S(a) = 1$. Therefore, the form of (3-24) is reduced to the well-known Rayleigh distribution

$$p(a) = \frac{a}{\sigma_x^2} e^{-\frac{a^2}{2\sigma_x^2}} \tag{3-27}$$

Note that the results apply without any bandwidth limitation.

Parallel to the definition (3-21), the energy envelope of the secondary system, $E_s(t)$, and the energy envelope of the primary system, $E_p(t)$, can be defined implicitly by

$$\begin{aligned} E_s^2 &= x_s^2 + \frac{\dot{x}_s^2}{\omega_s^2} \\ E_p^2 &= x_p^2 + \frac{\dot{x}_p^2}{\omega_p^2} \end{aligned} \tag{3-28}$$

Some advantages follow immediately. First, the envelopes defined in Eq. (3-28) can be physically interpreted as quantities whose squares are proportional to the total energy per unit mass of certain structures which may be thought of as the original primary and secondary structures but with fixed bases. Secondly, there is no bandwidth restriction imposed and they can be reduced identically to the envelopes used by Iwan and Smith (1987) for a narrow-banded response and broad-banded excitation. Furthermore, explicit solutions for the second moments of the envelope responses can be obtained in terms of the elements of the covariance matrix $\mathbf{Q}(t)$ without any difficulty. That is

$$\begin{aligned} E[E_s^2(t)] &= E[x_s^2(t)] + \frac{E[\dot{x}_s^2(t)]}{\omega_s^2} \\ E[E_p^2(t)] &= E[x_p^2(t)] + \frac{E[\dot{x}_p^2(t)]}{\omega_p^2} \end{aligned} \tag{3-29}$$

Finally, as mentioned previously, a closed form representation of the nonstationary probability density function of the energy envelope response is available if an additional assumption of Gaussian excitation is introduced and the form can be reduced to a Rayleigh distribution for the stationary case.

3.3 Dynamic Characteristics of P-S Systems Subjected to Nonstationary Excitations

Extensive studies have been conducted on the dynamic behavior of P-S systems. Some general conclusions have been drawn regarding several inherent dynamic characteristics, such as tuning, nonclassical damping, interaction effect, etc. As mentioned before, relatively few results have been obtained for the stochastic response and most of these are for stationary response. Bearing in mind the uncertainty and the nonstationarity of earthquake loads, it is necessary to discuss the dynamic behavior of P-S systems subjected to nonstationary excitation.

The nonstationarity of earthquake ground motion may be introduced by stationary models modulated by a deterministic time-varying envelope function which accounts for the characteristic build-up, stationary phase, and tail of earthquake generated ground motion. The term "envelope function" or "load envelope" will sometimes be used in order to distinquish the deterministic modulating envelope from the energy envelope process previously defined. The results are mainly presented for two types of load envelope: a boxcar envelope and the Shinozuka-Sato type envelope defined in Chapter 2. In some cases, a step envelope function is also used.

Three cases are examined to study the effect of interaction, tuning, and nonclassical damping on the mean square value of the response and the mean square value of the energy envelope. As discussed in section 3.2.1, the difference in analysis lies in the evaluation of the eigenvalues and eigenvectors $\mathbf{v}_k$ and $\mu_k, k = 1,2,3,4$ of the system (3-3). As long as the solution for $\mathbf{v}_k$ and μ_k is found, the system matrix $\mathbf{L}$ can be constructed by using Eqs. (2-12), (2-14) and (2-15), and explicit solutions can then be obtained.

3.3.1 Nonstationary Response of P-S Systems When Interaction Is Neglected

It is common practice to neglect the interaction between the primary system and the secondary system (ASME Boiler and Pressure Vessel Code, 1981) which allows the two systems to be analysed separately. The response of the primary system at supporting point(s) can be obtained first by neglecting the existence of the secondary system and, then, used as the input to the secondary system. The approach greatly facilitates the design process, but previous studies have shown that this approach is valid only for systems with a large difference in mass.

For the noninteraction case, Eq. (3-3) may be rewritten as

$$\begin{aligned} \ddot{x}_p(t) + 2\varsigma_p\omega_p\dot{x}_p(t) + \omega_p^2 x_p(t) &= -a(t) \\ \ddot{x}_s(t) + 2\varsigma_s\omega_s\dot{x}_s(t) + \omega_s^2 x_s(t) &= 2\varsigma_p\omega_p\dot{x}_p(t) + \omega_p^2 x_p(t) \end{aligned} \qquad (3-30)$$

The first equation above governs the dynamic behavior of the primary system and its solution can be expressed as

$$\begin{aligned} \mathbf{Z}_p(t) &= -\mathbf{L}_p\int_0^t \mathbf{P}_p(t-\tau)a(\tau)d\tau \\ \mathbf{Q}_p(t) &= S_0\mathbf{L}_p\int_0^t \eta^2(\tau)\mathbf{P}_p(t-\tau)\mathbf{P}_p^T(t-\tau)d\tau\mathbf{L}_p^T \\ \mathbf{G}_p(t,\omega) &= \frac{S_0}{\pi}\mathbf{L}_p\mathbf{w}_p(t,\omega)\mathbf{w}_p^{*T}(t,\omega)\mathbf{L}_p^T \end{aligned} \qquad (3-31)$$

where

$$\mathbf{Z}_p(t) = \begin{pmatrix} x_p(t) \\ \dot{x}_p(t) \end{pmatrix} \qquad \mathbf{P}_p(t) = e^{-\varsigma_p\omega_p t}\begin{pmatrix} \cos\omega_{pd}t \\ \sin\omega_{pd}t \end{pmatrix}$$
$$\mathbf{L}_p = \begin{pmatrix} 0 & \dfrac{1}{\omega_{pd}} \\ 1 & -\dfrac{\varsigma_p\omega_p}{\omega_{pd}} \end{pmatrix} \qquad (3-32)$$

in which the damped natural frequency ω_{pd} is defined by

$$\omega_{pd} = \omega_p\sqrt{1-\varsigma_p^2} \qquad (3-33)$$

The second equation in (3-30) gives the response of the secondary system subjected to a base excitation caused by the motion of the primary system. Define

$$\begin{aligned} \bar{f}(t) &= 2\varsigma_p\omega_p\dot{x}_p(t) + \omega_p^2 x_p(t) \\ &= \mathbf{b}\mathbf{Z}_p(t) \end{aligned} \qquad (3-34)$$

where $\mathbf{b}$ is defined by

$$\mathbf{b} = (2\varsigma_p\omega_p, \quad \omega_p^2) \qquad (3-35)$$

The response of the secondary system can be expressed in terms of the response of the primary system as

$$\begin{aligned}
\mathbf{Z}_s(t) &= \mathbf{L}_s \int_0^t \mathbf{P}_s(t-\tau)\mathbf{b}\mathbf{Z}_p(\tau)d\tau \\
\mathbf{Q}_s(t) &= \mathbf{L}_s \int_0^t \mathbf{P}_s(t-\tau)\mathbf{b}\mathbf{Q}_p(\tau)\mathbf{b}^T\mathbf{P}_s^T(t-\tau)d\tau\mathbf{L}_s^T \qquad (3-36)\\
\mathbf{G}_s(t,\omega) &= \mathbf{L}_s \int_0^t \int_0^t \mathbf{P}_s(t-\tau_1)\mathbf{b}\mathbf{G}_p(\tau_1,\tau_2,\omega)\mathbf{b}^T\mathbf{P}_s^T(t-\tau_2)d\tau_1 d\tau_2\mathbf{L}_s^T
\end{aligned}$$

where

$$\begin{aligned}
\mathbf{Z}_s(t) &= \begin{pmatrix} x_s(t) \\ \dot{x}_s(t) \end{pmatrix} \qquad \mathbf{P}_s(t) = e^{-\varsigma_s\omega_s t}\begin{pmatrix} \cos\omega_{sd}t \\ \sin\omega_{sd}t \end{pmatrix} \\
\mathbf{L}_s &= \begin{pmatrix} 0 & \dfrac{1}{\omega_{sd}} \\ 1 & -\dfrac{\varsigma_s\omega_s}{\omega_{sd}} \end{pmatrix} \qquad (3-37)\\
\mathbf{G}_p(t_1,t_2,\omega) &= \frac{S_0}{\pi}\mathbf{L}_p\mathbf{w}_p(t_1,\omega)\mathbf{w}_p^{*T}(t_2,\omega)\mathbf{L}_p^T
\end{aligned}$$

in which

$$\omega_{sd} = \omega_s\sqrt{1-\varsigma_s^2} \qquad (3-38)$$

$\mathbf{G}_p(t_1,t_2,\omega)$ above may be referred to as *the evolutionary floor spectral density* which specifies the input to the secondary system from the primary system. The evolutionary floor spectral density completely determines the secondary system response using Eq. (3-36).

Although the above approach is physically intuitive, an alternative approach is employed herein in the calculation of the secondary system response due to its mathematical convenience. Consider the combined P-S system and let $\epsilon = 0$ in Eq. (3-13). Then,

$$\mu^4 + (2\varsigma_p\omega_p + 2\varsigma_s\omega_s)\mu^3 + (\omega_p^2 + \omega_s^2 + 4\varsigma_s\varsigma_p\omega_s\omega_p)\mu^2 + (2\varsigma_s\omega_s\omega_p^2 + 2\varsigma_p\omega_p\omega_s^2)\mu + \omega_s^2\omega_p^2 = 0 \qquad (3-39)$$

The solutions for μ can be expressed as

$$\begin{aligned}
\mu_{1,2} &= -\varsigma_s\omega_s \pm i\omega_{ds} \\
\mu_{3,4} &= -\varsigma_p\omega_p \pm i\omega_{dp}
\end{aligned} \qquad (3-40)$$

The corresponding eigenvectors can be obtained from Eq. (3-16). Thus, the results can be used to construct the system matrix $\mathbf{L}$ and the solution can be expressed as in Eqs. (3-9)-(3-10), (3-18) and (3-29). These two methods give, as expected, identical solutions.

Some numerical results are shown in Figs. 3.2 - 3.5. Fig. 3.2 gives a comparison of the mean square values of the energy envelope and the envelope used by Iwan and Smith (1987) for a P-S system subjected to a modulated white noise excitation with the boxcar load envelope of duration $T_d = 10T_p$ where T_p is the natural period of the primary system. It is observed that two curves nearly overlap for small damping ratios $\zeta_s = \zeta_p = 0.05$, but differ considerably for large damping ratio $\zeta_s = \zeta_p = 0.5$. The results show that an underestimate of the envelope response may be obtained if using the Iwan-Smith envelope for broad-banded excitation and response.

Fig. 3.3 presents the results for the transient mean square response and the mean square value of the energy envelope of the secondary system when the uncoupled P-S system is subjected to a modulated white noise with the unit step function. Three different critical damping ratios $\zeta_p = \zeta_s = 0.05$, 0.1, and 0.2 are used in the calculation. As time increases all the solutions tend to converge to their stationary values and the larger the damping ratio, the faster the convergence. While the mean square values of the displacement and velocity exhibit a strongly oscillatory behaviour, the mean square value of the energy envelope varies nearly monotonically with the time. This property of the energy envelope response may lend itself to applications of the method of slowly varying parameters.

A comparison of the response of the secondary system and the primary system subjected to a white noise excitation modulated by the boxcar envelope function with the duration of $T_d = 10T_p$ is shown in Fig. 3.4. A *delay effect* is observed in the secondary system response. The mean square value of the envelope response of the secondary system reaches its maximum after its counterpart of primary system response. When the ground motion ceases, the primary system response immediately decreases to zero, while the secondary system response continues to increase for a while, and then begins decreasing. The delay effect is due to the fact that the secondary system is actually excited by the oscillation of the primary system which has longer significant duration than the ground motion. Smaller damping ratios result in higher peak response and longer delay. This phenomenon implies that a longer duration may need to be considered in the aseismic design and analysis of secondary systems as compared with primary systems.

The evolutionary frequency content of the responses is illustrated in Fig. 3.5. Fig. 3.5a and Fig. 3.5c show the evolutionary floor spectral density when $t_1 = t_2 = t$ and Fig. 3.5b and Fig. 3.5d give the evolutionary spectral density for the secondary system response which is determined by Eq. (3-36). Similar to the results for SDOF systems presented in Chapter 2, for small t, relatively flat curves are observed, which implies a broad-bandness of the response at the initial stage, and for large t, these curves gradually converge to their corresponding frequency response functions which are sharply peaked if the system has small damping ratios. As anticipated, for $\frac{\omega_s}{\omega_p}$ close to 1 the evolutionary spectral density function of the secondary system has only one peak, as shown in Fig. 3.5b, but for values of $\frac{\omega_s}{\omega_p}$ far away from 1 two peaks may be observed as in Fig. 3.5d.

3.3.2 Nonstationary Response of P-S Systems with Classical Damping When Interaction Is Taken into Account

It is noted that if the mass difference becomes small, neglecting the interaction between the primary and secondary systems would cause a significant error. Therefore, the interaction effect has to be taken into account in this case.

Consider the undamped system associated with Eq. (3-3). The natural frequencies of the combined P-S system can be obtained from

$$(\mathbf{K} - \omega^2\mathbf{M})\mathbf{u} = \mathbf{O} \qquad (3-41)$$

or,

$$\begin{vmatrix} (1+\epsilon)\omega_s^2 - \omega^2 & -\omega_p^2 \\ -\epsilon\omega_s^2 & \omega_p^2 - \omega^2 \end{vmatrix} = \mathbf{O} \qquad (3-42)$$

The results can be written as

$$\begin{aligned} \omega_1^2 &= \gamma_s^2\omega_s^2 \\ \omega_2^2 &= \gamma_p^2\omega_p^2 \end{aligned} \qquad (3-43)$$

where the parameters γ_s^2, γ_p^2 are defined as

$$\gamma_p^2 = \frac{1}{2}\left((1+\epsilon)(\frac{\omega_s}{\omega_p})^2 + 1 + sign(\omega_p^2 - \omega_s^2)\sqrt{[1-(1+\epsilon)(\frac{\omega_s}{\omega_p})^2]^2 + 4\epsilon(\frac{\omega_s}{\omega_p})^2}\right)$$
$$\gamma_s^2 = \frac{1}{2}\left((1+\epsilon) + (\frac{\omega_p}{\omega_s})^2 + sign(\omega_s^2 - \omega_p^2)\sqrt{[(\frac{\omega_p}{\omega_s})^2 - (1+\epsilon)]^2 + 4\epsilon}\right) \tag{3-44}$$

Note that

$$\gamma_s^2 \gamma_p^2 = 1 \tag{3-45}$$

Eq. (3-43) indicates that the parameters γ_s and γ_p, assumed be positive without loss of generality, describe changes in the natural frequencies of a coupled P-S system before and after the interaction is taken into account. Thus, either of the two parameters, referred to as interaction parameters, can be used to measure the importance of the interaction between the primary and secondary systems. Note that γ_p^2 and γ_s^2 are related to the parameter θ used by Iwan and Smith (1987) by the relationship

$$\gamma_p^2 = 1 + \theta$$
$$\gamma_s^2 = \frac{1}{1+\theta} \tag{3-46}$$

Fig. 3.6 shows variations in the interaction parameter γ_p with respect to mass ratio ϵ and the frequency rario $\frac{\omega_s}{\omega_p}$ respectively. For a fixed mass ratio the maximum of γ_p is achieved when $\frac{\omega_s}{\omega_p} = 1$ implying that the primary and secondary systems are tuned. For a fixed frequency ratio, the interaction parameter γ_p increases with increased mass ratio when $\frac{\omega_s}{\omega_p} \leq 1$ and decreases when $\frac{\omega_s}{\omega_p} > 1$. Note that γ_p is discontinuous at $\frac{\omega_s}{\omega_p} = 1$ for fixed mass ratio. It may be shown that the natural frequencies of the combined P-S system are always more widely spaced than those of the uncoupled P-S system, which implies that coupled P-S systems exhibit a tendency to detune.

The modal matrix of Eq. (3-41) can be found as

$$\mathbf{U} = \begin{pmatrix} 1 & 1 \\ \gamma_p^2 - 1 & \gamma_s^2(\frac{\omega_s}{\omega_p})^2 - 1 \end{pmatrix} \tag{3-47}$$

It can be verified that

$$\mathbf{U}^{-1}\mathbf{M}\mathbf{U} = \mathbf{I}$$

$$\mathbf{U}^{-1}\mathbf{K}\mathbf{U} = \begin{pmatrix} \gamma_s\omega_s^2 & 0 \\ 0 & \gamma_p^2\omega_p^2 \end{pmatrix}$$

$$\mathbf{U}\mathbf{C}\mathbf{U}^{-1} = \begin{pmatrix} 2\zeta_s\omega_s\gamma_s^2\left(1+(1-\frac{\zeta_p\omega_s}{\zeta_s\omega_p})\frac{\gamma_p^2-1}{\Delta}\right) & 2\zeta_s\omega_s\gamma_s^2\frac{\gamma_s^2(\frac{\omega_s}{\omega_p})^2-1}{\Delta}(1-\frac{\zeta_p\omega_s}{\zeta_s\omega_p}) \\ 2\zeta_p\omega_p\gamma_p^2\frac{1-\gamma_p^2}{\Delta}(\frac{\zeta_s\omega_p}{\zeta_p\omega_s}-1) & 2\zeta_p\omega_p\gamma_p^2\left(1+(1-\frac{\zeta_s\omega_p}{\zeta_p\omega_s})\frac{\gamma_p^2-1}{\Delta}\right) \end{pmatrix} \tag{3-48}$$

where

$$\Delta = \gamma_s^2(\frac{\omega_s}{\omega_p})^2 - \gamma_p^2 \tag{3-49}$$

It follows that the necessary and sufficient condition for classical damping is that the ratio of natural frequencies should be equal to the ratio of the fractions of critical damping, namely,

$$\frac{\zeta_s}{\omega_s} = \frac{\zeta_p}{\omega_p} \tag{3-50}$$

The quantity

$$\beta = \frac{\zeta_s}{\omega_s} - \frac{\zeta_p}{\omega_p} \tag{3-51}$$

is sometimes refered to as the *nonclassical damping parameter.*

In this section, systems with classical damping, i.e., $\beta = 0$, are studied. Substituting Eq. (3-50) into Eq.(3-48) yields

$$\mathbf{U}^{-1}\mathbf{C}\mathbf{U} = \begin{pmatrix} 2\zeta_s\omega_s\gamma_s^2 & 0 \\ 0 & 2\zeta_p\omega_p\gamma_p^2 \end{pmatrix} \tag{3-52}$$

Define

$$\begin{aligned} \bar{\omega}_s &= \gamma_s\omega_s \qquad & \bar{\zeta}_s &= \gamma_s\zeta_s \\ \bar{\omega}_p &= \gamma_p\omega_p \qquad & \bar{\zeta}_p &= \gamma_p\zeta_p \end{aligned} \tag{3-53}$$

Then, the eigenvalues μ_k can be expressed as

$$\begin{aligned} \mu_{1,2} &= -\bar{\zeta}_s\bar{\omega}_s \pm i\bar{\omega}_{ds} \\ \mu_{3,4} &= -\bar{\zeta}_p\bar{\omega}_p \pm i\bar{\omega}_{dp} \end{aligned} \tag{3-54}$$

where

$$\begin{aligned} \bar{\omega}_{ds} &= \bar{\omega}_s\sqrt{1-\bar{\zeta}_s^2} \\ \bar{\omega}_{dp} &= \bar{\omega}_p\sqrt{1-\bar{\zeta}_p^2} \end{aligned} \tag{3-55}$$

The corresponding eigenvectors $\mathbf{v}_k$ are

$$\mathbf{v}_{1,2} = \begin{pmatrix} 1 \\ \gamma_p^2 - 1 \end{pmatrix}$$
$$\mathbf{v}_{3,4} = \begin{pmatrix} 1 \\ \gamma_s^2 (\frac{\omega_s}{\omega_p})^2 - 1 \end{pmatrix} \qquad (3-56)$$

The system matrix $\mathbf{L}$ can then be obtained in terms of μ_k and $\mathbf{v}_k$.

As indicated by Iwan and Smith (1987), the coupled P-S system with classical damping can be replaced by an equivalent uncoupled P-S system by introducing the transformation

$$x_p = \bar{x}_p + (1 - \gamma_s^2)\bar{x}_s$$
$$x_s = \gamma_s^2 \bar{x}_s \qquad (3-57)$$

Recalling the definitions in Eq. (3-53), the equivalent uncoupled P-S system can be written as

$$\ddot{\bar{x}}_s + 2\bar{\zeta}_s \bar{\omega}_s \dot{\bar{x}}_s + \bar{\omega}_s^2 \bar{x}_s = -\ddot{G}(t) - \ddot{\bar{x}}_p$$
$$\ddot{\bar{x}}_p + 2\bar{\zeta}_p \bar{\omega}_p \dot{\bar{x}}_p + \bar{\omega}_p^2 \bar{x}_p = -\ddot{G}(t) \qquad (3-58)$$

Therefore, all the methods and results for the uncoupled P-S system will apply.

For each given frequency ratio, the nonstationary mean square value of the energy response of the coupled P-S system is calculated for four different mass ratios $\epsilon = 0.0, 0.01, 0.05$, and 0.1. The numerical results are presented in Figs. 3.7 - 3.9. Note that zero mass ratio corresponds to the case where interaction is neglected. The comparison shows that neglecting the interaction between the primary and the secondary systems always gives a conservative result. Larger mass ratios seem to reduce the secondary system response. The delay effect is again observed in the coupled P-S system. While the primary system response decreases immediately after the base excitation ceases, the secondary system keeps increasing for some time and then begins to decrease relatively slowly. For large mass ratios, an additional minor peak may be observed in the primary system response which may be interpreted as an interaction effect. Though the base excitation ceases beyond its duration, the motion of the secondary system acts as another input to the primary system which causes the second peak in the primary system response.

While it is a commonly accepted argument that in P-S systems with small damping the stationary secondary system response would be dominant when the primary system and the secondary system are tuned, it may not be true for nonstationary response. It has been found that for certain excitation models the secondary system response is also greatly affected by the parameters of the load model employed. Fig. 3.10 gives an example of the nonstationary response of the secondary system when the P-S system is subjected to a Shinozuka-Sato type model. It is observed that the nonstationary secondary system response is greater when $\frac{\omega_s}{\omega_p} = 0.8$ than it is for the case $\frac{\omega_s}{\omega_p} = 1$. This becomes clear by recalling Eq. (2-113). If $\bar{\zeta}_s \bar{\omega}_s$ takes values close to one of the excitation parameters α, β and $\alpha + \beta$, the response may increase dramatically for the lightly-damped system.

3.3.3 Nonstationary Response of P-S Systems with Nonclassical Damping

If the condition as shown in Eq. (3-50) does not hold, the system will have nonclassical damping, which means that the mass matrix, damping matrix and stiffness matrix cannot be uncoupled simultaneously. Therefore, a complete analysis is required. Eq. (3-14) can be solved for μ by the following theorem.

It has been found that the algebraic equation

$$x^4 + bx^3 + cx^2 + dx + e = 0 \qquad (3-59)$$

has the same four roots as those obtained from the following two quadratic equations

$$\begin{aligned} x^2 + (b + \sqrt{8y + b^2 - 4c})\frac{x}{2} + (y + \frac{by - d}{\sqrt{8y + b^2 - 4c}}) = 0 \\ x^2 + (b - \sqrt{8y + b^2 - 4c})\frac{x}{2} + (y - \frac{by - d}{\sqrt{8y + b^2 - 4c}}) = 0 \end{aligned} \qquad (3-60)$$

where y is any real root of the cubic equation

$$8y^3 - 4cy^2 + (2bd - 8e)y + e(4c - b^2) - d^2 = 0 \qquad (3-61)$$

which can explicitly be solved by certain standard technique available in many textbooks on algebra.

After μ_k is determined, the corresponding eigenvectors $\mathbf{v}_k$ can be obtained from Eqs. (3-16) and (3-17). Then, the system matrix $\mathbf{L}$ can be evaluated and the explicit solution for the covariance response, the evolutionary spectral density of the response, and the mean square energy envelope response can be found.

As seen from Eqs. (3-48) and (3-49), the classical damping criterion (3-50) only gives the condition for which the system is classically damped. It seems that the overall effect of nonclassical damping cannot be simply evaluated from the magnitude of the nonclassical damping parameter β as defined in Eq. (3-51). Instead, the nonclassical damping effect is a combined effect of the interaction parameter and tuning parameter as well as the nonclassical damping parameter when the system is nonclassically damped.

Figs. 3.11 -3.13 present some numerical results for the variation of the mean square value of the envelope response of both the primary and secondary systems for different damping, mass, and frequency ratios. In these figures, the solid lines show the results for the nonclassically damped P-S systems and the dashed lines represent the results for the corresponding P-S system with classical damping which is obtained by choosing an appropriate value for the critical damping ratio of the secondary system such that the nonclassical damping criterion is satisfied. For the cases considered, it has been found that the nonclassical damping effect is not sensitive to changes in frequency ratio, or damping ratio, even when the secondary system is tuned or nearly tuned with the primary system, as shown in the Figs. 3.11 and 3.12. However, the effect is strongly influenced by the mass ratio, especially for the primary system response. A considerable difference between the responses of classically and nonclassically damped systems is observed for the primary system response when the mass ratio equals 0.01, as shown in Fig. 3.13. In all of these cases, the existence of noncalssical damping has a negligible effect on the secondary system response. It is interesting that in the neighbourhood of the peaks of the primary system response, the peak value for the nonclassically damped P-S system is higher than that for the classically damped system. An opposite conclusion may be drawn for the peaks of the secondary system response.

3.4 Conclusions

The simplified state-variable method has been applied to the seismic analysis of combined primary-secondary systems subjected to a modulated white noise excitation to find an explicit solution for the covariance response of both the primary and secondary systems. The energy envelope process has been employed and its mean square value has been explicitly evaluated in terms of the solution for the mean square values of the displacement and velocity. No bandwidth restriction is imposed. The energy envelope can be reduced to the envelope used in Iwan and Smith (1987) for narrow-banded processes where an approximate solution for the mean square envelope response has been obtained based on the assumption of broad-banded excitation and narrow-banded responses. A closed form representation for the probability density can be derived if the excitation is also assumed to be Gaussian. The expression may be reduced to the well-known Rayleigh distribution in the stationary case.

An emphasis has been placed on the dynamic characteristics of P-S systems subjected to nonstationary excitation. Three different cases are discussed in detail and the explicit solutions obtained greatly facilitate the discussion. These cases are distinguished by two important parameters: the interaction parameter γ_ρ^2 and the nonclassical damping parameter β. The former is proposed to evaluate the importance of interaction between the primary and secondary systems and the latter provides a useful criterion for whether the P-S system is nonclassically damped. The interaction parameter proposed herein has a simple relationship with the parameter employed by Iwan and Smith (1987).

For sufficiently small interaction parameters, the interaction may be neglected and the primary and secondary systems become uncoupled. The evolutionary floor spectral density is introduced which can be used as the input to calculate the nonstationary response of the secondary system. When the interaction parameter becomes large, the interaction between these two systems is significant and, therefore, a combined P-S system has to be studied. If the nonclassical damping parameter is zero, i.e., the ratio of the damping to frequency of the secondary system is equal to that of the primary system, the system is classically damped. In this case, a transformation of variables is employed and, as result, an equivalent uncoupled P-S system can be analysed, as suggested by Iwan and Smith. Finally, if the classical damping requirement does not hold, a complete analysis of the nonclassically damped P-S system needs to be performed. An explicit solution may still be available by the simplified state-variable method, as shown in this chapter.

Some numerical results are presented for different mass, frequency, and damping ratios to illustrate the dynamic characteristics of the nonstationary response of P-S systems. A delay effect of the secondary system response is observed due to the nonstationary nature of the excitation. When the excitation ceases, the primary system response dramatically decreases to zero, but the secondary system response continues to increase for a while and then begins decreasing. It is observed that nonstationary response may depend on the parameters of certain types of excitation models such as the white noise modulated by the Shinozuka-Sato type envelope. The argument that the stationary solutions of the tuned P-S systems give conservative results may not be valid especially when the system parameters are close to some critical range of the load parameters. It is also concluded that nonclassical damping generally has a negligible effect on the secondary system response but has a considerable effect on the response of the primary system when the mass ratio is small.

The results have certain significance in the aseismic design of the secondary systems such as light equipment. For instance, such equipment should be designed to undergo longer duration than that of the excitation due to the delay effect observed. Even though the stationary solutions of the uncoupled P-S system usually give conservative results, the nonstationarity of the excitation brings some new features into the analysis, especially when the system parameters are close to the critical range of the load parameters. Therefore, a nonstationary analysis is suggested for the design of important equipment in critical structures.

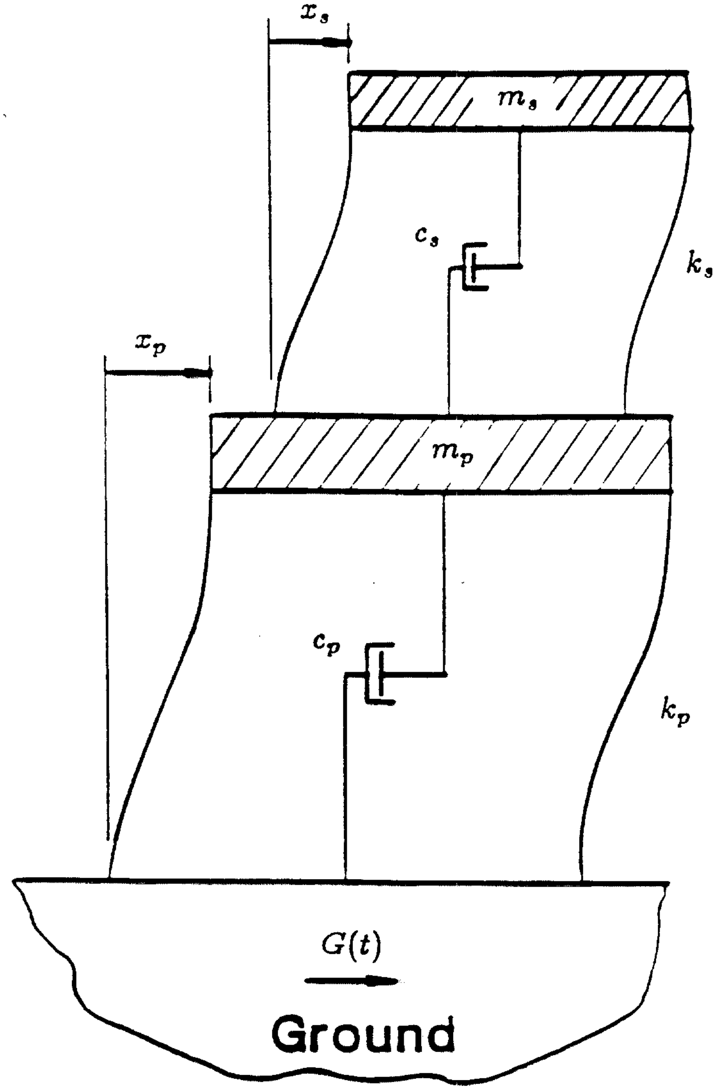


Figure 3.1. Single-degree-of-freedom secondary system attached to single-degree-of-freedom primary system subjected to a nonstationary base excitation.

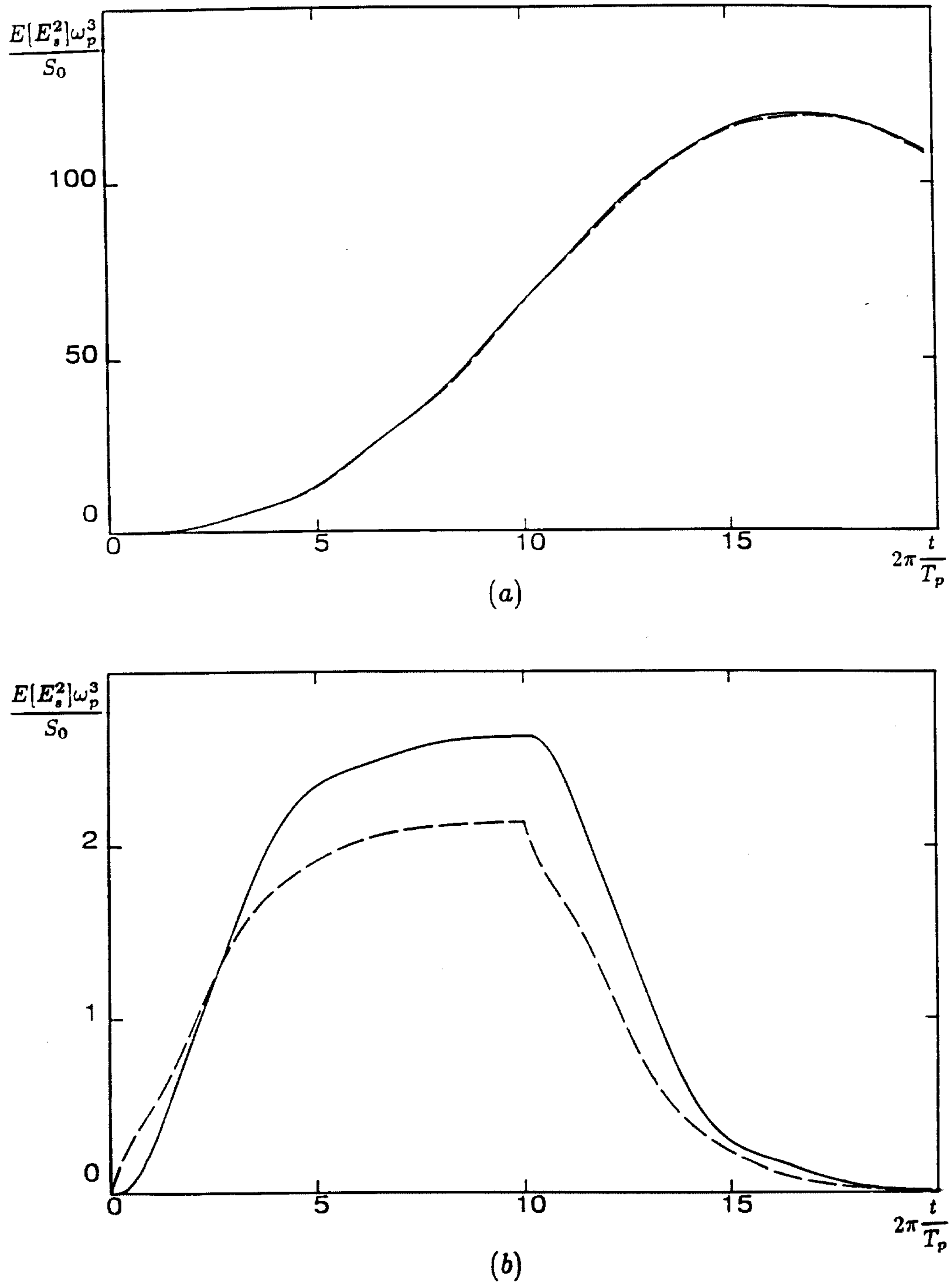


Figure 3.2. Comparison of mean square values of the energy envelope response and Iwan-Smith envelope response of the secondary system subjected to a base excitation of finite duration. The solid line is for energy envelope and the dashed one for the Iwan-Smith envelope. $\epsilon = 0$, $\frac{\omega_s}{\omega_p} = 0.8$, and the nondimensional duration $T_d = 10$. (a) $\zeta_s = \zeta_p = 0.05$; and (b) $\zeta_s = \zeta_p = 0.5$.

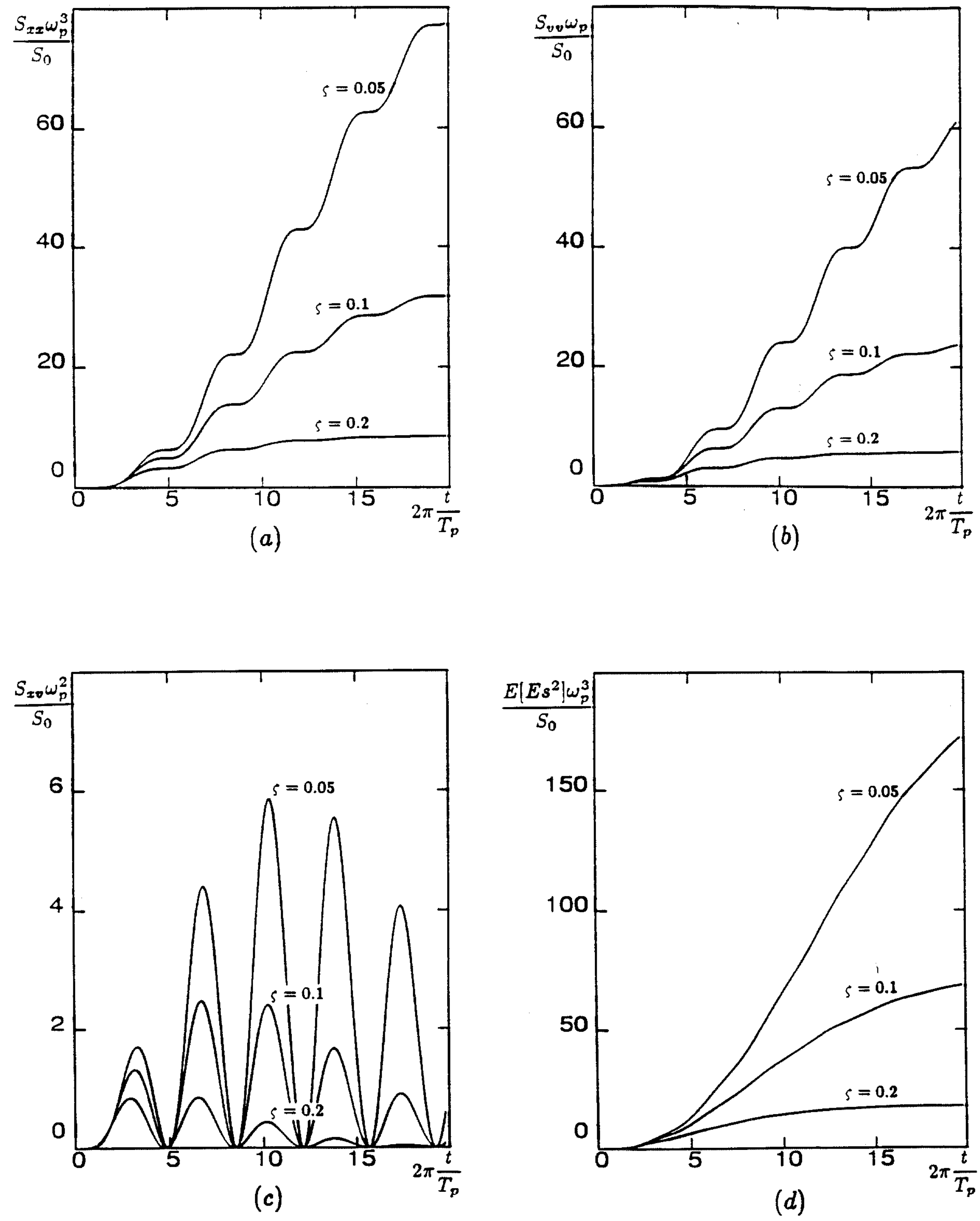
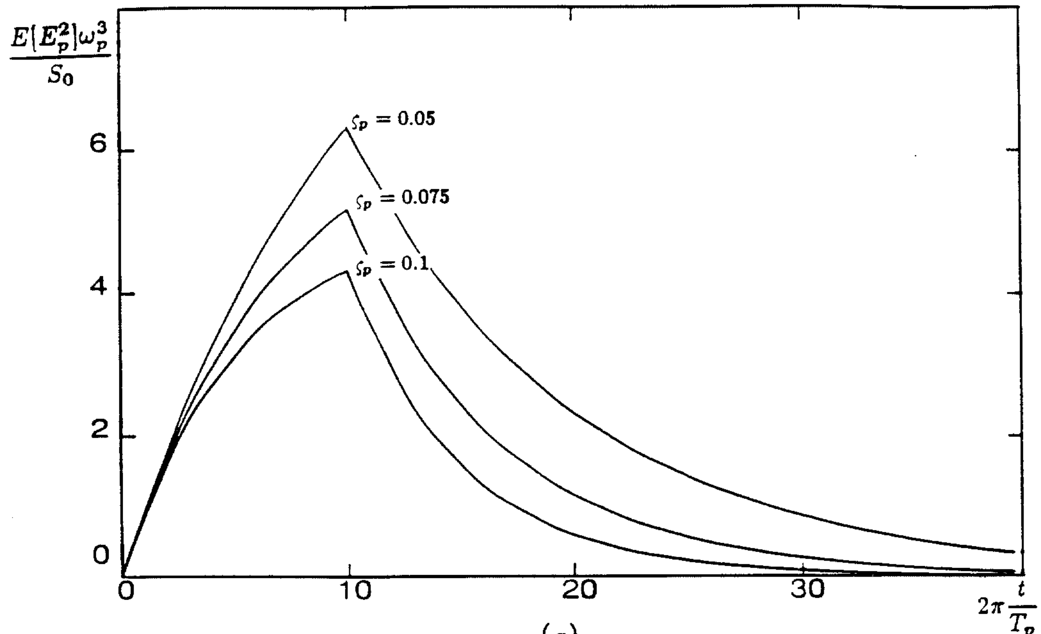
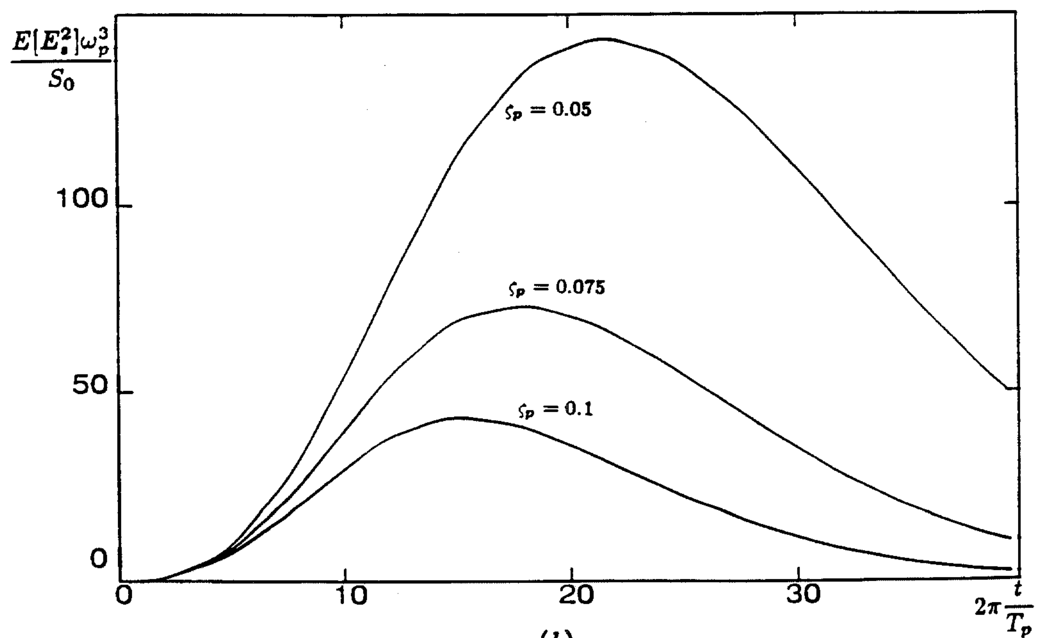


Figure 3.3. Nonstationary covariance of the response and mean square energy response of the secondary system for a uncoupled P-S system subjected to a white noise modulated by the unit step function. $\frac{\omega_s}{\omega_p} = 0.8$, $\varsigma_s = 0.005, 0.1$, and 0.2.



(a)



(b)

Figure 3.4. Nonstationary mean square energy envelope response for uncoupled P-S systems subjected to a white noise modulated by the rectangular function with a nondimensional duration $T_d = 10$. $\frac{\omega_s}{\omega_p} = 1.0$, $\frac{\varsigma_s}{\varsigma_p} = 0.9$, and $\varsigma_p = 0.05, 0.075, 0.1$. (a) The primary system response; and (b) the secondary system response.

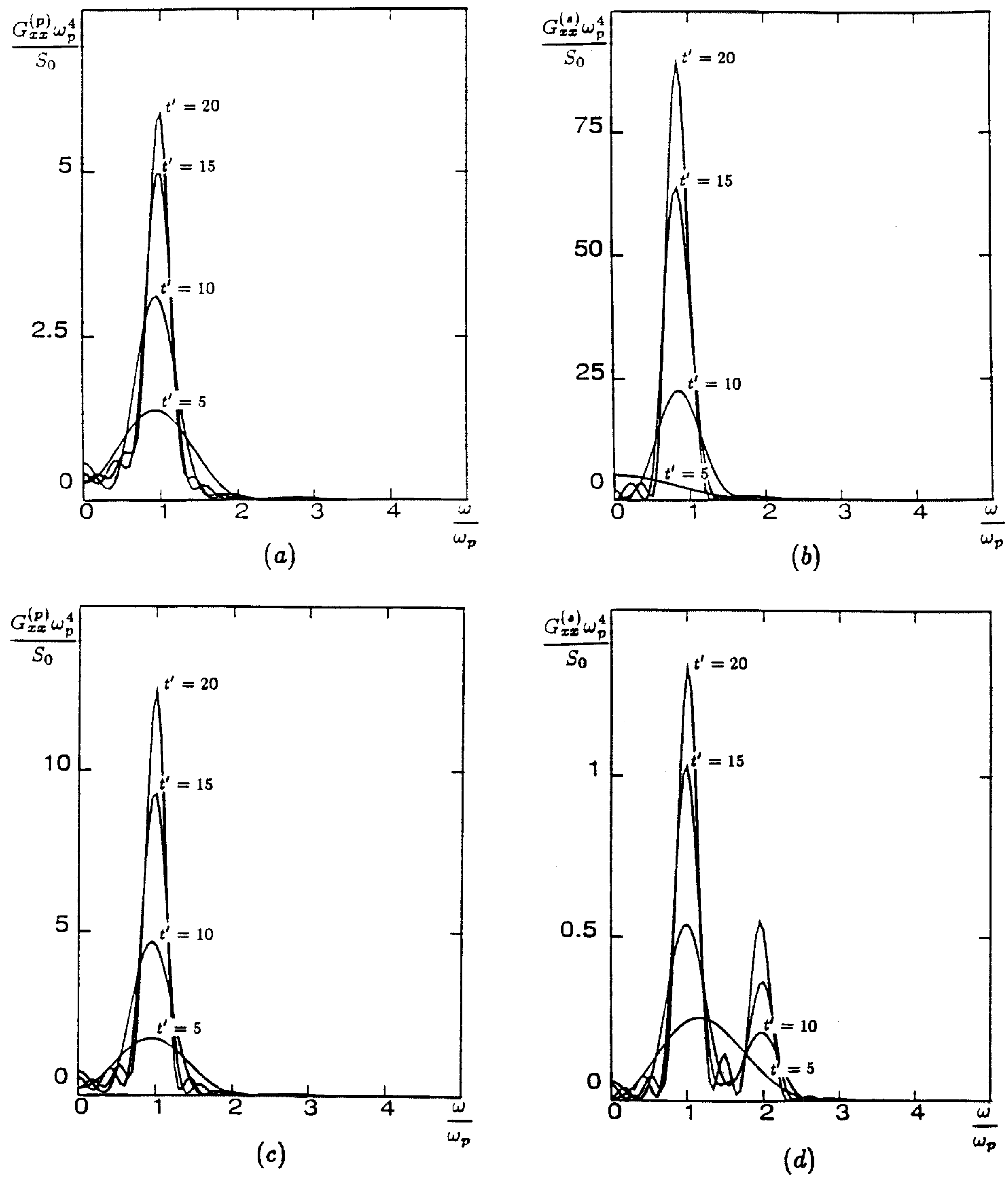


Figure 3.5. Evolutionary floor spectral densities $G_{xx}^{(p)}$ and evolutionary power spectral density of the secondary system response $G_{xx}^{(s)}$ for a uncoupled P-S system subjected to a modulated white noise with unit step envelope. $t' = 2\pi\frac{t}{T_p}$. $\zeta_s = \zeta_p = 0.1$. (a) $G_{xx}^{(p)}$ for $\frac{\omega_s}{\omega_p} = 0.8$, (b) $G_{xx}^{(s)}$ for $\frac{\omega_s}{\omega_p} = 0.8$; (c) $G_{xx}^{(p)}$ for $\frac{\omega_s}{\omega_p} = 2.0$, (d) $G_{xx}^{(s)}$ for $\frac{\omega_s}{\omega_p} = 2.0$.

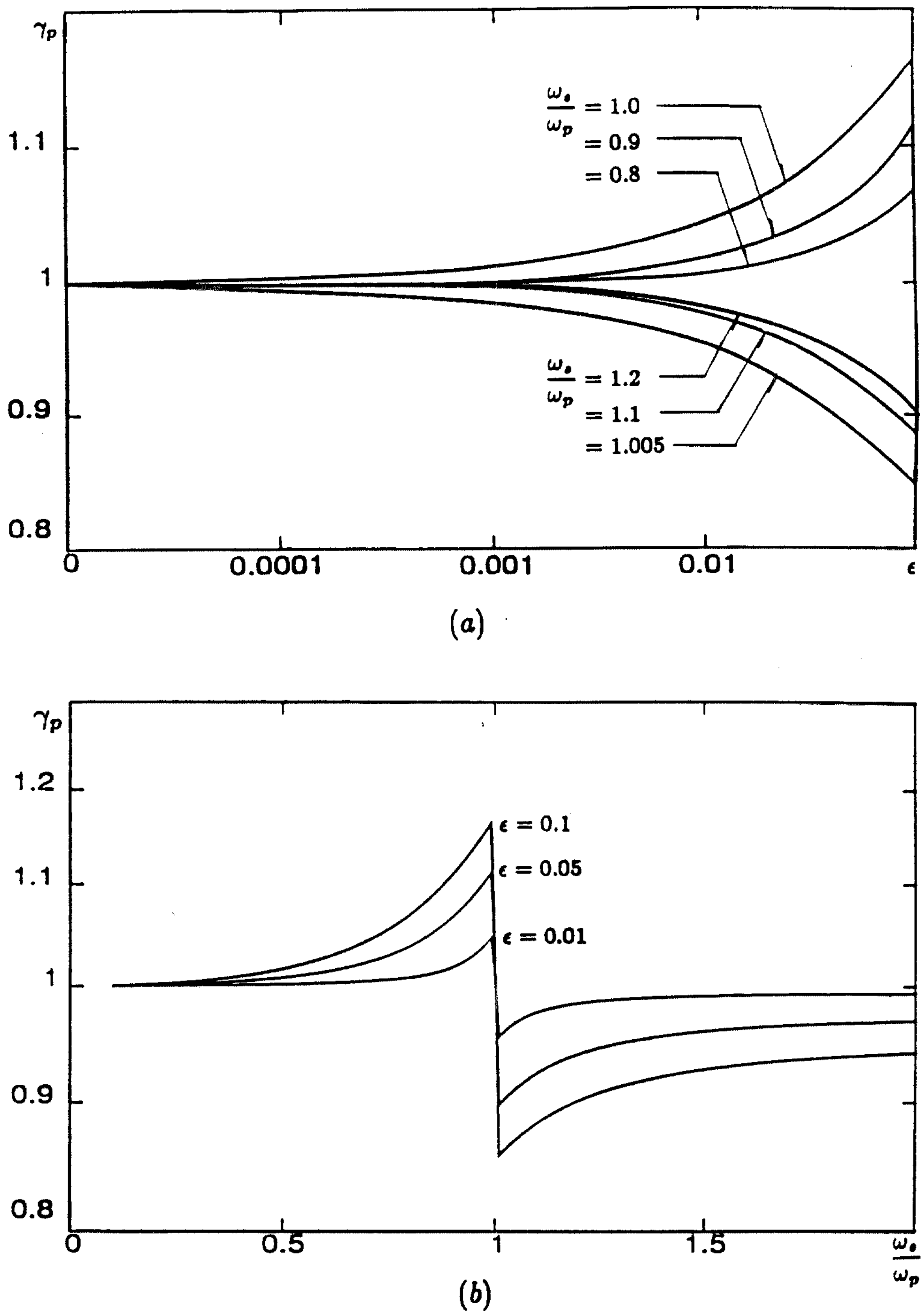


Figure 3.6. Interaction parameter R_p. (*a*) R_p versus mass ratio ϵ; (*b*) R_p versus frequency ratio $\frac{\omega_s}{\omega_p}$.

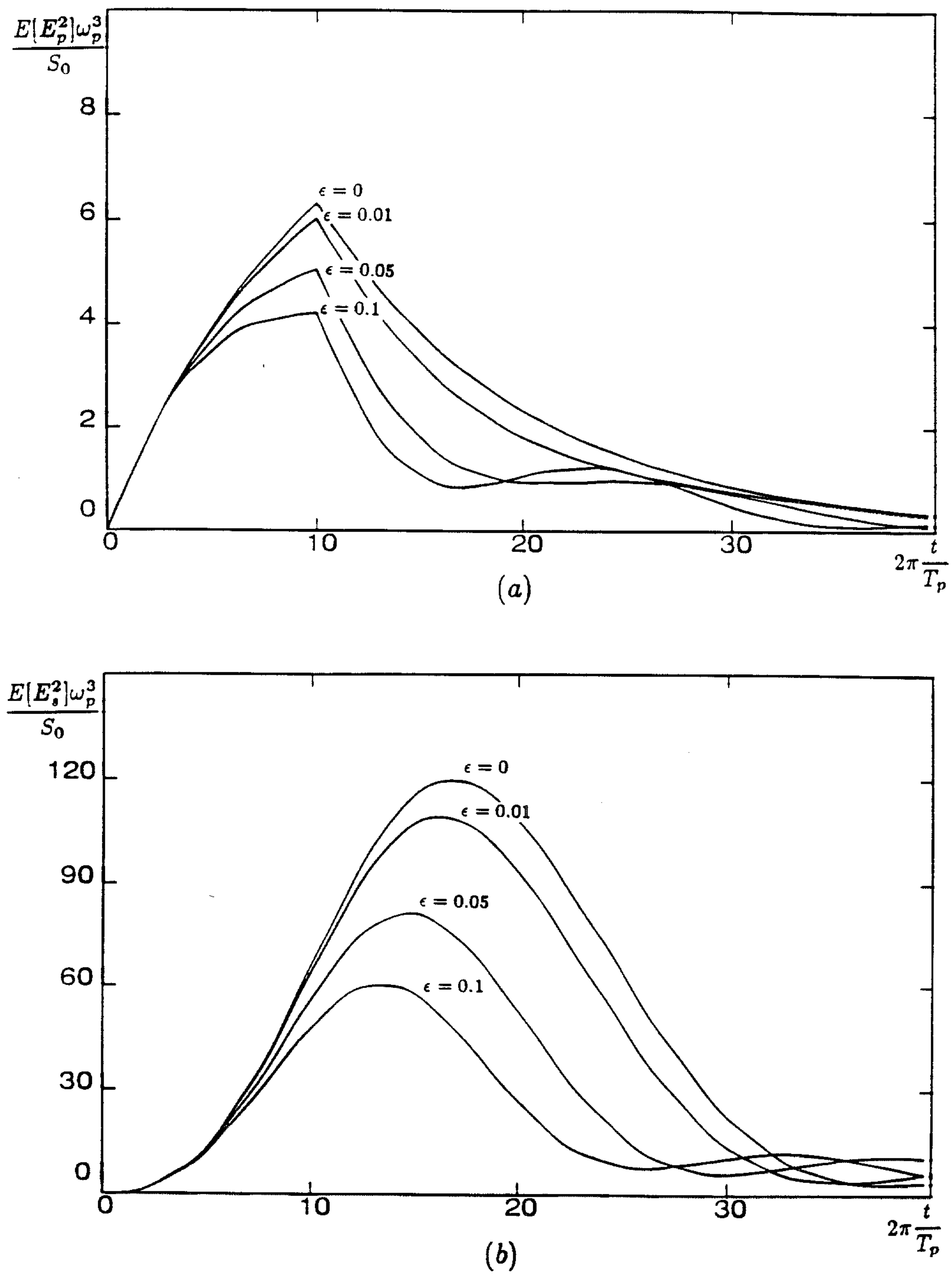


Figure 3.7. Nonstationary mean square energy envelope response of coupled P-S systems subjected to a modulated white noise with the rectangular envelope of a nondimensional duration $T_d = 10$. $\frac{\omega_s}{\omega_p} = 0.8$ and $\zeta_s = \zeta_p = 0.05$. (a) The primary system response; (b) the secondary system response.

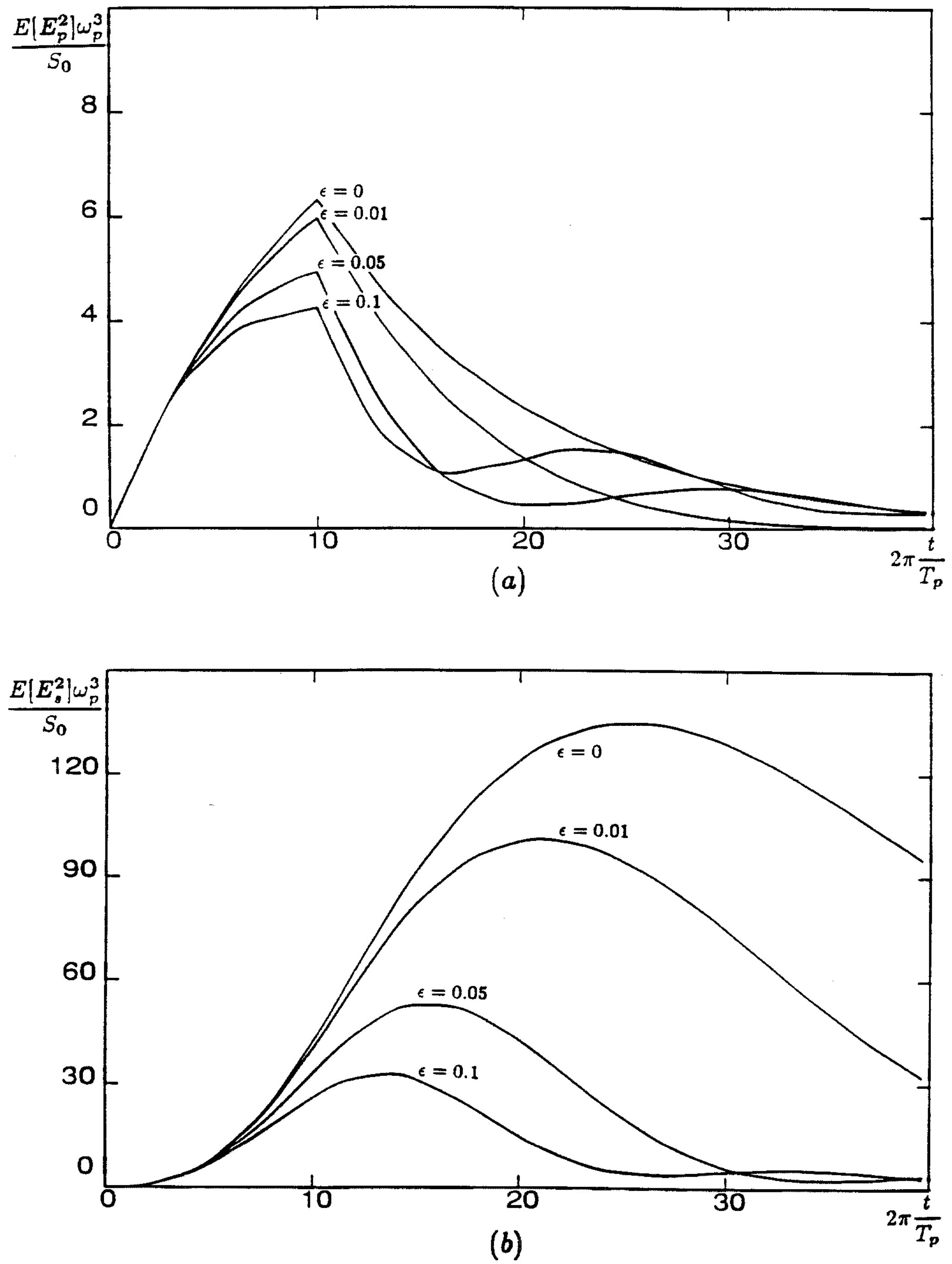


Figure 3.8. Nonstationary mean square energy envelope response of coupled P-S systems subjected to a modulated white noise with the rectangular envelope of a nondimensional duration $T_d = 10$. $\frac{\omega_s}{\omega_p} = 1.0$ (tuning) and $\zeta_s = \zeta_p = 0.05$. (a) The primary system response; (b) the secondary system response.

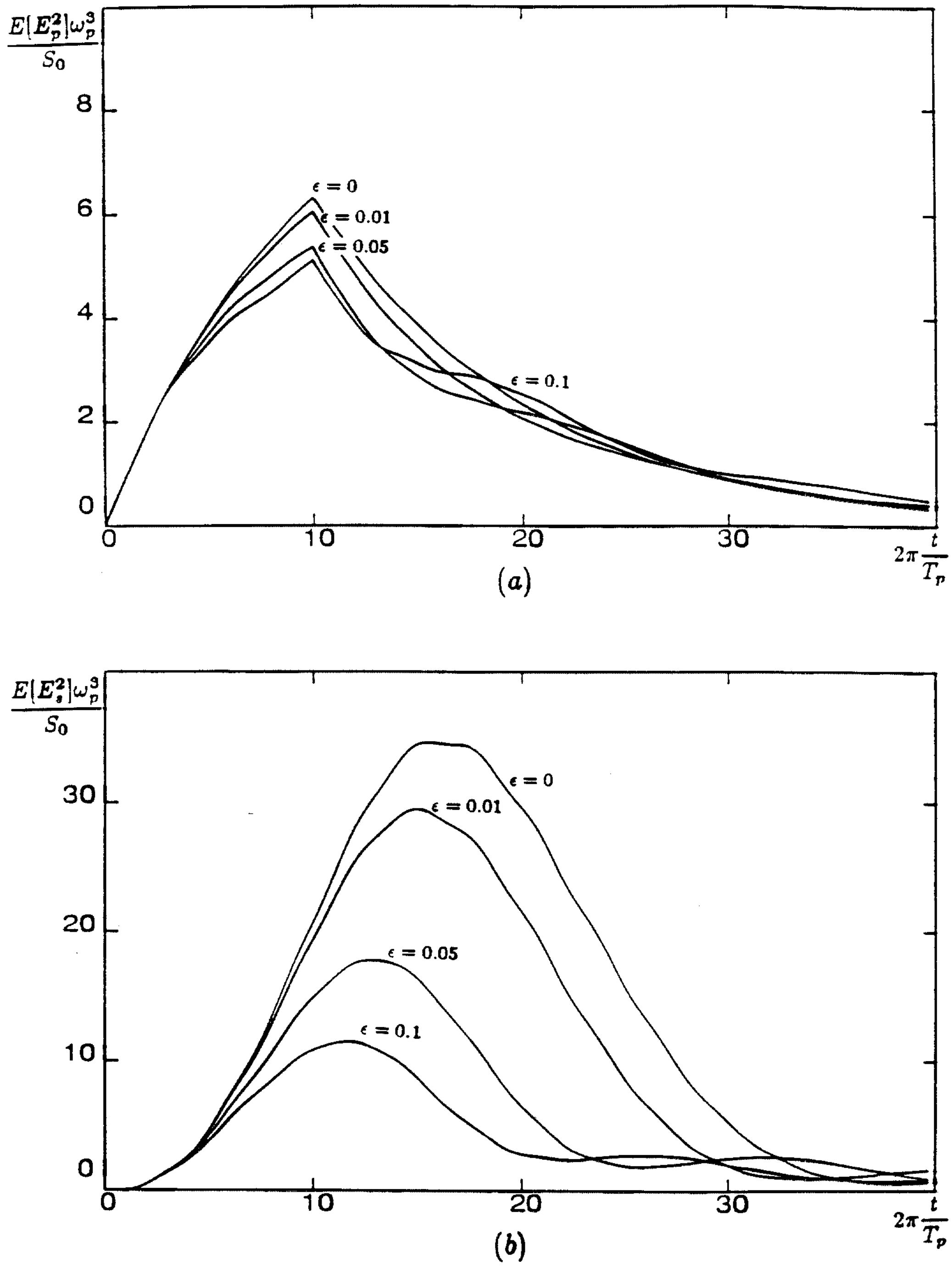


Figure 3.9. Nonstationary mean square energy envelope response of coupled P-S systems subjected to a modulated white noise with the rectangular envelope of a nondimensional duration $T_d = 10$. $\frac{\omega_s}{\omega_p} = 1.2$ (tuning) and $\varsigma_s = \varsigma_p = 0.05$. (*a*) The primary system response; (*b*) the secondary system response.

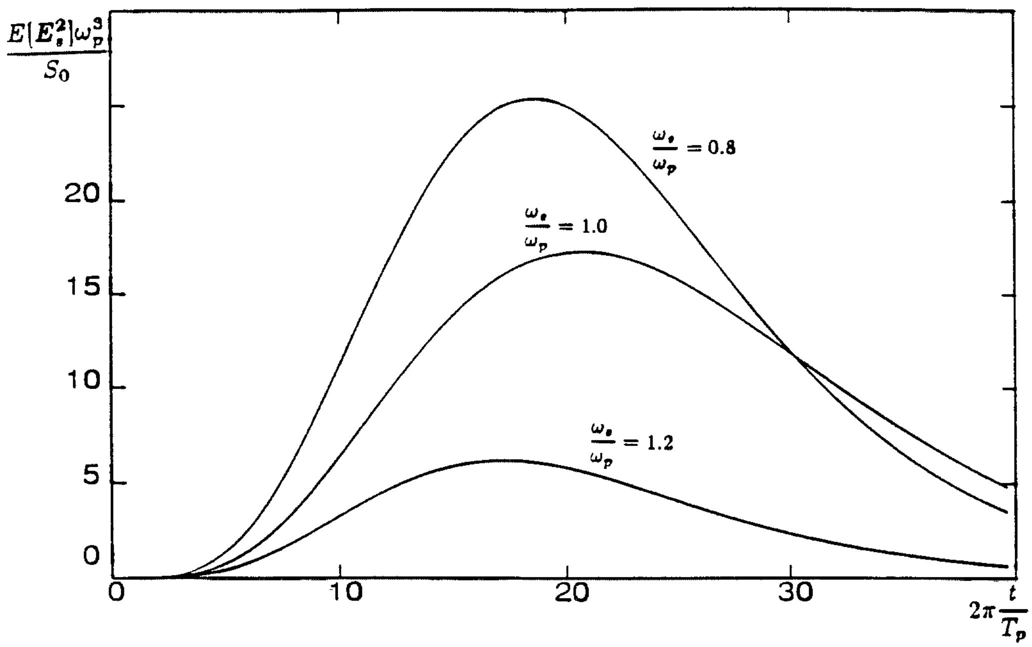


Figure 3.10. Nonstationary mean square energy envelope response of the secondary system for coupled P-S systems with different frequency ratios subjected to a modulated white noise with a Shinozuka-Sato type envelope of nondimensional parameters $A = 2.0, \alpha = 0.09382$, and $\beta = 0.218$. $\epsilon = 0.01$ and $\zeta_p = \zeta_s = 0.05$.

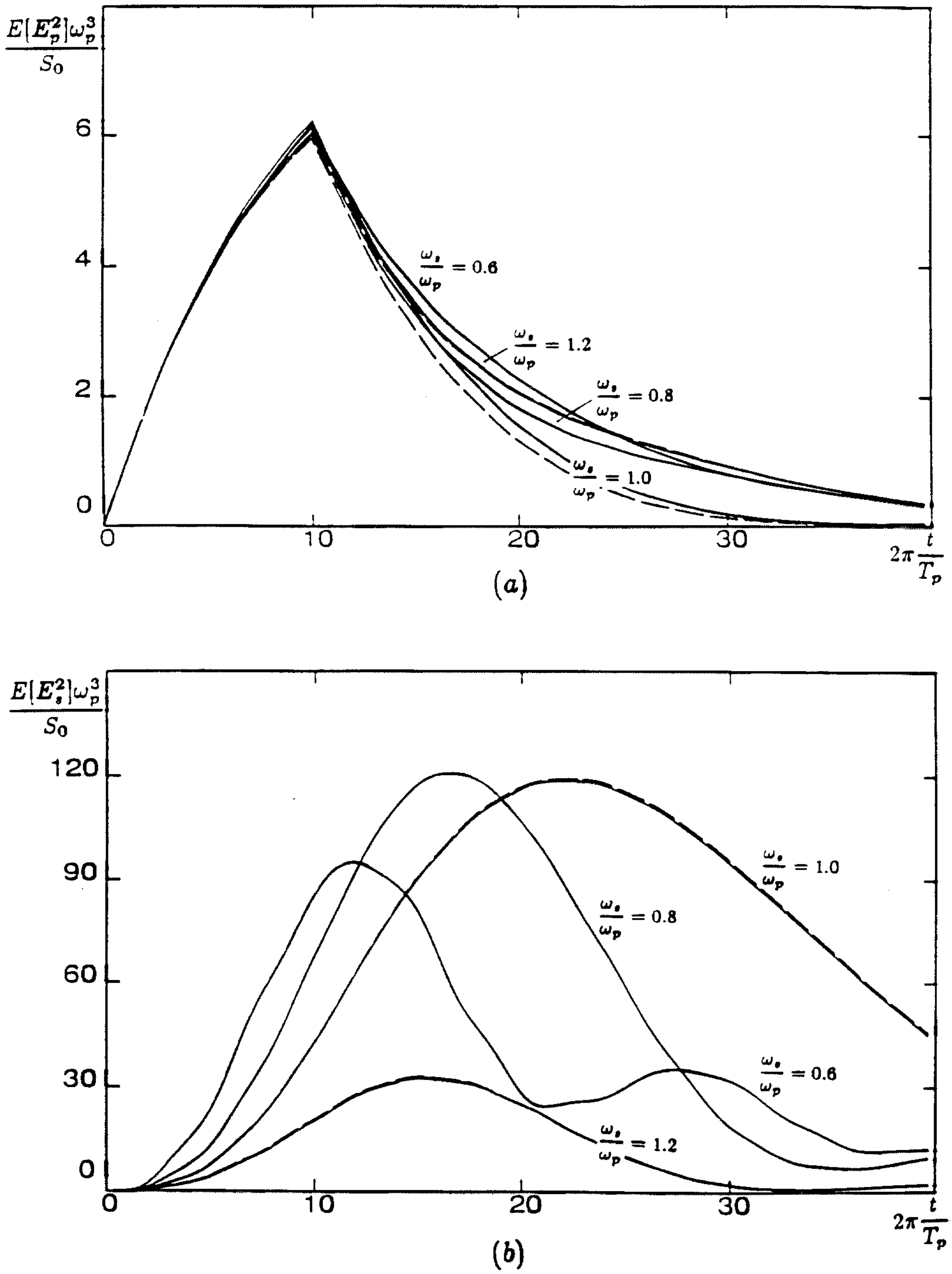


Figure 3.11. Mean square energy envelope response of coupled P-S systems with different frequency ratios subjected to a modulated white noise with the rectangular envelope of a nondimensional duration $T_d = 10$. $\zeta_s = 0.02$ and $\zeta_p = 0.05$. $\epsilon = 0.01$. Solid lines are for the system with nonclassical damping and dashed lines for classical damping. (*a*) the primary system response; (*b*) the secondary system response.

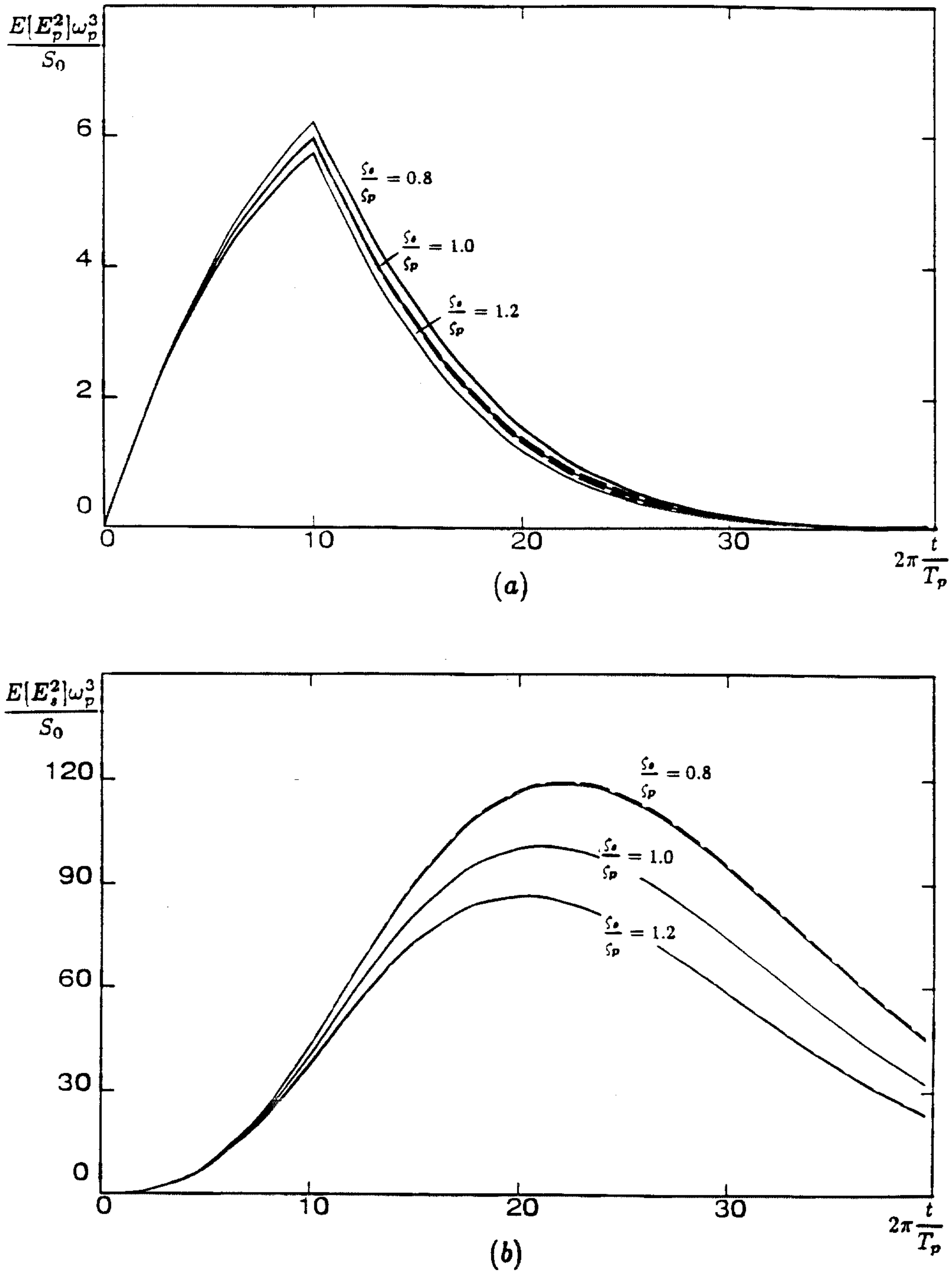


Figure 3.12. Mean square energy envelope response of coupled P-S systems with different damping ratios subjected to a modulated white noise with the rectangular envelope of a nondimensional duration $T_d = 10$. $\frac{\omega_s}{\omega_p} = 1.0$ and $\epsilon = 0.01$. Solid lines are for the system with non-classical damping and dashed lines for classical damping. (a) The primary system response; (b) the secondary system response.

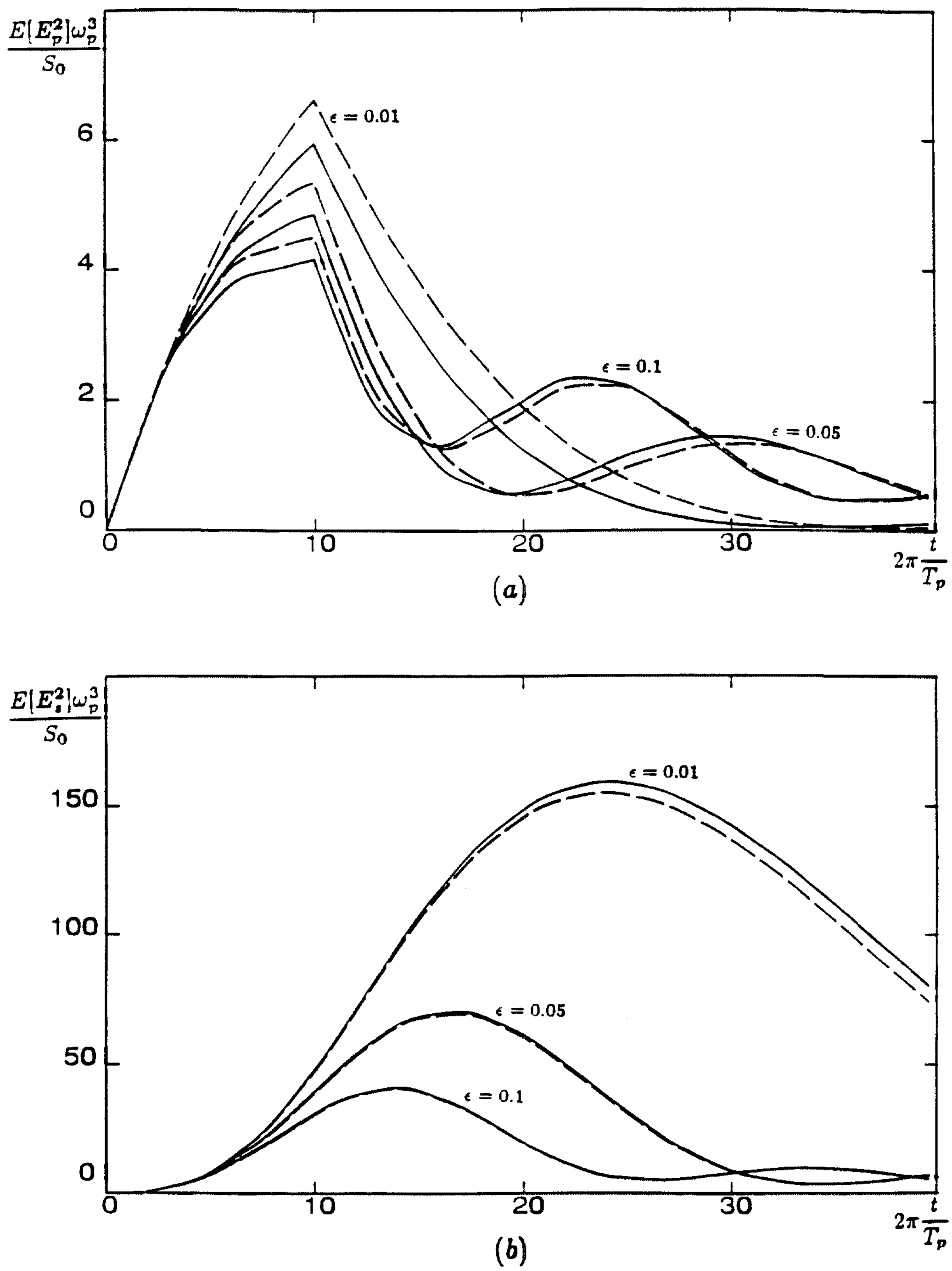


Figure 3.13. Mean square energy envelope response of coupled P-S systems with different mass ratios subjected to a modulated white noise with the rectangular envelope of a nondimensional duration $T_d = 10$. $\frac{\omega_s}{\omega_p} = 1.0, \zeta_p = 0.05$, and $\zeta_s = 0.025$. Solid lines are for the system with nonclassical damping and dashed lines for classical damping. (*a*) The primary system response; (*b*) the secondary system response.

Chapter 4

Nonstationary Response of Structures Subjected to a Class of Evolutionary Earthquake Models

4.1 Introduction

The well-known model for ground motion proposed by Kanai (1957) and Tajimi (1960) has been widely used in the seismic analysis of structures subjected to earthquake excitations. The Kanai-Tajimi model is a physically motivated three-parameter stochastic model. Using this model, the spectral density of the ground acceleration is expressed as

$$\Phi(\omega) = \frac{\omega_g^2 + (2\varsigma_g \omega_g \omega)^2}{(\omega_g^2 - \omega^2)^2 + (2\varsigma_g \omega_g \omega)^2} K \tag{4-1}$$

where the three parameters K, ς_g, and ω_g quantitatively characterize the spectrum level of a broad-band earthquake excitation at the bedrock, the dominant frequency of the site, and the attenuation of seismic waves in the ground.

The model can be physically interpreted as the stationary response of a mass supported by linear spring and dashpot, and subjected to a stationary white noise base excitation. In this case, K is the intensity of the white noise, and ς_g and ω_g are respectively the fraction of critical damping and the natural frequency of the mass-spring-dashport system, or filter. The governing equation of the filter will be

$$\ddot{R}(t) + 2\varsigma_g \omega_g \dot{R}(t) + \omega_g^2 R(t) = -n(t) \tag{4-2}$$

where $n(t)$ is a stationary white noise process with zero mean and the spectral density $\frac{K}{2\pi}$ and $R(t)$ is the ground surface response relative to the bedrock. The absolute ground acceleration is

$$G(t) = -2\varsigma_g \omega_g \dot{R}(t) - \omega_g^2 R(t) \tag{4-3}$$

The phenomenon of the filtering of ground motion by soft soils is well-known and cannot be ignored in earthquake ground motion modeling. The nature of ground resonance has been amply demonstrated in the records of a number of past earthquakes, including the recent Mexican Earthquake on September 19, 1985. The Mexico City records obtained at sites on thick clay deposits over an old lake bed show ground shaking energy to be concentrated around the resonance frequency of the clay layer (Beck and Hall, 1986).

One of the most attractive features of the Kanai-Tajimi model is its ability to simulate ground resonance in a very simple way and all the three model parameters involved have clear physical significance. As a consequence of its simplicity, the model can be easily incorporated into theoretical analysis and some analytical results can be obtained. For instance, in random vibration analysis, a simple relationship exists between the input spectral density and the output spectral density for linear systems subjected to the Kanai-Tajimi type ground motion. For nonlinear systems, the method of statistical linearization may sometimes be implemented, making linear spectral analysis applicable.

Note the the Kanai-Tajimi model is a stationary model. As pointed out in Appendix II, the stationary model may be used to model earthquakes of long duration with a significant stationary stage but is incapable of describing nonstationary characteristics which may be dominant in earthquakes of short or medium durations (Smith, 1985). This deficiency can be partially remedied by introducing a deterministic time-dependent envelope function to modify the original stationary model as follows:

$$\bar{G}(t) = \eta(t)G(t) \qquad (4-4)$$

where $\bar{G}(t)$ is the modified ground motion, $G(t)$ is the original stationary ground motion with a spectral density given by Eq. (4-1), and $\eta(t)$ is an envelope function which is assumed to be slowly varying in time. Nonstationarity can be described by choosing an appropriate envelope. Different envelope functions have been suggested by many investigators based on their understanding of the characteristics of the ground motions as well as mathematical convenience. A detailed review on the envelopes currently prevailing in earthquake engineering may be found in Appendix

II. The ground motion model expressed by Eq. (4-4) is sometimes referred to as the modulated filtered white noise model (Smith, 1985). It should be pointed out that this model accounts for the time variation of the intensity of the ground acceleration, but not the time variation of the frequency content. The latter is clearly visible in many available records.

To include the time variation of the frequency content into analysis, Lin (1987) suggested a random pulse train model. In this model, the earthquake source is modeled as an sequence of double-couples occuring at random time τ_j with magnitude Y_j, namely,

$$S(t) = \sum_{j=1}^{N(t)} Y_j \delta(t - \tau_j) \tag{4-5}$$

where $N(t)$ is a Poisson counting process with an average occurrence rate $\lambda(t)$. Then, the earthquake ground motion $G(t)$ can be obtained by passing the impulsive sequence as shown by Eq. (4-5) through a filter modeling the propagation of the seismic waves from the source to the site. The result can be expressed as a pulse train of the form

$$R(t) = \sum_{j=1}^{N(t)} Y_j h(t - \tau_j) \tag{4-6}$$

where $h(t)$ is the impulse response function of the filter. It can be shown that the pulse train $R(t)$ is an evolutionary random process defined by Priestly (1965). If the Kanai-Tajimi filter is used, the filtered pulse train model may be referred to as an evolutionary Kanai-Tajimi model (Lin, 1987).

Another limitation of the conventional Kani-Tajimi model is the assumption that the soil bahavior is simulated by an SDOF oscillator. When a dominant frequency exists, such as in the 1985 Mexican earthquake, a satisfactory result is expected by using such a simplification. However, if the ground motion is broad-banded, refined models are needed to give more realistic soil behavior. As an effort to improve the behavior of ground motion models in this direction, different filters have been suggested by previous studies including low- and high-pass filters in series as proposed by Clough and Pezien (1975), shear beam models used by Idriss and Seed (1968), and a "horn" filter by Lin (1987). These models may also be referred to as an evolutionary Kanai-Tajimi type model since they retain the same physical idea of a filter as in the Kanai-Tajimi model but different filters are employed. More information regarding soil filters is given in Appendix II.

While a great deal of work has been done on deterministic analysis of structures using these earthquake models, ralatively few results can be found in the literature for random vibration analysis, especially regarding explicit nonstationary solutions. As an application of the simplified state-variable method to linear continuous systems, this chapter presents a general formulation for the nonstationary response of a structure subjected to a class of earthquake models using a continuous filter. Explicit solutions are given for the case where the structure is modeled as an SDOF oscillator and the ground filter is represented by a shear beam with an input of a random pulse train at its base. Some results for an SDOF filter are also presented for comparison purposes. A structural reliability analysis for such earthquake models is included.

4.2 Formulation

Consider a simple linear structure with lumped mass m, stiffness k, and damping c excited by an earthquake ground motion. The ground motion can be thought of as the output of a linear shear beam filter excited by a sequence of random impulse, as shown in Fig. 4.1. The dynamic behavior of the filter simulates the performance of soil layers between the ground surface and bedrock.

Let $z(t)$ be the displacement of the structure relative to the ground surface, and $u(x,t)$ be the displacement of the beam at station x relative to bedrock where x is the axial coordinate of the beam originating at the base. Let l denote the total length of the beam, or the depth of the soil layers from surface to bedrock. u(l,t) will be the ground displacement relative to bedrock. For the sake of convenience in notation, define

$$y(t) = u(l,t) \tag{4-7}$$

The dynamic response of the structure is then governed by

$$\ddot{z}(t) + 2\varsigma_0\omega_0\dot{z}(t) + \omega_0^2 z(t) = -\ddot{y}(t) - a(t) \tag{4-8}$$

where ς_0 is the fraction of critical damping and ω_0 is the natural frequency of the structure defined by

$$\omega_0^2 = \frac{k}{m} \qquad \varsigma_0 = \frac{c}{2\sqrt{km}} \tag{4-9}$$

and $a(t)$ represents the base excitation.

The equation of motion for the beam filter is given by

$$\rho(x)(\frac{\partial^2}{\partial t^2}u(x,t) + a(t)) + \frac{\partial}{\partial t}(L_c u(x,t)) + L_k u(x,t) = 0 \tag{4-10}$$

where $\rho(x)$ denotes the distributed mass density of the beam, and L_c and L_k are some linear operators with respect to the spatial variable x. The three terms in above equation represent the inertial force, damping force, and elastic restoring force acting on a differential beam element at position x and time t.

Assume a set of orthogonal modes of the beam exists, the displacement $u(x,t)$ may then be expressed as

$$u(x,t) = \sum_{j=1}^{\infty} X_j(x)\alpha_j(t) \tag{4-11}$$

where $X_j(x)$ is jth model of the beam with orthogonal properties

$$\begin{gathered} \int_0^l X_i(x)X_j(x)dx = 0 \\ \int_0^l X_i(x)L_c(X_j(x))dx = 0 \\ \int_0^l X_i(x)L_k(X_j(x))dx = 0 \\ i,j = 1,2,...,i \neq j \end{gathered} \tag{4-12}$$

$\alpha_j(t)$ is then the corresponding time response of the jth mode.

Substituting Eq. (4-11) into Eqs. (4-8) and (4-10) and using the orthogonal relationship in Eq. (4-12) yields

$$\begin{aligned} &\ddot{z}(t) + 2\varsigma_0\omega_0\dot{z}(t) + \omega_0^2 z(t) = \sum_{j=1}^{\infty} f_j(l)\dot{\alpha}_j(t) + \sum_{j=1}^{\infty} g_j(l)\alpha_j(t) \\ &\ddot{\alpha}_j(t) + 2\varsigma_j\omega_j\dot{\alpha}_j(t) + \omega_j^2\alpha_j(t) = -r_j a(t) \qquad j = 1,2,... \end{aligned} \tag{4-13}$$

where

$$2\varsigma_j\omega_j = \frac{\int_0^l X_j(x)L_cX_j(x)dx}{\int_0^l \rho(x)X_j^2(x)dx}$$

$$\omega_j^2 = \frac{\int_0^l X_j(x)L_kX_j(x)dx}{\int_0^l \rho(x)X_j^2(x)dx} \tag{4-14}$$

$$r_j = \frac{\int_0^l \rho(x)X_j(x)dx}{\int_0^l \rho(x)X_j^2(x)dx}$$

and

$$f_j(x) = \frac{1}{\rho(x)}L_cX_j(x)$$

$$g_j(x) = \frac{1}{\rho(x)}L_kX_j(x) \tag{4-15}$$

Truncating the series expressions in Eq. (4-13) at n terms and rewriting the equations in matrix form yields

$$\mathbf{M}\ddot{\mathbf{Z}}(t) + \mathbf{C}\dot{\mathbf{Z}}(t) + \mathbf{K}\mathbf{Z}(t) = -a(t)\mathbf{r} \tag{4-16}$$

where M is a $(n+1)$ x $(n+1)$ unit matrix and

$$\mathbf{Z}(t) = \begin{pmatrix} z(t) \\ \alpha_1(t) \\ \alpha_2(t) \\ \vdots \\ \alpha_n(t) \end{pmatrix} \qquad \mathbf{r} = \begin{pmatrix} 0 \\ r_1 \\ r_2 \\ \vdots \\ r_n \end{pmatrix}$$

$$\mathbf{C} = \begin{pmatrix} 2\varsigma_0\omega_0 & -f_1(l) & -f_2(l) & \dots & -f_n(l) \\ 0 & 2\varsigma_1\omega_1 & 0 & \dots & 0 \\ 0 & 0 & 2\varsigma_2\omega_2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 2\varsigma_n\omega_n \end{pmatrix}$$

$$\mathbf{K} = \begin{pmatrix} \omega_0^2 & -g_1(l) & -g_2(l) & \dots & -g_n(l) \\ 0 & \omega_1^2 & 0 & \dots & 0 \\ 0 & 0 & \omega_2^2 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & \omega_n^2 \end{pmatrix} \tag{4-17}$$

The ground excitation $a(t)$ is assumed to be a random pulse train of the form

$$a(t) = \sum_{k=1}^{N(t)} Y_k \delta(t - \tau_k) \qquad (4-18)$$

where $N(t)$ is a Poisson counting process with an occurrence rate $\lambda(t)$, and Y_k are independent random variables with zero mean and uniform distribution which occur at time τ_k. As shown in Appendix II, the random pulse train, as in Eq. (4-18), is statistically equivalent to a modulated white noise with a well-defined envelope function $\eta(t)$ up to thr second moment. The envelope is defined as

$$\eta(t) = \sqrt{E[Y^2]\lambda(t)} \qquad (4-19)$$

Therefore, Eq. (4-18) can be replaced by

$$a(t) = \eta(t)n(t) \qquad (4-20)$$

in a second moment analysis.

The eigenvalue problem associated with Eq. (4-16) is

$$(\lambda^2\mathbf{M} + \lambda\mathbf{C} + \mathbf{K})\mathbf{v} = 0 \qquad (4-21)$$

which can be easily solved. The eigenvalues can be found as

$$\begin{aligned} \lambda_{1,2} &= -\varsigma_0\omega_0 \pm i\omega_{0d} \\ \lambda_{2k-1,2k} &= -\varsigma_k\omega_k \pm i\omega_{kd} \qquad k = 2, 3, ... \end{aligned} \qquad (4-22)$$

where i is the imaginary unit and

$$\omega_{kd} = \omega_k\sqrt{1 - \varsigma_k^2} \qquad k = 0, 1, 2, ... \qquad (4-23)$$

The corresponding eigenvectors $\mathbf{v}_k, k = 1, 2...$ can be found as

$$\mathbf{v}_1 = \mathbf{v}_2 = (1, 0, ..., 0)^T \qquad (4-24)$$

and

$$\mathbf{v}_{2k-1} = \mathbf{v}_{2k} = (\frac{g_k(l) + \lambda_k f_k(l)}{\lambda_k^2 + 2\xi_0\omega_0\lambda_k + \omega_0^2}, 0, ..., 1, ..., 0) \qquad k = 2, 3, ... \qquad (4-25)$$

where only the first and kth components are nonzero. In the above it is assumed that λ_1 or λ_2 does not coincide with any $\lambda_k, k = 3, 4...$ Otherwise, a modified result is obtained.

Once the eigenvalues and eigenvectors of Eq.(4-21) are obtained, corresponding eigensolutions for the associated state-variable form can be generated. Then the system matrix $\mathbf{L}$ can be constructed by a standard procedure as described in Chapter 2. The final solutions can then be expressed as

$$\begin{aligned} \mathbf{Q}(t) &= S_0 \mathbf{L} \int_0^t \eta^2(\tau) \mathbf{P}(t-\tau) \mathbf{P}^T(t-\tau) d\tau \mathbf{L}^T \\ \mathbf{G}(t,\omega) &= \frac{S_0}{\pi} \mathbf{L} \mathbf{w}(t,\omega) \mathbf{w}^{*T}(t,\omega) \mathbf{L}^T \end{aligned} \tag{4-26}$$

where $\mathbf{Q}(t)$ is the covariance matrix of the response, $\mathbf{G}(t,w)$ the one-sided evolutionary spectral density matrix, and all the other symbols have the same meaning as in Chapter 2. Since n, the number of beam modes considered in the analysis is choosen arbitrarily, the matrices in Eq. (4-16) can be thought of as infinite matrices with an infinite number of rows and columns. As n approaches infinity, the results are expected to converge to the exact solution.

It should be pointed out the $\ddot{y}(t) = \ddot{u}(l,t)$ is the ground surface acceleration relative to bedrock. Sometimes the absolute ground acceleration $G(t)$ is required in engineering applications. $G(t)$ may be found in terms of the modal displacements and velocities $\alpha_j(t)$ and $\dot{\alpha}_j(t)$ by

$$\begin{aligned} G(t) &= \ddot{y}(t) + a(t) \\ &= -\sum_{j=1}^{\infty} (f_j(l)\dot{\alpha}_j(t) + g_j(l)\alpha_j(t) \end{aligned} \tag{4-27}$$

Therefore, the covariance of the absolute ground acceleration can be obtained without difficulty in terms of the covariance of the relative ground motion or the modal displacements and velocities. The results in this chapter are presented for the absolute ground acceleration.

4.3 Solution for Shear Beam Models

In Section 4.2, a general formulation is presented for a continuous filter. Different filters may be employed by properly choosing the operators L_c and L_k. To model horizontal soil layers with elastic properties which vary linearly with depth, a nonuniform shear beam filter is sometimes used (Idriss and Seed, 1968). The operators L_c and L_k may be taken to be of the form

$$\begin{aligned} L_c(F) &= cF \\ L_k(F) &= -\frac{\partial}{\partial x}\left(k_0(1-x)^p \frac{\partial}{\partial x}F\right); \qquad p \leq \frac{1}{2} \\ &\forall F(x,t) \in C^2([0,l]X[0,\infty]) \end{aligned} \tag{4-28}$$

where c and k_0 are assumed to be constants and p is a parameter used to describe the variation of the elastic properties with respect to depth. ρ in Eq. (4-10) is also assumed be a positive constant.

Once the operators L_c and L_k are defined, the eigenmodes $X_k(x), k = 1,2,...$ can be found and, in turn, ω_k, ζ_k, r_k can be evaluated. Two different cases are considered in the following.

4.3.1 Nonuniform Beam

It has been shown (Idriss and Seed, 1968) that if $0 < p \leq \frac{1}{2}$, the eigenfuctions can be found as

$$X_k(x) = (\frac{\beta_k}{2})^b \Gamma(1-b)(\frac{l-x}{l})^{\frac{b}{\theta}} J_{-b}[\beta_k(\frac{l-x}{l})^{\frac{1}{\theta}}] \tag{4-29}$$

in which J_{-b} is the Bessel function of the first kind of order $-b$, β_k are the roots of $J_{-b} = 0, k = 1,2...$, and Γ is the gamma function. The constants b and θ are related to p by

$$\begin{aligned} p\theta - \theta + 2b &= 0 \\ p\theta - 2\theta + 2 &= 0 \end{aligned} \tag{4-30}$$

when $p > \frac{1}{2}$, a representatiom for the eigenfunctions in terms of Bessel functions is not available.

Substituting Eq. (4-29) into Eq. (4-14), the damping ratio, natural frequency, and participation factor of the kth mode can be expressed as

$$\begin{aligned} \omega_k &= \beta_k \sqrt{\frac{k_0}{\rho}} \frac{1}{\theta l^{\frac{1}{\theta}}} \\ \varsigma_k &= \frac{c}{2\rho\omega_k} \\ r_k &= \frac{1}{(\frac{\beta_k}{2})^{1+b}\Gamma(1-b)J_{1-b}(\beta_k)} \end{aligned} \qquad (4-31)$$

4.3.2 Uniform Beam

If $p = 0$, the model reduces to a uniform shear beam model. Eqs. (4-29)-(4-31) become

$$\begin{aligned} X_k(x) &= \cos[\frac{2k-1}{2}\pi(1-\frac{x}{l})] \\ \omega_k &= \frac{(2k-1)\pi}{2l}\sqrt{\frac{k_0}{\rho}} \\ \varsigma_k &= \frac{c}{2\rho\omega_k} \\ r_k &= (-1)^k \frac{4}{(2k-1)\pi} \end{aligned} \qquad (4-32)$$

In both cases, $f_k(l)$ and $g_k(l)$ in Eq. (4-15) can be found as

$$\begin{aligned} f_k(l) &= -2\varsigma_k\omega_k \\ g_k(l) &= -\omega_k^2 \end{aligned} \qquad (4-33)$$

Eqs. (4-29)-(4-33) can be used to create the system matrix **L** and Eq. (4-13) can then be used to evaluate the covariance response and the evolutionary spectral density. Some examples will be given later in this chapter.

4.4 Structural Reliability Analysis

Consider an ensemble of response histories $\{x(t)\}$ on the interval $[0, t)$ which all start from the same (deterministic or probabilistic) initial condition at $t = 0$. Let

$W(t)$ be the probability that the maximum value of $|x(t)|$ throughout the interval is smaller than some threshold value b, namely,

$$W(t) = Prob(|x(\tau)| < b; \qquad 0 \leq \tau < t) \tag{4-34}$$

If the maximum absolute value of structural response reaching the threshold b implies failure of structures, $W(t)$ represents the fraction of the initial ensemble remaining in the safe region at time t. Therefore, $W(t)$, referred to as the reliability function, is a measure of the safety of the structure.

Alternatively, the safety of a structure can be evaluated from the first-crossing probability density function $p(t)$. $p(t)dt$ denotes the probability that the structural response surpasses the threshold for the first time since $t = 0$ during the interval $(t, t + dt)$. A relationship between these two quantities is simply given by

$$\frac{dW}{dt} = -p(t) \tag{4-35}$$

The first-crossing density depends in a complicated manner on the characteristics of the dynamic system, the nature of the excitation, and the initial conditions imposed as well as on the magnitude of the threshold. No exact solution for $p(t)$ has yet been found, even for the stationary case. Instead, various approximations have been made to evaluate the so-called limiting decay rate, or hazard function, α by which $p(t)$ can be expressed as

$$p(t) = \alpha e^{-\alpha t} \tag{4-36}$$

It follows that

$$W(t) = e^{-\alpha t} \tag{4-37}$$

These expressions are justified for large times and high threshold levels. A detailed review can be found in Crandall (1970).

Eq. (4-36) has been extended to the nonstationary case by Corotis, Vanmarcke, and Cornell (1972) and Mason and Iwan (1983). The reliability function, $W(t)$, is expressed as

$$W(t) = Ae^{-\int_0^t \alpha(s)ds} \tag{4-38}$$

where A represents the structural reliability at $t = 0$. $A = 1$ is usually assumed, implying that the structure initially has no damage. If $\alpha(t)$ remains constant, Eq.

(4-38) reduces to the result for the stationary case. Note, however, the $W(\infty)$ is generally not equal to zero.

While relationship in Eq. (4-35) is still valid in the nonstationary case and, as a result,

$$p(t) = A\alpha(t)e^{-\int_0^t \alpha(s)ds} \tag{4-39}$$

it is dubious to call $p(t)$ the first passage probability density in the nonstationary case since the infinite integral of $p(t)$ is generally not equal to one thus contradicting the definition of a probability density function. As a remedy, $p(t)$ may be referred to as the first passage probability rate and $p(t)dt$ represents the portion of the ensemble which reaches the threshold in the interval $(t, t+dt)$. Alternatively, a new first passage probability density function may be defined as

$$\bar{p}(t) = cp(t) \tag{4-40}$$

where c is a normalizing factor given by

$$c = \frac{1}{W(0) - W(\infty)} \tag{4-41}$$

Note that $\bar{p}(t)$ denotes a conditional first passage probability density which gives the probability that the first passage occurs in the interval $(t, t+dt)$ given that the first passage happens in $(0, \infty)$. If $W(\infty) = 0$, as in the stationary case or in some nonstationary cases such as modulated white noise excitation with a step function envelope, the first passage probability rate is exactly the first passage probability density function, which is the case discussed in the previous studies. However, generally speaking, these two quantities are not the same, as shown in the illustrations later.

In this chapter, the estimate of the nonstationary limiting decay rate by Mason and Iwan (1983) is employed. Assuming the excitation to be Gaussian and the response to be narrow-banded, the nonstationary decaying rate $\alpha(t)$ may be expressed as

$$\alpha(t) = \frac{-2\mu_b(t)\ln[P^*(t)]}{[1 + \frac{\sigma^4(t)}{\sigma_s^4(t)}][1 - \frac{\mu_b(t)}{\mu_0}]} \tag{4-42}$$

where $\sigma(t)$ is the instantaneous variance of the structural response, $\sigma_s(t)$ is the stationary response variance associated with the instantaneous value of the excitation, and $\mu_b(t)$ is the threshold crossing rate given by

$$\mu_b(t) = \int_0^{\infty} \dot{x}(t) p(b, \dot{x}, t) d\dot{x} \tag{4-43}$$

Using the assumption of Gaussian excitation, $\mu_b(t)$ can be expressed by

$$\mu_b(t) = c_1 + c_2 \,\mathrm{erfc}(-\frac{\rho \mathrm{b}}{\sqrt{2(1-\rho^2)}\sigma_{\mathrm{x}}}) \tag{4-44}$$

where the two constants c_1 and c_2 are given by

$$\begin{aligned} c_1 &= \frac{\sqrt{1-\rho^2}}{2\pi} \frac{\sigma_{\dot{x}}}{\sigma_x} exp[-\frac{1}{2(1-\rho^2)}(\frac{b}{\sigma_x})^2] \\ c_2 &= \frac{\rho b \sigma_{\dot{x}}}{2\sqrt{2\pi}\sigma_x^2} exp[-\frac{1}{2}(\frac{b}{\sigma_x})^2] \end{aligned} \tag{4-45}$$

in which

$$\begin{aligned} \sigma_x &= \sqrt{E[x^2]} \\ \sigma_{\dot{x}} &= \sqrt{E[\dot{x}^2]} \\ \rho &= \frac{\sqrt{E[x\dot{x}]}}{\sigma_x \sigma_{\dot{x}}} \end{aligned} \tag{4-46}$$

and μ_0 is the zero crossing rate. $P^*(t)$ can be found as

$$P^*(t) = \int_b^{\infty} \frac{exp(-\frac{r^2}{2\sigma^2})}{\sigma\sqrt{2\pi}\,\mathrm{erfc}(\frac{\mathrm{b}}{\sqrt{2}\sigma})} \mathrm{erfc}[\frac{\mathrm{b - rc}}{\sqrt{2}\sigma(1-\mathrm{c}^2)^{\frac{1}{2}}}]\mathrm{dr} \tag{4-47}$$

where

$$c = exp(-\frac{\pi\varsigma\omega}{\omega_d}) \tag{4-48}$$

An estimate of $P^*(t)$ may be found in Mason and Iwan (1983). Substituting the expression for $\alpha(t)$ into Eqs. (4-38) and (4-39) yields the reliability function of the structure $W(t)$ and the first passage probability rate $p(t)$.

4.5 Examples

Using the above formulation, the responses of the soil layer and the structure are calculated for three different evolutionary earthquake models which simulate the soil layer as an SDOF oscillator, a uniform shear beam, and a nonuniform shear beam respectively. The SDOF filter has a natural frequency of 2 rad per second and a fraction of critical damping of 0.2. For the unifom beam model, its total thickness, is $l = 100$ ft, specific weight, $\gamma = 199.17$ pcf, shear modulus, $G = 1.0 \times 10^5$ psf, and damping ratio of 0.2 is assumed for all the modes. For the nonuniform shear beam model, the total thickness l is adjusted to 229.92 ft, constant $p = \frac{1}{3}$, and all the other parameters remain the same as those for the uniform beam. The beam parameters are chosen such that the natural frequency and the damping ratio of the first mode coincides with those of the SDOF filter for comparison purposes. In all three cases, the same structure having $\omega_0 = 1$ and a set of damping ratio $\zeta_0 = 0.05, 0.075$, and 0.15 is considered.

Figs. 4.2-4.4 present the results for the covariance responses of the structure and the covariance of the absolute ground acceleration using these three ground motion models excited by a suddenly applied white noise at bedrock. It is noted that the auto-covariance of the relative displacement S_{xx} and the auto-covariance of the relative velocity S_{vv} start from zero and increase to their stationary values while the cross-covariance S_{xv} decreases to zero. The results are presented for three different damping ratios, i.e., $\zeta_0 = 0.05, 0.075$, and 0.15. While the structural response is insensitive to the change in damping values in the initial stage, the damping effect becomes significant as time increases. The more damping the structure has, the less time is needed to achieve a stationary solution and a larger limiting value can be achieved. The covariance of the absolute ground acceleration shows essentially the same feature as that of the structure. In the shear beam models the first ten modes were used in the calculation.

Fig. 4.5 gives a comparison of the covariances of the structural response and the absolute ground acceleration for the three different soil models. In all the cases investigated, the damping ratio of the structures ζ_0 is assumed to be 0.05 and all the other parameters remain unchanged. A substantial increase of the absolute ground acceleraton is observed for the shear beam models whose first mode parameters are the same as those of the SDOF filter. Among the three models, the nonuniform beam model gives the largest estimation for the ground acceleration. As a consequence, the structural response shows the same trend. It may be concluded that using a Kanai-Tajimi model will result in a underestimate of the ground acceleration and the structural response and that the effect of nonuniform soil properties at the site should be considered in the aseismic design of critical structures.

Figs. 4.6-4.7 investigate the convergence of the solutions using the shear beam models. The results for total mode number $n = 1, 5, 10, 15$, and 16 are used. While the results for ground acceleration show a fairly rapid convergence, the results for structural response converge slower for the time interval considered. It is also observed that the results converge alternatively if more than four modes of the beam filter are used , i.e., the results approach the exact solutions from above when truncated at an odd number of beam modes, and from below if truncated at an even number of modes. Detailed studies of the convergence are beyond the scope of the thesis.

As shown in Chapter 2, the response of a structure under modulated white noise is an evolutionary processes defined by Priestly (1965), and the evolutionary spectral density matrix $\mathbf{G}(\omega, t)$ is given by Eq. (4-26). Figs. 4.8-4.11 present results for the evolutionary spectral density of the relative structural displacement and the absolute ground acceleration at times $t = 2, 5, 10$, and 30 sec. To study the asymptotic behavior of the responses, the unit step function envelope is employed. For small t, the curves appear flat, which implies a broad-bandness of the ground acceleration and the corresponding structural response at initial stage. As time approaches ∞, these curves become sharp and approach their stationary counterparts. It is obvious that not only the intensity changes with time but also the frequency content.

Significant differences in both structural response and ground acceleration can be seen for the three models used. A multi-peaked shape of the spectral density of the ground acceleration is observed in Figs. 4.9(b) and 4.10(b) for the beam models in contrast to the single-peaked shape shown in Fig. 4.8(b) for the SDOF model. Correspondingly, a structure may have much higher response using beam models than that predicted by the Kanai-Tajimi model, as shown in Figs. 4.8(a), 4.9(a), and 4.10(a).

Fig. 4.11 gives a comparison of the ground acceleration and structural response at a particular time using the three models in the frequency domain. Higher peak values of the evolutionary spectral density are observed for the beam models than for the Kanai-Tajimi model. The importance of the effect of local soil properties and the necessity of the introduction of improved ground motion models are again confirmed.

Some results of a reliability analysis of the structure using the three soil models are presented in Fig. 4.12. A Shinozuka-Sato type envelope, as shown in Fig. 4.12a, is used to model the nonstationarity of ground motion with finite energy. The envelope parameters are chosen such that $A = 2.32, \alpha = 0.09$, and $\beta = 1.49$ to simulate the S00E component of the May 18, 1940 Imperial Valley Earthquake, as employed by Corotis and Marshall (1977). For each model, the reliability function is calculated for three threshold levels $\frac{b}{\sigma} = 1, 1.5$, and 2 where σ is the corresponding stationary value of the square root of the covariance of structural response. The reliability decreases from an initial value of one to a limiting value. In contrast to the stationary case or the nonstationary case with a unit step envelope, the value is a constant between 0 and 1 instead of zero which verifies the argument made in the previous section about the deficiency in the simple definition of the first passage probability density function in the nonstationary case.

4.6 Conclusions

As an application of the simplified state-variable method to continuous systems, a unified formulation is presented to investigate the seismic response of structures as well as the absolute ground surface acceleration under a class of evolutionary ground motion models which simulate the dynamic behavior of soil subjected to a nonstationary excitation at the bedrock. These models include the uniform and nonuniform shear beams, and the SDOF oscillator Kanai-Tajimi model subjected to a nonstationary excitation at the base. In contrast to the previous studies including Lin (1987), explicit solutions are found for both structural response and absolute ground acceleration under the random impulse train earthquake model or the modulated white noise and their dynamic characteristics are investigated in detail.

The covariance response of the structure and the covariance of the absolute ground acceleration are calculated for all three soil models. A comparison shows that for the models with the same natural frequency and damping ratio in the first mode, a significant increase in both the ground motion and structural response are observed by using the beam models with the nonuniform model giving the largest absolute ground acceleration and structural response. Therefore, the conventional Kanai-Tajimi model may underestimate the ground motion and structural response and an improved ground motion model, such as the uniform or nonuniform shear beam model, which more properly describes the local soil properties, may be needed in the seismic analysis of critical structures. All of the three soil models considered, including the Kanai-Tajimi filter under a nonstationary excitation, show nonstationarity in both intensity and frequency content.

Some results for structural reliability are also presented. It is found that in the nonstationary case, the structural reliability generally does not approach zero which is justified by the fact that the structure may survive from a given earthquake event of finite energy. Therefore, the term of first passage probability density function, used to refer p(t) in the stationary case, will be improper in a general nonstationary case. As a remedy, a refined definition of the first passage probability density or more precise terminology as the "first passage probability rate" is suggested.

Since the formulation presented in this chapter is general, it can be applied to other ground motion models and different load envelopes. The approach may be applied without difficulty to the nonstationary analysis of structures, such as high-rising buildings or dams, which may be modeled as a beam structure in a simplified analysis.

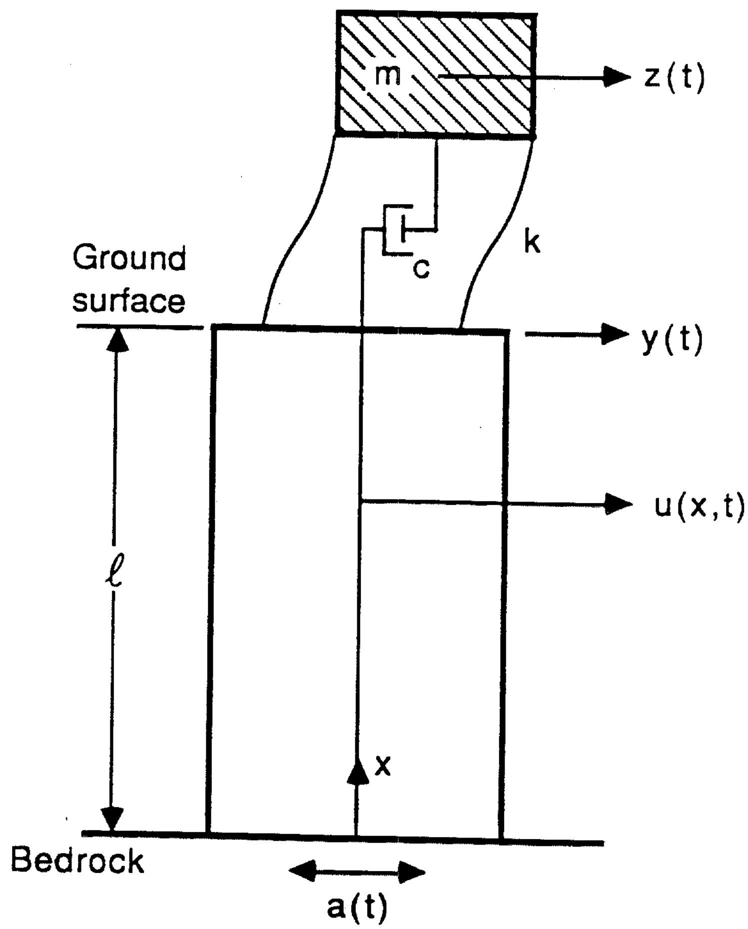


Figure 4.1. Single-degree-of-freedom structure subjected to a modulated white noise filtered by soil layers.

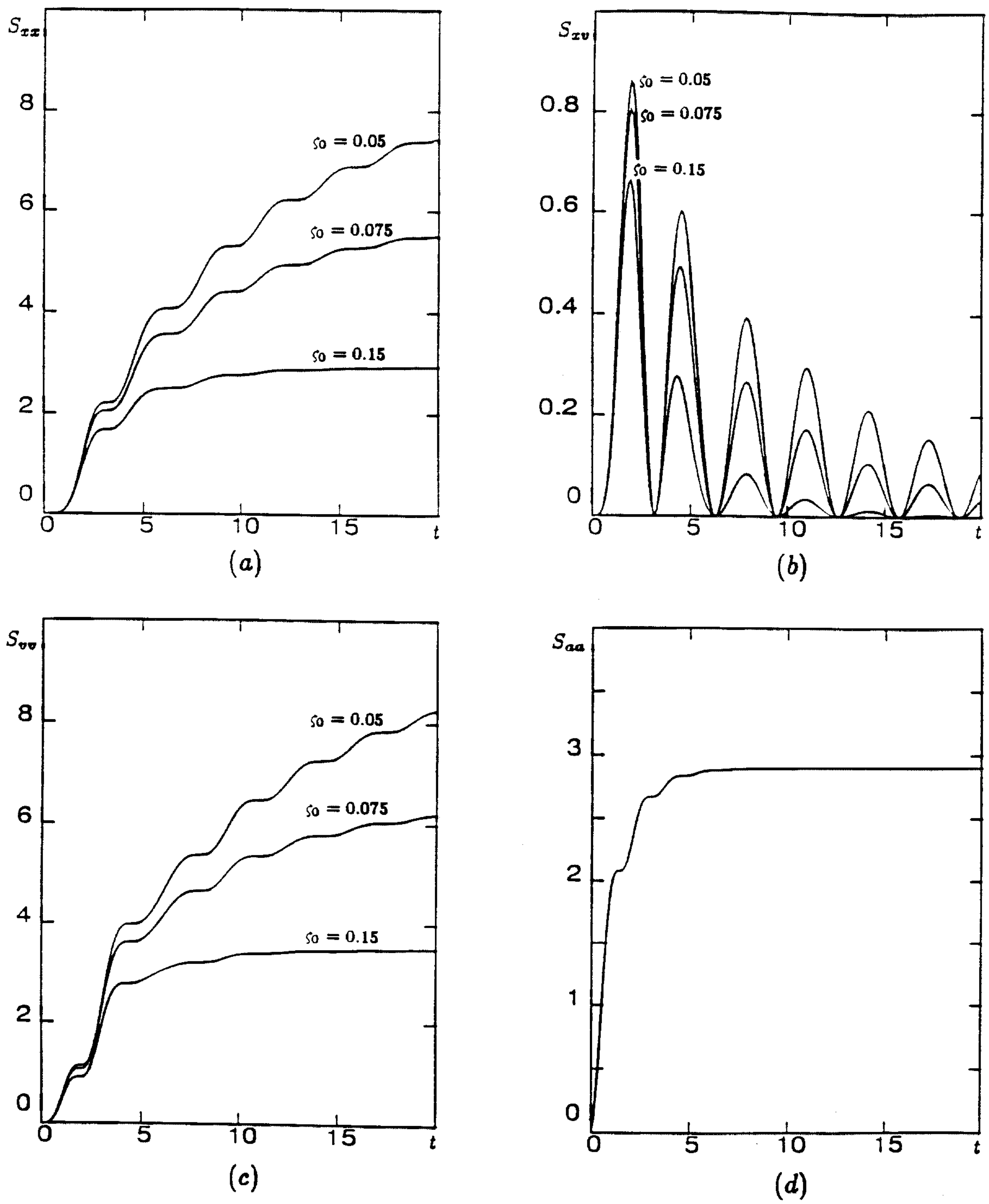


Figure 4.2. Covariances of the structural response and the absolute ground acceleration when using the evolutionary Kanai-Tajimi model. $\omega_0 = 1.0$, $\varsigma_0 = 0.05, 0.075$, and 0.15, $\omega_g = 2.0$, and $\varsigma_g = 0.2$.

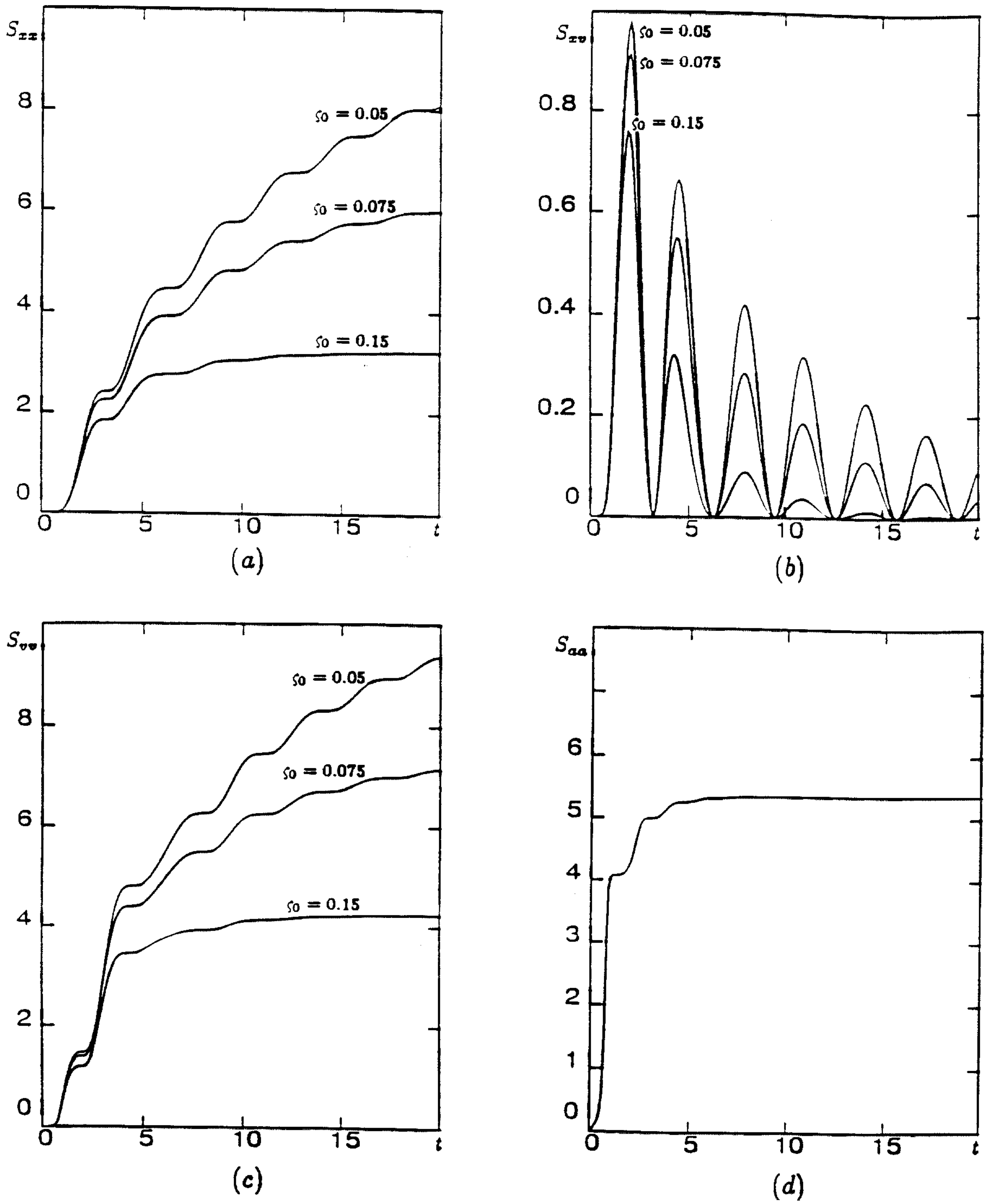


Figure 4.3. Covariances of the structural response and the absolute ground acceleration when using the evolutionary uniform shear beam model. $\omega_0 = 1.0$, $\varsigma_0 = 0.05, 0.075$, and 0.15. Beam parameters are chosen such that total thickness $l = 100$ ft, unit weight $\gamma = 199.17$ pcf, shear modulus $G = 1.0 \times 10^5$ psf, and for all the modes $\varsigma_k = 0.2$.

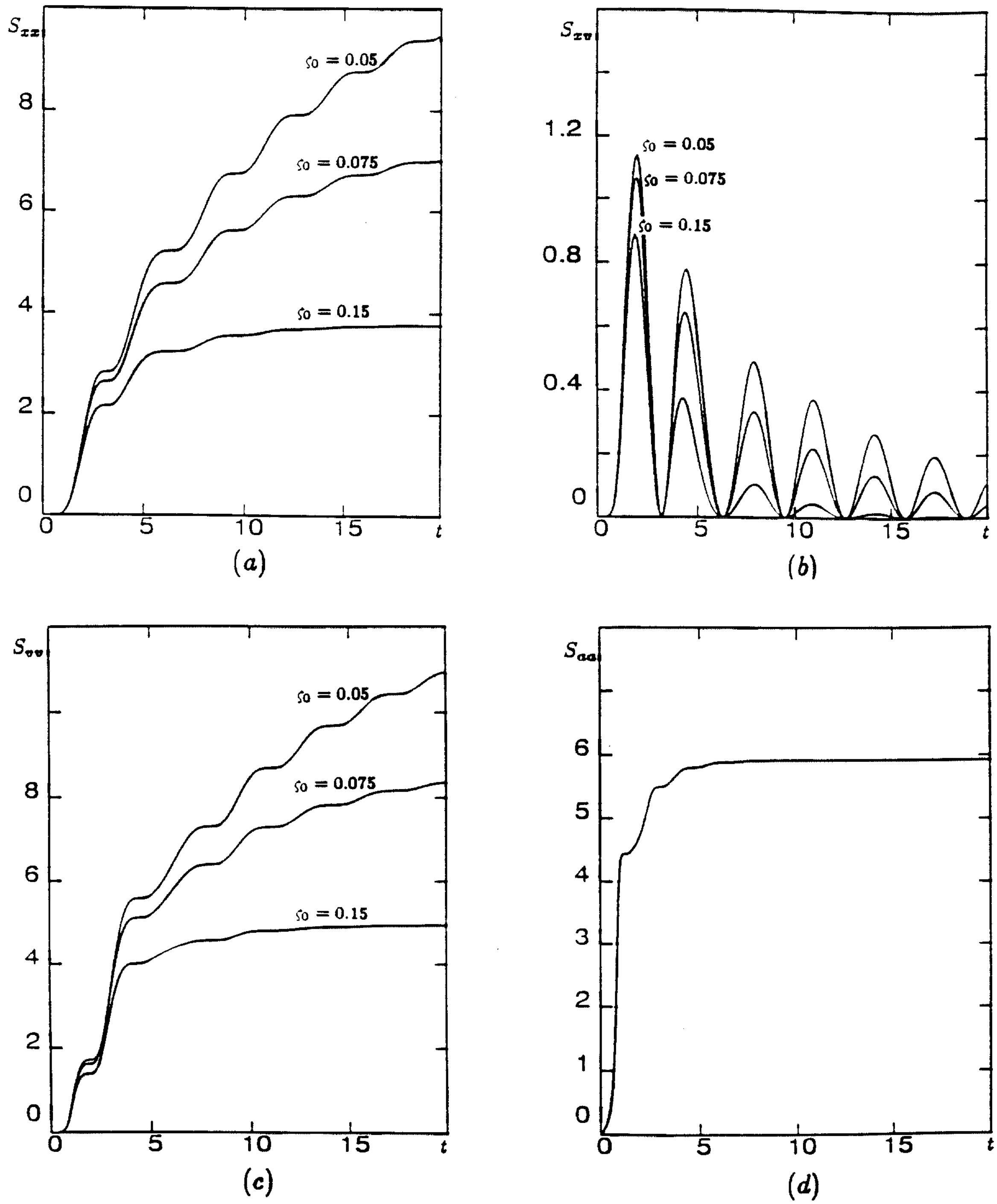


Figure 4.4. Covariances of the structural response and the absolute ground acceleration when using the evolutionary nonuniform shear beam model. $\omega_0 = 1.0$, $\varsigma_0 = 0.05, 0.075$, and 0.15. Beam parameters are chosen such that total thickness $l = 229.92$ ft, unit weight $\gamma = 199.17$ pcf, shear modulus $G = 1.0 \times 10^5$ psf, $p = \frac{1}{3}$, and for all the modes $\varsigma_k = 0.2$.

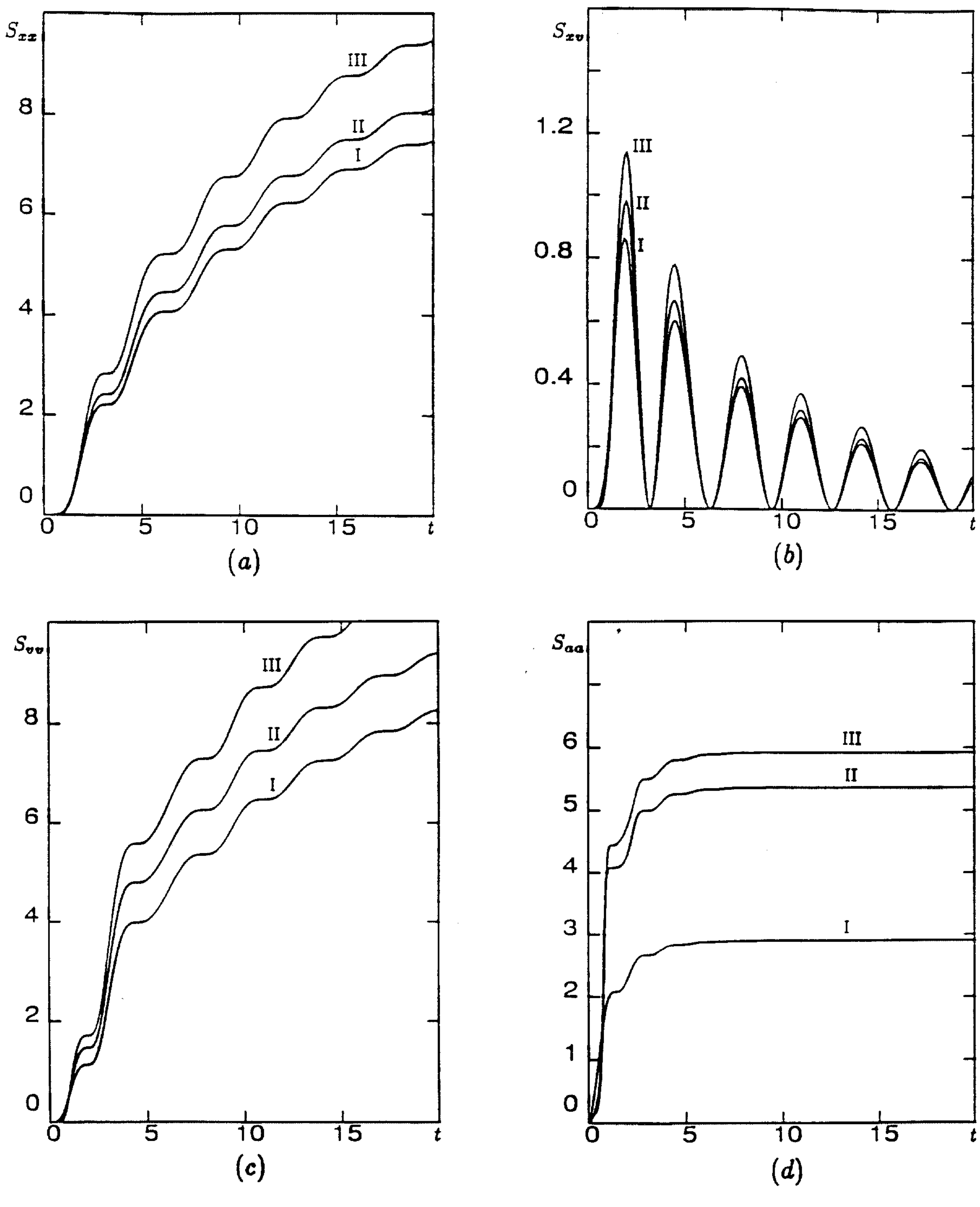


Figure 4.5. Comparison of the covariances of the structural response and the absolute ground acceleration for three evolutionary earthquake models: (I) Kanai-Tajimi model, (II) uniform beam model, and (III) nonuniform beam model. $\omega_0 = 1.0$ and $\zeta_s = 0.05$. Beam parameters are the same as in Figs. 4.3-4.4.

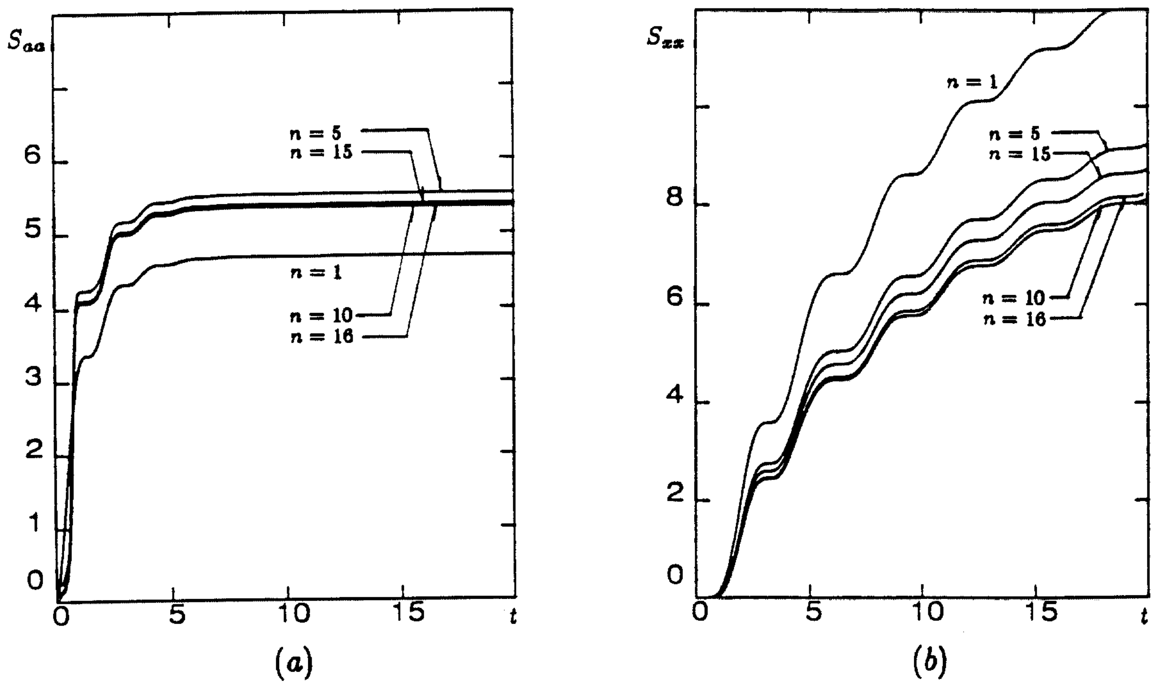


Figure 4.6. Convergence of the covariance S_{xx} and S_{aa} using uniform beam model. The beam parameters are the same as in Fig. 4.3.

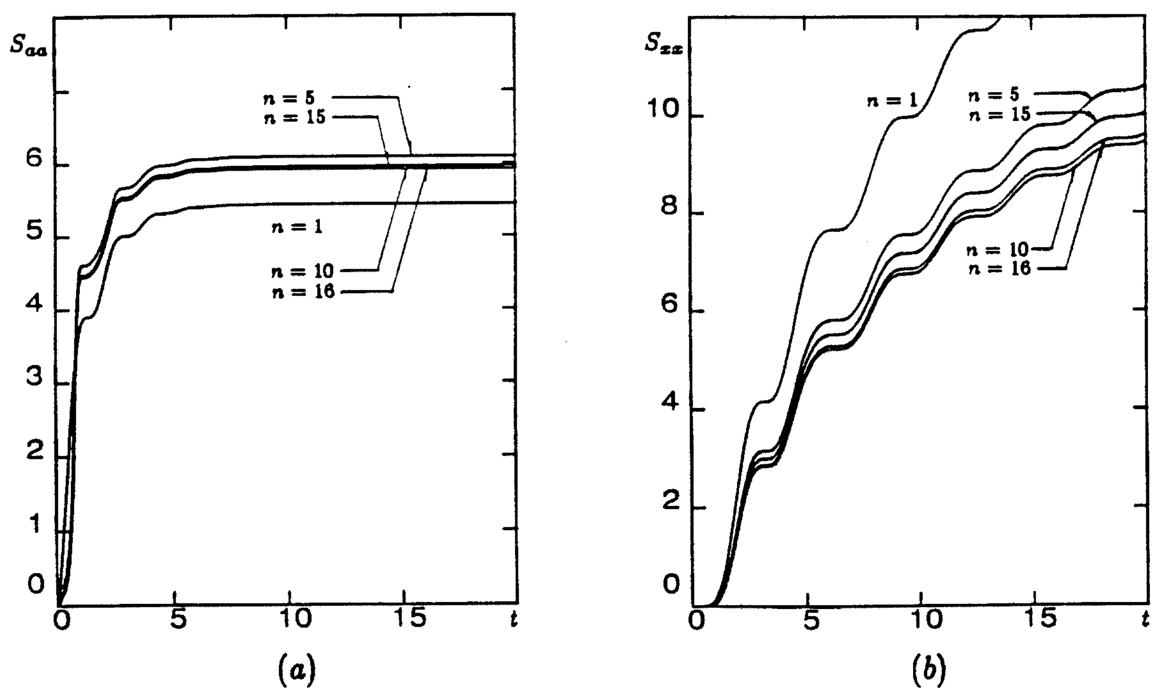


Figure 4.7. Convergence of the covariance S_{xx} and S_{aa} using nonuniform beam model. The beam parameters are the same as in Fig. 4.4.

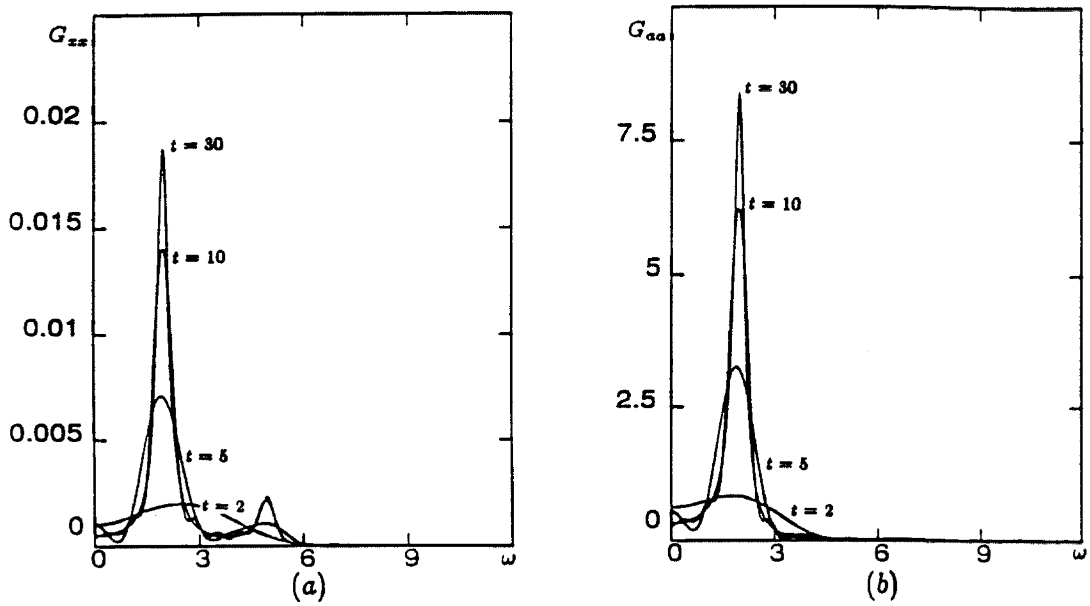


Figure 4.8. Evolutionary spectral density of the structural response G_{xx} and that of the absolute ground acceleration G_{aa} at $t = 2, 5, 10,$ and 30 second when using the evolutionary Kanai-Tajimi model with unit step envelope. $\omega_0 = 5.0, \zeta_0 = 0.05, \omega_g = 2.0,$ and $\zeta_g = 0.1$.

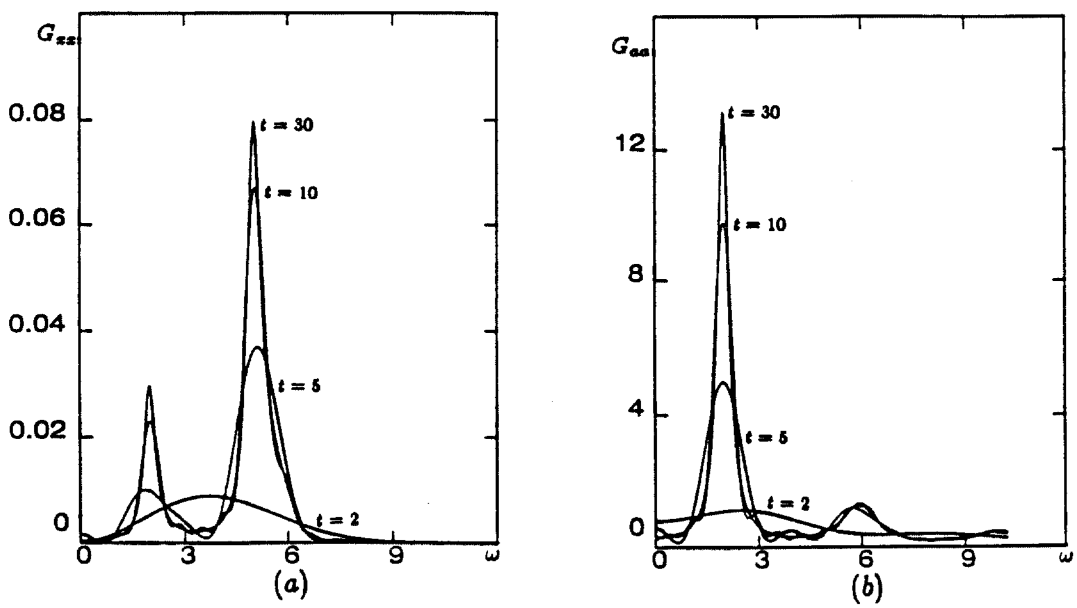


Figure 4.9. Evolutionary spectral density of the structural response G_{xx} and that of the absolute ground acceleration G_{aa} at $t = 2, 5, 10,$ and 30 second when using the uniform beam model with unit step envelope. $\omega_0 = 5.0, \zeta_0 = 0.05$. Beam parameters are the same as in Fig. 4.3.

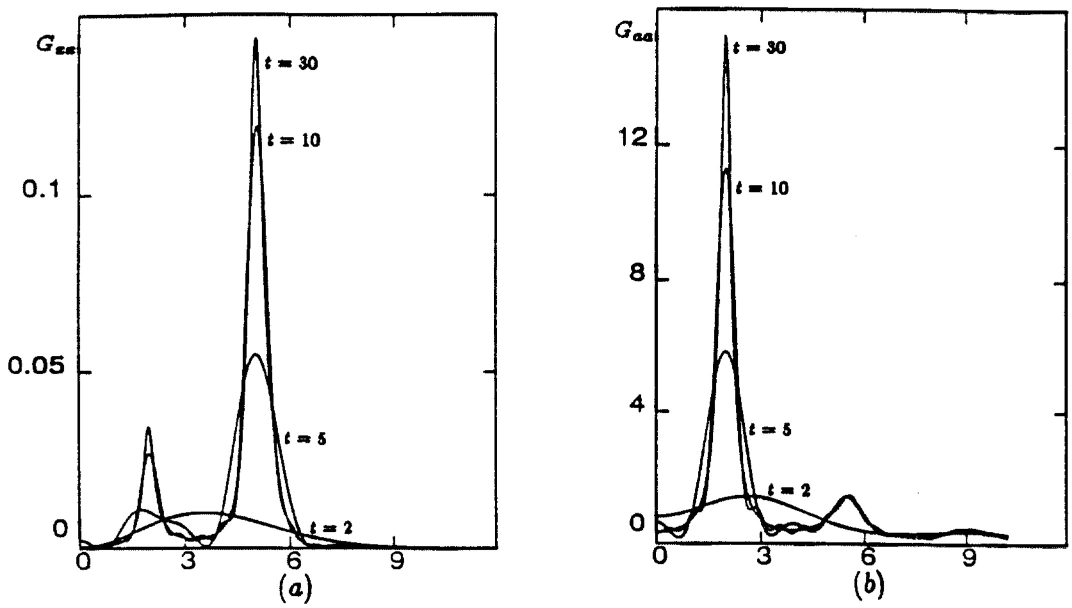


Figure 4.10. Evolutionary spectral density of the structural response G_{xx} and that of the absolute ground acceleration G_{aa} at $t = 2, 5, 10$, and 30 second when using the nonuniform beam model with unit step envelope. $\omega_0 = 5.0, \zeta_0 = 0.05$. Beam parameters are the same as in Fig. 4.4.

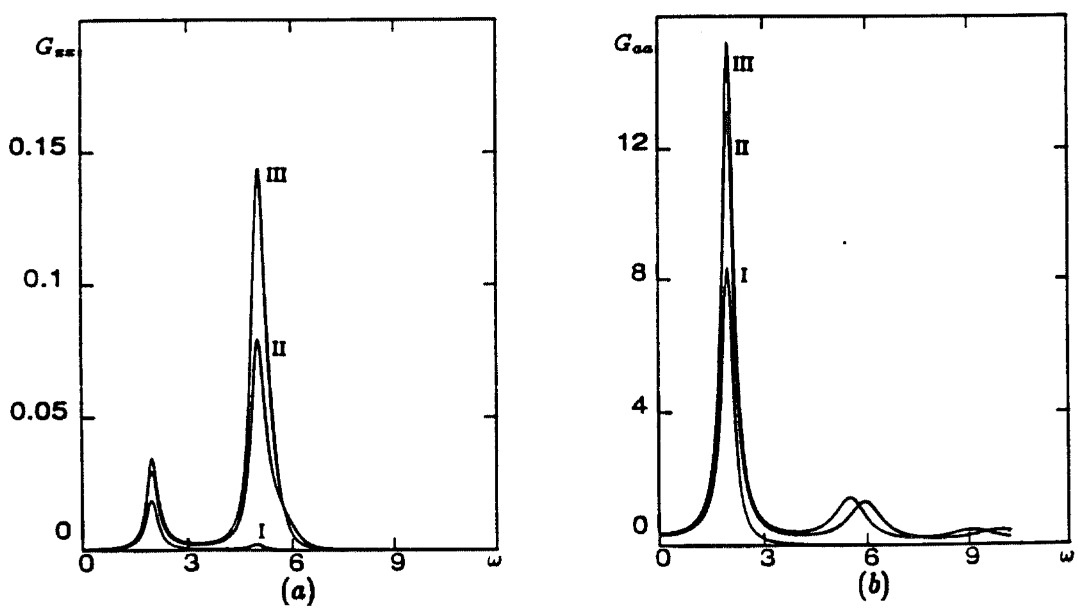


Figure 4.11. Comparison of evolutionary spectral densities of the structural response and the absolute ground acceleration at $t = 30$ second when using three evolutionary earthquake models: (I) Kanai-Tajimi model, (II) uniform beam model, and (III) nonuniform beam model with a unit step envelope. Model parameters are the same as in Figs. 4.8-4.10.

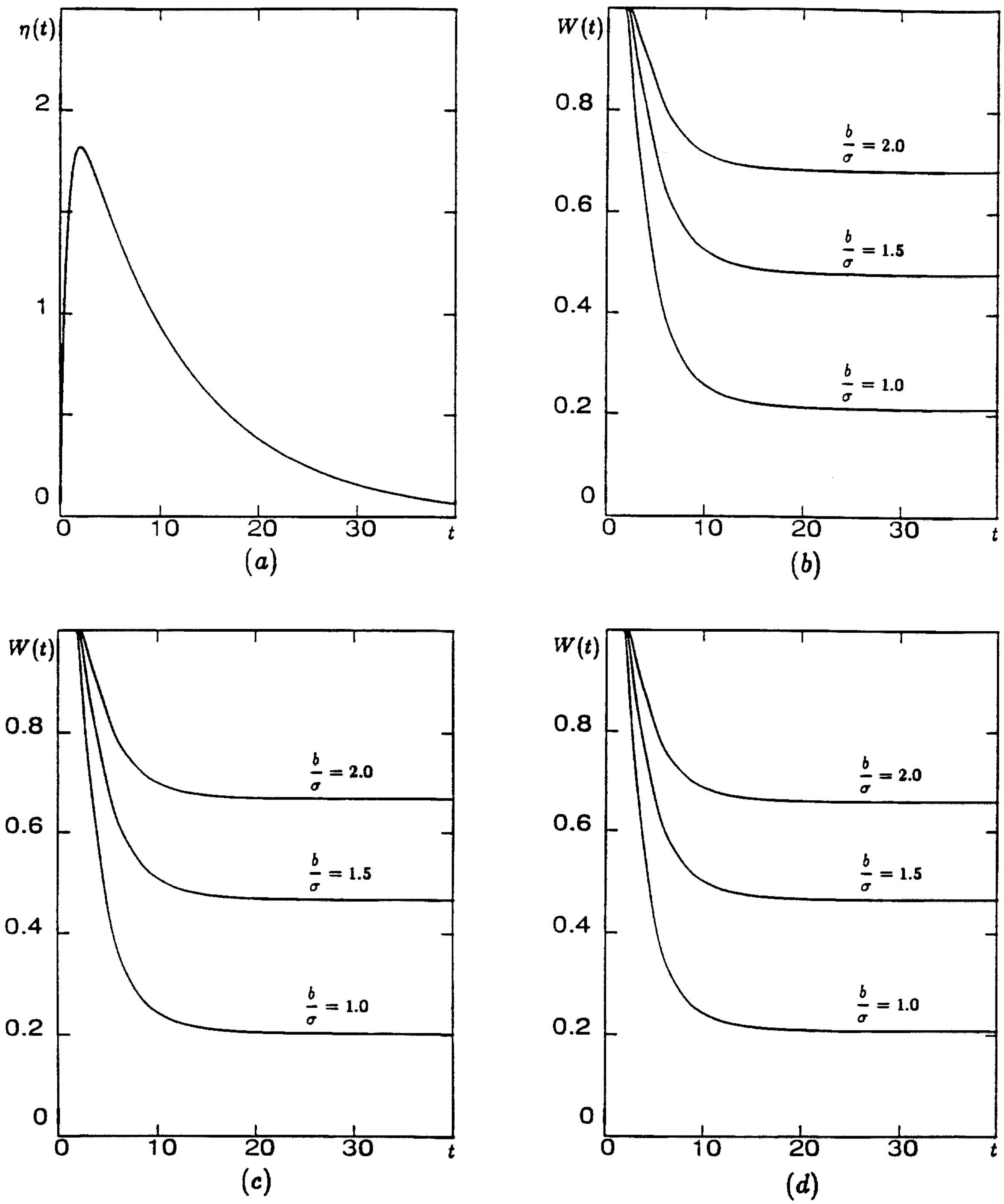


Figure 4.12. Reliability function of the structure for three different threshold levels $\frac{b}{\sigma} = 1.0, 1.5$, and 2.0 when using three evolutionary earthquake models: (b) Kanai-Tajimi model, (c) uniform beam model, and (d) nonuniform beam model with a Shinozuka-Sato envelope. Envelope parameters are $A = 2.32, \alpha = 0.09$, and $\beta = 1.49$. Model parameters are the same as in Figs. 4.8-4.10.

Chapter 5

Seismic Response of Structures Under Combined Horizontal and Vertical Ground Motions

5.1 Introduction

In the seismic analysis of structures, the ground motion is often modeled as a stochastic process in order to account for uncertainty. A modulated white noise model is commonly used due to its simplicity and ability to capture the main features of earthquake ground motion, such as intensity, duration and frequency content. Earthquakes generally produce both horizontal and vertical components of motion, but the effect of the vertical component is often neglected based on the commonly accepted argument that the vertical component is smaller than the horizontal component, and that structures generally have more inherent resistance in the vertical direction.

Building codes have given some special consideration to the effects of vertical ground motion for buildings assigned to the seismic performance categories C and D. Even though factors of safety provided for gravity load design, coupled with the small likelihood that maximum live loads and earthquake loads would occur simultaneously, introduce some protection against the effects of the vertical component of ground motion, special requirements are still needed. For instance, a variation of 20 percent is placed on the dead load for standard structures. More detailed specification for providing protection against the possible effects of the vertical component of ground motion may be found in ATC-3-06 (1984) and the Uniform Building Code (1988). This treatment of the vertical ground motion in the building codes has been questioned by some researchers since it ignores the fact that the response of systems under combined loads is a dynamic process and the effects of the vertical component cannot be judged only by its magnitude.

The analysis of structures under combined horizontal and vertical ground motion may be reduced to a parametric vibration problem where the vertical component enters the analysis as a parametric excitation. Existence of the parametric excitation may change the response significantly, especially near critical regions of instability. Extensive studies have been carried out in parametric random vibration analysis and some stability criteria have been found by Kozin (1963), Caughey and Gray (1965), Infante (1968), Benaroya and Rehak (1989), etc., for general linear systems where the excitation is modeled as a stationary random process. A detailed review of parametric random vibration can be found in Ibrahim (1985).

Some investigations have been performed to study the dynamic behavior of structures under combined horizontal and vertical loads. Shinozuka and Henry (1965) investigated the response of a vertical cantilever subjected to combined random horizontal motion at its base and a deterministic axial load at its top. Ariaratnam (1967) studied the dynamic stability conditions of a hinged column and some results were proposed as design guides. Wirsching and Yao (1971) conducted experiments to investigate the dynamic stability of a column which is subjected to a random axial load. Iyengar and Shinozuka (1971) presented Monte Carlo results for the effect of vertical acceleration on the behavior of tall columns. Lin and Shih (1980, 1982) obtained the second moment response of structures under combined loads by numerically solving the moment equations. More recently, Nielsen and Kiremidjian (1988) investigated the reliability of tall columns under combined loads which are modeled as filtered modulated white noise. While the explicit solution for the second moment response can be obtained in the stationary case, no such solution has, to the author's knowledge, yet been presented for the nonstationary case.

As an application of the simplified state-variable method, explicit solutions are herein presented for the nonstationary covariance response of a simple linear structure subjected to a suddenly applied white noise excitation. The stationary correlation matrix of the response can be obtained by considering the limiting case of the nonstationary solution as time approaches infinity. The stability of these solutions is then discussed. The discussion of stability is extended to the case where a general envelope function is used. Numerical results are presented to illustrate the dynamic behavior of the system with different dampings under different intensities of the parametric excitation. Some suggestions are made for design considerations based on the results obtained.

5.2 Formulation

Consider a simple structural model which consists of a massless beam-column supporting a concentrated mass. The column is assumed to be rigidly fixed to the ground at its lower end, as shown in Fig. 5.1. The system is excited by an earthquake ground motion in vertical and horizontal directions simultaneously.

5.2.1 Governing Differential Equation

The system is governed by the following equations:

$$\begin{aligned} &EIy_{\xi\xi} = P(x-y) + F(l-\xi) \\ &F = -\left[m(x_{tt} + H(t)) + cx_t\right] \\ &P = m(g + V(t)) \\ &y(0) = y_\xi(0) = 0 \qquad y(l) = x \end{aligned} \tag{5-1}$$

In the above equation, x is the horizontal displacement of the mass relative to the ground and $y(\xi)$ is that of the column relative to the ground. F and P denote the shear and the axial force at the cross section, $H(t)$ and $V(t)$ are the horizontal and vertical components of ground motion respectively. As usual, g is the acceleration of gravity, EI, m, and c represent the bending rigidity, mass, and viscous damping respectively. l gives the height of the column, and the subscripts $(\ )_{\xi\xi}, (\ )_{tt}, (\ )_t$ denote the spatial and time derivatives.

Based on the theory of elastic stability (Timoshenko and Gere, 1961), the shear force F can be determined as:

$$F = \frac{3EIx}{l^3}\frac{1}{\chi(u)} \tag{5-2}$$

where

$$\chi(u) = \frac{3(\tan u - u)}{u^3}$$
$$u = \frac{\pi}{2}\sqrt{\frac{P}{P_{cr}}} \qquad (5-3)$$

P_{cr} is the buckling load of the clamped column, i.e.,

$$P_{cr} = \frac{\pi^2 EI}{4l^2}$$

It is assumed that

$$\frac{1}{\chi(u)} \approx 1 - \frac{P}{P_{cr}} \qquad (5-4)$$

which is justified by tabulated data for $\frac{1}{\chi(u)}$ and $1 - \frac{P}{P_{cr}}$, as $\frac{P}{P_{cr}}$ varies from zero to one (Lin and Shih, 1980). A comparison shows that less than one percentage of error may be generated by this replacement.

Using the assumption in Eq. (5-4), the original nonlinear problem may be reduced to a linear SDOF system with time-variant stiffness as

$$mx_{tt} + cx_t + \frac{3EI}{l^3}(1 - \frac{P}{P_{cr}})x = -mH(t) \qquad (5-5)$$

Introduce

$$\omega^2 = \frac{3EI}{ml^3}(1 - \frac{mg}{P_{cr}})$$
$$\varsigma = \frac{c}{2m\omega} \qquad (5-6)$$
$$h(t) = -H(t)$$
$$v(t) = \alpha V(t)$$

where the constant α is defined as

$$\alpha = \frac{m}{P_{cr} - mg} \qquad (5-7)$$

Eq. (5-5) may then be reduced to

$$\frac{d^2}{dt^2}z(t) + 2\varsigma\omega\frac{d}{dt}z(t) + \omega^2 z(t) - v(t)z(t) = h(t) \qquad (5-8)$$

which will be used throughout the present study. Note that $h(t)$ and $v(t)$ have different dimensions in Eq. (5-8). The above derivation follows Lin and Shih (1980) and is included here for completeness.

Using state variable notation, Eq. (5-8) becomes

$$\frac{d}{dt}\mathbf{Y}(t) = \mathbf{A}\mathbf{Y}(t) + \mathbf{F}(t) + \mathbf{B}(t)\mathbf{Y}(t) \tag{5-9}$$

where

$$\begin{aligned} \mathbf{Y}(t) &= \begin{pmatrix} z(t) \\ \dot{z}(t) \end{pmatrix} \qquad \mathbf{A} = \begin{pmatrix} 0 & 1 \\ -\omega^2 & -2\varsigma\omega \end{pmatrix} \\ \mathbf{F}(t) &= \begin{pmatrix} 0 \\ h(t) \end{pmatrix} \qquad \mathbf{B}(t) = \begin{pmatrix} 0 & 0 \\ v(t) & 0 \end{pmatrix} \end{aligned} \tag{5-10}$$

Note that in Eq.(5-9), the random time-varying function $v(t)$, a quantity related to the vertical component of the ground motion, plays the role of a parametric excitation. Therefore, Eq. (5-9) governs the dynamic behavior of the structure under combined parametric and external random excitations.

5.2.2 Specification of Excitations

In Eq. (5-8), $h(t)$ and $v(t)$ are related to the horizontal and vertical components of ground acceleration respectively. As indicated in Appendix II, these two components may be modeled as evolutionary random processes of the type

$$a(t) = \eta(t)n(t) \tag{5-11}$$

where $a(t)$ is ground excitation, $n(t)$ is a stationary Gaussian white noise and $\eta(t)$ is a slowly varying deterministic envelope function which is used to account for the nonstationarity of the excitation. Different envelopes have been employed by different investigators to capture the main features of earthquakes such as intensity, frequency content, and duration.

Comparing the vertical and horizontal components of an earthquake ground motion, the former is less intensive and has a greater portion of energy distributed in the higher frequency region (Housner, 1961). Some statistical research has shown that the ratio of the maximum vertical acceleration and the maximum horizontal acceleration generally ranges from 0.2 to 0.8, but may be greater than 1 in some cases (Hattori, 1978). Some seismological studies have also shown that the vertical component of the ground motion can be highly correlated with the horizontal component at a measured station far from the source of disturbance and are nearly uncorrelated near the source (Bouchon, 1978; Hanks, 1975).

In the present study, the parametric and external excitations will be modeled as modulated white noise with the same envelope. That is

$$\begin{aligned} h(t) &= \eta(t)w_1(t) \\ v(t) &= \eta(t)w_2(t) \end{aligned} \tag{5-12}$$

where $w_1(t)$ and $w_2(t)$ are assumed to be white noise processes with the properties:

$$E[w_1(t)] = 0$$

$$E[w_2(t)] = 0$$

and

$$\begin{aligned} E[w_1(t)w_1(t+\tau)] &= S_{11}\delta(\tau) \\ E[w_2(t)w_2(t+\tau)] &= S_{22}\delta(\tau) \\ E[w_1(t)w_2(t+\tau)] &= S_{12}\delta(\tau) \end{aligned} \tag{5-13}$$

in which S_{11} and S_{22} are constants representing intensities of the two white noise processes and S_{12} specifies their correlation. The unit step envelope function

$$\eta(t) = \begin{cases} 1, & \text{if } t \geq 0; \\ 0, & \text{otherwise.} \end{cases} \tag{5-14}$$

is employed unless otherwise specified.

5.3 Solutions for Uncorrelated Vertical and Horrizontal Components

As mentioned above, seismological studies have shown that the horizontal and vertical components may be nearly uncorrelated near the source. Therefore, as the first example, uncorrelation of the two components is assumed in this section. That is

$$S_{12} = 0$$

A zero-mean initial condition is also assumed for simplicity.

As discussed in Chapter 2, the general solution of Eq. (5-9) with a zero-mean initial condition can be expressed as

$$\begin{aligned} \mathbf{Y}(t) &= \int_0^t \mathbf{\Phi}(t-s)(\mathbf{F}(s) + \mathbf{B}(s)\mathbf{Y}(s))ds \\ &= \mathbf{Y}^{(0)}(t) + \int_0^t \mathbf{\Phi}(t-s)\mathbf{B}(s)\mathbf{Y}(s)ds \end{aligned} \tag{5-15}$$

where $\mathbf{\Phi}(t)$ is the fundamental solution of the system and $\mathbf{Y}^{(0)}(t)$ is defined as

$$\mathbf{Y}^{(0)}(t) = \int_0^t \mathbf{\Phi}(t-s)\mathbf{F}(s)ds \tag{5-16}$$

It is clear that $\mathbf{Y}^{(0)}(t)$ is the solution when the parametric excitation does not exist.

It follows that the mean value of the response is given as

$$E[\mathbf{Y}(t)] = \int_0^t \mathbf{\Phi}(t-s)E[\mathbf{B}(s)\mathbf{Y}(s)]ds \tag{5-17}$$

and the mean square response is expressed as

$$\begin{aligned}
E[\mathbf{Y}(t_1)\mathbf{Y}^T(t_2)] &= E[\mathbf{Y}^{(0)}(t_1)\mathbf{Y}^{(0)T}(t_2)] \\
&+ \int_0^{t_1} \mathbf{\Phi}(t_1-s_1)E[\mathbf{B}(s_1)\mathbf{Y}(s_1)\mathbf{Y}^{(0)T}(t_2)]ds_1 \\
&+ \int_0^{t_2} E[\mathbf{Y}^{(0)}(t_1)\mathbf{Y}^T(s_2)\mathbf{B}^T(s_2)]\mathbf{\Phi}^T(t_2-s_2)ds_2 \\
&+ \int_0^{t_1}\int_0^{t_2} \mathbf{\Phi}(t_1-s_1)E[\mathbf{B}(s_1)\mathbf{Y}(s_1)\mathbf{Y}^T(s_2)\mathbf{B}^T(s_2)]\mathbf{\Phi}^T(t_2-s_2)ds_1ds_2
\end{aligned} \tag{5-18}$$

It has been shown by Samuels (1960) and later confirmed by Benaroya and Rehak (1989) that $\mathbf{Y}(t)$ and $\mathbf{B}(t)$ are uncorrelated in the case that the excitations are the stationary white noise processes and this conclusion can be extended to the case of nonstationary modulated white noise. Therefore, the following holds.

$$\begin{aligned}
&E[y_i(t)b_{jk}(t)] = E[y_i(t)]E[b_{jk}(t)] \\
&E[b_{ij}(t_1)b_{mn}(t_2)y_k(t_1)y_l(t_2)] = E[b_{ij}(t_1)b_{mn}(t_2)]E[y_k(t_1)y_l(t_2)]
\end{aligned} \tag{5-19}$$

Using the above properties in Eqs. (5-17) and (5-18) leads to

$$E[\mathbf{Y}(t)] = \mathbf{O} \tag{5-20}$$

and

$$\begin{aligned}
\mathbf{Q}(t_1,t_2) &= \mathbf{Q}^{(0)}(t_1,t_2) \\
&+ \mathbf{L}\int_0^{min(t_1,t_2)} S_{22}Q_{11}(\tau,\tau)\mathbf{P}(t_1-\tau)\mathbf{P}^T(t_2-\tau)d\tau\mathbf{L}^T
\end{aligned} \tag{5-21}$$

where

$$\mathbf{Q}(t_1,t_2) = E[\mathbf{Y}(t_1)\mathbf{Y}^T(t_2)]$$
$$\mathbf{Q}^{(0)}(t_1,t_2) = E[\mathbf{Y}^{(0)}(t_1)\mathbf{Y}^{(0)T}(t_2)] \qquad (5-22)$$

$\mathbf{L}$ and $\mathbf{P}(t)$ are the same as defined in Chapter 2. Note that $\mathbf{Q}^{(0)}(t_1,t_2)$ is the solution when the parametric excitation is ignored.

5.3.1 Stationary Correlation Matrix

The stationary correlation matrix of the response may be obtained by taking the limit in Eq. (5-21) as time approaches infinity. Assume that

$$t_2 = t_1 + \tau \qquad (5-23)$$

and denote the stationary correlation matrix as

$$\mathbf{R}(\tau) = \mathbf{R}(t_2 - t_1) = \mathbf{Q}(t_1,t_2) \qquad (5-24)$$

Letting t_1 approach infinity in the solution (5-21) yields

$$\mathbf{R}(\tau) = \mathbf{R}^{(0)}(\tau) + S_{22}R_{11}(0)\mathbf{L}\int_0^{+\infty} \mathbf{P}(s)\mathbf{P}^T(s+\tau)ds\mathbf{L}^T \qquad (5-25)$$

where $\mathbf{R}(\tau)$ is the correlation matrix of the response under combined external and parametric random excitations while $\mathbf{R}^{(0)}(\tau)$ is the correlation matrix for the external load alone. $R_{11}(\tau)$ denotes the first row, first column component of $\mathbf{R}(\tau)$.

Note that

$$\mathbf{R}^{(0)}(\tau) = S_{11}\mathbf{L}\int_0^{+\infty} \mathbf{P}(s)\mathbf{P}^T(s+\tau)ds\mathbf{L}^T \qquad (5-26)$$

Thus, substituting Eq. (5-26) into Eq. (5-25) yields

$$\mathbf{R}(\tau) = (1 + \frac{S_{22}}{S_{11}}R_{11}(0))\mathbf{R}^{(0)}(\tau) \qquad (5-27)$$

5.3.1.1 Solution for $\mathbf{R}^{(0)}(\tau)$

The explicit solution for $\mathbf{R}^{(0)}(\tau)$ may be obtained from Eq. (5-26) as

$$\begin{aligned}
\mathbf{R}^{(0)}(\tau) &= S_{11}\mathbf{L}\mathbf{I}(\tau)\mathbf{L}^T \\
\mathbf{I}(\tau) &= \int_0^{+\infty}\mathbf{P}(s)\mathbf{P}^T(s+\tau)ds \\
\mathbf{P}(\tau) &= e^{-\varsigma\omega\tau}\begin{pmatrix}\cos\omega_d\tau \\ \sin\omega_d\tau\end{pmatrix} \\
\mathbf{L} &= \begin{pmatrix} 0 & \dfrac{1}{\omega_d} \\ 1 & -\dfrac{\varsigma\omega}{\omega_d}\end{pmatrix}
\end{aligned} \tag{5-28}$$

The components of $\mathbf{I}(\tau)$ are expressed as

$$\begin{aligned}
I_{11}(\tau) &= \frac{e^{-\varsigma\omega\tau}}{4}\left(\frac{1+\varsigma^2}{\varsigma\omega}\cos\omega_d\tau - \frac{\omega_d}{\omega^2}\sin\omega_d\tau\right) \\
I_{12}(\tau) &= \frac{e^{-\varsigma\omega\tau}}{4}\left(\frac{\omega_d}{\omega^2}\cos\omega_d\tau + \frac{1+\varsigma^2}{\varsigma\omega}\sin\omega_d\tau\right) \\
I_{21}(\tau) &= \frac{e^{-\varsigma\omega\tau}}{4}\left(\frac{\omega_d}{\omega^2}\cos\omega_d\tau - \frac{1-\varsigma^2}{\varsigma\omega}\sin\omega_d\tau\right) \\
I_{22}(\tau) &= \frac{e^{-\varsigma\omega\tau}}{4}\left(\frac{1-\varsigma^2}{\varsigma\omega}\cos\omega_d\tau + \frac{\omega_d}{\omega^2}\sin\omega_d\tau\right)
\end{aligned} \tag{5-29}$$

Eqs. (5-28) and (5-29) give the solution for the stationary correlation matrix of response under the external excitation alone.

5.3.1.2 Solution for $\mathbf{R}(\tau)$

The solution for $\mathbf{R}(\tau)$ can be obtained by first solving for $R_{11}(0)$. Letting $\tau = 0$ in equation (5-27) gives

$$\left(1 - \frac{S_{22}}{S_{11}}R_{11}^{(0)}(0)\right)R_{11}(0) = R_{11}^{(0)}(0) \tag{5-30}$$

In order to have a finite positive solution for $R_{11}(0)$, it is required that

$$1 - \frac{S_{22}}{S_{11}}R_{11}^{(0)}(0) > 0 \tag{5-31}$$

In the case that the above inequality holds,

$$R_{11}(0) = \frac{S_{11}}{4\varsigma\omega^3 - S_{22}} \qquad (5-32)$$

Therefore,

$$\mathbf{R}(\tau) = \frac{1}{1-\lambda}\mathbf{R}^{(0)}(\tau) \qquad (5-33)$$

where

$$\lambda = \frac{S_{22}}{4\varsigma\omega^3} \qquad (5-34)$$

It follows from inequality (5-31) that $0 \leq \lambda < 1$. Eqs. (5-33)-(5-34) complete the stationary correlation matrix of the response under the combined parametric and external excitations.

Eq. (5-33) indicates that the stationary covariance solution under the combined external and parametric excitations is equal to the solution under the external load alone multiplied by a constant factor, referred to as the *parametric amplification factor*, which depends only on the nondimensional parameter λ. Rewritting Eq. (5-33) yields

$$\mathbf{R}(\tau) = A_p\mathbf{R}^{(0)}(\tau) \qquad (5-35)$$

where the parametric amplification factor, A_p, is defined as

$$\begin{aligned} A_p &= \frac{1}{1-\lambda} \\ &= \frac{4\varsigma\omega^3}{4\varsigma\omega^3 - S_{22}} \end{aligned} \qquad (5-36)$$

Note that λ and, therefore, A_p are functions of system parameters and the intensity of the parametric excitation, and is independent of the intensity of the external excitation. $A_p \geq 1$ implying that existence of parametric excitation will magnify the correlation matrix and, therefore, the mean square response of the structure. Rewrite Eq. (5-31) as

$$\mathbf{R}(\tau) = \mathbf{R}^{(0)}(\tau) + \frac{\lambda}{1-\lambda}\mathbf{R}^{(0)}(\tau) \qquad (5-37)$$

An alternative interpretation is that an extra part will be introduced due to the existence of the parametric excitation in addition to the solution under the external excitation alone. The above conclusions hold for the spectral densities of the response due to the well-known Wiener-Khintchine relationship.

5.3.1.3 Stability Criterion

The inequality (5-31) gives the stability criterion for the stationary solution. Substituting

$$R_{11}^{(0)}(0) = \frac{S_{11}}{4\varsigma\omega^3} \qquad (5-38)$$

into (5-31), the stability condition becomes

$$S_{22} < 4\varsigma\omega^3 \qquad (5-39)$$

which agree with the result from the previous studies such as Ibrahim (1985) and Benaroya and Rehak (1989).

It is noted that the stability condition (5-39) depends only on the system damping ς, natural frequency ω, and the intensity of the parametric excitation S_{22}. A significant increase in the mean square response of the structure may be observed if these parameters are close to the region of instability. Therefore, the effect of the vertical ground motion may not be neglected even if its magnitude is comparatively smaller than that of the vertical counterpart. System nonlinearity could prevent the response from increasing infinitely, but a significant effect would still be expected. Structures with higher damping and frequency will exhibit better dynamic performance under the combined loads. If a structure suffers severe damage during an earthquake, the structural response may become unstable due to the decrease of natural frequency of the structure caused by the damage.

5.3.1.4 Examples

Some numerical results for the stationary solution are presented in Figs. 5.2 and 5.3. Figure 5.2 gives the variation of the parametric amplification factor A_p versus the nondimensional parameter λ which is a function of damping ratio, natural frequency, and the intensity of the vertical ground motion. Note that $A_p \geq 1$. This figure shows that the vertical motion can have a considerable effect on the correlation matrix of the response, especially near the region of instability. Therefore, even when the vertical ground motion itself is small, the response can still be significantly increased as λ approaches 1.

Figure 5.3 shows the stationary auto- and cross-correlation of the response for three different values of the intensity of the parametric excitation, namely, $S_{22} = 0, 0.12$, and 0.24. The system parameters are chosen such that $\varsigma = 0.1$ and $\omega = 1.0$. The intensity of the external excitation is assumed to be 1.0. It is clear that the larger the vertical ground motion, the greater will be the auto- and cross-correlation response of the structure. For the auto-correlation response, the curves start from the stationary values of the mean square response corresponding to $\tau = 0$ which are maxima, and then exhibit an oscillatory decrease to zero as τ approaches infinity. The cross-correlation response has the same trend as the auto-correlation response except that it starts from zero. Note that $R_{xv}(\tau) = -R_{vx}(\tau)$.

5.3.2 Nonstationary Covariance Matrix

In the nonstationary case, the main emphasis is placed on the solution for $\mathbf{Q}(t,t)$ by setting $t_1 = t_2 = t$ in Eg. (5-21). $\mathbf{Q}(t_1, t_2)$ may be obtained by the same consideration as in Chapter 2 without difficulty. An explicit solution for the correlation matrix is presented for the case of a unit step envelope function. However, in the stability analysis a more general form of envelope function is employed.

5.3.2.1 Solution for the Nonstationary Covariance Matrix $\mathbf{Q}(t)$

Recall equation (5-21) and let $t_1 = t_2 = t$. Then

$$\mathbf{Q}(t) = \mathbf{Q}^{(0)}(t) + S_{22}\mathbf{L} \int_0^t \mathbf{P}(t-\tau)\mathbf{P}^T(t-\tau) Q_{11}(\tau) d\tau \mathbf{L}^T \qquad (5-40)$$

which is the Volterra Integral Equation of the second type. Eq. (5-40) implies that the covariance response for uncorrelated combined loads is a superposition of the response for the external excitation alone and a correction term. Performing a Laplace transform of Eq. (5-40) yields

$$\mathbf{Q}(s) = Q^{(0)}(s) + S_{22} Q_{11}(s) \mathbf{L}\mathbf{I}(s)\mathbf{L}^T \qquad (5-41)$$

where $\mathbf{L}$ is defined in Eq. (5-28), and $\mathbf{I}(s)$ is the Laplace tranform of $\mathbf{P}(t)\mathbf{P}^T(t)$ which can be expressed as

$$\mathbf{I}(s) = Laplace\left[\mathbf{P}(t)\mathbf{P}^T(t)\right] \qquad (5-42)$$

In component form, it may be shown that

$$
\begin{aligned}
I_{11}(s) &= \frac{(s+2\varsigma\omega)^2 + 2\omega_d^2}{(s+2\varsigma\omega)\left[(s+2\varsigma\omega)^2 + (2\omega_d)^2\right]} \\
I_{12}(s) = I_{21}(s) &= \frac{\omega_d}{(s+2\varsigma\omega)^2 + (2\omega_d)^2} \\
I_{22}(s) &= \frac{2\omega_d^2}{(s+2\varsigma\omega)\left[(s+2\varsigma\omega)^2 + (2\omega_d)^2\right]}
\end{aligned}
\tag{5-43}
$$

Note that Eq. (5-41) is a set of algebraic equations which can be solved by first solving for $Q_{11}(s)$.

$Q_{11}(s)$ can be expressed as

$$
Q_{11}(s) = \frac{2S_{11}}{s(s+a)\left[(s+b)^2 + c^2\right]} \tag{5-44}
$$

where $-a$ and $-b \pm ic$ are the three roots of the cubic equation:

$$
(s+2\varsigma\omega)^3 + p(s+2\varsigma\omega) + q = 0 \tag{5-45}
$$

with

$$
p = 4\omega_d^2, \qquad q = -2S_{22} \tag{5-46}
$$

It can be shown that there exists one real root and two complex conjugate roots of Eq. (5-45). The stability criterion is obtained by requiring that the real parts of these three roots be negative. This gives

$$
S_{22} < 4\varsigma\omega^3 \tag{5-47}
$$

which agrees with the stability criterion (5-39) for the stationary case.

Rearrange equation (5-41) as follows

$$
\mathbf{Q}(s) = \mathbf{L}\mathbf{J}(s)\mathbf{L}^T \tag{5-48}
$$

where

$$
\mathbf{J}(s) = \left(\frac{S_{11}}{s} + S_{22}Q_{11}(s)\right)\mathbf{I}(s) \tag{5-49}
$$

Then,

$$
\begin{aligned}
J_{mn}(s) &= \frac{f_{mn}(s)}{g(s)} \\
&= \frac{\alpha_{mn}}{s+a} + \frac{\delta_{mn}}{s} + \frac{\beta_{mn}(s+b) + \gamma_{mn}c}{(s+b)^2 + c^2}
\end{aligned}
$$

$$(m, n = 1, 2) \qquad (5-50)$$

where

$$\begin{aligned} g(s) &= s(s+a)\left[(s+b)^2 + c^2\right] \\ f_{11}(s) &= S_{11}\left[(s+2\varsigma\omega)^2 + 2\omega_d^2\right] \\ f_{12}(s) &= f_{21}(s) = S_{11}\omega_d(s+2\varsigma\omega) \\ f_{22}(s) &= 2S_{11}\omega_d^2 \end{aligned} \qquad (5-51)$$

and

$$\alpha_{mn} = \left.\frac{f_{mn}(s)}{s((s+b)^2+c^2)}\right|_{s=-a}$$

$$i\beta_{mn} + \gamma_{mn} = \left.\frac{1}{c}\frac{f_{mn}(s)}{s(s+a)}\right|_{s=-b+ic} \qquad (5-52)$$

$$\delta_{mn} = \left.\frac{f_{mn}(s)}{(s+a)((s+b)^2+c^2)}\right|_{s=0}$$

$$(m, n = 1, 2)$$

where i represents the imaginary unit.

The inverse Laplace transform gives the final nonstationary second moment response as follows:

$$\mathbf{Q}(t) = \mathbf{L}\mathbf{J}(t)\mathbf{L}^T \qquad (5-53)$$

in which the components of $J(t)$ are:

$$J_{mn}(t) = \alpha_{mn}e^{-at} + e^{-bt}(\beta_{mn}\cos ct + \gamma_{mn}\sin ct) + \delta_{mn};$$

$$(m, n = 1, 2) \qquad (5-54)$$

and the matrix $\mathbf{L}$ is as previously defined. Note that $\mathbf{Q}(t)$ is also the covariance matrix of the response since the mean of the response is zero.

5.3.2.2 Stability Analysis

As mentioned in the above section, the same stability criterion as that for the stationary correlation matrix of the response can be obtained for the nonstationary correlation matrix when the unit step envelope function is employed. A useful conclusion may be drawn for a more general form of envelope defined as

$$\eta(t) = \begin{cases} at^b e^{(-ct)}, & \text{if } t \geq 0; \\ 0, & \text{otherwise.} \end{cases} \tag{5-55}$$

where a, c are positive and b is nonnegative. Eq. (5-55) includes many envelope functions currently prevailing in earthquake engineering. Although an explicit solution is generally not available for the nonstationary correlation matrix, and, therefore, numerical integration must be employed to find the solution for a given set of parameters, the stability analysis can still be conducted on the general form of envelope (5-55).

Differentiating Eq. (5-40) with respect to time and rearranging terms yields

$$\frac{d}{dt}\mathbf{Q}(t) = \mathbf{A}\mathbf{Q}(t) + \mathbf{F} + \mathbf{B}(t)\mathbf{Q}(t) \tag{5-56}$$

where

$$\mathbf{Q}(t) = \begin{pmatrix} E[y_1^2(t)] \\ E[y_1(t)y_2(t)] \\ E[y_2^2(t)] \end{pmatrix} \qquad \mathbf{F} = \begin{pmatrix} 0 \\ 0 \\ \eta_1^2(t)S_{11} \end{pmatrix} \tag{5-57}$$

$$\mathbf{A} = \begin{pmatrix} 0 & 2 & 0 \\ -\omega^2 & -2\varsigma\omega & 1 \\ 0 & -2\omega^2 & -4\varsigma\omega \end{pmatrix} \tag{5-58}$$

$$\mathbf{B}(t) = \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ \eta_2^2(t)S_{22} & 0 & 0 \end{pmatrix} \tag{5-59}$$

Eq. (5-56) is the same as that used by Lin and Shih (1980) for uncorrelated combined loads where it was derived by using Ito's calculus. The solution of Eq. (5-56) is stable if the type of envelope expressed as Eq. (5-55) is used. This is a direct conclusion of the following theorem.

Theorem: The solution of the nonautonomous system

$$\frac{d}{dt}\mathbf{Y}(t) = \mathbf{A}\mathbf{Y}(t) + \mathbf{F}(t) + \mathbf{B}(t)\mathbf{Y}(t) \tag{5-60}$$

is stable if the following conditions are satisfied:

(i) The solution of $\frac{d}{dt}\mathbf{Y}(t) = \mathbf{A}\mathbf{Y}(t)$ is stable;

(ii) $\mathbf{B}(t)$ is impulsively small, i.e., $\exists K > 0$ such that

$$\int_0^{+\infty} \|\mathbf{B}(t)\| dt < K$$

where $\|\cdot\|$ is any matrix norm;

(iii) $\mathbf{F}(t)$ is sufficiently small as t approaches zero, i.e., $\exists a > 0, b > 0$ such that

$$|\mathbf{F}(t)| < ae^{-bt}$$

where $|\cdot|$ stands for the associated vector norm. A proof of this theorem is given in Appendix III.

5.3.2.3 Examples

Some numerical results for the nonstationary covariance response of structures subjected to *uncorrelated* suddenly applied stationary white noises are presented in Fig. 5.4 for three different intensities of vertical motion. All the parameters have the same values as those for the stationary solution. $S_{22} = 0$ corresponds to the case where the structure is subjected to the external solution alone. It may be observed that the covariance solution for the combined loads exhibits a similar trend as that for the external load alone. These responses all approach their stationary values as t approaches $+\infty$. The time required to achieve stationarity depends on the damping, frequency of the system and, in addition, the intensity of the parametric excitation. The existence of the vertical ground motion generally magnifies the covariance response. The larger the intensity of the parametric excitation, the greater the response. The proportionality between the responses due to the pure external load and the combined loads generally does not hold during the transient stage of the solution but is still valid for the stationary values.

5.4 Solution for Correlated Vertical and Horizontal Ground Motions

As mentioned previously, seismological studies have shown that the horizontal and vertical components of earthquake ground motion can be highly correlated at a station far from the source of disturbance. Therefore, it is necessary to study the effect of the correlation between these two components on the seismic response of structures. Unfortunately, the derivation for the uncorrelated ground motions as in section 5.3 generally does not hold since the solution $\mathbf{Y}(t)$ and the vertical components $v(t)$ would become correlated if these two components have a nonzero correlation S_{12} and, as a consequence, Eq. (5-19) would fail. A moment equation which accounts for the effect of the correlation has been established based on Ito's calculus by Lin and Shih (1980), but no results are presented there for the correlated case when the mean value of the response is not zero. In addition, their formulation is not suitable for explicit solution.

In this section, an alternative integral reprentation for the second moment response due to correlated external and parametric excitations is presented. The representation facilitates the explicit solution and leads to Lin and Shih's form by differentiation. The difficulty caused by the correlation between the response and the parametric excitation is overcome by introducing an additional variable which transforms the original problem to a new problem which deals with parametric excitations only and, therefore, Samuels's results can be directly used.

5.4.1 Formulation

Recall Eq. (5-9)

$$\frac{d}{dt}\mathbf{Y}(t) = \mathbf{A}\mathbf{Y}(t) + \mathbf{F}(t) + \mathbf{B}(t)\mathbf{Y}(t)$$

with the initial condition

$$\mathbf{Y}(0) = \mathbf{Y}_0 = \begin{pmatrix} x(0) \\ \dot{x}(0) \end{pmatrix}$$

which governs the dynamic behavior of linear systems under combined external and parametric excitations. Introducing a new variable y_3 such that $y_3 = 1$ with the probability 1, Eq. (5-9) may be replaced by a new equation

$$\frac{d}{dt}\mathbf{Z}(t) = \tilde{\mathbf{A}}\mathbf{Z}(t) + \tilde{\mathbf{B}}(t)\mathbf{Z}(t) \qquad (5-61)$$

with the corresponding initial condition

$$\mathbf{Z}(0) = \mathbf{Z}_0 \qquad (5-62)$$

where the augmented vectors $\mathbf{Z}(t)$, $\mathbf{Z}_0$ and the augmented matrices $\tilde{\mathbf{A}}$, $\tilde{\mathbf{B}}(t)$ are expressed as follows:

$$\mathbf{Z}(t) = \begin{pmatrix} x(t) \\ \dot{x}(t) \\ y_3 \end{pmatrix} \qquad \tilde{\mathbf{A}} = \begin{pmatrix} 0 & 1 & 0 \\ -\omega^2 & -2\varsigma\omega & 0 \\ 0 & 0 & 0 \end{pmatrix}$$
$$\mathbf{Z}_0 = \begin{pmatrix} x(0) \\ \dot{x}(0) \\ 1 \end{pmatrix} \qquad \tilde{\mathbf{B}}(t) = \begin{pmatrix} 0 & 0 & 0 \\ v(t) & 0 & h(t) \\ 0 & 0 & 0 \end{pmatrix} \tag{5-63}$$

Note that Eq. (5-61) is a governing equation for the motion of a linear system subjected to parametric excitations only.

The solution of Eq. (5-61) is given by

$$\mathbf{Z}(t) = \tilde{\mathbf{\Phi}}(t)\mathbf{Z}_0 + \int_0^t \tilde{\mathbf{\Phi}}(t-\tau)\tilde{\mathbf{B}}(\tau)\mathbf{Z}(\tau)d\tau \tag{5-64}$$

The first two moment responses can be found as

$$E[\mathbf{Z}(t)] = \tilde{\mathbf{\Phi}}(t)E[\mathbf{Z}_0] + \int_0^t \tilde{\mathbf{\Phi}}(t-\tau)E[\tilde{\mathbf{B}}(\tau)\mathbf{Z}(\tau)]d\tau \tag{5-65}$$

and

$$\begin{aligned}\tilde{\mathbf{Q}}(t_1,t_2) &= E[\mathbf{Z}(t_1)\mathbf{Z}^T(t_2)] \\ &= \tilde{\mathbf{\Phi}}(t_1)E[\mathbf{Z}_0\mathbf{Z}_0^T]\tilde{\mathbf{\Phi}}^T(t_2) + \tilde{\mathbf{L}}\int_0^{min(t_1,t_2)} q(\tau,\tau)\tilde{\mathbf{P}}(t_1-\tau)\tilde{\mathbf{P}}^T(t_2-\tau)d\tau\tilde{\mathbf{L}}^T\end{aligned} \tag{5-66}$$

where the augmented vectors and matrices are

$$\mathbf{Z}(t) = \begin{pmatrix} \mathbf{Y}(t) \\ y_3 \end{pmatrix} \qquad \tilde{\mathbf{P}}(t) = \begin{pmatrix} \mathbf{P}(t) \\ 1 \end{pmatrix}$$
$$\tilde{\mathbf{\Phi}}(t) = \begin{pmatrix} \mathbf{\Phi}(t) & 0 \\ 0 & 1 \end{pmatrix} \qquad \tilde{\mathbf{L}} = \begin{pmatrix} \mathbf{L} & 0 \\ 0 & 1 \end{pmatrix} \tag{5-67}$$
$$\tilde{\mathbf{Q}}(t_1,t_2) = \begin{pmatrix} \mathbf{Q}(t_1,t_2) & E[\mathbf{Y}(t_1)] \\ E[\mathbf{Y}^T(t_2)] & E[y_3(t_1)y_3(t_2)] \end{pmatrix}$$

and $q(t_1,t_2)$ is derived from the expression

$$\begin{aligned}&E[(v(t_1)y_1(t_1) + h(t_1)y_3(t_1))(v(t_2)y_1(t_2) + h(t_2)y_3(t_2))] \\ =&E[v(t_1)y_1(t_1)v(t_2)y_1(t_2) + v(t_1)y_1(t_1)h(t_2)y_3(t_2) \\ &\quad + h(t_1)y_3(t_1)v(t_2)y_1(t_2) + h(t_1)y_3(t_1)h(t_2)y_3(t_2)] \\ =&q(t_1,t_2)\eta(t_1)\eta(t_2)\delta(t_2-t_1)\end{aligned} \tag{5-68}$$

Directly using Eq. (5-19) which is applicable to the new augmented formulation, and noticing that y_3 is unity with the probability 1 and is independent of $h(t)$ and $v(t)$, $q(t_1,t_2)$ in Eq. (5-68) reduces to

$$q(t_1,t_2) = S_{22}\tilde{Q}_{11}(t_1,t_2) + S_{12}(E[y_1(t_1)] + E[y_1(t_2)]) + S_{11} \qquad (5-69)$$

Substituting Eq. (5-69) into Eq. (5-66), and returning to the original unaugmented expression, one obtains the first two moment responses as

$$E[\mathbf{Y}(t)] = \mathbf{\Phi}(t)E[\mathbf{Y}_0] \qquad (5-70)$$

and

$$\mathbf{Q}(t_1,t_2) = \mathbf{\Phi}(t_1)E[\mathbf{Y}_0\mathbf{Y}_0^T]\mathbf{\Phi}^T(t_2)]+$$
$$\mathbf{L}\int_0^{min(t_1,t_2)} (S_{22}Q_{11}(\tau) + 2S_{12}E[y_1(\tau)] + S_{11})\eta^2(\tau)\mathbf{P}(t_1-\tau)\mathbf{P}^T(t_2-\tau)d\tau\mathbf{L}^T \qquad (5-71)$$

In the nonstationary case discussed in this chapter, the second moment response is desired, which can be obtained by setting $t_1 = t_2 = t$ and $\eta(\tau) = 1$ in Eq. (5-71). That is

$$\mathbf{Q}(t) = \mathbf{Q}^{(0)}(t) + \mathbf{L}\int_0^t (S_{22}Q_{11}(\tau) + 2S_{12}E[Y_1(\tau)] + S_{11})\mathbf{P}(t-\tau)\mathbf{P}^T(t-\tau)d\tau\mathbf{L}^T \qquad (5-72)$$

where

$$\mathbf{Q}^{(0)}(t) = \mathbf{\Phi}(t)E[\mathbf{Y}_0\mathbf{Y}_0^T]\mathbf{\Phi}^T(t) \qquad (5-73)$$

Differentiating Eq. (5-72) leads to the following differential equation for the second moment response, which is essentially the same as the vector form derived by Lin and Shih (1980) from Ito's calculus.

$$\frac{d}{dt}\mathbf{Q}(t) = \mathbf{A}\mathbf{Q}(t) + \mathbf{Q}(t)\mathbf{A}^T + \mathbf{C}(t) \qquad (5-74)$$

where

$$\mathbf{C}(t) = \begin{pmatrix} 0 & 0 \\ 0 & S_{22}Q_{11}(t) + 2S_{12}E[Y_1(t)] + S_{11} \end{pmatrix} \qquad (5-75)$$

Note that it is not trivial to derive the integral form in Eq. (5-72) from the differential equation (5-74). The integral form is employed in this chapter since it is more appropriate for the purpose of finding an analytical solution and studying the intrinsic behavior of the solution.

5.4.2 Solution for the Case Where the Mean Value of the Displacement Is Zero

It is observed from Eq. (5-72) that the correlation information of the two components of ground motion will enter into the analysis *if and only if* both the mean of the displacement response and the cross-spectral density S_{12} are nonzero. Obviously, Eq. (5-21) is a special case of Eq. (5-72) where $S_{12} = 0$ and/or $E[y_1(t)] = 0$. Therefore, all the results presented in the previous section can be directly applied to the case where a correlation between the two components of ground motion exists, provided that the mean value of the displacement remains zero.

5.4.3 Solution for the Case Where the Mean Value of the Displacement Is Nonzero

A nonzero mean of the displacement can result from various factors, two of which will be considered here. In the first case, the nonzero mean of the displacement is caused by a nonzero mean of the initial displacement and/or velocity, while in the second case, the nonzero mean of the displacement is caused by a preexisting additional external excitation which can be modeled as a stationary random process. In both cases, the nonzero initial second moment response, in addition to the nonzero mean of initial displacement and/or velocity, is required in order to ensure that $\mathbf{Q}(t)$ is semi-positive definite.

The following discussion is restricted to the nonstationary solution. The stationary solution can be obtained by setting $t_2 = t_1 + \tau$ and $t_1 = +\infty$ in (5-72). In this case,

$$\mathbf{R}(\tau) = (S_{22}R_{11}(0) + 2S_{12}E[y_1] + S_{11})\mathbf{L}\int_0^{+\infty} \mathbf{P}(s)\mathbf{P}^T(s+\tau)d\tau\mathbf{L}^T \qquad (5-76)$$

where $R(\tau)$ is the correlation matrix of the response. Note that the stationary mean value of the response is zero, as observed from Eq. (5-70). Substituting $E[y_1] = 0$

into Eq. (5-76) yields

$$\mathbf{R}(\tau) = (S_{22}R_{11}(0) + S_{11})\mathbf{L}\int_0^{+\infty} \mathbf{P}(s)\mathbf{P}^T(s+\tau)d\tau\mathbf{L}^T \qquad (5-77)$$

Note that the above equation is the same as Eq. (5-25), which means that the correlation between two components has no effect on the stationary value of correlation response even if the nonstationary mean value of the displacement may not be zero.

5.4.3.1 Case 1: Nonzero Mean Displacement Caused by the Nonzero Mean of the Initial Conditions

The nonstationary solution of equation (5-72) contains some additional terms caused by the nonzero mean of the initial displacement or velocity, and nonzero correlation of the two components of ground motion. Assuming the mean of the initial displacement of the system $E[x(0)] = x_0$ and/or the mean of the initial velocity $E[\dot{x}(0)] = v_0$ is nonzero, the nonstationary mean of the displacement can be found as

$$E[y_1(t)] = e^{-\varsigma\omega t}(\gamma\cos\omega_d t + \delta\sin\omega_d t) \qquad (5-78)$$

where

$$\begin{aligned} \gamma &= x_0 \\ \delta &= \frac{\varsigma\omega x_0 + v_0}{\omega_d} \end{aligned} \qquad (5-79)$$

Performing a Laplace transform on both sides of equation (5-72) gives

$$\mathbf{Q}(s) = \mathbf{Q}^{(0)}(s) + \left(\frac{S_{11}}{s} + \frac{2S_{12}\left(\gamma(s+\varsigma\omega) + \delta\omega_d\right)}{(s+\varsigma\omega)^2 + \omega_d^2} + S_{22}Q_{11}(s)\right)\mathbf{L}\mathbf{I}(s)\mathbf{L}^T \quad (5-80)$$

where $\mathbf{I}(s)$ is defined in Eq. (5-43). As before, solving first for $Q_{11}(s)$ yields

$$Q_{11}(s) = \frac{Q_{11}^{(0)}(s)\lambda(s) + 2\mu(s)}{\nu(s)} \qquad (5-81)$$

where

$$\begin{aligned} \mu(s) &= \frac{S_{11}}{s} + \frac{2S_{12}\left[\gamma(s+\varsigma\omega) + \delta\omega_d\right]}{(s+\varsigma\omega)^2 + \omega_d^2} \\ \nu(s) &= (s+a)\left[(s+b)^2 + c^2\right] \\ \lambda(s) &= (s+2\varsigma\omega)\left[(s+2\varsigma\omega)^2 + (2\omega_d)^2\right] \end{aligned} \qquad (5-82)$$

and a, b, c are determined from Eqs. (5-45) and (5-46).

Substituting Eq. (5-81) into Eq. (5-80) and rearranging terms yields

$$\mathbf{Q}(s) = \mathbf{Q}^{(0)}(s) + \mathbf{L}\tilde{\mathbf{J}}(s)\mathbf{L}^T \tag{5-83}$$

where

$$\tilde{J}(s) = \lambda(s)\frac{\mu(s) + S_{22}Q^{(0)}(s)}{\nu(s)}I(s) \tag{5-84}$$

$I(s)$ is defined as in Eq. (5-43), and $\lambda(s), \mu(s), \nu(s)$ are defined as in Eq. (5-82). Finally, the Laplace transform of the solution, $\mathbf{Q}(s)$, can be written as

$$\mathbf{Q}(s) = \mathbf{Q}^{(1)}(s) + \mathbf{Q}^{(2)}(s) + \mathbf{Q}^{(3)}(s) \tag{5-85}$$

where $\mathbf{Q}^{(1)}(s)$ corresponds to the solution for two uncorrelated components, and $\mathbf{Q}^{(2)}(s)$, $\mathbf{Q}^{(3)}(s)$ are the correction terms for the nonzero initial first and second moments respectively.

$$\begin{aligned}
\mathbf{Q}^{(1)}(s) &= \frac{\lambda(s)}{s\nu(s)}S_{11}\mathbf{L}\mathbf{I}(s)\mathbf{L}^T \\
\mathbf{Q}^{(2)}(s) &= S_{12}\mathbf{L}(x_0\mathbf{G}^{(1)}(s) + v_0\mathbf{G}^{(2)}(s))\mathbf{L}^T \\
\mathbf{Q}^{(3)}(s) &= \mathbf{Q}^{(0)}(s) + \mathbf{L}\mathbf{G}^{(3)}(s)\mathbf{L}^T
\end{aligned} \tag{5-86}$$

The components of the matrices $\mathbf{G}^{(k)}(s), k = 1, 2, 3$ can be expressed as

$$\tilde{G}_{mn}^{(k)}(s) = \frac{\tilde{f}_{mn}^{(k)}(s)}{\tilde{g}^{(k)}(s)} \tag{5-87}$$

where

$$\begin{aligned}
\tilde{f}_{11}^{(k)}(s) &= r^{(k)}(s)\left[(s + 2\varsigma\omega)^2 + 2\omega_d^2\right] \\
\tilde{f}_{12}^{(k)}(s) &= \tilde{f}_{21}^{(k)}(s) = r^{(k)}(s)\omega_d(s + 2\varsigma\omega) \\
\tilde{f}_{22}^{(k)}(s) &= 2\omega_d^2 r^{(k)}(s) \qquad k = 1, 2, 3.
\end{aligned} \tag{5-88}$$

$\tilde{g}^{(k)}(s)$ and $r^{(k)}$ are determined by

$$\begin{aligned}
\tilde{g}^{(k)}(s) &= (s + a)\left[(s + b)^2 + c^2\right]\left[(s + \varsigma\omega)^2 + \omega_d^2\right] \\
r^{(k)}(s) &= 2\left[\gamma^{(k)}(s + \varsigma\omega) + \delta^{(k)}\omega_d\right] \qquad k = 1, 2 \\
\tilde{g}^{(3)}(s) &= (s + a)[(s + b)^2 + c^2](s + 2\varsigma\omega)[(s + 2\varsigma\omega)^2 + (2\omega_d)^2] \\
r^{(3)}(s) &= S_{22}\left[\alpha(s + 2\varsigma\omega)^2 + \beta(s + 2\varsigma\omega) + \theta(2\omega_d)^2\right]
\end{aligned} \tag{5-89}$$

and

$$\begin{aligned}\alpha &= Q_{11}^{(0)} \\ \beta &= 2\varsigma\omega Q_{11}^{(0)} + 2Q_{12}^{(0)} \\ \theta &= \frac{Q_{11}^{(0)}}{2(1-\varsigma^2)} + \frac{\varsigma\omega}{\omega_d}Q_{12}^{(0)} + \frac{Q_{22}^{(0)}}{2\omega_d^2}\end{aligned} \tag{5-90}$$

$$\begin{aligned}\gamma^{(1)} &= 1 \quad \delta^{(1)} = \frac{\varsigma\omega}{\omega_d} \\ \gamma^{(2)} &= 0 \quad \delta^{(2)} = \frac{1}{\omega_d}\end{aligned} \tag{5-91}$$

where $Q_{mn}^{(0)}(0), m, n = 1, 2$ are the initial second moments given.

To facilitate the inverse Laplace transformation, a form of partial fraction is preferred. Thus,

$$\tilde{G}_{mn}^{(k)}(s) = \frac{\tilde{a}_{mn}^{(k)}}{s+a} + \frac{\tilde{b}_{mn}^{(k)}(s+b) + \tilde{c}_{mn}^{(k)}c}{(s+b)^2+c^2} + \frac{\tilde{e}_{mn}^{(k)}(s+\varsigma\omega) + \tilde{h}_{mn}^{(k)}\omega_d}{(s+\varsigma\omega)^2+\omega_d^2}$$

$$k = 1, 2 \tag{5-92}$$

where

$$\begin{aligned}\tilde{a}_{mn}^{(k)} &= \left.\frac{\tilde{f}_{mn}^{(k)}(s)}{[(s+b)^2+c^2]\,[(s+\varsigma\omega)^2+\omega_d^2]}\right|_{s=-a} \\ i\tilde{b}_{mn}^{(k)} + \tilde{c}_{mn}^{(k)} &= \left.\frac{1}{c}\,\frac{\tilde{f}_{mn}^{(k)}(s)}{(s+a)\,[(s+\varsigma\omega)^2+\omega_d^2]}\right|_{s=-b+ic} \\ i\tilde{e}_{mn}^{(k)} + \tilde{h}_{mn}^{(k)} &= \left.\frac{1}{\omega_d}\,\frac{\tilde{f}_{mn}^{(k)}(s)}{(s+a)\,[(s+b)^2+c^2]}\right|_{s=-\varsigma\omega+i\omega_d}\end{aligned}$$

$$m, n = 1, 2; \qquad k = 1, 2 \tag{5-93}$$

and

$$\begin{aligned}\tilde{G}_{mn}^{(3)}(s) = \frac{\tilde{a}_{mn}^{(3)}}{s+a} &+ \frac{\tilde{b}_{mn}^{(3)}(s+b) + \tilde{c}_{mn}^{(3)}c}{(s+b)^2+c^2} + \frac{d_{mn}^{(3)}}{(s+2\varsigma\omega)} \\ &+ \frac{\tilde{e}_{mn}^{(3)}(s+2\varsigma\omega) + \tilde{h}_{mn}^{(k)}(2\omega_d)}{(s+2\varsigma\omega)^2+(2\omega_d)^2}\end{aligned}$$

$$m, n = 1, 2 \tag{5-94}$$

where

$$\begin{aligned}
\tilde{a}^{(3)}_{mn} &= \left.\frac{\tilde{f}^{(3)}_{mn}(s)}{[(s+b)^2+c^2]\,(s+2\varsigma\omega)\,[(s+2\varsigma\omega)^2+(2\omega_d)^2]}\right|_{s=-a} \\
i\tilde{b}^{(3)}_{mn}+\tilde{c}^{(3)}_{mn} &= \left.\frac{1}{c}\frac{\tilde{f}^{(3)}_{mn}(s)}{(s+a)(s+2\varsigma\omega)\,[(s+2\varsigma\omega)^2+(2\omega_d)^2]}\right|_{s=-b+ic} \\
i\tilde{e}^{(3)}_{mn}+\tilde{h}^{(3)}_{mn} &= \left.\frac{1}{2\omega_d}\frac{\tilde{f}^{(3)}_{mn}(s)}{(s+a)\,[(s+b)^2+c^2]\,(s+2\varsigma\omega)}\right|_{s=-2\varsigma\omega+i2\omega_d} \\
\tilde{d}^{(3)}_{mn} &= \left.\frac{\tilde{f}^{(3)}_{mn}(s)}{(s+a)[(s+b)^2+c^2][(s+2\varsigma\omega)^2+(2\omega_d)^2]}\right|_{s=-2\varsigma\omega}
\end{aligned} \qquad (5-95)$$

The final solution will be of the form

$$\mathbf{Q}(t) = \mathbf{Q}^{(1)}(t) + \mathbf{Q}^{(2)}(t) + \mathbf{Q}^{(3)}(t) \qquad (5-96)$$

where $\mathbf{Q}^{(1)}(t)$ is the solution for the uncorrelated case as in Eqs. (5-52)-(5-54). The correction terms $\mathbf{Q}^{(k)}, k = 2, 3$ due to the correlation of the two components of the ground motion can be expressed as

$$\begin{aligned}
\mathbf{Q}^{(2)}(t) &= S_{12}\mathbf{L}(x_0\mathbf{G}^{(1)}(t) + v_0\mathbf{G}^{(2)}(t))\mathbf{L}^T \\
\mathbf{Q}^{(3)}(t) &= \mathbf{\Phi}(t)\mathbf{Q}(0)\mathbf{\Phi}^T(t) + \mathbf{L}\mathbf{G}^{(3)}(t)\mathbf{L}^T
\end{aligned} \qquad (5-97)$$

where the components of $\tilde{\mathbf{G}}^{(k)}(t), k = 1, 2, 3$ are

$$\begin{aligned}
\tilde{G}^{(k)}_{mn}(t) =&\tilde{a}^{(k)}_{mn}e^{-at} + e^{-bt}(\tilde{b}^{(k)}_{mn}\cos ct + \tilde{c}^{(k)}_{mn}\sin ct) \\
&+ e^{-\varsigma\omega t}(\tilde{e}^{(k)}_{mn}\cos\omega_d t + \tilde{h}^{(k)}_{mn}\sin\omega_d t) \\
&m,n = 1,2 \qquad k = 1,2
\end{aligned} \qquad (5-98)$$

and

$$\begin{aligned}
\tilde{G}^{(3)}_{mn}(t) =& \tilde{a}^{(3)}_{mn}e^{-at} + e^{-bt}(\tilde{b}^{(3)}_{mn}\cos ct + \tilde{c}^{(3)}_{mn}\sin ct) \\
&+ e^{-2\varsigma\omega t}(\tilde{d}^{(3)}_{mn} + \tilde{e}^{(3)}_{mn}\cos 2\omega_d t + \tilde{h}^{(3)}_{mn}\sin 2\omega_d t) \\
&m,n = 1,2
\end{aligned} \qquad (5-99)$$

Eqs. (5-96)-(5-99) complete the solution.

It is interesting to see that the final solution consists of three parts. Among these, $\mathbf{Q}^{(1)}(t)$ denotes the solution for uncorrelated components of the ground motion, $\mathbf{Q}^{(2)}(t)$ represents a correction term for nonzero cross-spectral density and nonzero mean of the initial displacement and/or velocity, and $\mathbf{Q}^{(3)}(t)$ represents another correction term for nonzero initial second moments. If $\mathbf{Q}^{(2)}(t)$ is not zero, $\mathbf{Q}^{(3)}(t)$ must appear in the final solution, which means that an appropriate initial second moment, in addition to the mean of the initial displacement and velocity, must be given to ensure the final solution is semi-positive definite. It is easily seen from the final solution that the effect of correlation can be neglected if the two components are uncorrelated and/or the mean of initial conditions are zero with probability 1, the latter is the case usually considered in earthquake engineering.

Some numerical results are presented in Fig. 5.4 for the system with $\omega = 1$ and $\varsigma = 0.1$ under the initial conditions $x_0 = 1, v_0 = 1, Q_{11}^{(0)} = 1.5, Q_{22}^{(0)} = 1.5, Q_{12}^{(0)} = 0.5$ and subjected to combined horizontal and vertical ground motions with spectral densities $S_{11} = 1.0, S_{22} = 0.1$, and $S_{12} = 0, 0.15, 0.3$. It is clear that in this case the correlation between the two components of ground motion has a considerable effect on the transient stage of the second moment response and no effect on the stationary value. The larger the cross-spectral density, the stronger the effect will be.

Note that the same stability criterion as used before for the uncorrelated parametric and external excitations applies in this case. That is clear for the stationary solution since the correlation has no effect in this case. For the nonstationary solution, although the final solutions seem different, the same criterion still holds since no new restrictions are required during the derivation, and $E[y_1(t)]$ is impulsively small if the general form of envelope (5-55) is employed.

5.4.3.2 Case 2: Nonzero Mean Displacement Caused by an Additional Stationary External Excitation

Assume that in addition to the original external and parametric excitations there is an additional external load which can be modeled as a stationary white noise applied to the same system in the direction of the external excitation, and which causes a nonzero initial mean, and therefore, a nonzero second moment of the response. Probably this is a more interesting case than the first case from the engineering point of view, since it may be used to deal with the random analysis of simple structures subjected to combined horizontal and vertical ground motions in a stationary environment, such as wind, current, etc. The analysis can be conducted by assuming

$$F(t) = \begin{pmatrix} 0 \\ f(t) \end{pmatrix} \tag{5-100}$$

where $f(t) = h(t) + w(t)$ and $h(t)$ is the horizontal component of ground motion and $w(t)$ is the additional external load which is modeled as a stationary white noise with the spectral density S_{ww}.

It is assumed that earthquake loads $h(t)$ and $v(t)$ start from $t = 0$, while the load $w(t)$ starts from $t = -\infty$ and the system has achieved its stationary response before $t = 0$. Let the stationary mean and second moments of the response of the system caused by $w(t)$ be $Y^{(w)}$ and $Q^{(w)}$ respectively. Then the solution can be written as

$$\mathbf{Q}(t) = \int_{-\infty}^{t} \mathbf{\Phi}(t-\tau)\mathbf{C}(\tau)\mathbf{\Phi}^T(t-\tau)d\tau \tag{5-101}$$

where

$$\mathbf{C}(t) = \begin{pmatrix} 0 & 0 \\ 0 & c(t) \end{pmatrix} \tag{5-102}$$

By analogy to Eq. (5-68), $c(t)$ is obtained from

$$\begin{aligned} & E[(f(t)y_3(t) + v(t)y_1(t))^2] \\ =& E[v^2(t)y_1^2(t)] + 2E[f(t)v(t)y_1(t)y_3(t)] + E[f^2(t)y_3^2] \\ =& E[v^2(t)y_1^2(t)] + 2E[h(t)v(t)]E[y_1(t)] + E[h^2(t)] + E[w^2(t)] \\ & \quad + 2E[h(t)w(t)](1 + E[y_1(t)]) \\ =& c(t)\delta(0) \end{aligned} \tag{5-103}$$

which can be reduced to

$$\begin{aligned} c(t) &= E[w^2(t)] + (E[v^2(t)]E[y_1^2(t)] + 2E[h(t)v(t)]E[y_1(t)] + E[h^2(t)]) \\ &= E[w^2(t)] + (S_{22}Q_{11}(t) + 2S_{12}E[y_1(t)] + S_{11}) \end{aligned} \tag{5-104}$$

since the earthquake loads and the additional load $w(t)$ are uncorrelated and the mean of $h(t)$ equals zero. Note that the earthquake loads are zero before $t = 0$ and the response of the system due to $w(t)$ has already achieved its stationary stage, the final solution can be expressed as

$$\mathbf{Q}(t) = \mathbf{Q}^{(w)} + \mathbf{L}\int_0^t (S_{22}Q_{11}(\tau) + 2S_{12}Y_1^{(w)} + S_{11})\mathbf{P}(t-\tau)\mathbf{P}^T(t-\tau)d\tau\mathbf{L}^T \quad (5-105)$$

where constant $\mathbf{Q}^{(w)}$ and $Y_1^{(w)}$ are the second moment and the mean response caused by $w(t)$.

The solution can be found in terms of the solution for uncorrelated external and parametric excitations without the additional load $w(t)$. Let the latter solution be $Q^{(u)}(t)$ and notice that $Q^{(u)}$ is proportional to the spectral density of the external excitation. Thus, final solution for the covariance response can be expressed as

$$\mathbf{Q}(t) = \mathbf{Q}^{(w)} + \alpha\mathbf{Q}^{(u)}(t) \quad (5-106)$$

where

$$\alpha = \frac{2S_{12}Y_1^{(w)} + S_{11}}{S_{11}} \quad (5-107)$$

Eqs. (5-106) and (5-107) complete the nonstationary covariance response for the correlated combined loads where the nonzero mean displacement is caused by the presence of an additional stationary excitation.

It is clear from Eq. (5-106) that in this case the second moment response can be obtained by scaling the corresponding solution for the uncorrelated parametric and external excitations without the additional load $w(t)$, and then translating these scaled curves up or down by an amount which is equal to the stationary second moment response $\mathbf{Q}^{(w)}$ caused by $w(t)$ alone. The scaling factor α depends on $Y_1^{(w)}$, the correlation S_{12} and intensity S_{11}. The effect of correlation between parametric and external components on the covariance response increases for increasing values of cross-spectral density S_{12}. Some numerical results are shown in Fig. 5.6.

It may be concluded from Eq. (5-106) that the previous stability criterion still applies to this case but with a slight revision. Due to the presence of the additional stationary load, a weakly stable solution, instead of a stable solution, is obtained for the general form of envelope (5-55).

5.5 Conclusions

The simplified state-variable method has been used to obtain the explicit solution for both the stationary and nonstationary second moment responses to study the dynamic behavior of a structure under combined horizontal and vertical ground excitations which may be uncorrelated or correlated. Stability of the solutions has also been discussed and a general stability criterion for the covariance response has been given. Some numerical results are presented.

In the stationary case it has been found that the correlation response under combined loads is simply equal to that under horizontal ground motion alone multiplied by a Parametric Amplification Factor, A_p, defined in Eq. (5-36). This factor depends only on the system parameters and the intensity of vertical motion. The results show that the effect of the vertical component of earthquake ground motion on the second moment response is significant in certain cases, especially near the region of instability where the intensity of the vertical component could be very small relative to that of horizontal motion. These conclusions are valid for both the correlated and noncorrelated combined excitations.

In the nonstationary case, if the parametric and external excitations are uncorrelated, the covariance solution may be obtained by a superposition of the solution for the external excitation alone and an additional part which depends on the intensity of the parametric excitation. Proportionality between the solutions for the external load alone and the combined loads, as exhibited in the stationary correlation solution, generally does not hold at the initial nonstationary stage of the solutions obtained, but may still be valid for their stationary values. The same stability criterion as that for the stationary solution applies when a step function envelope is used. However, it is concluded that the second moment responses are always stable for envelopes of the type in Eq. (5-55) which covers the most commonly used envelopes in earthquake engineering. These results hold for correlated combined loads, provided the mean value of the displacement remains zero.

The effect of the correlation of the horizontal and vertical ground motions has been studied in detail. It has been shown that correlation enters the analysis if and only if both the correlation and the mean of the displacement response are nonzero. Two cases which may cause a nonzero mean of the displacement have been discussed.

For the case that the nonzero mean of the displacement is caused by a nonzero initial displacement and/or velocity, the solution consists of three parts, namely the solution corresponding to uncorrelated components, and two correction terms from nonzero initial mean of the displacement and/or velocity and nonzero initial second moment response respectively. The correlation may have considerable effect on the transient stage of the solution but little effect on the stationary values. The significance depends on the magnitude of the correlation.

If the nonzero mean of the displacement is caused by the presence of an additional stationary external load, the dynamic behavior of the system is essentially the same as that for the case where the additional load is absent and the final solution can be obtained by scaling the solution for no extra load and then translating the curve up or down by an amount which is equal to the initial covariance caused by the stationary excitation. The effect of the correlation of the loads on the second moment response depends on the magnitude of the correlation and the given nonzero initial conditions resulting from the presence of the additional external load.

In both cases, the same stability criterion as that for uncorrelated components of ground motion applies except that a weakly stable solution, in stead of stable solution, is obtained for the nonstationary solution using the general form of envelope in Eq. (5-55), if the nonzero mean displacement results from the presence of an additional stationary external load.

The results suggest that in earthquake resistant design, the system parameters should be chosen such that the system is far away from the region of instability in order to reduce the effect of the accompanying vertical motion on the second moment response. If the system parameters are near those for instability, either by design or as a result of damage of the structure, the effect of the vertical motion can be significant. Increasing the damping in a structure or its natural frequency will improve the earthquake resistance under combined horizontal and vertical excitations.

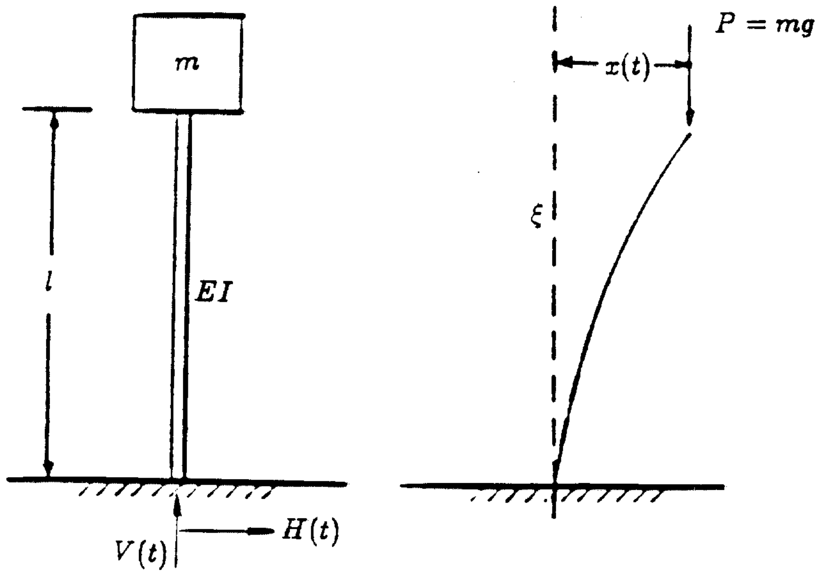


Figure 5.1. A structure model subjected to combined vertical and horizontal earthquake ground motions.

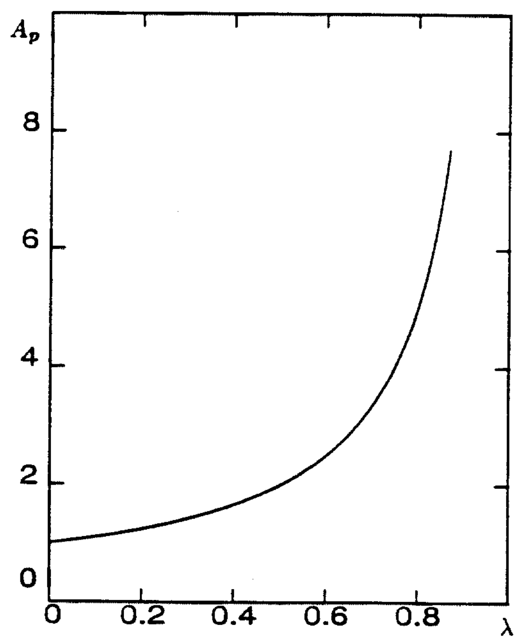


Figure 5.2. Parametric Amplification Factor A_p.

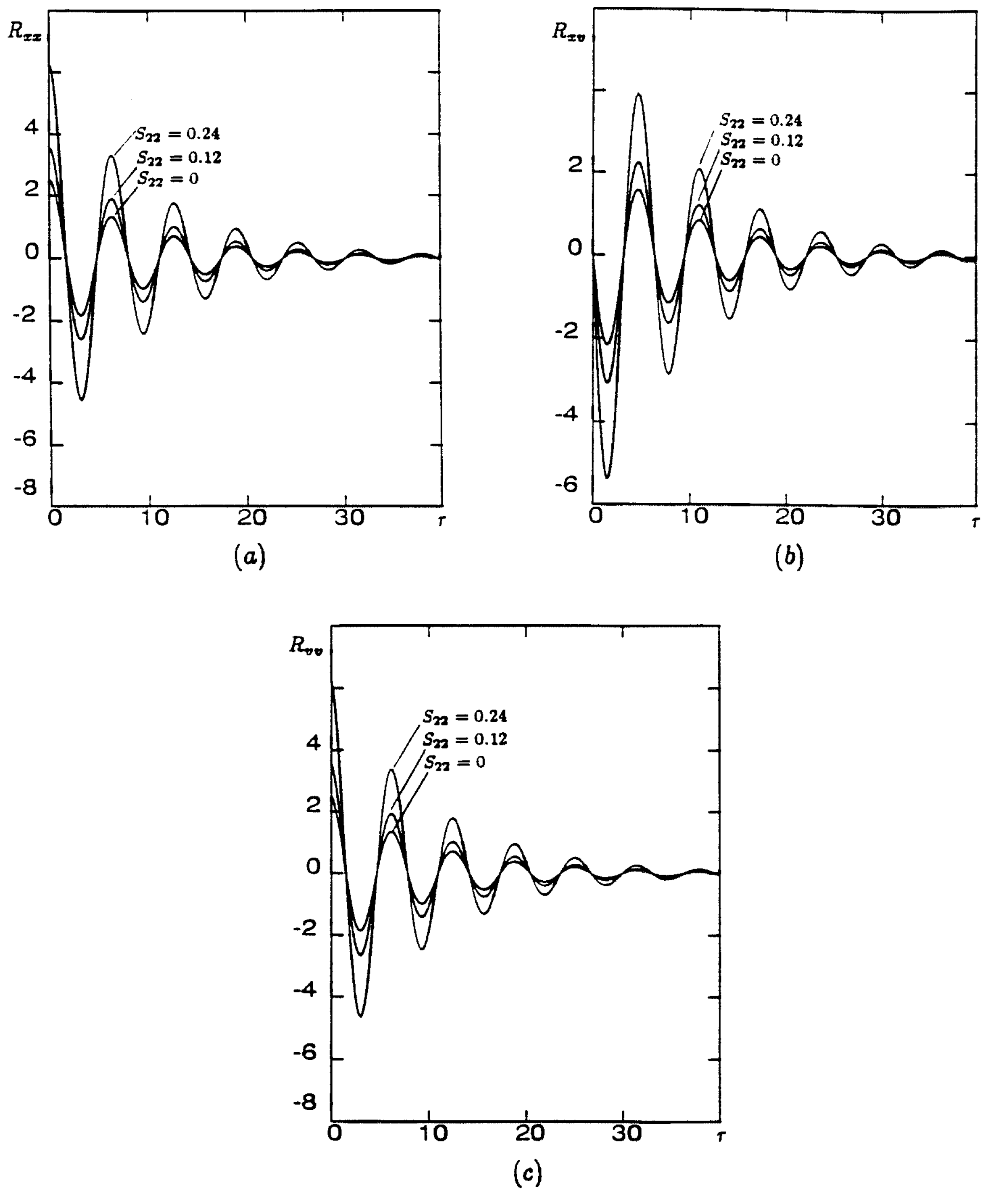


Figure 5.3. Stationary correlation of the structural response under uncorrelated parametric and external excitations. Structural parameters $\omega = 1.0$ and $\varsigma = 0.1$. Excitation parameters $S_{11} = 1.0$, $S_{12} = 0$, and $S_{22} = 0, 0.12$, and 0.24.

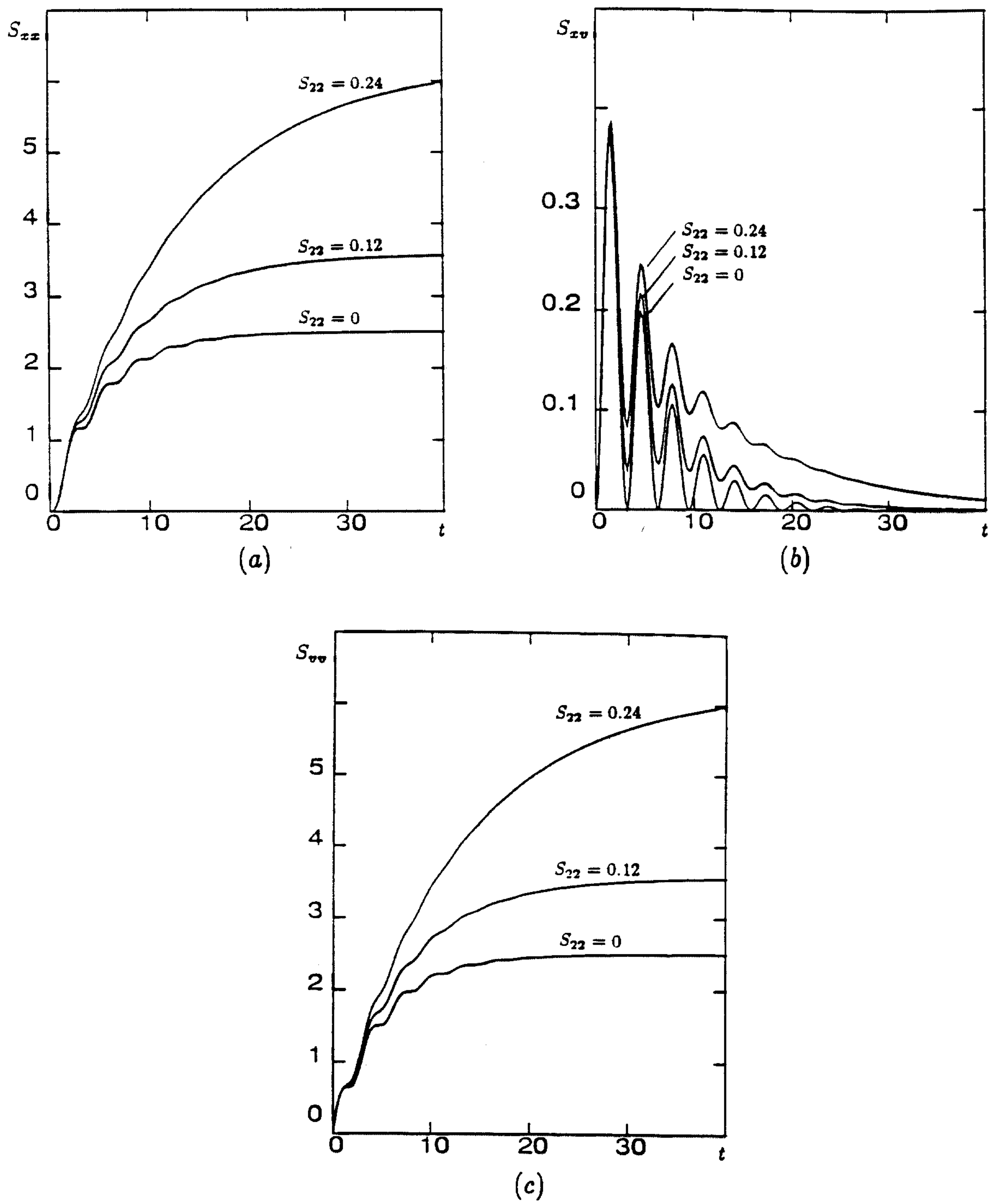


Figure 5.4. Nonstationary covariances of the structural response under uncorrelated parametric and external excitations using the unit step envelope. Structural parameters $\omega = 1.0$ and $\varsigma = 0.1$. Excitation parameters $S_{11} = 1.0$, $S_{12} = 0$, and $S_{22} = 0, 0.12$, and 0.24.

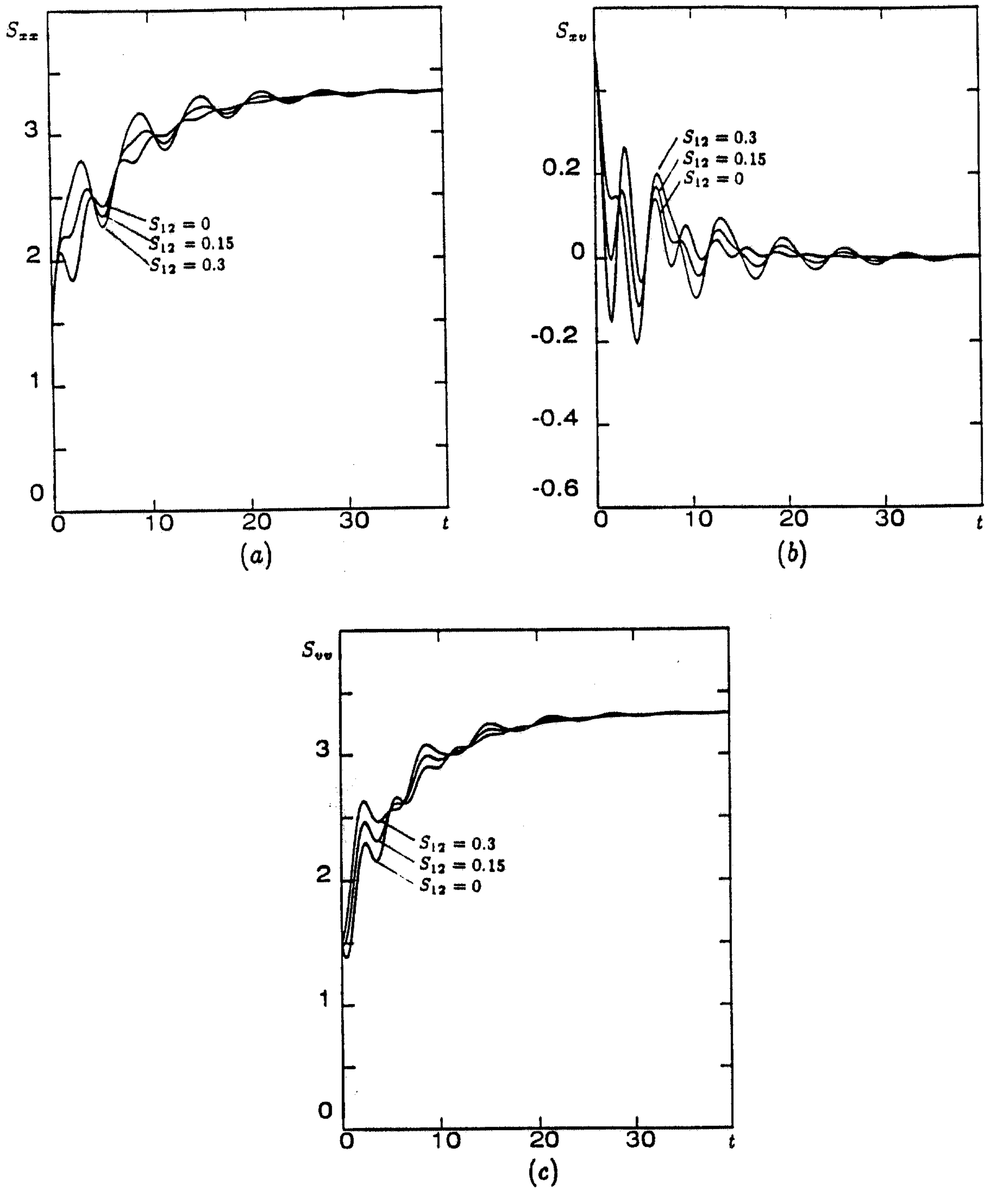


Figure 5.5. Nonstationary second moments of the structural response under correlated parametric and external excitations using a unit step envelope when nonzero mean of the initial conditions is given. Structural parameters $\omega = 1.0$ and $\varsigma = 0.1$. Excitation parameters $S_{11} = 1.0$, $S_{22} = 0.1$, and $S_{12} = 0, 0.15$, and 0.3. The initial conditions $x_0 = 1.0$, $v_0 = 1.0$, $Q_{11}^{(0)} = 1.5$, $Q_{22}^{(0)} = 1.5$, and $Q_{12}^{(0)} = 0.5$.

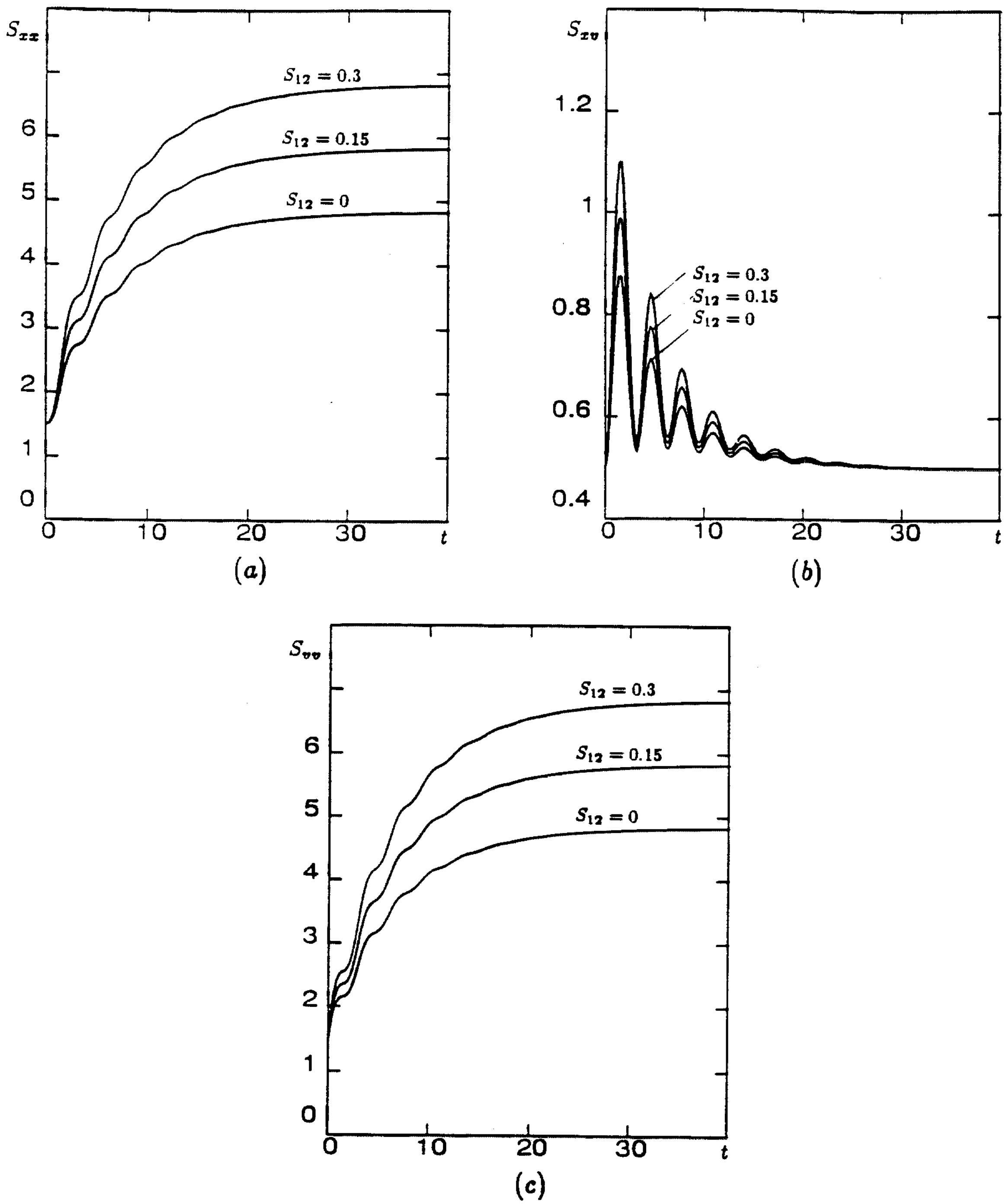


Figure 5.6. Nonstationary covariances of the structural response under correlated parametric and external excitations using a unit step envelope in the presence of an additional stationary external excitation. Structural parameters $\omega = 1.0$ and $\varsigma = 0.1$. Excitation parameters $S_{11} = 1.0$, $S_{22} = 0.1$, and $S_{12} = 0, 0.15$, and 0.3. The initial conditions caused by the additional external excitation $x_0 = 1.0$, $v_0 = 1.0$, $Q_{11}^{(0)} = 1.5$, $Q_{22}^{(0)} = 1.5$, and $Q_{12}^{(0)} = 0.5$.

Chapter 6

Summary and Conclusions

A simplified state-variable method has been proposed to solve for the non-stationary response of linear MDOF systems subjected to a modulated stationary excitation. The method is applied in both the time and frequency domains. The resulting covariance matrix and evolutionary spectral density matrix of the response may be expressed as a product of a constant system matrix and a time-dependent matrix, the latter can be evaluated explicitly for most envelopes currently prevailing in engineering. As a bonus, the stationary correlation matrix of the response may be found by taking the limit of the covariance response when a unit step envelope is used. Reliability analysis can then be performed based on the first two moments of the response so obtained.

While it is frequently reported that the explicit solution is cumbersome to manipulate, mainly due to the algebraic difficulty even for simple systems, the new formulation makes the explicit solution much easier to obtain for general MDOF systems. Whether or not the explicit solution can be obtained depends, to a large extent, on the type of envelope function. Fortunately, it succeeds for a class of envelopes which include most envelopes commonly used in engineering as special cases. This approach is flexible enough to be applied to different linear systems including SDOF, MDOF, and continuous systems, to different stochastic models of excitation such as the stationary models, modulated stationary models, filtered stationary models, and filtered modulated stationary models and their stochastic equivalents including the random pulse train model, filtered shot noise, and some ARMA models. If the stationary model is white noise, the solution becomes much easier. If not, the version in the frequency domain is easily employed to obtain the evolutionary spectral density matrix first and then the covariance response by an integration.

In addition to a set of explicit solutions presented for the response of simple linear structures subjected to modulated white noise earthquake models with four different envelopes, the method has been applied to three selected topics in earthquake engineering which have drawn considerable attention in recent studies and are particularly chosen herein to show the application of the proposed method to SDOF, MDOF, and continuous systems. In contrast to previous studies, all the solutions presented are explicit solutions and, as a consequence, the solutions are more accurate, more efficient and especially suitable for theoretical analysis. The results obtained may have significance in earthquake resistant design.

The first application is given to the seismic analysis of primary-secondary systems under nonstationary excitation. The system may be either classically damped or nonclassically damped. Interaction effects have been taken into account. The mean square values of the energy envelope of both the primary and the secondary system have been explicitly evaluated in terms of the solution for the covariance matrix of the structural response. No bandwidth restriction is imposed. The energy envelope can be reduced to the envelope used by Iwan and Smith (1987) for narrow-banded processes where an approximate solution for the mean-square envelope response has been obtained based on the assumption of broad-banded excitation and narrow-banded response. A closed-form representation for the probability density can be derived if the excitation is also assumed to be Gaussian. The expression may be reduced to the well-known Rayleigh distribution in the stationary case.

It has been shown that the dynamic characteristics of primary-secondary systems depends on two important parameters: the interaction parameter and the nonclassical damping parameter. The former evaluates the importance of interaction between the primary and secondary systems, and the latter gives a necessary and sufficient condition for whether a system is classically damped. The interaction parameter proposed herein has a simple relationship with the parameter employed by Iwan and Smith (1987).

For sufficiently small interaction parameters, the interaction may be neglected and the primary and secondary systems become uncoupled. The concept of evolutionary floor spectral density is introduced which can be used as the input to calculate the nonstationary response of a secondary system. When the interaction parameter becomes large, the interaction between the two systems is significant and, therefore, a combined primary-secondary system must be studied. A delay effect of the secondary system response is observed due to the nonstationarity of the excitation, which implies that longer duration should be considered in the aseismic design of secondary systems. It is noticed that nonstationary response may depend on the parameters of certain types of excitation models such as a white noise modulated by the Shinozuka-Sato type envelope. The covariance response of a system may exhibit a significant increase when the system parameters are close to some critical range determined by the load parameters. It is also observed that nonclassical damping generally has a negligible effect on the secondary system response but has a considerable effect on the response of the primary system when the mass ratio is small.

As the second application, a unified formulation is presented to investigate the seismic response of structures as well as the absolute ground surface acceleration under a class of evolutionary ground motion models which simulate the dynamic behavior of soil subjected to a nonstationary excitation at bedrock. The models include uniform and nonuniform shear beams in addition to the Kanai-Tajimi model subjected to a nonstationary excitation at the base. Explicit solutions are found for both structural response and absolute ground acceleration under the random impulse train earthquake model or a modulated white noise. All the three soil models considered, including the Kanai-Tajimi filter under a nonstationary excitation, and the resulting structural response, show nonstationarity in both intensity and frequency content.

A comparison shows that for the models with the same natural frequency and damping ratio in the first mode, a significant increase in both the ground motion and structural response are observed by using the beam models with the nonuniform model giving the largest absolute ground acceleration and structural response. Therefore, the conventional Kanai-Tajimi model may underestimate the ground motion and structural response and an improved ground motion model, such as the uniform or nonuniform shear beam model, which more properly describes the local soil properties may be needed in the seismic analysis of critical structures.

Some results for structural reliability are also presented. It is found that in the nonstationary case, the structural reliability generally does not approach zero as time increases, which is attributed by the fact that the structure may survive a given earthquake event of finite energy. Therefore, the term of first passage probability density function, frequently used in the literature to refer to p(t) in the stationary case, will be improper in the general nonstationary case. As a remedy, a refined definition of the first passage probability density, or more precise terminology such as the "first passage probability rate" is suggested.

The last application addresses the effect of the vertical component of ground motion on the seismic performance of structures. The simplified state-variable method is used to obtain the explicit solution for both the stationary and nonstationary second moment responses of a linear simple structure under combined horizontal and vertical ground excitations which may be uncorrelated or correlated. A general stability criterion for the covariance response has been given.

In the stationary case, it has been found that the correlation response under combined loads is simply equal to that under horizontal ground motion alone multiplied by the Parametric Amplification Factor defined in Chapter 5. This factor depends only on the system parameters and the intensity of vertical motion.

In the nonstationary case, if the excitations are uncorrelated, the covariance solution may be obtained by a superposition of the solution for the external excitation alone and an additional part which depends on the intensity of the vertical excitation. Proportionality between the solutions for the external load alone and the combined loads, as exhibited in the stationary correlation solution, generally does not hold during the initial nonstationary stage of the nonstationary solutions but may still be valid for the stationary values. If the excitations are correlated, the correlation enters the analysis if and only if both the correlation and the mean of the displacement response are nonzero.

Two cases which can cause a nonzero mean of the displacement have been discussed. For the case that the nonzero mean of the displacement is caused by a nonzero initial displacement and/or velocity, the solution consists of three parts, namely the solution corresponding to uncorrelated components, and two correction terms for nonzero initial mean of the displacement and/or velocity and nonzero initial second moment response respectively. The correlation may have considerable effect on the transient stage of the solution but little effect on the stationary values. The significance depends on the magnitude of the correlation.

If the nonzero mean of the displacement is caused by the presence of a pre-existing additional stationary external load, the dynamic behavior of the system is essentially the same as that for the case where the additional load is absent. The final solution can be obtained by scaling the solution for no extra load and then increasing or decreasing it by an amount of the initial covariance caused by the stationary excitation. The effect of the correlation of the loads on the second moment response depends on the magnitude of the correlation and the given nonzero initial conditions resulting from the presence of the additional external load.

The mean square stability of the structural response has been discussed. It has been shown that the covariance response is conditionally weakly stable for the step envelope and unconditionally stable for the exponentially decaying envelope. If the nonzero mean of the displacement is caused by the existence of an additional stationary load, the latter is also weakly stable. The effect of the vertical component of earthquake ground motion on the second moment response may be significant in certain cases, especially near the region of instablity even when the intensity of the vertical component is very small relative to that of horizontal motion. Increasing the damping ratio and the natural frequency of a structure may reduce this effect.

Finally, the proposed method may be applied to many other engineering problems which can be formulated as a linear MDOF or continuous system subjected to modulated stationary excitation. Though the formulation in this thesis is basically for a single random input, the method may be extended to the case where multiple random inputs must be considered, such as in the seismic analysis of long-span structures. The method can also be readily incorporated into existing finite element codes for random analysis of linear structures. The application of the method to analysis of linear systems with system uncertainty and nonlinear structures is under investigation.

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Appendix I
A Selected Review

A selected review of the methods determining the second moment response of linear structures subjected to a nonstationary random excitation is given in this appendix. The review shows the motivation of this research and provides a background for comparison of the existing methods and the simplified state-variable method proposed in Chapter 2. Two parts are included in this review. Part 1 summarizes the methods currently available for the nonstationary covariance analysis of simple systems, which gives the basic concepts of the analysis. Part 2 addresses the analysis of multi-degree-of freedom systems where different techniques have been proposed to overcome special difficulties caused by the size of the problem. Merit and limitation of each method have been discussed. The review excludes the nonstationary analysis of uncertain structures and that of linear structures under parametric excitations. For further information on these aspects, the reader is refered to Vanmarcke et al. (1986), Benaroya and Rehak (1988), Ibrahim (1985), and To (1989).

A1.1 Covariance Response of Simple Linear Systems

Great progress has been made to study the characteristics of the response of simple structures subjected to nonstationary excitation, including the works by Bogdanoff et al. (1961), Caughey and Stumpf (1961), Barnoski and Meurer (1969, 1973), Bucciarelli and Kuo (1970), Corotis and Marshall (1977), and Igusa (1989, 1989). The equation of motion of the system is usually expressed as

$$\ddot{x}(t) + 2\varsigma_0\omega_0\dot{x}(t) + \omega_0^2 x(t) = f(t) \qquad (A1-1)$$

where ς_0 and ω_0 denote the ratio of critical damping, and natural frequency of the system, f(t) is a zero-mean random load per unit mass whose statistical properties may be characterized by its covariance $Q_{ff}(t_1, t_2)$ defined by

$$Q_{ff}(t_1,t_2) = E[f(t_1)f(t_2)] \qquad (A1-2)$$

where $E[.]$ denotes the ensemble average. In many cases, $f(t)$ may be expressed in the form

$$f(t) = \eta(t)n(t) \qquad (A1-3)$$

where $\eta(t)$ is a deterministic modulating envelope and $n(t)$ is a stationary process characterized by its covariance $R(\tau)$ or spectral density $S(\omega)$. A detailed discussion of the modeling of random loads is given in Appendix 2.

Basically, the methods available may be classified into two catagories: time domain approaches and frequency domain approaches as discussed below.

A1.1.1 Time Domain Approaches

A time domain approach is often employed when the transient covariance response is sought directly and the characteristics of the input are readily given in the time domain.

i. Impulse Response Approach

This approach can be thought of as a direct extention of deterministic analysis. For any realization of the load process $f(t)$, the corresponding sample of response $x(t)$ can be expressed by a Duhamel convolution integral of the form

$$x(t) = \int_0^t h(t-\tau)f(\tau)d\tau \qquad (A1-4)$$

where the impulse response $h(t)$ can be found as

$$h(t) = \begin{cases} \frac{1}{\omega_d}e^{-\varsigma_0\omega_0 t}\sin\omega_d t, & \text{if } t \geq 0; \\ 0, & \text{otherwise.} \end{cases} \qquad (A1-5)$$

in which ω_d is the damped natural frequency of the system.

The covariance response can be obtained for Eq. (A1-4) as

$$Q_{xx}(t_1,t_2) = \int_0^{t_1}\int_0^{t_2} h(t_1-\tau_1)h(t_2-\tau_2)Q_{ff}(\tau_1,\tau_2)d\tau_1 d\tau_2 \qquad (A1-6)$$

The mean square response and the spectral density of the response can then be determined from Eq. (A1-6). If $f(t)$ is modeled as a modulated white noise, the double integral may be reduced to a single integral of the form

$$Q_{xx}(t_1, t_2) = S_0 \int_0^{min(t_1,t_2)} h(t_1 - \tau)h(t_2 - \tau)\eta^2(\tau)d\tau \qquad (A1-7)$$

where S_0 is the intensity of the white noise. Similar expressions exist for the autocorrelation of the velocity of the response and the cross-correlation of the displacement and velocity.

The approach may be found in most of textbooks on the random vibration. Explicit solutions are available only for simple envelopes. Algebraic difficulty increases if auto- and cross-covariance of both displacement and velocity are needed, as in reliability analysis.

ii. Incremental Load Method

The method also stems from its counterpart in deterministic analysis where the general solution of Eq. (A1-1) may be expressed by

$$x(t) = \int_0^t H(t - \tau)df(\tau) \qquad (A1-8)$$

where $H(t)$, referred to as the Heaviside response, is the response due to a unit step load applied at time $t = 0$. Eq. (A1-8) says that the solution may be thought as a superposition of all the responses caused by the incremental loads $df(\tau)$ applied at $t = \tau$ prior to the current time. The idea was extended to random analysis by Madsen and Krenk (1982), Krenk et al. (1983), Di Paolr (1985), Langley (1986), Muscolino (1988), Borino et al. (1988), and Igusa (1989a, 1989b).

Consider an SDOF system as described by Eq. (A1-1) subjected to a modulated white noise

$$f(t) = \eta(t)w(t) \qquad (A1-9)$$

where $w(t)$ is a stationary white noise process. Then, the nonstationary response may be expressed as

$$x(t) = \int_0^t \dot{\eta}(s)\kappa(t, s)ds \qquad (A1-10)$$

where

$$\kappa(t,s) = x_0(t) - g(t-s)x_0(s) - h(t-s)\dot{x}_0(s) \qquad (A1-11)$$

in which $x_0(t)$ is the stationary solution of the system under stationary white noise excitation, and $h(t)$ and $g(t)$ are the impulse response functions of the system for initial unit velocity and displacement respectively. The expressions for $g(t)$ and $h(t)$ can be found in Madsen and Krenk (1982).

Note that substituting $s = 0$ in Eq. (A1-11) yields

$$\kappa(t,0) = x_0(t) - g(t)x_0(0) - h(t)\dot{x}_0(0) \qquad (A1-12)$$

which is, in fact, the nonstationary solution for a unit step envelope. Eq. (A1-10) gives a way to express a nonstationary solution in terms of a corresponding stationary solution.

The nonstationary covariance can then be expressed as

$$\begin{aligned} R(t_1,t_2) &= E[x(t_1)x(t_2)] \\ &= \int_0^{t_1}\int_0^{t_2} \dot{\eta}(s_1)\dot{\eta}(s_2)r(t_1,t_2,s_1,s_2)ds_1ds_2 \end{aligned} \qquad (A1-13)$$

where

$$\begin{aligned} r(t_1,t_2,s_1,s_2) = c(t_1-t_2) &- g(t_1-s_1)c(s_1-t_2) - g(t_2-s_2)c(t_1-s_2) \\ &- h(t_1-s_1)\dot{c}(s_1-t_2) + h(t_2-s_2)\dot{c}(t_1-s_2) \\ &+ g(t_1-s_1)g(t_2-s_2)c(s_1-s_2) - h(t_1-s_1)h(t_2-s_2)\ddot{c}(s_1-s_2) \\ &+ [g(t_2-s_2)h(t_1-s_1) - g(t_1-s_1)h(t_2-s_2)]\dot{c}(s_1-s_2) \end{aligned} \qquad (A1-14)$$

$c(\tau)$ is the stationaty covariance response defined as

$$\begin{aligned} c(\tau) &= E[x_0(t+\tau)x_0(t)] \\ &= \int_0^{\infty} \cos\omega\tau |H(\omega)|^2 d\omega \end{aligned} \qquad (A1-15)$$

Similar results can be obtained for $\bar{x}(t)$, the Hilbert transform of the response, defined by

$$\bar{x}(t) = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{x(\tau)}{t-\tau}d\tau \qquad (A1-16)$$

Those results are used to evaluate the first passage probability of the envelope of the response (Cramer and Leadbetter, 1967) defined by

$$\epsilon(t) = \sqrt{x^2(t) + \bar{x}^2(t)} \qquad (A1-17)$$

It is extremely difficult to obtain explicit solutions from the lengthy expression in Eq. (A1-14) and the only exact solution found by this method is for the unit step modulation. An approximate expression has been proposed for general nonstationary modulations under certain assumptions which greatly limit the applicability of the method.

iii. Kolmogorov-Fokker-Plank (KFP) Equation Approach

If the response is assumed to be a Markov process, which is the case when the system is excited by modulated white noise, the forward Kolmogorov equation, or Fokker-Plank Equation may be employed. To limit the derivatives in the equation to second order, some additional assumptions need to be introduced, such as when the excitation is Gaussian (Lin, 1967), or when a lightly damped system is excited by a broad-band excitation (Huang and Spanos, 1984; Solomos and Spanos, 1984). For instance, the KFP equation which governs the transition probability density $q(x, \dot{x}, t|x_0, \dot{x}_0, t_0)$ for a simple linear system subjected to a Gaussian shot noise with intensity $I(t)$ will be

$$\frac{\partial q}{\partial t} + \dot{x}\frac{\partial q}{\partial x} - \frac{\partial}{\partial \dot{x}}[(2\zeta_0\omega_0\dot{x} + \omega_0^2 x)q] - \frac{I(t)}{2}\frac{\partial^2 q}{\partial \dot{x}^2} = 0 \qquad (A1-18)$$

Eq. (A1-18) was indirectly solved for linear systems subjected to a Gaussian shot noise by first obtainining the mean and covariance of the response (Lin, 1964). Generally, the KFP equation is difficult to solve, especially in the nonstationary case. Instead of directly solving for the transition probability density, the KFP equation is often used to derive certain moment equations for the response. The solution for these moment equations generally requires a numerical technique.

A1.1.2 Frequency Domain Approaches

Recently, a great deal of interest has been devoted to the spectral analysis of nonstationary processes. The primary objective of the effort is to extend the relationship between the power spectral density and mean square value of a stationary process to nonstationary processes and, therefore, provide a mapping of the energy content of a nonstationary process in the frequency-time domain. The time-dependant covariance of the response may be obtained by integrating the corresponding power spectral density. As pointed out by Corotis and Vanmarcke (1975), there is no unique definition of power spectral density for nonstationary processes. Several popular approaches are summarized in the following.

i. Double Fourier Transform Approach

The well-known Wienner-Khintchine relationship for stationary processes may be formaly extended to nonstationary processes by means of a double Fourier transform. Let $R(t_1, t_2)$ be the correlation of a nonstationary processes $x(t)$, i.e.,

$$R(t_1, t_2) = E[x(t_1)x(t_2)] \tag{A1-19}$$

Then, one may obtain the following Fourier transform pair

$$\begin{aligned} S(\omega_1, \omega_2) &= \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} R(t_1, t_2) e^{-i(\omega_1 t_1 - \omega_2 t_2)} dt_1 dt_2 \\ R(t_1, t_2) &= \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} S(\omega_1, \omega_2) e^{i(\omega_1 t_1 - \omega_2 t_2)} d\omega_1 d\omega_2 \end{aligned} \tag{A1-20}$$

$S(\omega_1, \omega_2)$ has been called as the *generalized power spectrum.*

Let $H(\omega)$ be the transfer function of a linear system, $f(t)$ be the random excitation, and $x(t)$ be the response of the system, then

$$S_{xx}(\omega_1, \omega_2) = H(\omega_1) H^*(\omega_2) S_{ff}(\omega_1, \omega_2) \tag{A1-21}$$

where $S_{ff}(\omega_1, \omega_2)$ and $S_{xx}(\omega_1, \omega_2)$ are the generalized power spectra of the input and output. The covariance $R_{xx}(t_1, t_2)$ can then be evaluated as

$$R_{xx}(t_1, t_2) = \frac{1}{4\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} H(\omega_1) H^*(\omega_2) S_{ff}(\omega_1, \omega_2) e^{i(\omega_1 t_1 - \omega_2 t_2)} d\omega_1 d\omega_2 \tag{A1-22}$$

The above approach was employed by Holman and Hart (1971, 1972, 1974) to obtain the covariance response of structures under segmented nonstationary excitation. Though mathematically Eq. (A1-20) is strictly true, the expression for

$S(\omega_1, \omega_2)$ is difficult to interpret physically and, in addition, double integration is generally needed in evaluating the covariance, which greatly restricts the application of the technique.

ii. Page's instantaneous power spectrum

Page (1952) was the first to define a time-dependent power spectrum and use the term "instantaneous power spectrum." The so-called periodogram

$$g_t^*(\omega) = E[|\int_0^t x(\tau)e^{-i\omega\tau}d\tau|^2] \tag{A1-23}$$

was used to define the spectrum $f^*(\omega)$ of $x(t)$ as

$$f^*(\omega) = \lim_{t\to\infty} g_t^*(\omega) \tag{A1-24}$$

Then, the instantaneous power spectrum $S_p(t,\omega)$ was defined as

$$\begin{aligned} S_p(t,\omega) &= \frac{d}{dt}g_t^*(\omega) \\ &= 2\int_0^t R(t,\tau)\cos\omega(t-\tau)d\tau \end{aligned} \tag{A1-25}$$

Note that

$$f^*(\omega) = \int_0^\infty S_p(t,\omega)dt \tag{A1-26}$$

Thus, the instantaneous power spectrum $S_p(t,\omega)$ represents roughly the difference between the energy distributions over the interval $(0,t)$ and the interval $(0,t+dt)$. Therefore, the term "instantaneous" refers only to the time-dependence and has nothing to do with the local energy distribution of the process in the neighborhood of the instant t. The concept was used by Liu (1972) in his covariance response analysis.

iii. Instantaneous Power Spectrum Used by Bendat et al. (1971)

An alternative approach, used by Bendat and Piersol (1971), defines the time-dependent auto-correlation function for a zero-mean process as

$$R(t,\tau) = E[x(t-\frac{\tau}{2})x(t+\frac{\tau}{2})] \tag{A1-27}$$

where t denotes an avarage time and τ represents a time lag. That is

$$\begin{aligned} t &= \frac{t_1 + t_2}{2} \\ \tau &= \frac{t_1 - t_2}{2} \end{aligned} \qquad (A1-28)$$

A time-dependent spectral density function may be obtained from the Fourier transform of the above equation with respect to the variable τ as follows.

$$G(t,\omega) = \frac{2}{\pi} \int_0^\infty R(t,\tau) e^{-i\omega\tau} d\tau \qquad (A1-29)$$

The inverse gives the covariance

$$R(t,\tau) = \int_0^\infty G(t,\omega) e^{i\omega\tau} d\omega \qquad (A1-30)$$

Note that by taking the time lag τ over all the values between $\pm\infty$, the spectral density (A1-29) represents an average rather than an instantaneous value.

Corotis and Vanmarcke (1975) started from this definition and obtained a general closed form solution for the spectral density of the response of a simple linear system subjected to a modulated stationary process as

$$G(t,\omega) = \int_0^t \int_0^t \eta(u)\eta(v)h(t-u)h(t-v)G_\phi(\omega)\cos\omega(u-v)dudv \qquad (A1-31)$$

where $G_\phi(\omega)$ is the spectral density of the stationary process. Due to the mathematical difficuty involved, an explicit expression for $G(t,\omega)$ was given only for the unit Heaviside envelope. It should be pointed out that the solution (A1-31) does not agree with the definition (A1-29). In fact, the spectral density defined by the form (A1-31) is the evolutionary spectrum defined by Priestley (1965).

iv. Mark's Physical Spectrum

Instead of using the history of a process prior to instant t, Mark (1970) used the values of the process in the neighborhood of instant t by introducing a window function $w(t)$ which is nonnegative, sharply-peaked, and normalized as

$$\int_{-\infty}^{\infty} w^2(t) = 1 \qquad (A1-32)$$

Parseval's Formula gives

$$\int_{-\infty}^{\infty}[w(t-u)x(u)]^2du = \int_{-\infty}^{\infty}|\int_{-\infty}^{\infty} w(t-u)x(u)e^{-i\omega u}du|^2 d\omega \qquad (A1-33)$$

Integrating both sides over t and using the normalizing condition of $w(t)$ yields

$$\int_{-\infty}^{\infty} E[x^2(t)]dt = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} S_M(t,\omega)d\omega dt \qquad (A1-34)$$

where the so-called physical spectrum $S_M(t,\omega)$ is defined as

$$S_M(t,\omega) = E\left[|\int_{-\infty}^{\infty} w(t-u)x(u)e^{-i\omega u}du|^2\right] \qquad (A1-35)$$

which characterizes the frequency content of the process in the neighborhood of time t. An example was given using the normalized rectangular function as the window function. The resulting expressions require judicious smoothing to be physically consistent, but the definition appears to be useful, especially for actual process measurement (Corotis and Vanmarcke, 1975). To the author's knowledge, no result for covariance response by this approach is presented in the literature.

v. Evolutionary Power Spectrum

Pristley (1965, 1967) developed the theory of the evolutionary power spectrum to describe the local energy distribution of a random process $x(t)$ in the neighborhood of the time instant t. It is found that a class of nonstationary processes, such as the response of structures subjected to a modulated stationary excitation, may be expressed as

$$x(t) = \int_{-\infty}^{\infty} A(t,\omega)e^{i\omega t}dZ(\omega) \qquad (A1-36)$$

where $A(t,\omega)$ is slowly-varying function and $dZ(\omega)$ is an orthogonal process with

$$E[dZ(\omega_1)dZ(\omega_2)] = G_f(\omega_1)\delta(\omega_1-\omega_2)d\omega_1 d\omega_2 \qquad (A1-37)$$

Eq. (A1-36) is a nonstationary generalization of the Wiener representation of stationary processes.

The evolutionary spectral density can be defined as

$$S_E(t,\omega) = G_f(\omega)|A(t,\omega)|^2 \qquad (A1-38)$$

which is related to the nonstationary covariance of the process by

$$E[x^2(t)] = \int_{-\infty}^{\infty} S_E(t,\omega)d\omega \qquad (A1-39)$$

Eq. (A1-38) is valid even if $A(t,\omega)$ varies rapidly. However, in that case the evolutionary spectral density will lose its physical meaning. The concept of evolutionary spectral density has been extended to vector processes by Priestley and Tong (1973), and Hammond (1973).

Priestley's evolutionary spectrum seems very useful in the spectral analysis of nonstationary processes. It provides a local energy distribution which has the same physical interpretation as the power spectral density function of a stationary random process. The transient covariance may be evaluated by integrating the evolutionary spectral density over the frequecy domain. Several results for covariance analysis are available using this method, including Hammond (1968), Barnoski and Maurer (1969, 1973), To (1982), Ahmadi (1986), Borino et al. (1988), and Shihab and Preumont (1989). Numerical integration is generally needed in the evaluation of the covariance response.

A1.2 Covariance Response of MDOF Systems

It is seen from the previous section that it is cumbersome to solve for the nonstationary covariance response even for simple systems. The problem becomes more serious when the number of degree-of-freedom of the system increases. An efficient method is needed to find the nonstationary solution of MDOF systems. Some effort has been devoted to this aspect.

A1.2.1 Modal Superposition Approach

Modal analysis is a fundamental approach in the dynamic analysis of linear MDOF systems. Assume the equation of motion to be

$$\mathbf{M}\ddot{\mathbf{x}}(t) + \mathbf{C}\dot{\mathbf{x}}(t) + \mathbf{K}\mathbf{x}(t) = \mathbf{f}(t) \qquad (A1-40)$$

where $\mathbf{M}, \mathbf{C}$, and $\mathbf{K}$ are the mass, damping, and stiffness matrices respectively, $\mathbf{f}(t)$ is a nonstationary random load vector, and $\mathbf{x}(t)$ is the structure response vector.

By the modal superposition approach, the structural response can be written as

$$\mathbf{x}(t) = \mathbf{\Phi}\alpha(t) \qquad (A1-41)$$

where $\mathbf{\Phi}$ is the modal matrix and $\alpha(t)$ is the corresponding modal response vector. Eq. (1-42) implies that the total response is the superposition of contributions from all the modes. The modal responses $\alpha_i(t), i = 1, ..., n$ are determined from

$$\ddot{\alpha}_i(t) + 2\varsigma_i\omega_i\dot{\alpha}_i(t) + \omega_i^2\alpha_i(t) = f_i(t) \qquad (A1-42)$$

where $f_i(t)$ is the modal force.

The statistics of the modal responses can be obtained by any approaches mentioned in the previous section. For example, if the Duhamel integral approach is used, the solution of Eq. (A1-40) takes the form

$$\alpha_i(t) = \int_0^{\infty} h_i(t-\tau)f_i(\tau)d\tau \qquad (A1-43)$$

where $h_i(t)$, the impulsive response of ith mode, is expressed as

$$h_i(t) = \begin{cases} \frac{1}{m_i\omega_{id}}e^{-\varsigma_i\omega_i t}\sin\omega_{id}t, & \text{if } t \geq 0; \\ 0, & \text{otherwise.} \end{cases} \qquad (A1-44)$$

The modal auto- and cross-covariances can be found as

$$\begin{aligned} E[\alpha_i(t_1)\alpha_j(t_2)] &= \int_0^{t_1}\int_0^{t_2} h_i(t_1-\tau_1)h_j(t_2-\tau_2)E[f_i(\tau_1)f_j(\tau_2)]d\tau_1 d\tau_2 \\ E[\dot{\alpha}_i(t_1)\alpha_j(t_2)] &= \int_0^{t_1}\int_0^{t_2} \dot{h}_i(t_1-\tau_1)h_j(t_2-\tau_2)E[f_i(\tau_1)f_j(\tau_2)]d\tau_1 d\tau_2 \\ E[\dot{\alpha}_i(t_1)\dot{\alpha}_j(t_2)] &= \int_0^{t_1}\int_0^{t_2} \dot{h}_i(t_1-\tau_1)\dot{h}_j(t_2-\tau_2)E[f_i(\tau_1)f_j(\tau_2)]d\tau_1 d\tau_2 \\ & i, j = 1, ..., n \end{aligned} \qquad (A1-45)$$

The statistics of physical quantities can be obtained by assembling the statistics of the modal responses. For example, the covariance of displacements can be evaluated by

$$E[x_i(t_1)x_j(t_2)] = \sum_{l=1}^{n}\sum_{m=1}^{n} \Phi_{il}\Phi_{jm}\alpha_l(t_1)\alpha_m(t_2) \qquad (A1-46)$$

The modal superposition method has been a fundamental tool for the analysis of MDOF systems and has been used by many investigators including Hommand (1968), Hart (1970), Masri (1978, 1979), Madsen and Krenk (1982), To (1984, 1986, 1987). Once the eigenvalues and eigenvectors are obtained, the problem may be solved by first finding the modal response. However, this method applies only to the dynamic systems of second-order with classical dampings, which leads to a set of uncoupled modal equations. Though the method has a clear physical interpretation, it has been found that it is cumbersome to manipulate mathematically. Exact solutions have been obtained only in limited cases and numerical solution is time-consuming due to the convolutions required.

A1.2.2 State-Variable Approach

Due to the difficulty of mathematical manipulation in the modal superposition approach, some effort has been directed towards reformulating the problem in a state space. Gasparini (1979) formulated the problem in the modal state space. Choosing the modal displacements and modal velocities as the basic variables, the modal state variable vector can be constructed as

$$\mathbf{a}(t) = \{\alpha_1(t), \dot{\alpha}_1(t), \ldots, \alpha_n(t), \dot{\alpha}_n(t)\}^T \tag{A1-47}$$

An explicit matrix expression for the statistics of the modal state variables may be obtained as

$$\mathbf{Q}_{aa}(t_1, t_2) = E[\mathbf{a}(t_1)\mathbf{a}^T(t_2)] \tag{A1-48}$$

whose entries can be evaluated by Eq. (A1-45). A synthesis is then needed for the statistics of desired physical quantities, such as displacements and velocities. For instance, let $\mathbf{y}(t)$ be a vector of desired responses expressed in terms of $\mathbf{a}(t)$ as

$$\mathbf{y}(t) = \mathbf{D}\mathbf{a}(t) \tag{A1-49}$$

where $\mathbf{D}$ is a constant matrix. Then the corresponding covariance response can be found as

$$\mathbf{Q}_{yy} = \mathbf{D}\mathbf{Q}_{aa}(t)\mathbf{D}^T \tag{A1-50}$$

A similar formulation was adopted by Grigoriu (1981) for stationary excitations.

The matrix expression brings great convenience to the solution of the covariance response, and a variety of problems have been studied using this approach by Gasparini(1979, 1980), Gasparini and DebChaudhury (1980), DebChaudhury and Gasparini (1982), DebChaudhury and Gazis (1988), To (1987). Essentially, the method is nothing but modal superposition as described previously, since all the entries in the matrix are individually evaluated on a modal basis. Therefore, the method has the same restriction as the modal analysis does. A post-synthesis is needed to obtain the statstics of physical quantities.

An alternative formulation has been given directly in terms of the physical state variables, i.e., displacement and velocity, see Lin (1967), Langley (1986). Eq. (A1-44) may be rewritten as

$$\frac{d\mathbf{Y}(t)}{dt} = \mathbf{A}\mathbf{Y}(t) + \mathbf{F}(t) \qquad (A1-51)$$

where

$$\mathbf{Y}(t) = \begin{pmatrix} \mathbf{x}(t) \\ \dot{\mathbf{x}}(t) \end{pmatrix} \quad \mathbf{A} = \begin{pmatrix} \mathbf{O} & \mathbf{I} \\ -\mathbf{M}^{-1}\mathbf{K} & -\mathbf{M}^{-1}\mathbf{C} \end{pmatrix} \quad \mathbf{F} = \begin{pmatrix} \mathbf{O} \\ \mathbf{f}(t) \end{pmatrix} \qquad (A1-52)$$

The mean square response is then given by

$$\mathbf{Q}(t) = \int_0^t \int_0^t \mathbf{\Phi}(t-\tau_1)\mathbf{B}(\tau_1,\tau_2)\mathbf{\Phi}^T(t-\tau_2)d\tau_1 d\tau_2 \qquad (A1-53)$$

where

$$\mathbf{B}(t_1,t_2) = \begin{pmatrix} \mathbf{O} & \mathbf{O} \\ \mathbf{O} & \mathbf{M}^{-1}E[\mathbf{F}(t_1)\mathbf{F}^T(t_2)]\mathbf{M}^{-T} \end{pmatrix} \qquad (A1-54)$$

and $\mathbf{\Phi}(t)$ is the fundamental matrix solution of the system. The explicit solution for MDOF systems is very difficult to obtain by this method even in the case of modulated white noise. Numerical techniques are generally used to find the solution based on the differential version of Eq. (A1-53), as shown in the next section.

Most recently, Yang, Sarkani, and Long (1988) used the canonical modal analysis to solve Eq. (A1-51). Once the eigenvalues and eigenvectors of the matrix $\mathbf{A}$ is obtained, a real matrix $\mathbf{T}$ is constructed as

$$\mathbf{T} = [\mathbf{a}_1, \mathbf{b}_1, \mathbf{a}_2, \mathbf{b}_2, ..., \mathbf{a}_j, \mathbf{b}_j, ..., \mathbf{a}_n, \mathbf{b}_n] \qquad (A1-55)$$

where $\mathbf{a}_j$ and $\mathbf{b}_j$ are the real and imaginary parts of the jth pair of eigenvectors respectively. Using the transformation defined dy

$$\mathbf{Y} = \mathbf{T}\mathbf{v} \tag{A1-56}$$

Eq. (A1-51) may be reduced to N pairs of coupled differential equations as

$$\begin{aligned} \dot{v}_{2j-1} &= -\zeta_j\omega_j v_{2j-1} + \omega_{dj} v_{2j} + \bar{F}_{2j-1} \\ \dot{v}_{2j} &= -\omega_{dj} v_{2j-1} - \zeta_j\omega_j v_{2j} + \bar{F}_{2j} \\ j &= 1, 2, ..., n \end{aligned} \tag{A1-57}$$

where $\zeta_j\omega_j$ and ω_{dj} are the real and imaginary parts of the jth pair of the complex conjugate eigenvalues and

$$\bar{\mathbf{F}} = \mathbf{T}^{-1}\mathbf{F} \tag{A1-58}$$

The final solution for the mean square response vector of $\mathbf{Y}(t)$ is given by

$$\sigma_Y^2(t) = \int_{-\infty}^{\infty} |\mathbf{M}_Y(t,\omega)|^2 \Phi_{nn}(\omega) d\omega \tag{A1-59}$$

where $\mathbf{M}_Y(t,\omega)$ is found as

$$\mathbf{M}_Y(t,\omega) = \int_0^t \mathbf{T}\mathbf{h}_v(\tau)\eta(t-\tau)e^{-i\omega\tau} d\tau \tag{A1-60}$$

in which $\mathbf{h}_v(t)$ is the impulsive response vector whose components may be solved from Eq. (A1-57).

This method may be applied to the analysis of systems with nonclassical damping. The solution is expressed in terms of the the evolutionary spectral density. Generally, explicit solutions are difficult to find and, instead, a numerical technique such as FFT is employed to find the final solution.

A1.2.3 Lyapunov Direct Method

A differential equation governing the covariance response may be found by either manipulating Eq. (A1-51) or differentiating Eq. (A1-53) with respect to t. This yields

$$\dot{\mathbf{Q}}(t) = \mathbf{A}\mathbf{Q}(t) + \mathbf{Q}(t)A^T + \mathbf{\Theta}(t) \tag{A1-61}$$

where

$$\mathbf{\Theta}(t) = \int_0^t \mathbf{\Phi}(t-\tau)\mathbf{R}_{FF}(t,\tau)d\tau + \int_0^t \mathbf{R}_{FF}(t,\tau)\mathbf{\Phi}^T(t-\tau)d\tau \qquad (A1-62)$$

which can be evaluated explicitly if the excitation is a modulated white noise process.

In the stationary case, the above equation may be reduced to the well-known Liapunov's matrix equation

$$\mathbf{AQ} + \mathbf{QA}^T = \mathbf{B} \qquad (A1-63)$$

which can be solved by various techniques as discussed in Yang and Iwan (1972). In the nonstationary case, the differential Eq. (A1-61) can be solved numerically by some standard method such as Runge-Kutta integration, the predictor-corrector method, etc.

The method is quite flexible for the nonstationary solution for both autonomous and nonautonomous systems, and different types of excitation. It is obvious that it would be computationally time-consuming if the excitation is not a modulated white moise since in each time step a convolution must be calculated. Anothor problem can arise in the first few time steps. The numerical error might destroy the semi-positiveness of the covariance metrix, which can be seen from the following example.

Let us consider the case of an SDOF system subjected to a suddenly applied statonary white noise with a constant intensity S_0. If a forward difference scheme is used, the difference equation of the system will be

$$\mathbf{Q}^{(i+1)} = \mathbf{Q}^{(i)} + \left(\mathbf{AQ}^{(i)} + \mathbf{Q}^{(i)}\mathbf{A}^T + \mathbf{B}\right)\Delta t \qquad (A1-64)$$

where

$$\mathbf{B} = \begin{pmatrix} 0 & 0 \\ 0 & S_0 \end{pmatrix} \qquad (A1-65)$$

If higher order terms are neglected in each entry, the results are

$$\mathbf{Q}^{(1)} = \begin{pmatrix} 0 & 0 \\ 0 & S_0\Delta t \end{pmatrix}$$

$$\mathbf{Q}^{(2)} = \begin{pmatrix} 0 & S_0\Delta t^2 \\ S_0\Delta t^2 & 2S_0\Delta t + o(\Delta t) \end{pmatrix}$$

$$\mathbf{Q}^{(i)} = \begin{pmatrix} S_0\dfrac{i(i-1)(i-2)}{3}\Delta t^3 + o(\Delta t^3) & S_0\dfrac{i(i-1)}{2}\Delta t^2 + o(\Delta t^2) \\ S_0\dfrac{i(i-1)}{2}\Delta t^2 + o(\Delta t^2) & S_0 i\Delta t + o(\Delta t) \end{pmatrix} \quad i \geq 2 \tag{A1-66}$$

The determinent of the covariance matrix will be

$$|\mathbf{Q}^{(i)}| = \frac{i^2(i-1)(i-5)}{12}\Delta t^4 + o(\Delta t^4) \tag{A1-67}$$

Therefore, in the first few steps for sufficiently small Δt, the determinant of the covariance matrix is negative no matter how small Δt might be. The same problem may be observed in some other integration schemes. The conclusion contradicts the semi-positiveness of the covariance matrix. If the emphasis is only on the covariance matrix, this may not cause serious problems. However, this problem will affect a reliability analysis where the semi-positiveness of the covarince matrix is always assumed. Some adjustment is needed, but the effect of the error is not clear.

A1.2.4 Approximate Techniques

Considerable algebraic difficulties will be involved in evaluation of the covariance response of MDOF systems. Explicit solutions exist only in a very few simple cases. Some approximate techniques has been suggested to faciliate the solution.

One simplification may be achieved in the case that the structure is lightly damped and the load is wide-banded in which case all the quantities of higher order in the critical damping ratio may be neglected. The technique has been used in some studies such as those by Caughey and Stumpf (1961), Solomos and Spanos (1984), and Igusa (1989). Explicit solutions are obtained at the cost of restrictions on the range of applicability of the solutions.

Another attention has been given to the case where the load can be modeled as a modulated stationary process expressed by Eq. (A1-3)

$$f(t) = \eta(t)n(t)$$

If the envelope function $\eta(t)$ takes a complicated form, considerable algebraic difficulties will be encountered. Some approximation of the load may bring great simplification to the calculation, as shown by Roberts (1971), Hasselman (1972), Holman and Hart (1974), Sun and Kareem (1989).

Roberts (1971) suggested a simple approximate method which has a direct physical significance. Noticing that real earthquakes have a finite duration, instead of using a single pulse of white noise to model the ground motion, a train of such pulses is constructed. Each pulse in the train is statistically identical to the original one and is spaced at regular intervals. If the spacing between these pulses is sufficiently large, the system will respond to each pulse in the train in a manner almost identical to that in which it would respond to an isolated pulse. The "overlapping error" will be reduced as the spacing between the impulses increases. This consideration leads to a series expansion of the excitation which makes integration easier. Some numerical examples have been given to show the accuracy of the method.

Hasselman (1972), Holman and Hart (1974), Sun and Kareem (1989) employed another type of approximation of the load by replacing the original envelope with a staircase unit function expansion.

$$\eta(t) \approx \sum_{n=1}^{N} \eta_n[U_n(t) - U_{n+1}(t)] \qquad (A1-68)$$

where the step function $U_n(t)$ is defined as

$$U_n(t) = \begin{cases} 1, & \text{if } t \geq t_n; \\ 0, & \text{otherwise.} \end{cases} \qquad (A1-69)$$

in which $t_1 < t_2 < ... < t_{N+1}$ are some intervals appropriately chosen, and

$$\eta_n = \eta(t^*) \qquad t_n \leq t^* \leq t_{n+1} \qquad (A1-70)$$

Substituting Eq. (A1-68) into the closed-form solution for the covariance response yields a series solution in which each term can be evaluated explicitly. Numerical examples have been presented to explore the efficiency and accuracy of this method.

Another approximation may be called the quasi-stationary approximation. By using certain stationary relationship in the Liapunov equation (A1-61) governing the transient covariance responses (Bucher 1988), a set of uncoupled differential equations governing the auto- and cross-correlation of modal responses can be obtained. A post-synthesis is needed to obtain the covariance of physical quantities such as displacement and velocity. The method may be extended to the analysis of nonlinear systems, as reported.

A1.3 Conclusions

The nonstationarity of environmental loads requires a nonstationary analysis of the response of structures, which brings new features as well as difficulties into analysis. Many different formulations for the analysis of the second moment response of linear MDOF systems have been developed in the both the time and frequency domains, and a variety of techniques has been proposed to obtain the solutions. However, explicit solutions are still difficult to obtain by the existing methods mainly due to the algebraic difficulties caused by the nonstationarity of the loads and the size of the system. Considering the importance of the explicit solution in both theoretical and practical studies on the dynamic behavior of structures under a nonstationary excitation, it is necessary to develop a new efficient method to overcome these difficulties and facilitate obtaining the explicit solution for the covariance response.

Appendix II

Earthquake Ground Motion Models

This appendix is devoted to the modeling of environmental loads. Since the applications in this thesis are primarily related to earthquake engineering, the discussion will be mainly limited to the stochastic models of earthquake ground motion.

The review serves two purposes. First, the development of the methodology used in the covariance analysis, as summarized in Appendix I, may be traced to the evolution of load models such as earthquake ground motion models. Occurrence of every new stochastic model for excitation may motivate a new method to solve for the response of structures subjected to such a load model, if old methods are inappropriate. Therefore, it is felt that the previous review would not be complete if a brief summary of stochastic models of earthquake ground motion were not given. Secondly, the review may reveal the relationship between different models and, therefore, an appropriate model can then be chosen for the formulation of the method proposed in this thesis. Different reviews on earthquake ground motion models may be found in Smith (1985), Shinozuka and Deodatis (1989), Kozin (1989) etc. It is hoped that this review will have a little different flavour.

Ground motion models may be separated into two catagories: stationary models and nonstationary models. The development of these models is governed by the current knowledge about earthquake motions and mathematical convenience.

A2.1 Stationary Models

Stationary models are often employed in random analysis to model certain earthquake ground motions, especially those of long duration, and the well-developed theory of stationary processes makes these models easy to be incorporated into theoretical analysis.

The stochastic properties of a stationary model may be given by specifying its auto-correlation function, or equivalently by its spectral density function. The two quantities are associated to each other by the well-known Winner-Khintchine relationship.

$$R(\tau) = \frac{1}{2\pi} \int_{-\infty}^{\infty} S(\omega) e^{i\omega\tau} d\omega$$
$$S(\omega) = \int_{-\infty}^{\infty} R(\tau) e^{-i\omega\tau} d\tau \qquad (A2-1)$$

Housner (1947) was the first to use a stochastic process to model earthquake ground motion. In his model, the ground motion was modeled as a large number of impulses arriving at random times. White noise models were suggested by Bycroft (1960), and Rosenblueth and Bustamante (1962), to study the structural behavior under earthquake loads. The stationary white noise model is a mathematical idealization, since its frequency content is uniformly distributed and its mean square value is unbounded. However, in certain circumstances such as for lightly damped system under wide-band excitation, where the frequency content of the load is less important, the model may give a satisfactory result.

To better characterize the frequency content of the ground motion, some improved stationary models have been proposed. Housner (1955) used a superposition of one-cycle sine pulses arriving at random times, with the average number of pulses depending on frequency. Kanai (1957) and Tajimi (1960) suggested a filtered stationary white noise model whose spectral density is given by

$$S(\omega) = \frac{\omega_g^4 + 4\varsigma_g^2 \omega_g^2 \omega^2}{(\omega_g^2 - \omega^2)^2 + 4\varsigma_g^2 \omega_g^2 \omega^2} K \qquad (A2-2)$$

where ω_g, ς_g, and K characterize the dominant frequency, the energy dissipation at the site and the intensity of incident seismic waves at bedrock. An improved model of this type has been proposed by Singh and Wen (1977). The spectral density in their model is expressed as

$$S_N(\omega) = \sum_{i=1}^{N} \frac{\omega_{gi}^4 + 4\varsigma_{gi}^2 \omega_{gi}^2 \omega^2}{(\omega_{gi}^2 - \omega^2)^2 + 4\varsigma_{gi}^2 \omega_{gi}^2 \omega^2} K_i \qquad (A2-3)$$

which may have multiple peaks in contrast to the single peak in Kanai-Tajimi model. Another model has been employed by Clough and Penzien (1975) by incorporating a high-pass filter model into the low-pass Kanai-Tajimi filter. The resulting spectral density takes the form

$$S(\omega) = S_0 \quad \frac{1+(2\varsigma_g\frac{\omega}{\omega_g})^2}{[1-(\frac{\omega}{\omega_g})^2]^2+(2\varsigma_g\frac{\omega}{\omega_g})^2}\cdot\frac{1+(2\varsigma_k\frac{\omega}{\omega_k})^2}{[1-(\frac{\omega}{\omega_k})^2]^2+(2\varsigma_k\frac{\omega}{\omega_k})^2} \qquad (A2-4)$$

where ς_k and ω_k are the parameters of the high-pass Clough-Penzien filter. Lutes (1979) has suggested first-order filters for the low- and/or high-pass filters.

Sometimes, a stationary model may also be specified by its correlation function in the time domain, such as

$$R(\tau) = e^{-\alpha|\tau|}\cos p|\tau| \qquad (A2-5)$$

used by Barnoski and Maurer (1969) and

$$R(\tau) = Ae{-}\delta|\tau| + 2Ae^{-\frac{\delta}{2}|\tau|}\sin\omega_0(|\tau|\phi) \qquad (A2-6)$$

by Masri and Udwadia (1977). These models may be directly incorporated into the time domain analysis.

Another group of models, referred to as seismological models, may be used as stationary models in covariance analysis (Safak, 1988). These models have been recently developed by seismologists based on the physical aspects of earthquake source and wave propagation. One such model is given as (Boore, 1985)

$$S_a(f) = \frac{A}{T}CS_1(f)S_2(f)S_3(f) \qquad (A2-7)$$

where f denotes frequency in Hz, $S_a(f)$ is the one-side power spectral density, A is a normalizing factor, T is the effective duration, and C, S_1, S_2, S_3 denote the scaling factor, source spectrum, amplification factor, and attenuation factor, respectively. Detailed expressions for these parameters may be found in Safak (1988).

The stationary covariance response may be obtained in the time domain by using a modulated stationary input with a Heaviside envelope and letting the time approach infinity, or alternatively in the frequency domain by

$$E[x^2(t)] = \int_{-\infty}^{\infty} H(i\omega)S(\omega)H^{*T}(i\omega)d\omega \qquad (A2-8)$$

where $H(i\omega)$ is the frequency response function of the system.

The usage of stationary models is greatly limited due to its inability to describe nonstationary phenomena which may be dominant in earthquakes of short or medium durations.

A2.2 Nonstationary Models

Most environmental loads are nonstationary in nature. For instance, it is widely accepted that earthquake ground motion has well-defined buildup, stationary, and tail stages which suggest that real ground motion is a nonstationary process. Different nonstationary models have been developed to account for this feature.

A2.2.1 Modulated Stationary Models

The transient nature of the earthquake process may be modeled explicitly by modulating a stationary process $n(t)$ with a deterministic function of time $\eta(t)$ as

$$f(t) = \eta(t)n(t) \tag{A2-9}$$

Therefore, the stochastic properties of the resulting response process will depend on both the envelope and the stationary process.

The stochastic properties of the stationary process may be given in terms of its correlation function in the time domain or its spectral density function in the frequency domain. A detailed discussion on stationary processes is given in the previous section.

The deterministic envelope is usually assumed to be slowly varying and is generally chosen empirically to account for both its capability of matching real records and mathematical convenience. Various definitions of the envelope function have been introduced by different investigators including the following:

a. Heaviside Unit Step Function

$$\eta(t) = U(t) = \begin{cases} 1, & \text{if } t \geq 0; \\ 0, & \text{otherwise.} \end{cases} \qquad (A2-10)$$

The step function is used to model a suddenly applied stationary process as the first attempt to account for nonstationarity. Due to its simplicity, this envelope has been used in numerous studies including Caughey and Stumpf (1961).

b. Rectangular Function (Boxcar Envelope)

$$\eta(t) = U(t) - U(t-T) \qquad (A2-11)$$

where T is the duration.

The rectangular envelope is employed to account for the effect of the finite duration of an earthquake. The solution may be obtained by the superposition of two solutions by using two unit step functions initiating at different times. The envelope was used by Housner and Jennings (1964), Mosberg and Yieldiz (1976), Smith (1985) etc.

c. Exponential Envelopes

The simplest exponential envelope is of the form

$$\eta(t) = e^{-\alpha t} U(t) \qquad (A2-12)$$

which was used by Bolotin (1960) and Cornell (1960). The envelope accounts for the tail stage of an earthquake groung motion.

The envelope was revised by Shinozuka and Sato (1967) to include the build-up stage of the motion as well. The revised envelope is expressed as

$$\eta(t) = A(e^{-\alpha t} - e^{-\beta t}) U(t) \qquad (A2-13)$$

where A is a normalizing factor and $\beta > \alpha > 0$. Eqs. (A2-10) and (A2-12) may be treated as the special cases of Eq. (A2-12) where β approaches ∞ and/or $\alpha = 0$. The envelope has been widely accepted due to its simplicity and flexibility.

d. Product of Polynomial and Exponential Functions

Representative expressions of this type of envelope were given by Iyengar and Iyengar (1969) as

$$\eta(t) = (a_1 + a_2 t)e^{-\alpha t^n}; \qquad n = 1 \quad or \quad 2 \qquad (A2-14)$$

and by Saragoni and Hart (1974), and Hsu and Bernard (1978) as

$$\eta(t) = At^{\gamma}e^{-\alpha t} \qquad (A2-15)$$

By choosing appropriate parameters, these models can characterize the nonstationary phases of ground motion.

e. Piecewise Envelopes

To model realistic earthquake ground motion, some piecewise expressions have been suggested. Jennings, Housner, and Tsai (1968) used

$$\eta(t) = \begin{cases} 0 & \text{for } t < 0 \\ \frac{t^2}{16} & \text{for } 0 \leq t < 4 \\ 1.0 & \text{for } 4 \leq t < 35 \\ e^{-0.0357(t-35)} & \text{for } 35 \leq t < 80 \\ 0.05 + 0.0000938(120-t)^2 & \text{for } 80 \leq t < 120 \end{cases} \qquad (A2-16)$$

as the envelope for a type A earthquake and similar expressions for types B, C, and D. The model parameters were selected from an analysis of real earthquake data.

Piecewise envelopes offer a more realistic description of ground motion, but, as a penalty, bring more algebraic difficuties into the solution.

A2.2.2 Random Pulse Train Model

A random pulse train model was proposed by Lin (1963). The model can be expressed as

$$S(t) = \sum_{k=1}^{N(t)} Y_k \delta(t - \tau_k) \qquad (A2-17)$$

where $N(t)$ is a Poisson counting process, and Y_k are the independant random variables with zero-mean uniform distribution which occur at time τ_k. Lin indicated that if the times at which the impulses occur are uncorrelated events, or if the amplitudes Y_k have a zero mean, $S(t)$ is a shot noise process. That is

$$\begin{aligned} &E[S(t)] = 0 \\ &E[S(t_1)S(t_2)] = \theta(t_1)\delta(t_1 - t_2) \end{aligned} \qquad (A2-18)$$

where

$$\theta(t) = E[Y^2]\lambda(t) \qquad (A2-19)$$

in which $\lambda(t)$ is the impulse occurrence rate. Note that $\theta(t)$ is nonnegative.

It can be shown that the random pulse train process is stochastically equivalent to a modulated white noise up to the first two moments. A brief proof is given below.

Assume an associated modulated white noise process defined as

$$\bar{S}(t) = \bar{\eta}(t)n(t) \qquad (A2-20)$$

where $n(t)$ is a normalized white noise with zero mean and unit spectral intensity, and the envelope $\bar{\eta}(t)$ is given by

$$\begin{aligned} \bar{\eta}(t) &= \sqrt{\theta(t)} \\ &= \sqrt{E[Y^2]\lambda(t)} \end{aligned} \qquad (A2-21)$$

which is always possible since $\theta(t)$ is nonnegative.

The first two moments of $\bar{S}(t)$ can be found as

$$\begin{aligned} &E[\bar{S}(t)] = 0 \\ &E[\bar{S}(t_1)\bar{S}(t_2)] = \bar{\eta}(t_1)\bar{\eta}(t_2)\delta(t_1 - t_2) \end{aligned} \qquad (A2-22)$$

It can be shown by distribution theory that

$$\bar{\eta}(t_1)\bar{\eta}(t_2)\delta(t_1 - t_2) = \bar{\eta}^2(t_1)\delta(t_1 - t_2) \qquad (A2-23)$$

Therefore, the first two moments of $\bar{S}(t)$ are the same as those of $S(t)$ which implies that the random pulse train model can be replaced by a modulated white noise model without changing the first two moments if an appropriate envelope function is chosen. Furthermore, if the random pulse train process is also Gaussian, it is equivalent to the corresponding modulated Gaussian white noise. The relationship between the two models was indicated by Shinozuka and Sato (1967), but a much simpler and strict proof is given here without involving the filter response. Lin (1986) also demonstrated that if $E[Y_n] = 0$ this model is equivalent, at least up to the second moment, to an evolutionary process with a properly defined evolutionary power spectral density, but here a more specific conclusion is obtained. As a consequence of the equivalence, if only first two moments are of interest, the random pulse train model may be replaced by a modulated white noise process, which brings great convenience to the covariance analysis if a random pulse train model is employed.

A2.2.3 Shot Noise Models

Shot noise models were used by Amin and Ang (1969) to study nonstationary behavior of ground motion and structural response. A random process $S(t)$ is called a shot noise if its mean and covariance are given by (Lin, 1967)

$$\begin{aligned} &E[S(t)] = 0 \\ &E[S(t_1)S(t_2)] = I(t_1)\delta(t_1 - t_2) \end{aligned} \qquad (A2-24)$$

where $I(t)$ is the intensity function of the shot noise. It can be shown that the white noise, modulated white noise, and Poisson impulse train mentioned above are all the special cases of the shot noise. Conversely, if $I(t)$ is nonnegative at any time, the shot noise model is statistically equivalent up to second moment to a modulated white noise with properly defined envelope.

The second moment of a shot noise process are completely determined by its intensity function. If the shot noise is a modulated white noise process, the intensity function equals the square of the envelope. Other forms are also used including the half-sine impulse intensity used by Lin (1963, 1987)

$$I(t) = \begin{cases} \lambda_0 \sin \frac{\pi t}{b}, & \text{if } 0 \le t \le b; \\ 0, & \text{otherwise.} \end{cases} \qquad (A2-25)$$

and the piecewise expression proposed by Amin and Ang (1968)

$$2\pi SI(t) = \begin{cases} 0, & \text{for } t < 0 \\ I_0(\frac{t}{a})^2 & \text{for } 0 \leq t \leq T_1 \\ I_0 & \text{for } T_1 \leq t \leq T_2 \\ I_0 e^{-ct} & \text{for } T_2 \leq t \end{cases} \qquad (A2-26)$$

and a similar form by Ruiz and Penzien (1971). An explicit solution for covariance response of simple systems under a shot noise model with the intensity defined by Eq. (A2-26) was given by Amin and Ang (1968).

A2.2.4 Filtered Modulated Stationary Processes

The modulated stationary ground motion models describe only the change in the intensity of the ground acceleration with time, but not its frequency content. This is in conflict with the observed fact that nonstationarity should exist in both the intensity and frequency content of ground motions. As a remedy, various filters have been introduced to make the models more realistic.

The dynamic behavior of a linear filter can be characterized by its impulsive response function $h(t)$, or equivalently, its frequency response function $H(\omega)$. A realistic acceleration process $a(t)$ can be obtained by passing a modulated stationary process through the filter. Mathematically, the output of the filter may be written as

$$a(t) = \int_0^t h(t-\tau)\eta(\tau)n(\tau)d\tau \qquad (A2-27)$$

It can be shown that the filtered modulated stationary process is an evolutionary process as defined by Priestley.

The choice of the filter depends on knowledge of the earthquake source mechanism, transmission path, site condition, etc., and on mathematical convenience. Some typical filters are described below:

a. First-order Filters

The behavior of this type of filter is governed by a first-order ordinary differential equation such as that used by Amin and Ang (1968)

$$\frac{da(t)}{dt} + la(t) = f(t) \qquad (A2-28)$$

whose impulsive response function is found as

$$h(t) = e^{-lt}U(t) \qquad (A2-29)$$

The principal advantage is its computational simplicity.

b. Second-order Filters

The dynamic behavior of the filters may be described by a second-order differential equation such as

$$\frac{d^2G(t)}{dt^2} + 2\varsigma_g\omega_g\frac{dG(t)}{dt} + \omega_g^2G(t) = -2\varsigma_g\omega_g\frac{df(t)}{dt} - \omega_g^2f(t) \qquad (A2-30)$$

where $G(t)$ is the absolute ground motion. The behavior of the filter is usually given by its frequency response function as

$$|H(\omega)|^2 = \frac{\omega_g^4 + 4\varsigma_g^2\omega_g^2\omega^2}{(\omega_g^2-\omega^2)^2 + 4\varsigma_g^2\omega_g^2\omega^2} \qquad (A2-31)$$

This physically motivated filter is referred to as the Kanai (1957)-Tajimi (1961) filter and is widely used in earthquake engineering. A more detailed discussion is given in Chapter 4.

c. Continuous Filters

The dynamic behavior of this type of filter is governed by a partial differential equation. Assume the Green function of the system to be $G(\mathbf{r},t)$. Then the resulting output is

$$a(t) = \int_\Omega\int_0^t G(\mathbf{r}-\mathbf{x}, t-\tau)f(\mathbf{x},\tau)d\mathbf{x}d\tau \qquad (A2-32)$$

One of the continuous filters is the shear beam filter (Lin, 1987) governed by

$$\frac{\partial^2 w}{\partial t^2} - \beta^2\frac{\partial^2 w}{\partial y^2} = f(t) \qquad (A2-33)$$

which is intented for modeling soil behavior at the site.

Lin (1987) prposed another continuous filter given by

$$\frac{\partial^2 G(t)}{\partial t^2} - \beta^2\frac{\partial^2 G(t)}{\partial y^2} - \beta^2\frac{d}{dy}[lnA(y)]\frac{\partial G(t)}{\partial y} = f(t) \qquad (A2-34)$$

which accounts for geometrical spreading of seismic energy.

A combination of the above three filters is sometimes useful. Clough and Penzien (1975) recommended a series of low-pass and high-pass filters both of which are second-order filters and Lutes (1979) suggested first-order filters for low-pass and/or high-pass filter instead, as discussed in section A2.1.

The covariance response of structures under filtered modulated stationary models may be obtained in two ways. A two-step procedure may be adopted by first finding the statistical properties of the output of the filter and then determining the covariance response by using the resulting output as the ground motion input to structures. Alternatively, noting that the filters are governed by linear differential equations, a filter can be treated as a substructure of the original structures. The methods for MDOF systems subjected to modulated stationary excitation may be used to directly find the covariance response of the augmented structures. The second approach is prefered in this research.

A2.2.5 Autoregressive Moving Average (ARMA) Models

The ARMA model is a general linear model for time series analysis. In contrast to so-called continous models described by a linear differential equation, the model is governed by a discrete finite difference equation of the form

$$y_i + a_1 y_{i-1} + a_2 y_{i-2} + ... + a_n y_{i-n} = u_i + b_1 u_{i-1} + ... + b_m u_{i-m} \qquad (A2-35)$$

where a_i and $b_i, i = 1, ..., n$ are constant coefficients and the random sequence $\{u_i\}$ consists of independent identically distributed Gaussian random variables. y_i represents the data observed at time $i\Delta t$. Model (A2-35) is referred to as an ARMA model of order (n,m) where n and m are autoregressive order and the moving average order respectively. Much research has been done in this area and detailed reviews may be found in Box and Jenkins (1970), Chang, et al. (1979) and Kozin (1988).

The ARMA models may be thought of as the finite difference versions of some filtered modulated white noise models. Correspondence between the ARMA model of order (2,1) and the filtered white noise model where a second-order oscillator filter is used was discussed by Chang, et al. (1979). The correspondence exists, but may not be unique. If the correspondence is defined, the method for filtered modulated stationary models may be employed to find the solution.

A2.3 Remarks

In all the models mentioned previously, the model parameters should be determined from real earthquake data. This process is called calibration. A conflicting situation exists in the modeling of earthquake loads. While the capability of taking account for more features of the earthquake ground motion requires more parameters, calibration and analysis prefer simple models with less parameters. A good model should be justified by its capability to capture the main features of earthquakes, the possibility of being easily calibrated from real records, and the feasibility of being used in analysis. It is assumed that the model parameters are known throughout the thesis.

This review has emphasized single random load processes. For long-span structures such as long bridges, pipelines, and dams, spatial correlation effects of ground motion as well as traveling wave effects may be important, which necessitates so-called space-time ground motion models.

For a homogeneous excitation, the cross spectrum of ground excitation can be written as (Hindt and Novak, 1980)

$$S_a(y_1, y_2) = S_a(f)R(r, f) \qquad (A2-36)$$

where $r = |y_1 - y_2|$ is the horizontal distance between two points y_1 and y_2, $S_a(f)$ is the local spectrum of ground acceleration common for all the stations. Eq. (A2-36) implies that the cross spectrum depends on the seperation r rather than the positions.

The power spectral density from any stationary model may be a candidate for the local spectrum $S_a(f)$, among which the white noise filtered by the combination of low-pass Kanai-Tajimi filter and/or high-pass Clough-Penzien filter is prefered by Hindy and Novak (1980), Datta and Mashaly (1986), and Harichandran and Wang (1988).

One suggested form for $R(r, f)$ is (Hindy and Novak 1980)

$$R(r, f) = e^{-c\left(\frac{rf}{V_s}\right)^\gamma} \qquad (A2-37)$$

where V_s is the shear wave velocity of the soil, f is frequency, c is a constant depending on some seismological parameters, and γ is taken to be unity until more refined data becomes available. The model was later refined by Harichandran and Vanmarcke (1986) based on the data from a far-field event at the SMART-1 seismograph in Lotung, Taiwan.

Eq. (A2-36) may be used to determine the auto- and cross-spectral densities of multi-input random processes. Essentially, the model is a stationary model.

Based on the above review of the available earthquake ground motion models, the modulated stationary model may be chosen as a basic stochastic model for excitation to be used in the analysis. This choice gives the simplified state-variable method a maximum flexibility to be used to many other models, as indicated in Chapter 2.

Appendix III

A Proof of the Stabilitity Theorem in Chapter 5

The stability theorem in Chapter 5 can be proved as follows: Since the solution of

$$\frac{d}{dt}Y(t) = AY(t) \tag{A3-1}$$

is stable, there exists positive numbers $c > 0$ and $d > 0$ such that its fundamental solution $\Phi(t)$ satisfies

$$\|\Phi(t)\| \leq ce^{-dt} \quad \forall t > 0 \tag{A3-2}$$

where $\|\cdot\|$ is any matrix norms. Without loss of generality, it is assumed that $d < b$ where b is interpreted in condition (iii).

The general solution of the corresponding nonautonomous system is

$$Y(t) = \Phi(t)Y_0 + \int_0^t \Phi(t-\tau)(F(\tau) + B(\tau)Y(\tau))d\tau \tag{A3-3}$$

Therefore, for any $t > 0$,

$$\|Y(t)\| \leq \|\Phi(t)\||Y(0)| + \int_0^t \|\Phi(t-\tau)\||F(\tau)|d\tau + \int_0^t \|\Phi(t-\tau)\|\|B(\tau)\||Y(\tau)|d\tau \tag{A3-4}$$

where $|\cdot|$ denotes the associated vector norm.

Assuming that

$$|Y(0)| = g \geq 0 \tag{A3-5}$$

and using the conditions (ii) and (iii), inequality (A3-4) becomes

$$\|Y(t)\| \leq cge^{-dt} + \frac{ca}{b-d}(1-e^{-(b-d)t})e^{-dt} + c\int_0^t e^{-d(t-\tau)}\|B(\tau)\||Y(\tau)|d\tau \tag{A3-6}$$

or,

$$\|Y(t)\|e^{dt} \leq A + \int_0^t c\|B(\tau)\||Y(\tau)|e^{d\tau}d\tau \tag{A3-7}$$

where A is defined as

$$A = cg + \frac{ca}{b-d} \tag{A3-8}$$

Using the well-known Gronwell's Lemma in Eq. (A3-7) yields

$$\begin{aligned} \|Y(t)\| e^{dt} &\leq A e^{\int_0^t c\|B(\tau)\| d\tau} \\ &\leq A e^{cK} \end{aligned} \qquad (A3-9)$$

Thus,

$$\|Y(t)\| \leq A e^{cK} e^{-dt} \qquad (A3-10)$$

It follows immediately that the solution of (A3-1) is stable if the conditions (i)-(iii) hold.

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