CALIFORNIA INSTITUTE OF TECHNOLOGY

EARTHQUAKE RESEARCH LABORATORY

Intensity of Ground Motion During Strong Earthquakes

by

G. W. Housner

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INTENSITY OF GROUND MOTION DURING STRONG EARTHQUAKES

by

G. W. HOUSNER

SECOND TECHNICAL REPORT

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INTENSITY OF GROUND MOTION DURING
STRONG EARTHQUAKES

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ABSTRACT

A measure of the surface intensity of the ground motion during an earthquake is defined which is proportional to the maximum stresses produced in structures. Intensities are computed for fourteen strong-motion shocks and these are compared with the Modified-Mercalli Intensities for the same earthquakes. A method is developed for computing the magnitude of an earthquake from the intensity and vice versa which permits the intensities to be calculated over the area affected by the earthquake. The maximum intensity and the maximum ground acceleration likely to be experienced by a California city are estimated. The frequency with which strong earthquakes are likely to occur in California is determined and an estimate is made of the frequency with which a California city is likely to experience severe ground motion.

I. INTRODUCTION. The basic problem of engineering seismology concerns the design of structures to resist earthquakes and there are three significant aspects of this problem. First, there is the problem of designing a structure so that all of its parts have equal strengths to resist the stresses produced by earthquakes, that is, there should be a uniform factor of safety against failure. Second, there is the problem of designing different structures to have the same factors of safety that is, to insure that a tall building will have the same degree of strength as a low building, that a flexible building has the same degree of resistance against earthquakes as a rigid building, etc. Third, there is the problem of determining the factor of safety, or the required strength to resist earthquakes so that structures will be able to withstand the most severe ground motion to which they are likely to be subjected, without experiencing serious damage.

The spectrum analysis of strong-motion earthquake records[1] is pertinent to the solution of all three of the aforementioned problems. The application of the earthquake spectrum to the problem of determining the stresses produced in structures is indicated in the previous report.[1] The present report deals with the third of the above problems, namely what is the most severe ground motion to which structures are likely to be subjected. This requires a method of measuring the intensity of the ground motion that is related to the maximum stresses produced in structures which means that the measure of intensity should be such that if it is said that the intensity of one earthquake is twice the intensity of another it implies that the maximum stresses produced by the first earthquake are twice those produced by the second. There is now no measure of intensity which has this property so that the spectrum intensity defined in this report, which does have this property, is very meaningful for engineering purposes. The spectrum intensities have been computed for fourteen shocks for which accelerograms are available, and it will be desirable to compute the spectrum intensities for succeeding earthquakes to accumulate data on the range of the intensity of the ground motion to which structures are likely to be subjected.

The intensity of an earthquake is a measure of the severity of the ground motion at a point on the surface of the ground. It will vary over the affected area, and to give a complete picture

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[1] Alford, Housner and Martel, Spectrum Analyses of Strong-Motion Earthquakes, August 1951, ONR Contract N6onr-244, Task Order 25 Project Designation NR-081-095.

the intensities must be associated with the magnitude[2] of the earthquake or the total energy release during the shock. It can be shown that the magnitude of an earthquake can be computed from the spectrum intensity and vice versa, so that given the magnitude of an earthquake it is possible to compute with reasonable accuracy, the intensity of the ground motion over the area affected by the shock.

Data on the frequency of occurrence of earthquakes show that there is an upper limit for the magnitude of a shock. With this information plus the data on spectrum intensities, it is possible to establish a trend which indicates the maximum intensity and maximum ground acceleration which is likely to occur during earth shocks in California.

With the preceding data it is also possible to estimate the frequency with which strong earthquakes are likely to occur in California and the frequency with which California cities are likely to be subjected to severe ground motion.

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[2]Gutenberg and Richter, Earthquake Magnitude, Intensity, Energy, and Acceleration, Bulletin of Seismological Society of America, Vol. 32, No. 3, 1942.

II. THE SPECTRA. Only a brief description of the technique of calculating spectra will be presented here and for further details the reader should refer to the previous report. If an elastic one-degree-of-freedom structure, with small damping, is subjected to a base acceleration 'a' the displacement y at any time t may be written:

$$y = \frac{T}{2\pi}\int_0^t a\, e^{-n\frac{2\pi}{T}(t-\tau)} \sin\frac{2\pi}{T}(t-\tau)\, d\tau \qquad (1)$$

where y = displacement at time t

T = natural period of vibration of the structure

a = base acceleration

n = fraction of critical damping

If 'a' is taken to be the earthquake ground acceleration, then equation (1) describes the resulting motion of the structure. The maximum value of equation (1) which occurs during the time 'a' is acting is then the maximum displacement of the structure.

If the structure is more complex, as for example having six stories instead of one, the displacement at each story can be written:

$$y = \sum_{i=1}^{6} C_i \int_0^t a\, e^{-n_i\frac{2\pi}{T_i}(t-\tau)} \sin\frac{2\pi}{T_i}(t-\tau)\, d\tau \qquad (2)$$

where C_i is a factor dependent on the physical properties of the structure and the location of the point where y is measured. The remainder of the symbols have the same significance as in equation (1) with the subscript i referring to the particular mode of vibration.

The integrals appearing in equations (1) and (2) are the same and, in general, the linear vibrations of any structure can be expressed in such a way that the effect of the base acceleration is contained in integrals of this form. The velocity spectrum of an earthquake record is defined to be the maximum value of the integral when plotted as a function of natural period of vibration T, that is,

$$S(T,n) = \left\{\int_0^t a\, e^{-n\frac{2\pi}{T}(t-\tau)} \sin\frac{2\pi}{T}(t-\tau)\, d\tau\right\}_{max.} \qquad (3)$$

Figures 1, 2 and 3 show examples of earthquake accelerograms and Figures 4, 5 and 6 show the corresponding spectra calculated for various amounts of damping. A total of 88 such spectrum curves have been computed and they will be found in the previous report.

III. SPECTRUM INTENSITIES. Except for some slight frequency modulation the maximum kinetic energy of vibration stored in the structure of equation (1) is

$$K.E. = \frac{1}{2} m S^2 \tag{4}$$

where m is the mass of the structure and S is the ordinate of the spectrum curve corresponding to the period of vibration of the structure. Actually, the spectrum curves were computed by a technique such that equation (4) is correct and the curves are in slight disagreement with equation (3). The amount of discrepancy is discussed in the previous report. It is seen, therefore, from equation (4) that the ordinate S of the spectrum curve is equal to the square root of twice the maximum kinetic energy per unit mass, that is,

$$S = (2 \times \text{K.E. per unit mass})^{\frac{1}{2}} \tag{5}$$

If equation (1) is written in the form

$$y_{max} = \frac{T}{2\pi} S$$

the maximum lateral force experienced by the structure is

$$F_{max} = K y_{max} = K \frac{T}{2\pi} S$$

where K is the modulus of rigidity. This can also be written:

$$F_{max} = K \frac{T}{\sqrt{2}\,\pi} (\text{K.E. per unit mass})^{\frac{1}{2}}$$

It is thus seen that the maximum lateral force experienced by the structure is directly proportional to S. The modulus of rigidity K and the period of vibration T depend upon the physical properties of the structure. For moderate amounts of damping, say $n \leq 0.4$, the effect of the damping on the period is negligible and T can be taken to be a function of the mass and rigidity only, $T = 2\pi\sqrt{\frac{m}{K}}$ so that

$$F_{max} = \sqrt{mK}\; S \tag{6}$$

Similarly, the maximum value of each term in equation (2) is equal to $C_i S_i$, where C_i is a function of mass, rigidity and proportions of the structure, and S_i is the ordinate of the spectrum curve at the appropriate frequency. Corresponding terms give the maximum lateral shears, etc.

The spectrum curve is thus a measure of the intensity of the earthquake[3] in the sense that a given structure subjected to

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[3] This is along the lines of a proposal made by H. Benioff, "The Physical Evaluation of Seismic Destructiveness", Bull. Seism. Soc. Amer., (1934), Vol. 24, pp. 398-403.

different earthquakes will experience maximum stresses directly proportional to the values of S for the different shocks, as shown by equation (6). In an actual city there will be a large number of structures with periods of vibration ranging from 0.1 seconds to perhaps 2.5 seconds or more and this range of periods must be taken into account when defining the intensity of the shock. If a city were composed of structures with periods of vibration ranging uniformly from 0.1 to 2.5 seconds, then the average value of the spectrum between 0.1 seconds and 2.5 seconds would be a measure of the intensity of an earthquake in the sense that on the average the maximum stresses experienced by the structures would be in the ratios of the average values of the spectra for different earthquakes. Since the intensity is a comparative measure for expressing the relative severity of earthquakes, the area under the spectrum curve can be used instead of the average ordinate. Accordingly, the <u>spectrum intensity</u> of an earthquake is defined to be the area under the velocity spectrum curve between 0.1 seconds and 2.5 seconds period, that is,

$$S.I. = \int_{0.1}^{2.5} S(n,T)\, dT \qquad (7)$$

As defined by equation (7) the spectrum intensity, S.I., is a function of the fraction of critical damping, n, and it will be necessary to specify whether the undamped spectrum intensity is being referred to, the 0.2 damped intensity, etc. This must be done since it is not possible to separate the effect of the damping from the effect of the ground motion, that is, it is not possible to write equation (6) in the form

$$F_{max} = \sqrt{mk}\; f(n)\; S_o$$

where f(n) is independent of the base acceleration and S_o is the undamped spectrum. Therefore, when speaking of the intensity of ground motion with regard to its effect on structures, it will be necessary to specify the amount of damping in the structures. Measurements have shown that buildings have damping ranging from n = 0.07 to n = 0.40 depending upon the type of construction.

TABLE I

Spectrum Intensities
(0.1 to 2.5 second period)

No.	Location	Component	Damping (fraction critical) 0.0	0.02	0.1	0.2	0.4
1a	El Centro, Calif.	N.S.	8.94	5.72		3.36	2.44
1b	May 18, 1940	E.W.	7.77	4.56		2.06	1.34
2a	El Centro, Calif.	N.S.	5.93			2.55	2.00
2b	December 30, 1934	E.W.	5.83			1.62	1.22
3a	Olympia, Wash.	S.80W.	6.05			2.74	2.30
3b	April 13, 1949	S.10E.	5.59			1.68	1.24
4a	Vernon, Calif.	S.82E.	4.90		2.15	1.77	
4b	March 10, 1933	N.08E.	4.35		2.03	1.64	
5a	Santa Barbara, Calif.	S.45E.	3.43			1.84	1.45
5b	June 30, 1941	N. 45E.	3.15			1.75	1.47
6a	Ferndale, Calif.	N.45E.	3.2			1.27	1.01
6b	October 3, 1941	S.45E.	2.78			1.56	1.27
7a	L.A. Subway Terminal	N.51W	3.21		1.23	0.96	
7b	March 10, 1933	N. 39E.	2.67		0.96	0.67	
8a	Seattle, Wash.	N.88W.	2.81			1.05	0.77
8b	April 13, 1949	S.02W.	2.46			1.14	0.91
9a	Hollister, Calif.	S.01W.	2.44			1.33	1.07
9b	March 9, 1949	N.89W.	2.29			1.20	0.93
10a	Helena, Montana	E.W.	2.49		1.72	1.39	
10b	October 31, 1935	N.S.	1.16		0.79	0.65	
11a	Ferndale, Calif.	N.45E.	1.64		0.94	0.82	
11b	September 11, 1938	S.45E.	1.27	0.85	0.55	0.46	
12a	Vernon, Calif.	S.82E.	1.65			0.80	0.62
12b	October 2, 1933	N.08E.	0.99			0.56	0.44
13a	Ferndale, Calif.	N.45E.	1.31			0.54	0.41
13b	February 9. 1941	S.45E.	0.88	0.59	0.38	0.27	
14a	L.A. Subway Terminal	N.39E.	1.14			0.55	0.41
14b	October 2, 1933	N.51W.	0.78			0.35	0.28

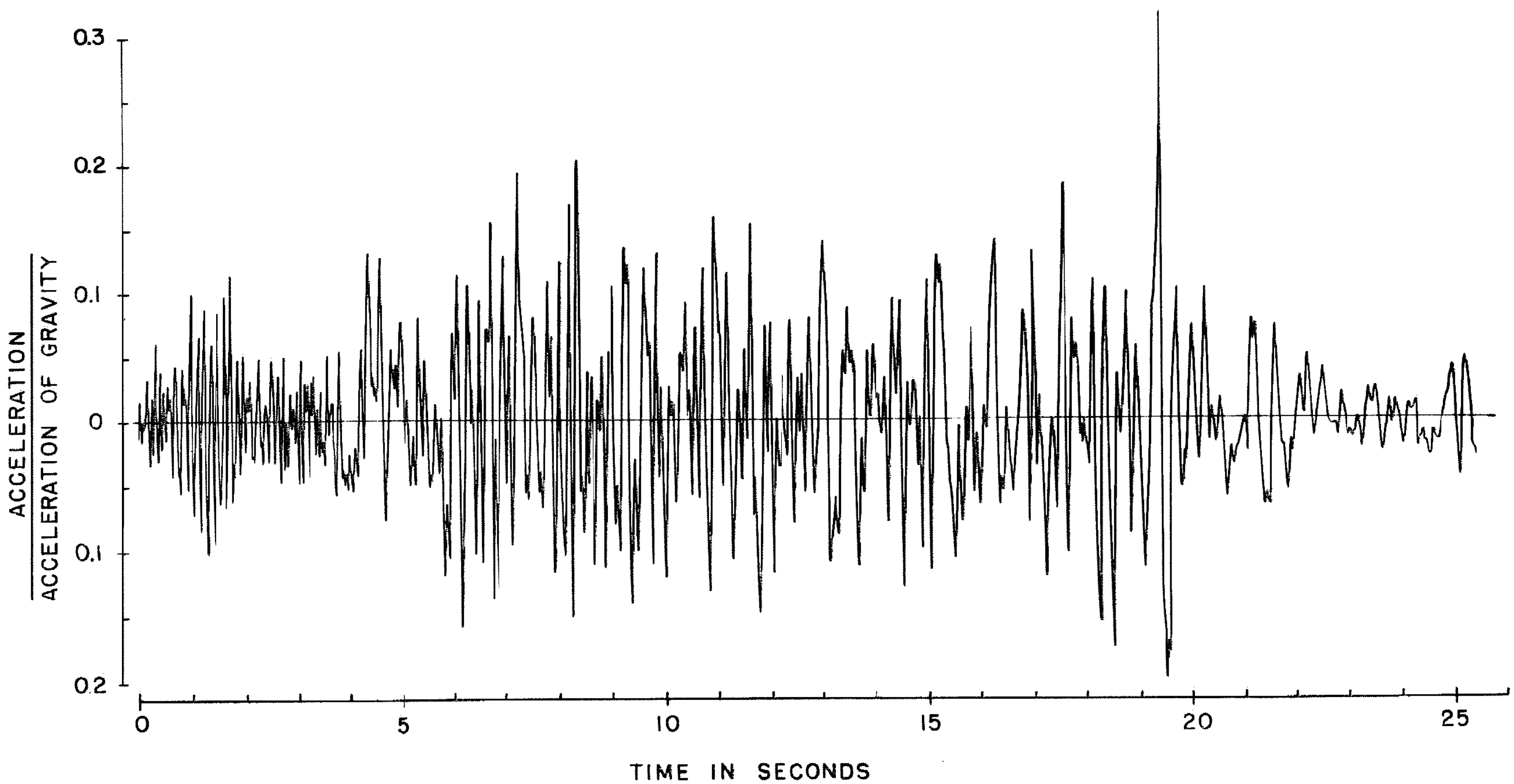


Figure 2. Accelerogram for Olympia, Washington; earthquake of April 13, 1949. Component S 80 W.

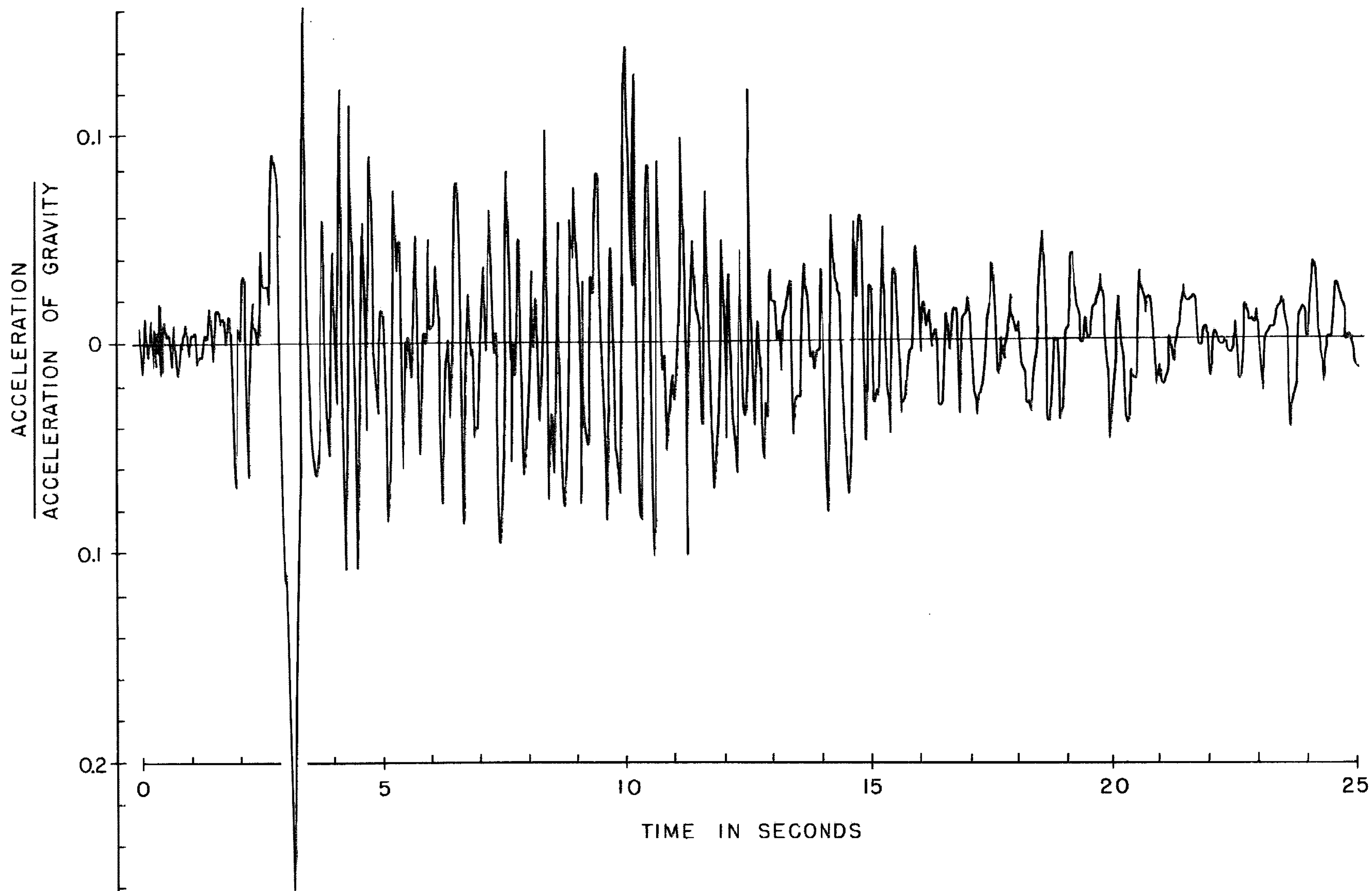


Figure 3 Accelerogram for El Centro, California: earthquake of Dec 30, 1934. Component N-S.

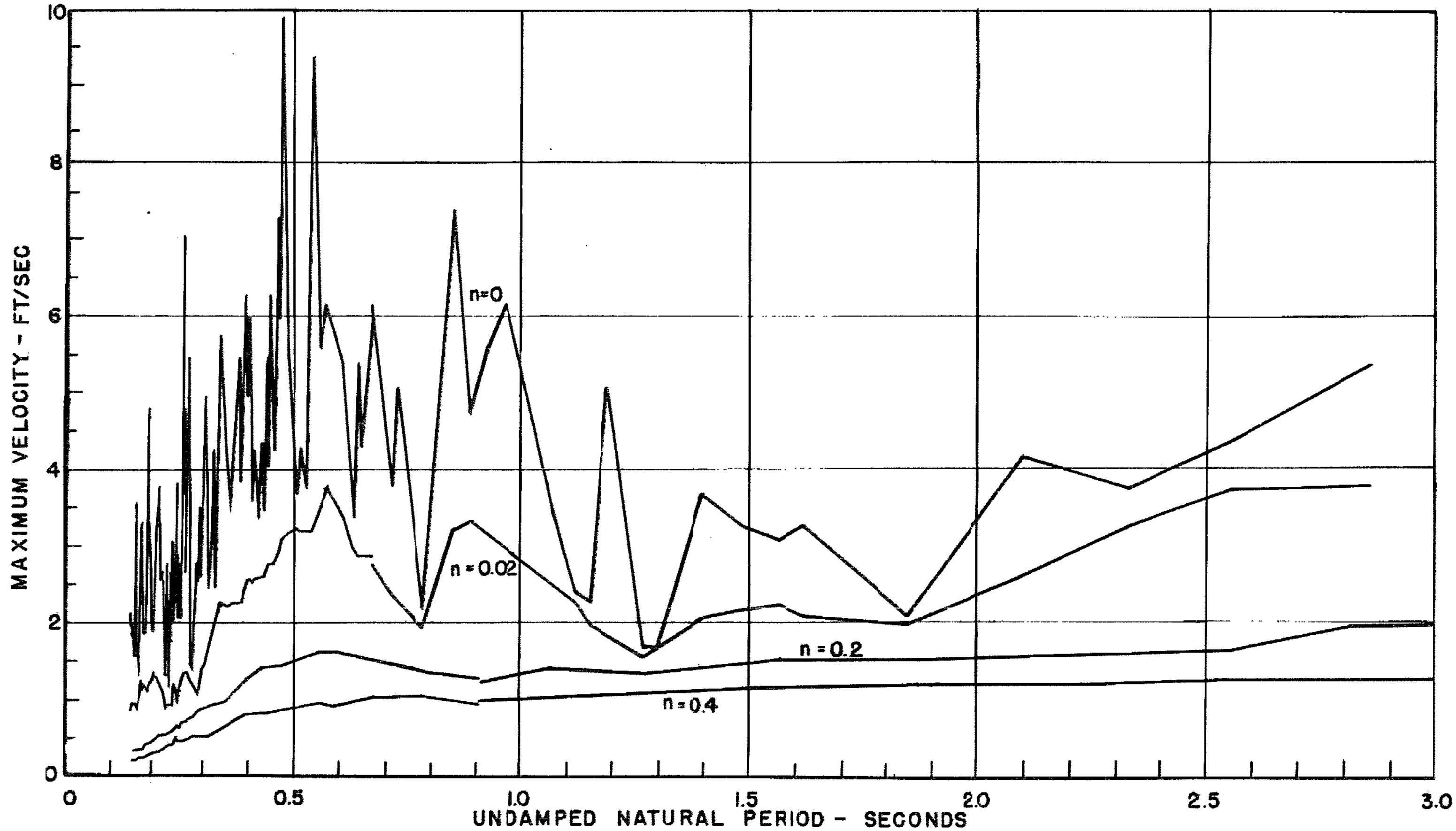


Figure 4. Velocity spectrum for El Centro, California; earthquake of May 18 1940 Component N-S.

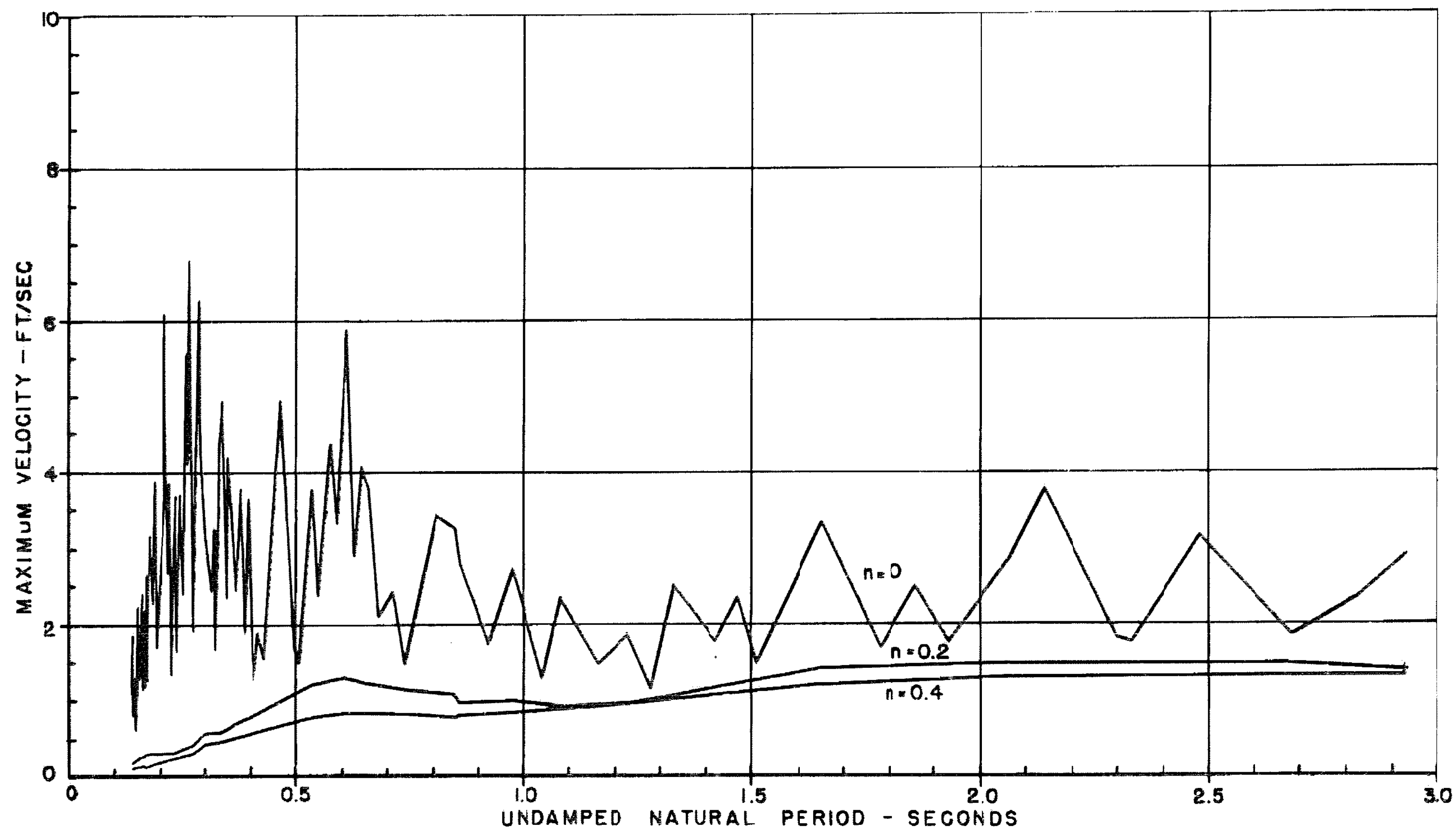


Figure 5. Velocity spectrum for Olympia, Washington; earthquake of April 13, 1949. Component S 80 W.

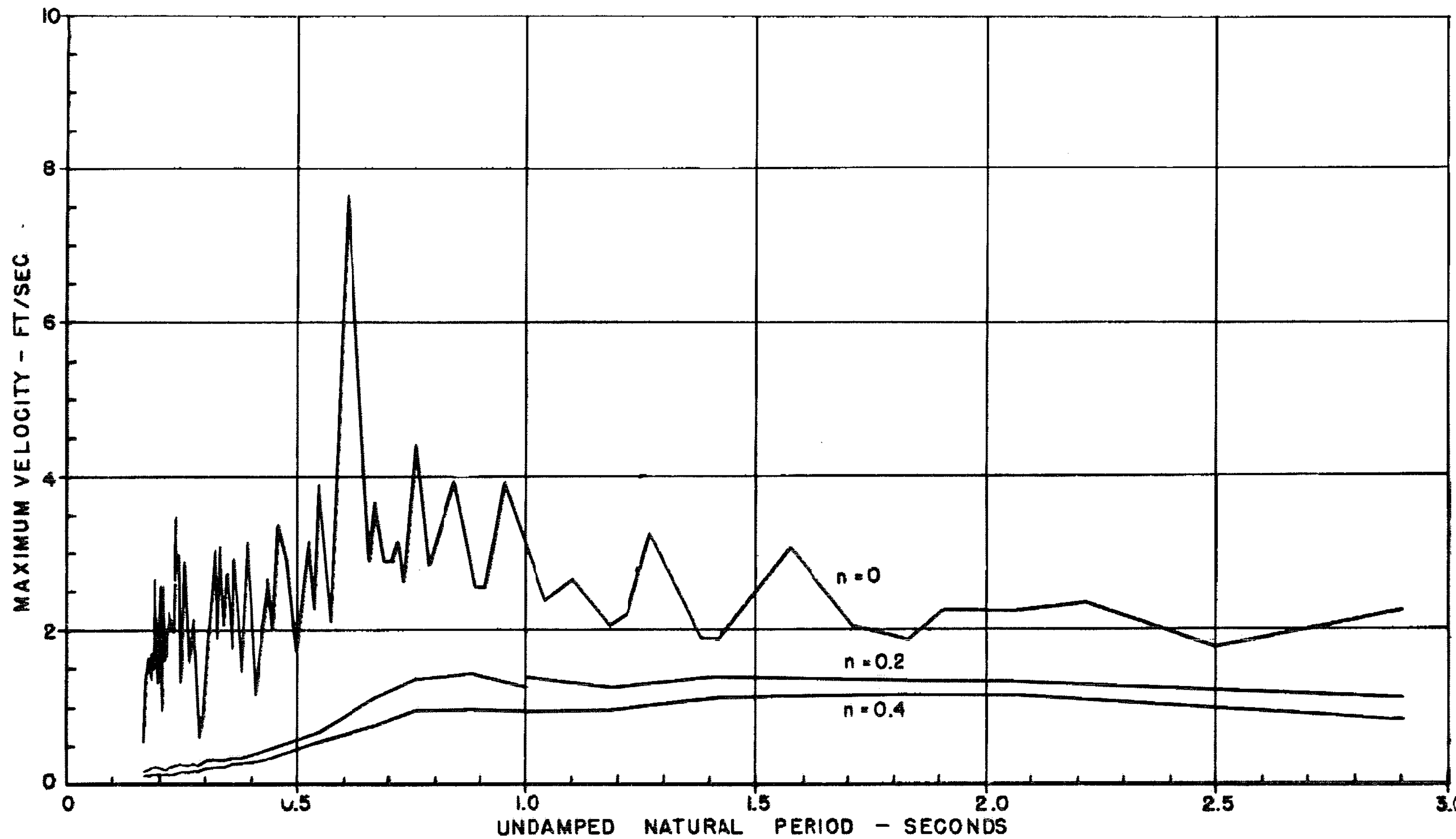


Figure 6. Velocity spectrum for El Centro, California; earthquake of Dec. 30, 1934 Component N-S.

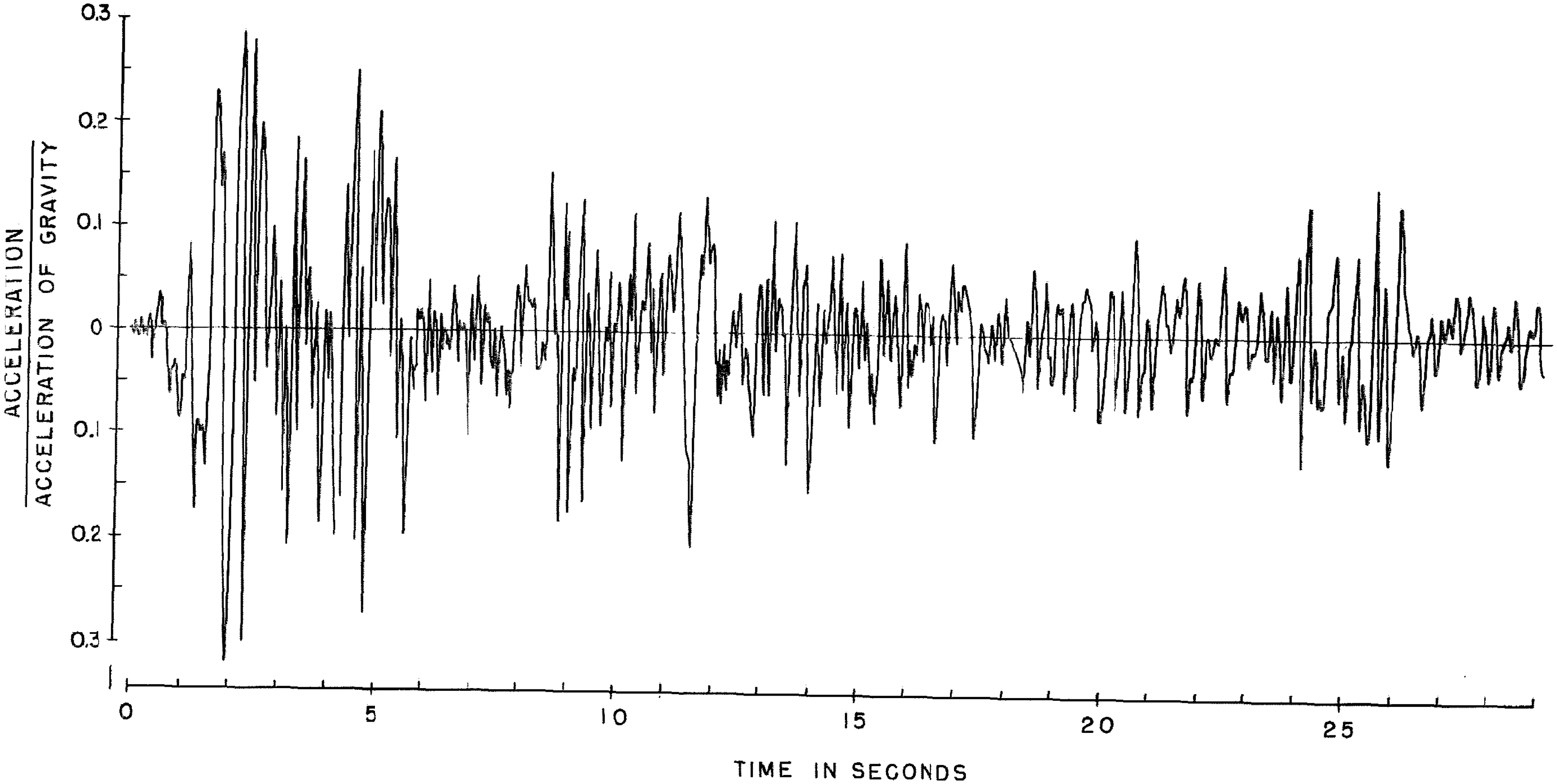


Figure 1 Accelerogram for El Centro, California; earthquake of May 18 1940 Component N-S.

The spectrum intensities as computed from the spectra presented in the previous report are shown in Table I. It should be noted that the intensities have been computed as the areas under the spectrum curves between T = 0.1 seconds and T = 2.5 seconds. Furthermore, as defined, the intensity is expressed in units of length and the numerical values in Table I are 8.94 feet, etc. If the intensities were divided by 2.4 seconds they would represent the average value of S over the range 0.1 second period to 2.5 second period, and they would then have the dimensions of a velocity.

The intensities shown in Table I are plotted in Figure 7 for zero damping and 0.2 damping. In Figure 8 the average intensities of the two components of each earthquake are plotted for 0.0, 0.2 and 0.4 damping. It is seen that the most intense ground motion is that of El Centro, May 18, 1940 at S.I. = 8.35 for zero damping and the least intense is Los Angeles Subway Terminal, October 2, 1933 at S.I. = 0.96 for zero damping. Table II shows the relative intensities taking the El Centro 1940 as 100%. The figures in parentheses are the average intensities of the two components of each shock.

These relative intensities may be interpreted as follows. A set of buildings with zero damping would, on the average, have experienced higher maximum stresses in El Centro, May 18, 1940 than the same buildings would have experienced in the vicinity of the Los Angeles Subway Terminal Building, October 2, 1933. These maximum stresses would have been greater in El Centro by the ratio $\frac{100}{11}$ or approximately nine times as large. Buildings with 0.2 critical damping would on the average have experienced greater maximum stresses in El Centro, May 18, 1940 than in the vicinity of the Subway Terminal Building, October 2, 1933 in the ratio of $\frac{100}{17}$, or approximately six times as large. From Table I it is seen that in El Centro, May 18, 1940, similar buildings differing only in the amount of damping would have experienced maximum N.S. stresses, for 0.4, 0.2, 0.02 and 0.0 damping, in the ratios of 2.44, 3.36, 5.72 and 8.94; that is, taking 0.2 damped intensity as 1 the maximum stresses are in the ratios 0.73, 1.0, 1.7, 2.65.

If the spectrum intensities for Vernon and L.A. Subway Terminal of March 10, 1933 are extrapolated to the City of Long Beach by the method discussed later, there is obtained a zero damped intensity ranging from 8.0 to 9.0 over the city. This compares with the 8.35 S.I. (average of two components) for El Centro, May 18, 1940, and 4.62 for Vernon, March 10, 1933, and 2.95 for Subway Terminal, March 10, 1933. This indicates that the intensity of the ground motion in Long Beach on March 10, 1933 was approximately the same as in El Centro on May 18, 1940. Taking the intensity of the ground motion at Subway Terminal as 1, there are obtained the ratios: 1, 1.56, 2.9, for the relative maximum stresses experienced by structures with zero damping, and 1, 2.1, 3.3 for the relative maximum stresses experienced by buildings with 0.2 damping, at Subway Terminal, Vernon and Long Beach, respectively.

TABLE II

RELATIVE INTENSITIES

		Damping		
		0.0	0.2	0.4
1.	El Centro, 1940	100% (8.35)	100% (2.71)	100% (1.89)
2.	El Centro, 1934	71% (5.88)	78% (2.09)	85% (1.61)
3.	Olympia, 1949	70% (5.82)	81% (2.21)	92% (1.77)
4.	Vernon, Mar. 1933	55% (4.62)	62% (1.70)	
5.	Santa Barbara, 1941	40% (3.29)	65% (1.80)	77% (1.46)
6.	Ferndale, Oct. 1941	36% (2.99)	53% (1.41)	60% (1.14)
7.	L.A. Subway Terminal, Mar. '33	35% (2.94)	37% (0.82)	
8.	Seattle, 1949	31% (2.63)	40% (1.10)	43% (0.84)
9.	Hollister, 1949	28% (2.36)	47% (1.27)	33% (1.00)
10.	Helena, 1935	22% (1.82)	37% (1.02)	
11.	Ferndale, 1938	17% (1.45)	23% (0.64)	
12.	Vernon, Oct. 1933	16% (1.32)	27% (0.69)	27% (0.53)
13.	Ferndale, Feb. '41	13% (1.10)	15% (0.40)	
14.	L.A. Subway Terminal, Oct. '33	11% (0.96)	17% (0.45)	18% (0.35)

If the surface intensities in San Francisco for the 1906 earthquake are estimated by the method described later, there are obtained S.I.s for 0.2 damping ranging from 3.0 to 6.0 over the city.

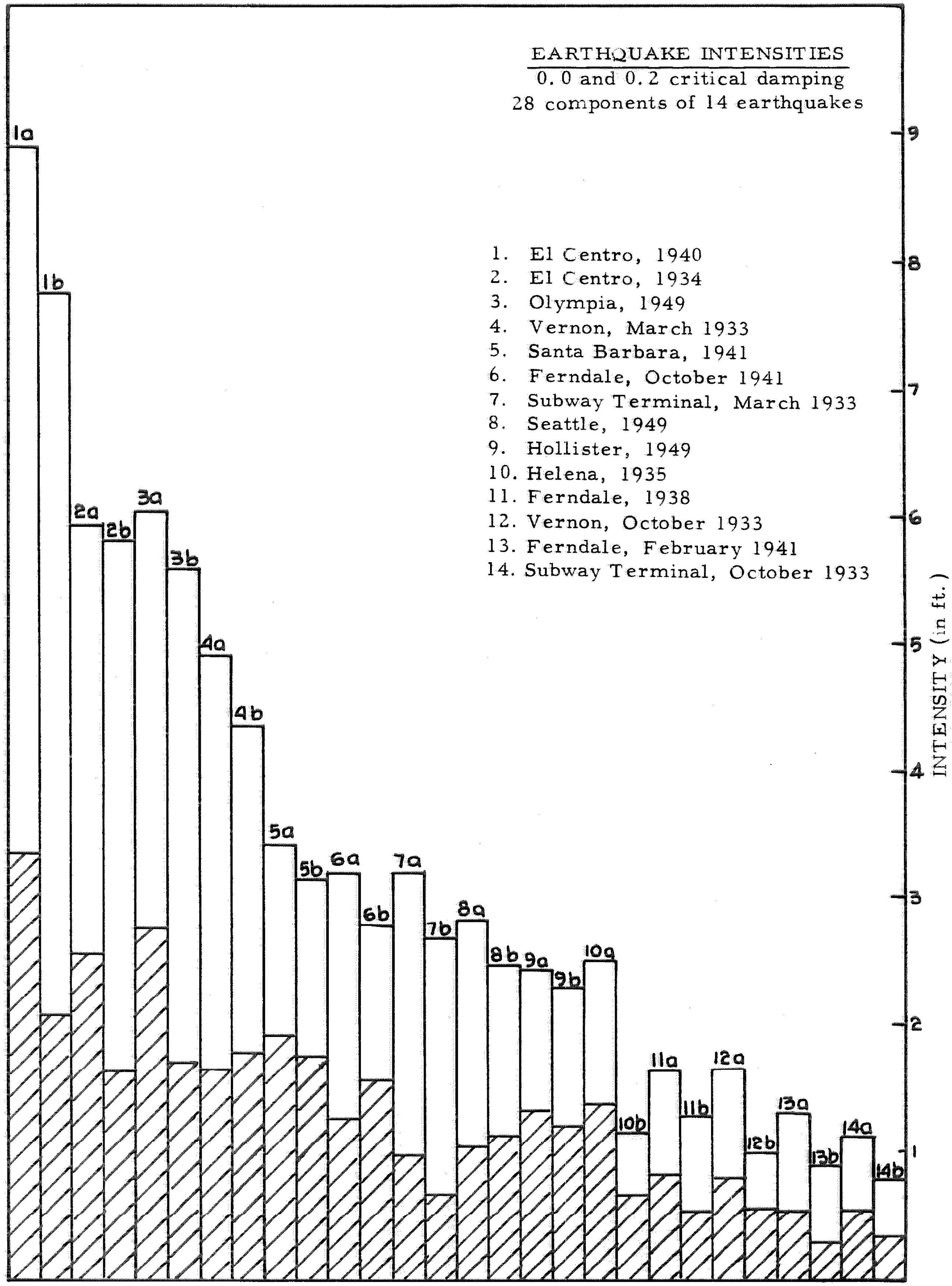

EARTHQUAKE INTENSITIES
0.0 and 0.2 critical damping
28 components of 14 earthquakes
1. El Centro, 1940
2. El Centro, 1934
3. Olympia, 1949
4. Vernon, March 1933
5. Santa Barbara, 1941
6. Ferndale, October 1941
7. Subway Terminal, March 1933
8. Seattle, 1949
9. Hollister, 1949
10. Helena, 1935
11. Ferndale, 1938
12. Vernon, October 1933
13. Ferndale, February 1941
14. Subway Terminal, October 1933
INTENSITY (in ft.)
1a
1b
2a
2b
3a
3b
4a
4b
5a
5b
6a
6b
7a
7b
8a
8b
9a
9b
10a
10b
11a
11b
12a
12b
13a
13b
14a
14b
1
2
3
4
5
6
7
8
9

FIGURE 7

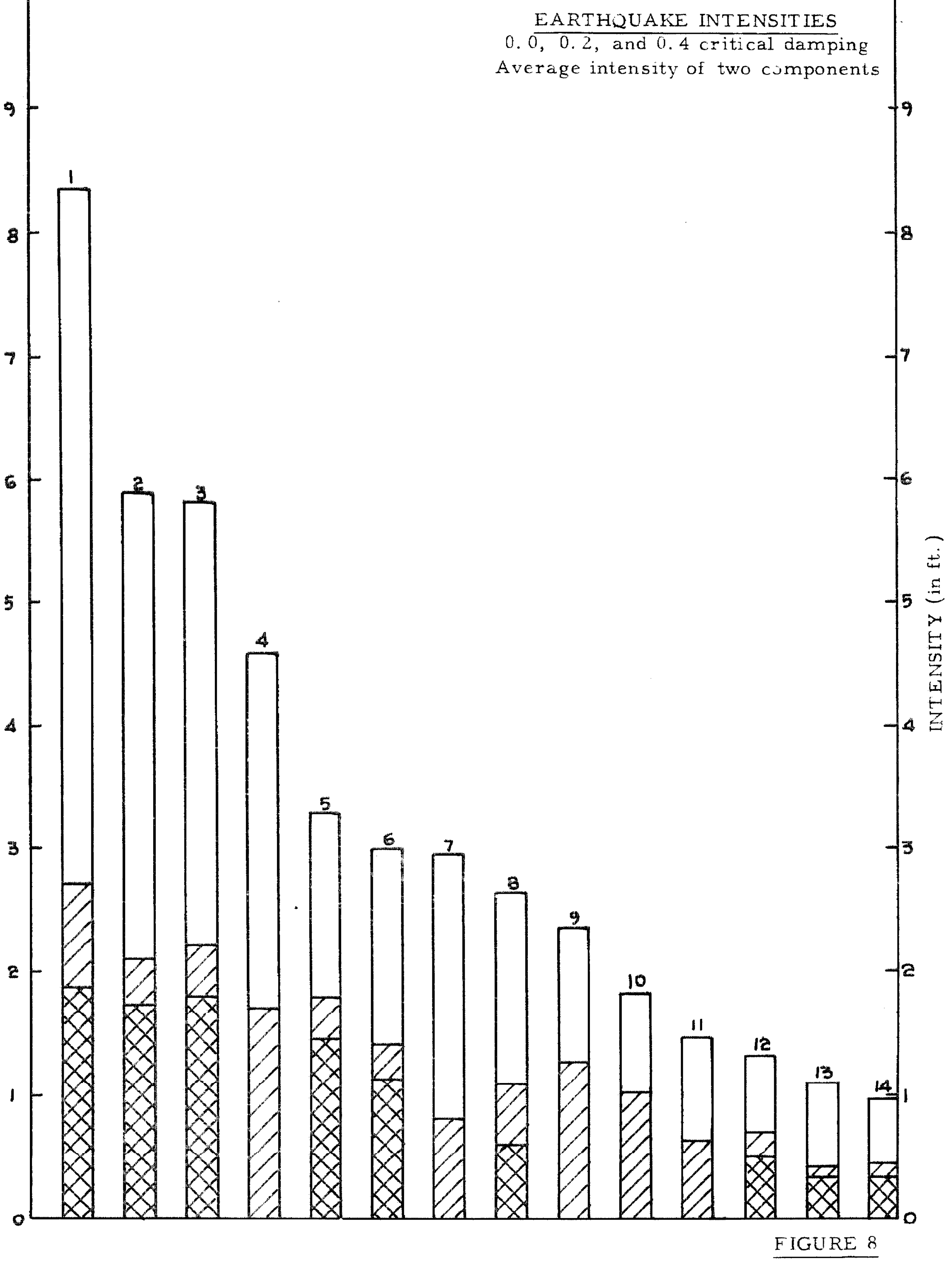

EARTHQUAKE INTENTSITIES
0.0, 0.2, and 0.4 critical damping
Average intensity of two components
INTENSITY (in ft.)
0
1
2
3
4
5
6
7
8
9
1
2
3
4
5
6
7
8
9
10
11
12
13
14

FIGURE 8

These compare with the 2.71 for El Centro, May 18, 1940 and Long Beach, March 10, 1933. This indicates that for the San Francisco shock the maximum stresses that would have been experienced by similar structures with 0.2 damping range from approximately the equal of El Centro and Long Beach to approximately double.

The foregoing remarks apply to structures which are not damaged by the earthquake. If a structure is damaged so that its structural action is changed, its response will be different and the preceding analysis is no longer applicable. In particular, if cracking, stressing beyond the yieldpoint, etc., occurs, large amounts of energy may be dissipated with a consequent relief of the structure.

IV. RELATION OF DAMPED TO UNDAMPED INTENSITIES. It is seen in Tables I and II that the ratios of the intensities of the 14 earthquakes are different depending upon the amount of damping. In general, the effect of damping is more pronounced on the long-duration earthquakes than on the short-duration shocks. For example, the addition of 0.2 damping reduces the average intensity of El Centro 1940 from 8.35 to 2.71; that is, the intensity is reduced to 32.5% of the undamped intensity, whereas Subway Terminal, October 2, 1933 is reduced from 0.96 to 0.45, which is a reduction to 47% of the undamped value.

This difference in effect can be explained by considering the behavior of the oscillator by means of which the surface intensity was defined (equation 1). If the oscillator is undamped, the kinetic energy imported to it during the initial part of the earthquake will not be dissipated and the energy imported to it by the later part of the ground acceleration is added to it with a consequent increase in the amplitude of the oscillator. Thus the undamped intensity will depend upon both the magnitude of the ground acceleration and the duration of the earthquake. If the oscillator is damped, the energy imported to it is continually dissipated, and with relatively high damping there is no tendency to build up in time. A highly damped intensity, therefore, will depend essentially only on the magnitude of the ground acceleration and not upon duration of the shock.

Although the damped spectrum intensity is relatively insensitive to the duration of the earthquake, and thus the maximum stresses experienced by damped structures are very little affected by the length of the shock, it does not follow that the total damage to structures is independent of the duration of the earthquake. Before failure occurs in an average structure there is a period of preliminary failure, during which there is cracking, stressing beyond the yieldpoint, etc. If the shock lasts sufficiently long, the preliminary failure will weaken the structure sufficiently so that major damage will occur.

The effect of the damping on the spectrum intensities is shown in Figure 9 where the intensities of the 28 components have been plotted. In Figure 10 the average intensities of the 14 earthquakes have been plotted; the equation of the curve fitted to these points is

$$y = 0.444x - 0.0124x^2 \tag{8}$$

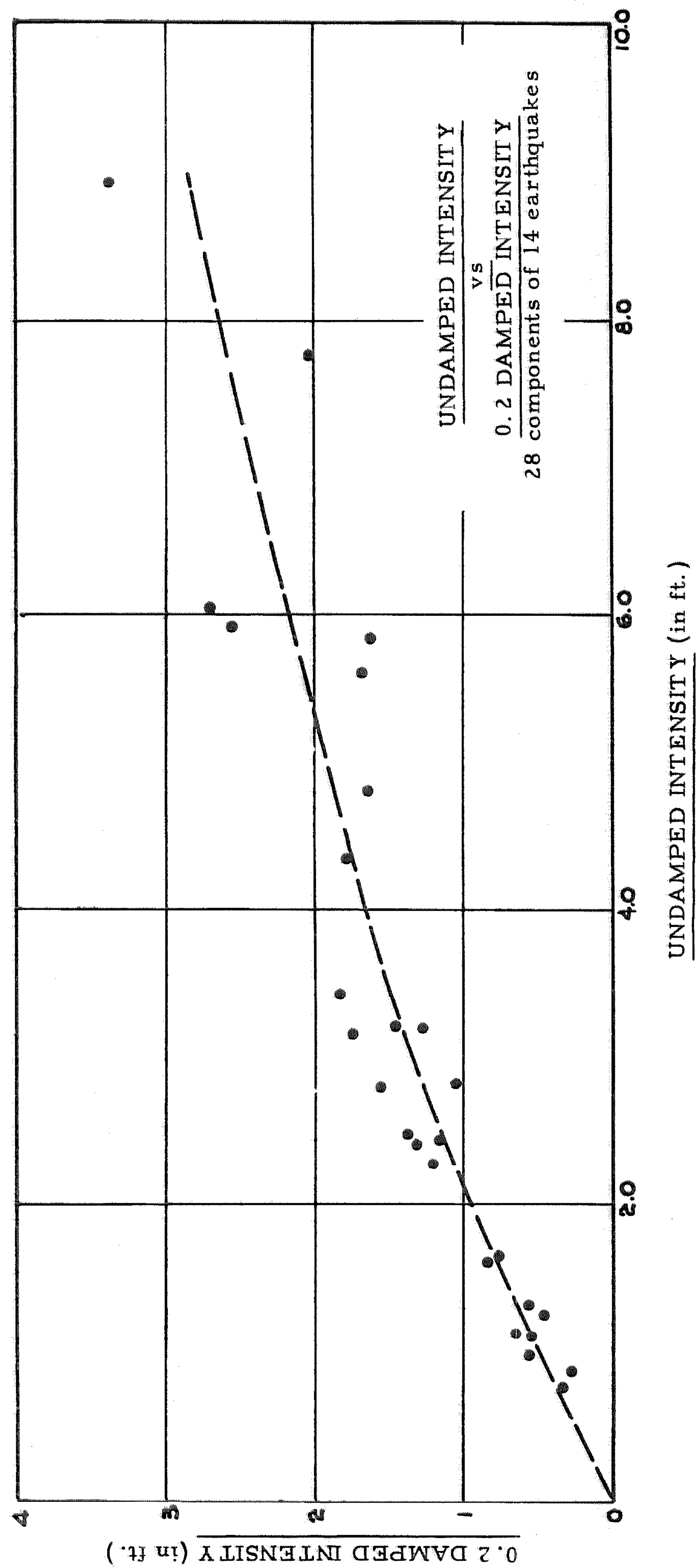
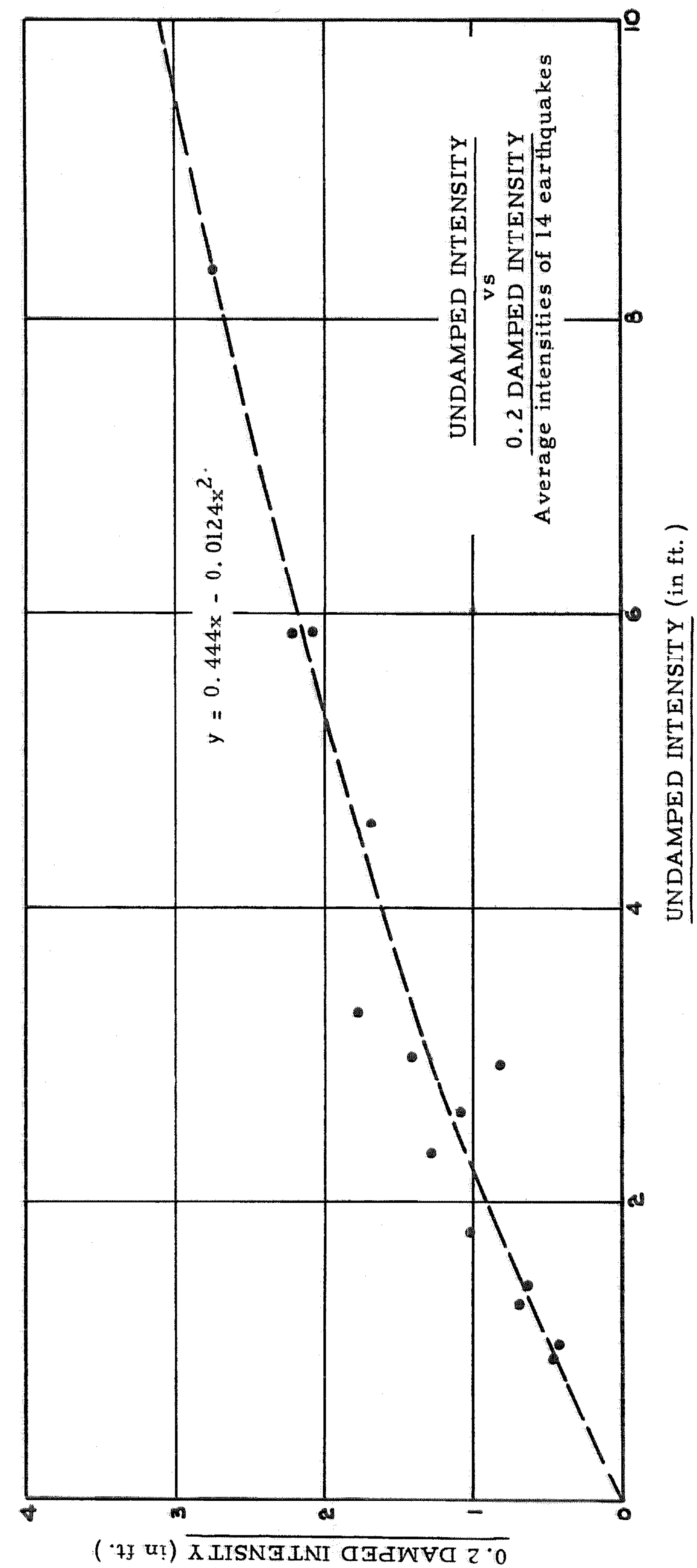

UNDAMPED INTENSITY
vs
0.2 DAMPED INTENSITY
28 components of 14 earthquakes
UNDAMPED INTENSITY (in ft.)
0.2 DAMPED INTENSITY (in ft.)
0
2.0
4.0
6.0
8.0
10.0
0
1
2
3
4

FIGURE 9



UNDAMPED INTENSITY
vs
0.2 DAMPED INTENSITY
Average intensities of 14 earthquakes
y = 0.444x - 0.0124x^2
UNDAMPED INTENSITY (in ft.)
0.2 DAMPED INTENSITY (in ft.)
0
2
4
6
8
10
1
2
3
4

FIGURE 10

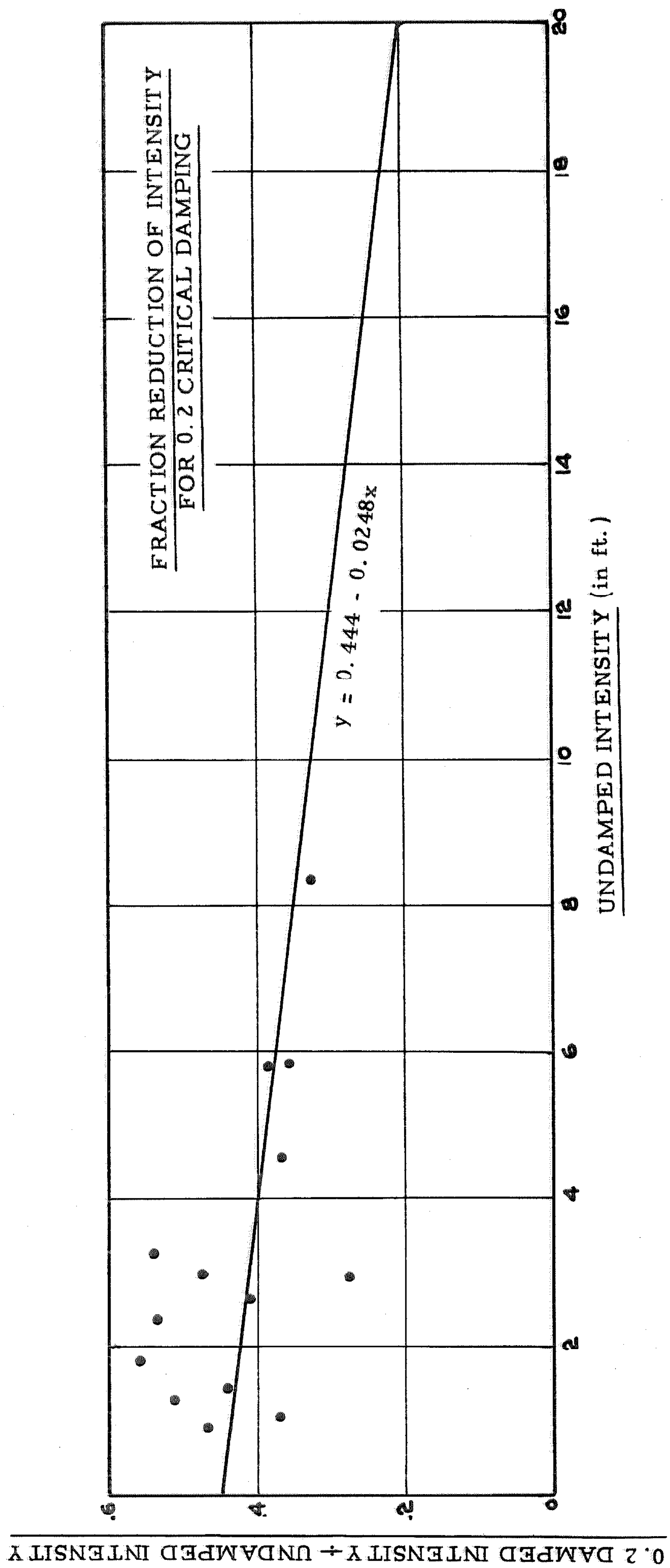

FRACTION REDUCTION OF INTENSITY
FOR 0.2 CRITICAL DAMPING
y = 0.444 - 0.0248x
UNDAMPED INTENSITY (in ft.)
2
4
6
8
10
12
14
16
18
20
0
.2
.4
.6
0.2 DAMPED INTENSITY ÷ UNDAMPED INTENSITY

FIGURE 11

where y is the 0.2 damped intensity and x is the undamped intensity. An alternate form of plotting the data is shown in Figure 11 where the ratio of the 0.2 damped to the undamped intensity is plotted against the undamped intensity. The straight line is the corresponding graph of equation (8).

Equation (8) can be put in a form to fit the data for various values of the damping if it is written

$$\begin{aligned} y &= ax - bx^2 \qquad (0 < x < 10) \\ a &= 0.4 + 0.6\, e^{-(34n)^{1/2}} \\ b &= 0.0125\left(1 - e^{-24n}\right) \qquad (0 < n < 0.5) \end{aligned} \tag{8a}$$

where y is the damped spectrum intensity corresponding to the fraction of critical damping "n", and x is the undamped spectrum intensity.

It should be noted that the computed spectrum intensities are for various distances from the centers of the earthquakes (see Section VIII). The average distance from the center of the shock to the point where the seismogram was recorded is approximately 30 miles, so that equations (8) and (8a) are indicative of the relative intensities at distances of approximately 30 miles from the centers of shocks. If equation (8) is used to extrapolate to large intensities than those shown in Figure 10, there is obtained a maximum 0.2 damped intensity of 4.0 at x = 17.8, so that this is an estimate of the maximum 0.2 damped intensity to be expected at a distance of approximately 30 miles from the center of a shock. This maximum value of 4.0 is 50% greater than the 2.7 that was obtained at El Centro May 18, 1940, which was approximately 30 miles from the center of the shock. This indicates that although greater earthquakes may have considerably longer time durations than the El Centro May 18, 1940 shock, the magnitude of the accelerations at 30 miles from the center will not be greater than approximately 1.5 times those actually recorded at El Centro.

V. CORRELATION OF INTENSITIES WITH MAXIMUM ACCELERATIONS. The maximum acceleration recorded on the accelerogram is correlated to a certain degree with the spectrum intensity computed from the accelerogram. The maximum accelerations are shown in Table III together with the corresponding intensities.

When the maximum accelerations are plotted against the undamped intensities, as in Figure 12, there is considerable scatter with the points lying about a curve of decreasing slope, but when the maximum accelerations are plotted against the 0.2 damped intensities the scatter is reduced and the points lie about a line that is very nearly straight. The solid line in Figure 13 is a straight line, given for reference, and the points lie about the dotted line whose equation is

$$y = \frac{1}{20}\left(1 + 1.85x - \frac{1}{1+x}\right) \tag{9}$$

The approximately linear trend for the 0.2 damped intensities is to be expected since the damped intensities should vary approximately as the accelerations, as was noted in Section IV.

In the preceding section it was shown that at 30 miles from the center of a shock the maximum 0.2 damped intensity to be expected is approximately 4.0. If this value is used in equation (9) to extrapolate to the corresponding maximum acceleration, there is obtained a value of 0.41 g for the maximum acceleration to be expected at a distance of 30 miles from the center of the shock. The maximum acceleration actually recorded at El Centro, May 18, 1940, at a distance of 30 miles from the center of the shock, was 0.33 g. The question of the maximum accelerations to be expected at closer distances than 30 miles is examined in Section XII.

TABLE III

Maximum Ground Accelerations

In Fraction Gravity

Location	Component	Maximum Acceleration	Undamped Intensity	0.2 Damped Intensity
El Centro, Calif.	NS	0.33	8.94	3.36
May 18, 1940	EW	0.23	7.77	2.06
El Centro, Calif.	NS	0.26	5.93	2.55
December 30, 1934	EW	0.20	5.83	1.62
Olympia, Wash.	S80W	0.31	6.05	2.74
April 13, 1949	S10E	0.18	5.59	1.68
Vernon, Calif.	S82E	0.19	4.90	1.77
March 10, 1933	N08W	0.13	4.35	1.64
Santa Barbara, Calif.	S45E	0.24	3.43	1.84
June 30, 1941	N45E	0.23	3.15	1.75
Ferndale, Calif.	N45E	0.13	3.2	1.27
October 3, 1941	S45E	0.12	2.78	1.56
L.A. Subway Terminal	N51W	0.065	3.21	0.96
March 10, 1933	N39E	0.04	2.67	0.67
Seattle, Wash.	N88W	0.075	2.81	1.05
April 13, 1949	S02W	0.058	2.46	1.14
Hollister, Calif.	S01W	0.23	2.44	1.33
March 9, 1949	N89W	0.11	2.29	1.20
Helena, Montana	E-W	0.16	2.49	1.39
October 31, 1935	N-S	0.14	1.16	0.65
Ferndale, Calif.	N45E	0.082	1.64	0.82
September 11, 1938	S45E	0.16	1.27	0.46
Vernon, Calif.	S82E	0.12	1.65	0.80
October 2, 1933	N08E	0.085	0.99	0.56
Ferndale, Calif.	S45E	0.075	1.31	0.54
February 9, 1941	N45E	0.04	0.88	0.27
L.A. Subway Terminal	N39E	0.065	1.14	0.55
October 2, 1933	N51W	0.060	0.78	0.35

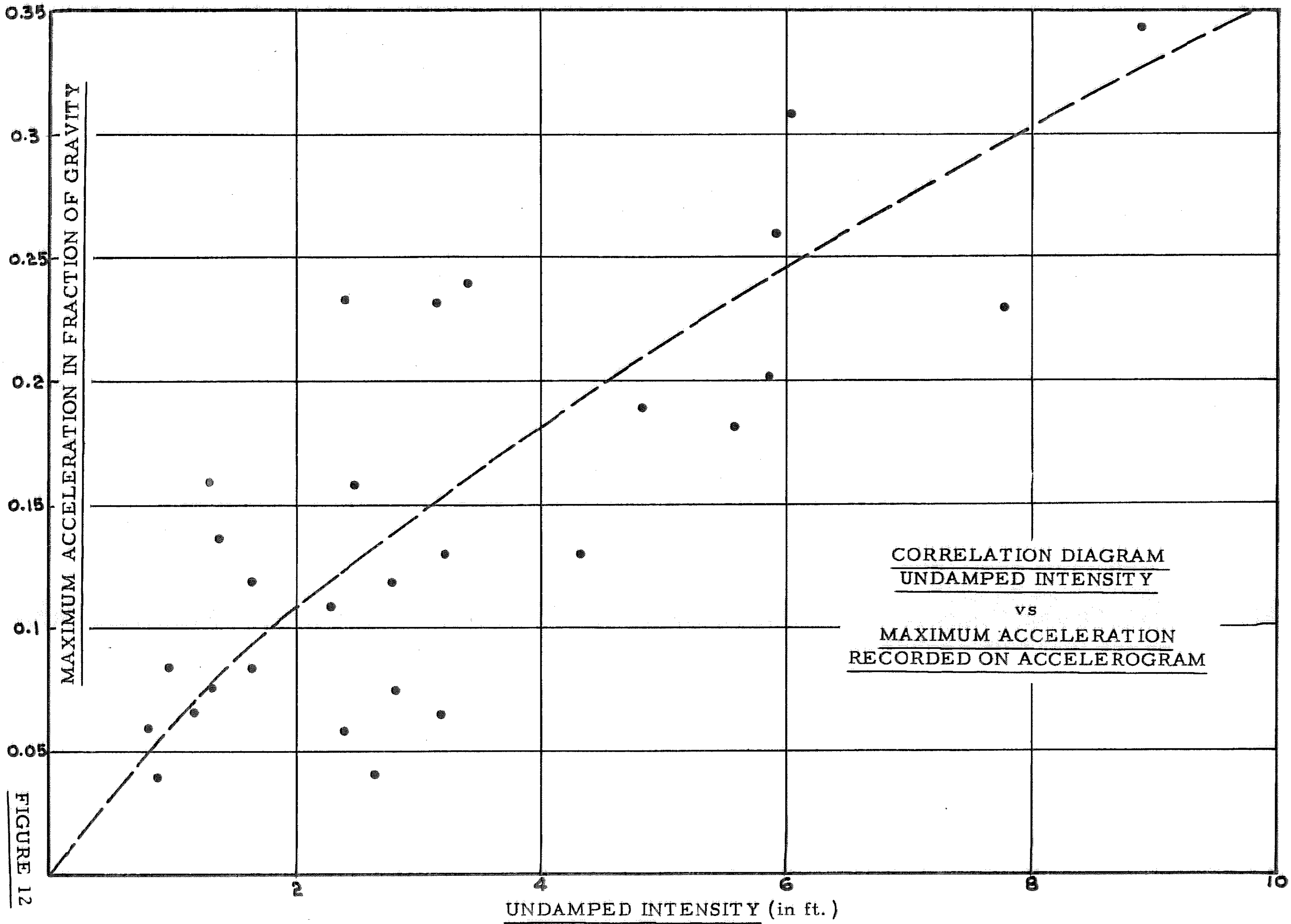

CORRELATION DIAGRAM
UNDAMPED INTENSITY
vs
MAXIMUM ACCELERATION
RECORDED ON ACCELEROGRAM
MAXIMUM ACCELERATION IN FRACTION OF GRAVITY
UNDAMPED INTENSITY (in ft.)
0.35
0.3
0.25
0.2
0.15
0.1
0.05
2
4
6
8
10

FIGURE 12

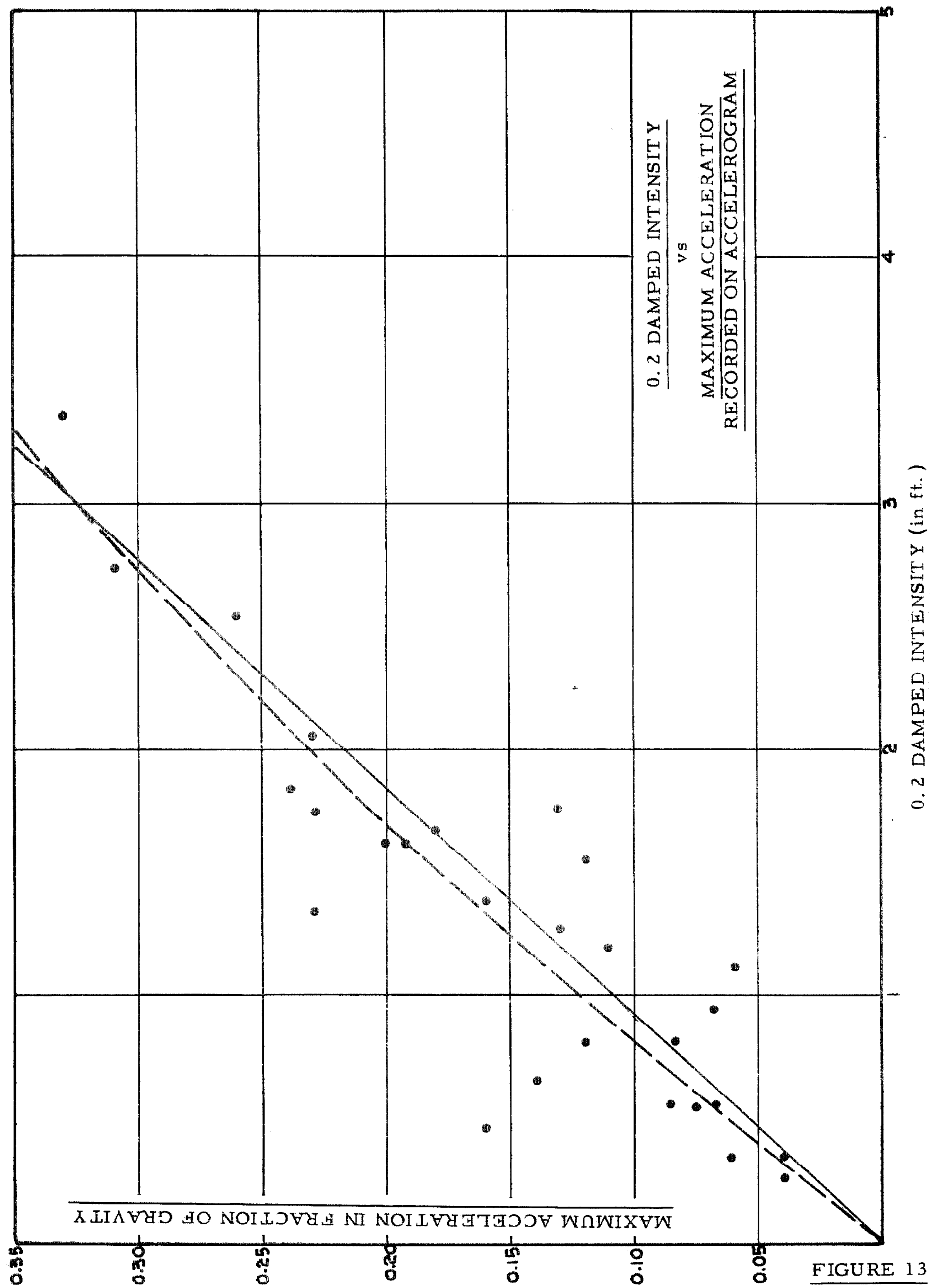

0.2 DAMPED INTENSITY
vs
MAXIMUM ACCELERATION
RECORDED ON ACCELEROGRAM
0.2 DAMPED INTENSITY (in ft.)
MAXIMUM ACCELERATION IN FRACTION OF GRAVITY
2
3
4
5
0.05
0.10
0.15
0.20
0.25
0.30
0.35

FIGURE 13

VI. Relation of Spectrum Intensities to Modified-Mercalli Intensities. The most common method of assessing the intensity of the ground motion is by the Modified-Mercalli scale of intensities. This is a non-instrumental method of measuring intensities based on the observations of persons feeling the ground motion and upon the damage to buildings caused by the earthquake. For convenient reference an abridged form of the Modified-Mercalli scale is given.

Modified-Mercalli Scale

1. Not felt except by a very few under especially favorable conditions.

2. Felt only by a few persons at rest, especially on upper floors of buildings. Delicately suspended objects may swing.

3. Felt quite noticeably indoors, but many persons do not recognize it as an earthquake.

4. During the day felt indoors by many, outdoors by few. Dishes, windows, doors disturbed.

5. Felt by nearly everyone; some dishes, windows, and so forth, broken.

6. Felt by all; some heavy furniture moved, a few instances of fallen plaster or damaged chimneys.

7. Damage negligible in buildings of good design and construction; slight to moderate in well-built ordinary structures; considerable in poorly built or badly designed structures.

8. Damage slight in specially designed structures; considerable in ordinary substantial buildings with partial collapse; great in poorly-built structures.

9. Damage considerable in specially designed structures; great in substantial buildings.

10. Some well-built wooden structures destroyed; most masonry and frame structures destroyed.

11. Few, if any, masonry structures remain standing.

12. Damage total.

The Modified-Mercalli intensities, as assessed by the U.S. Coast and Geodetic Survey, are listed in Table IV for the 14 localities where the accelerograms were obtained.

TABLE IV

MODIFIED-MERCALLI INTENSITIES

		Assessed Intensity
1.	El Centro, 1940	7.5
2.	El Centro, 1934	6
3.	Olympia, 1949	8
4.	Vernon, March 1933	7.5
5.	Santa Barbara, 1941	7
6.	Ferndale, October 1941	6
7.	Subway Terminal, March 1933	6.5
8.	Seattle, 1949	8
9.	Hollister, 1949	7
10.	Helena, 1935	8
11.	Ferndale, 1938	6
12.	Vernon, October 1933	6
13.	Ferndale, February 1941	6
14.	Subway Terminal, October 1933	5.75

In Figures 14 these Modified-Mercalli intensities are plotted against the undamped spectrum intensities. The equation of the dotted curve shown in this diagram is

$$y = \frac{1}{800} x^4 \tag{10}$$

where y is the undamped spectrum intensity and x is the Modified-Mercalli intensity. It is seen that there is considerable variability in the assessment of M-M intensities but the relative intensities of points 1 and 2, of 4 and 7, of 12 and 14 are self-consistant. Each pair of these represent two instances where M-M ratings were made in the same locality so that the relative intensities of the two do not reflect subjective judgments as to quality of building construction, etc. Some of the scatter of the points in Figure 14 can be explained by special circumstances which were in effect. For example, No. 10, Helena, Montana, October 31, 1935 was preceded by a stronger shock for which accelerograms were not obtained. The first shock caused considerable damage and apparently weakened many buildings so that the shock of October 31 caused more damage that it would have caused had it been the only earthquake to

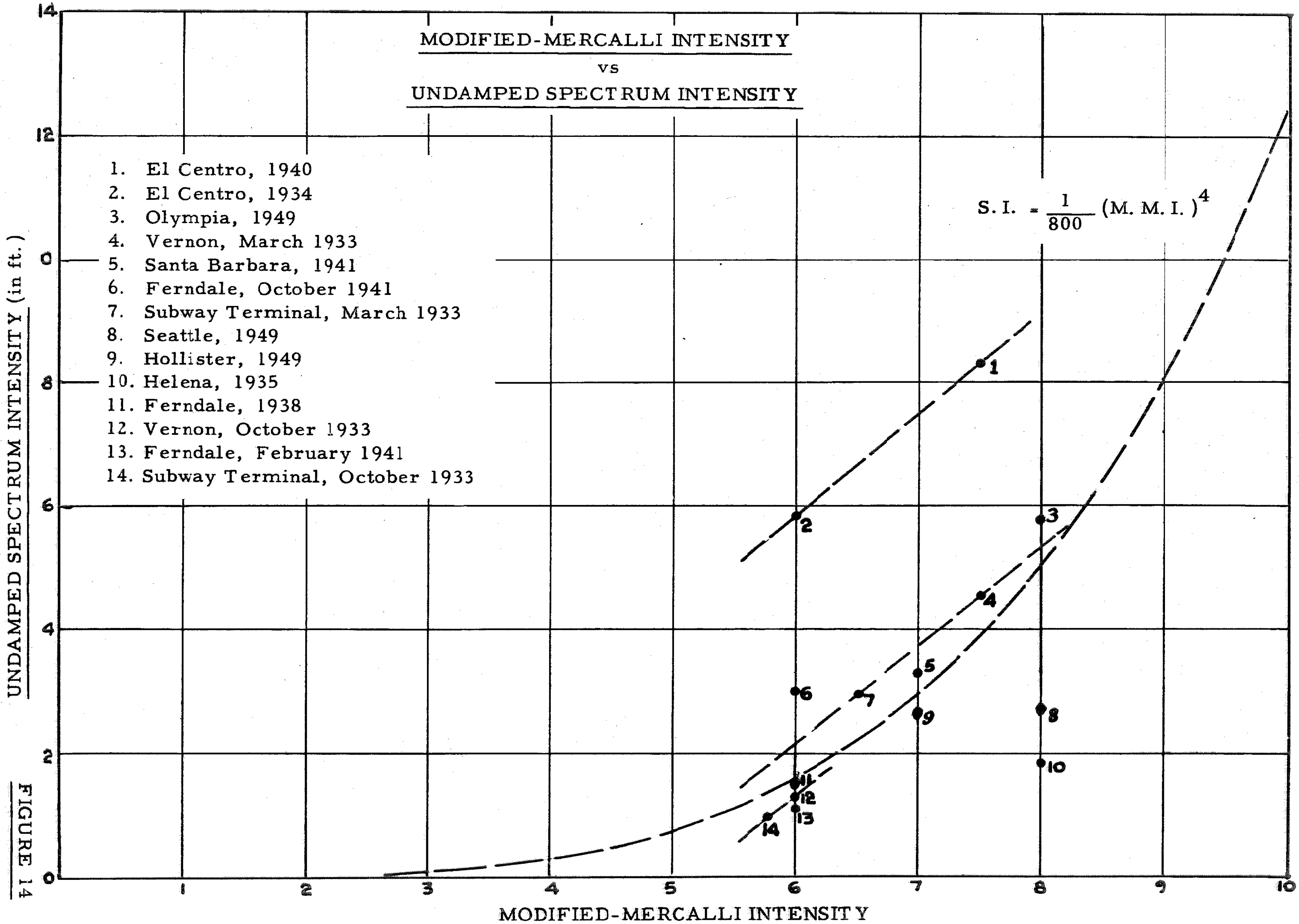

MODIFIED-MERCALLI INTENSITY
vs
UNDAMPED SPECTRUM INTENSITY
1. El Centro, 1940
2. El Centro, 1934
3. Olympia, 1949
4. Vernon, March 1933
5. Santa Barbara, 1941
6. Ferndale, October 1941
7. Subway Terminal, March 1933
8. Seattle, 1949
9. Hollister, 1949
10. Helena, 1935
11. Ferndale, 1938
12. Vernon, October 1933
13. Ferndale, February 1941
14. Subway Terminal, October 1933
S. I. = $\frac{1}{800}$ (M. M. I.)^4
UNDAMPED SPECTRUM INTENSITY (in ft.)
MODIFIED-MERCALLI INTENSITY
14
12
0
8
6
4
2
0
1
2
3
4
5
6
7
8
9
10

FIGURE 14

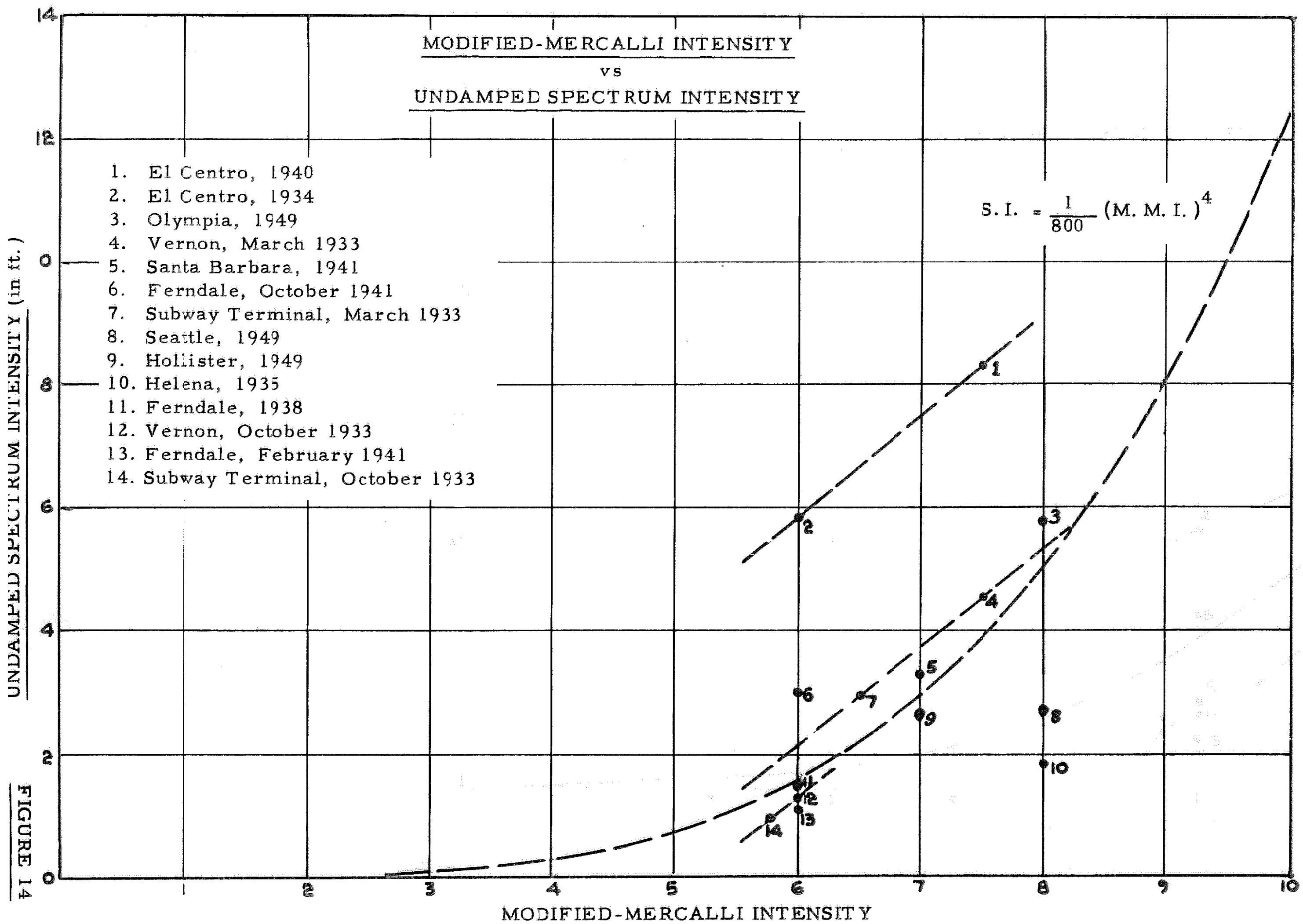
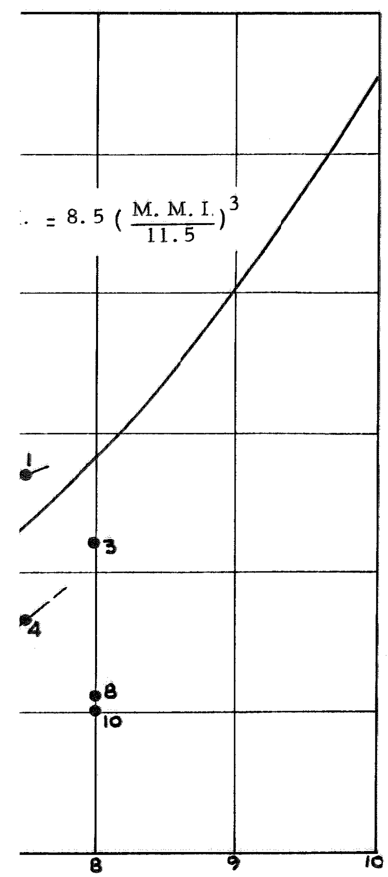

MODIFIED-MERCALLI INTENSITY
vs
UNDAMPED SPECTRUM INTENSITY
1. El Centro, 1940
2. El Centro, 1934
3. Olympia, 1949
4. Vernon, March 1933
5. Santa Barbara, 1941
6. Ferndale, October 1941
7. Subway Terminal, March 1933
8. Seattle, 1949
9. Hollister, 1949
10. Helena, 1935
11. Ferndale, 1938
12. Vernon, October 1933
13. Ferndale, February 1941
14. Subway Terminal, October 1933
S.I. = $\frac{1}{800}$ (M. M. I.)^4
UNDAMPED SPECTRUM INTENSITY (in ft.)
MODIFIED-MERCALLI INTENSITY

FIGURE 14


. = 8.5 ($\frac{\text{M. M. I.}}{11.5}$)^3

which the city had been subjected. The deviation of No. 8, Seattle, Washington, April 13, 1949 is perhaps explained by the fact that the accelerograph which recorded the motion was located on filled ground, adjacent to a sea wall and this apparently had an effect on the ground motion. The spectrum intensity for Seattle is thus indicative of the intensity of ground motion of the filled ground adjacent to the sea wall but it is thought to underestimate the intensity of motion in other parts of the city.

Since all buildings have damping, and ordinary buildings not specially designed to resist lateral forces can be expected to have relatively large amounts of damping, the spectrum intensities for 0.2 damping are perhaps more indicative of the destructiveness of an earthquake than are the undamped intensities. When the Modified-Mercalli intensities are plotted against the 0.2 damped spectrum intensities, there is obtained the diagram shown in Figure 15. The equation of the curve shown in this figure is

$$y = 8.5\left(\frac{x}{11.5}\right)^3 \tag{11}$$

where y is the 0.2 damped intensity and x is the Modified-Mercalli intensity. It is seen that the scatter is somewhat reduced in Figure 15 over what it was in Figure 14 and, considering the subjective factors entering into an assessment of Modified-Mercalli intensities, the fit of the points in Figure 15 is as good as can be expected.

It should be noted that the Modified-Mercalli intensity does not actually measure the same thing as the spectrum intensity. The latter is a measure of the maximum stresses produced in undamaged buildings, whereas the former is a measure of the actual damage incurred. To explain how this difference can lead to discrepancies between the two measures of intensity suppose the following state of affairs. Let there be two cities composed of similar buildings and let each be subjected to a shock of the same spectrum intensity. Then, if no failures occurred, on the average the maximum stresses in both cities would be the same. Suppose, next that the intensity of the shocks were sufficient to cause damage and suppose that the details of building construction in city A were such that the buildings could withstand a prolonged process of cracking, stressing key and the yield point, etc., whereas in city B the details of construction were such that serious shattering, brittle failures, etc., occurred without a prolonged period of minor cracking, yielding, etc. There would then be a large dissipation of vibrational energy in city A before serious failure occurred which would not be the case in city B. This dissipating energy would relieve the stresses with a consequent protection against severe damage. The assessed Modified-Mercalli intensity would then be much larger in city B than in city A, despite the fact that the spectrum intensities were the same.

VII. Magnitudes of Earthquakes. The magnitude of an earthquake was originally defined by C. F. Richter for shocks in Southern California, as the logarithm of the maximum trace amplitude expressed in thousandths of a millimeter with which the standard short-period torsion seismometer (free period 0.8 sec., static magnification 2800, damping nearly critical) would register that earthquake at an epicentral distance of 100 kilometers. The relation between earthquake magnitude, energy and acceleration is discussed by B. Gutenberg and C. F. Richter[4].

It is shown in the paper by Gutenberg and Richter that when the depth of shock is 18 kilometers the relation between the energy released by the shock and the magnitude of the earthquake may be expressed satisfactorily by the expression

$$E = (10)^{11.3} (10)^{1.8M} \tag{12}$$

where the energy E is expressed in ergs. It is also shown in the same paper that the data agrees satisfactorily with the relation

$$\sqrt{E} = C \frac{a}{h} (D^2 + h^2) \tag{13}$$

where C is a constant, a is the magnitude of an acceleration pulse recorded on an accelerogram, D is the distance from the epicenter of the shock to the point where the acceleration is measured, and h is the depth of the shock below the surface of the ground. Equation (13) thus gives the diminution of acceleration with increasing distance from the epicenter of the earthquake, that is,

$$a = C\sqrt{E} \frac{1}{h(1 + \frac{D^2}{h^2})} \tag{14}$$

If a_1 is the maximum acceleration at a distance D_1 from the epicenter and a_2 is the corresponding acceleration measured at a distance D_2, then equation (14) gives

$$a_1 = a_2 \frac{1 + (\frac{D_2}{h})^2}{1 + (\frac{D_1}{h})^2} \tag{15}$$

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4 B. Gutenberg and C. F. Richter, Earthquake magnitude, Intensity, Energy and Acceleration, Bulletin of the Seismological Society, Vol. 32, No. 3, July 1942.

This permits the measured acceleration to be projected to the standard epicentral distance $D_1 = 100$ kilometers and the magnitude M is then given by

$$M = \log_{10} K a_1$$

where the factor K takes care of the instrumental constants.

The magnitude is thus a measure of the size of the earthquake in the sense that it is a measure of the energy released. The spectrum intensity, on the other hand is a measure of the severity of the ground motion at a particular point on the surface of the ground. For a given earthquake the intensity will vary over the region affected by the shock. The intensity will be relatively high near the center of the shock and will diminish with increasing distance from the center.

VIII. Relation of Spectrum Intensities to Earthquake Magnitudes. If all earthquakes had the same general character that is if the accelerograms of all earthquakes were composed of the same proportion of waves of different periods with the same relative amplitudes, the undamped spectrum intensity would be a measure of the energy passing through the ground at the point where the accelerograph is located; the undamped spectrum intensity would be proportional to the square root of this energy. If the earth were completely homogenous, the energy passing through the ground under the accelerograph would bear a specific relation to the energy released by the earthquake. From equation (12), which relates energy released to the earthquake magnitude, it would then be possible to compute the magnitude of an earthquake from its spectrum intensity. In what follows the magnitudes are computed on this basis. It is clear that if these computed magnitudes agree with the magnitudes given by Gutenberg and Richter[5], it implies that these earthquakes have the same general character and that the spectrum intensities are not particularly sensitive to geological inhomogeneities. Conversely, if there is poor agreement, it implies either that earthquakes do not have the same general characteristics or that geological inhomogeneities have a strong effect on the spectrum intensities.

To compute the magnitudes from the spectrum intensities, allowance must be made for the fact that the accelerograms are obtained at varying distances from the origin of the shock. The spectrum intensity is proportional to the magnitude of the acceleration so that from equation (15) there is obtained

$$(S.I.)_o = (S.I.)\left[1 + \left(\frac{D}{h}\right)^2\right] \tag{16}$$

where $(S.I.)_o$ is the spectrum intensity projected back to the "apparent" center, D is the distance from the point where the accelerogram was obtained to the apparent center and h is the depth of the "apparent" origin of the shock beneath the surface of the ground (beneath the apparent center).

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[5] B. Gutenberg and C. F. Richter, Seismicity of the Earth, Princeton University Press, Princeton, New Jersey, 1949.

For most of the earthquakes considered here the depths of shock are assessed by seismologists to be approximately 20 kilometers, except the April 13, 1949 shock which has an assessed depth of approximately 60 kilometers. In the following calculations the apparent origins of most of the shocks are taken to be at depths of 15 miles. The apparent origin of the shock is defined to be the mean point of energy release. Actually the energy is released over a finite length of fault but for some of the computational procedures it is convenient to assume that all of the energy was released at a point (the apparent origin). The apparent center of the shock is defined to be the point on the surface of the ground directly above the apparent origin.

The locations of the apparent centers of the earthquakes are not known precisely since the lengths and positions of the faults are not known exactly. It is clear that the apparent center must be in a region near the epicenter for the instrumentally determined epicenter is essentially that point on the ground directly above the point at which the initial release of energy occurred. The epicenter does not in general coincide with the center of the fault but is often near one end of the fault. For a small earthquake, with a short fault the epicenter, the center of fault and the apparent center must be close together. For earthquakes with long faults the apparent center must be near the center of the fault but may be a number of miles from the epicenter. In the present calculations the apparent center was located by working from the instrumental epicenter. For the smaller earthquakes the apparent center was taken to coincide with the epicenter. For the larger earthquakes the apparent center was taken from 0 to 15 miles from the epicenter at the point which gave the best fit. For example, the epicenter of the 18 May 1940, El Centro shock was approximately 15 miles from El Centro, but the apparent center was selected 30 miles from El Centro more nearly at the center of the fault. The Ferndale shocks had their epicenters located off the coast of California with an estimated accuracy of perhaps 30 - 40 miles and for these the apparent centers were located at distances up to 40 miles from the reported epicenters.

In Table V are listed the distances D and depths h used in the calculations, the magnitudes calculated from the spectrum intensities, and the Gutenberg-Richter magnitudes of the earthquakes. The procedure used in calculation was to determine $(S.I.)_o$ from equation (16), then take the energy released to be given by

$$E = C\,(S.I.)_o^{\,2}$$

This determines the energy except for the unknown constant C. The value of this constant was taken so that the magnitude computed from the Vernon, October 2, 1933 record (most accurate location of epicenter) agreed with the Gutenberg-Richter magnitude of this earthquake. For those shocks whose depths were different from 15 miles the calculated energy was adjusted according to the ratio of the square of the depths. These conditions are equivalent to taking the undamped spectrum intensity to be

$$S.I. = (5.1)\,10^{(0.9M-5)}\left(\frac{h_o}{h}\right)^2 \frac{1}{1+\left(\frac{D}{h}\right)^2} \qquad (17)$$

where h_o = 15 miles.

To show how the values of S.I. vary with D there is plotted Figure 16 which is for h = 15 miles. The magnitudes computed from equation (17), that is, from

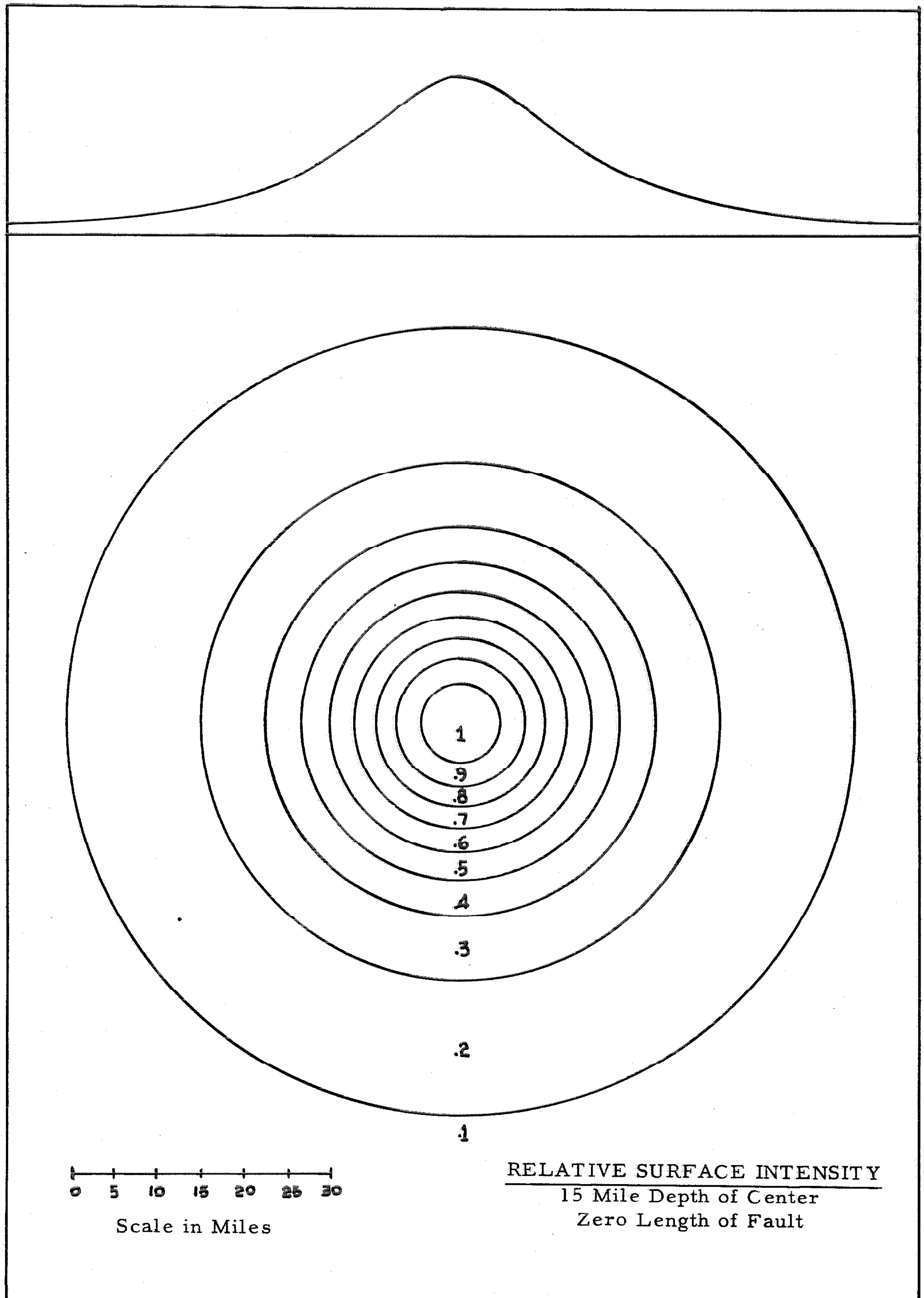
$$M = \frac{1}{0.9}\left\{5 + \log\left(\frac{S.I.}{5.1}\left[\frac{h}{h_o}\right]^2\left[1+\frac{D^2}{h^2}\right]\right)\right\} \qquad (17a)$$

are listed in Table V. The locations of the epicenters used for determining the distance D were furnished by C.F. Richter of the Pasadena Seismological Laboratory.

Table V

COMPUTED EARTHQUAKE MAGNITUDES

		D	h	Computed Magnitude	G-R Magnitude
1.	El Centro, 1940	30	15	6.58	6.7
2.	El Centro, 1934	35	15	6.54	6.5
3.	Olympia, 1949	45	45	7.02	7.1
4.	Vernon, March 1933	28	15	6.13	6.25
5.	Santa Barbara, 1941	15	19	5.79	5.9
6.	Ferndale, October 1941	50	15	6.50	6.4
7.	Subway Terminal, Mar 1933	33	15	6.15	6.25
8.	Seattle, 1940	55	45	6.75	7.1
9.	Hollister, 1949	10	15	5.35	5.3
10.	Helena, 1935	15	25	5.70	6.0
11.	Ferndale, 1938	35	10	5.8	5.5
12.	Vernon, October 1933	17	15	5.30	5.30
13.	Ferndale, February 1941	75	15	6.54	6.6
14.	Subway Terminal, Oct 1933	22	15	5.31	5.3



RELATIVE SURFACE INTENSITY
15 Mile Depth of Center
Zero Length of Fault

FIGURE 16

When the magnitudes computed from the spectra are plotted against the G-R magnitudes as in Figure 17, it is seen that all the points except three lie satisfactorily close to the 45° line. The three for which the agreement is not close are:

Ferndale, 1938	5.8	- 5.5
Helena, 1935	5.7	- 6.0
Seattle, 1940	6.75	- 7.1

The others all agree with an average difference of 0.07 of a magnitude whereas these three differ by 0.30 of a magnitude. It is not known to what to attribute these discrepancies except possibly in the case of Seattle, 1949. The accelerometer which recorded this shock was located on filled ground and was adjacent to a sea wall. The accelerogram showed relatively few short period waves and proportionately more longer period waves and it seems likely that the local geological conditions had an effect upon the ground accelerations which resulted in a decrease in spectrum intensity. It is perhaps possible that the discrepancy for the Helena shock may also reflect special geological conditions.

Perfect agreement is not to be expected between the two ways of calculating the magnitude of an earthquake since one method uses only the maximum acceleration whereas the other uses the cumulative effect of all of the acceleration pulses. In general, the agreement between the computed magnitudes and the G-R magnitudes is very good, thus indicating that the general character of these earthquakes is substantially the same, and that local geological conditions did not have a strong effect upon the spectrum intensities. The agreement is sufficiently good so that the G-R magnitudes could be used to estimate the spectrum intensities. A procedure for making such estimates is discussed in the following section.

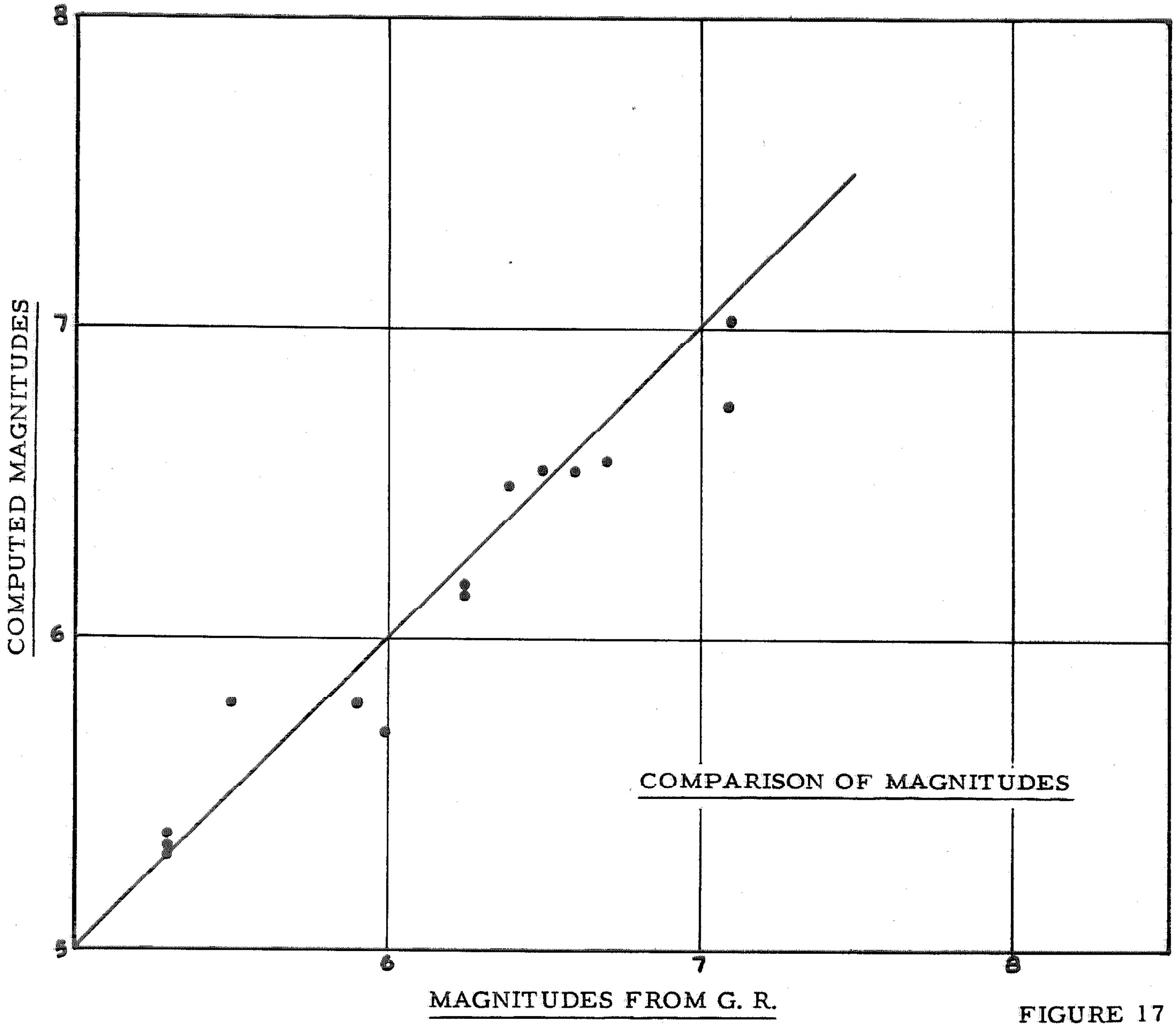

8
7
6
5
6
7
8
COMPUTED MAGNITUDES
MAGNITUDES FROM G. R.
COMPARISON OF MAGNITUDES

FIGURE 17

XIV. Estimation of Spectrum Intensities from G-R Magnitudes. As shown in the preceding section, there is good agreement between the magnitude of an earthquake and the spectrum intensity computed from a strong-motion accelerogram. It should be noted, however, that the agreement was demonstrated only for spectrum intensities computed from accelerograms obtained 15 miles or more from the center of the shock. If spectrum intensities are estimated using the foregoing procedures, equation (17) and Figure 16 are probably not reliable in the immediate vicinity of the center. The fact that use of equation (17) implies that the energy was released at a point whereas it actually is released over a length of fault would require a modification of equation (17). In addition, it has been observed in some instances that the area in the immediate vicinity of the fault was not shaken as severely as would be implied by projecting to the center by methods such as the use of equation (16). This flattening off of intensity in the immediate fault area is no doubt connected with the faulting mechanism and local geological conditions. An allowance can be made for the fact that energy is released over a finite length of fault but it must be noted that this may still give too high values of intensity in the immediate vicinity of the fault.

Assuming the application of equation (17) to be valid, the finite length of fault may be taken into account approximately by considering the energy to be released uniformly along a line of length l at a depth h. The spectrum intensity is then given by

$$S.I. = K \int_{-l/2}^{+l/2} \frac{1}{1 + \frac{(x-z)^2 + y^2}{h^2}} \, dz$$

where x is the horizontal coordinate measured from the center parallel to the fault, y is the horizontal coordinate measured perpendicular to the fault, h is the depth, z is measured horizontally along the fault, l is the length of the fault and K is a constant. When the foregoing expression is integrated, there is obtained

$$S.I. = \frac{K}{\sqrt{1+(\frac{y}{h})^2}} \left\{ \tan^{-1} \frac{\frac{x}{h} + \frac{l}{2h}}{\sqrt{1+(\frac{y}{h})^2}} - \tan^{-1} \frac{\frac{x}{h} - \frac{l}{2h}}{\sqrt{1+(\frac{y}{h})^2}} \right\} \qquad (18)$$

When this equation is evaluated for h = 15 miles, and l = 30 miles there result lines of equal intensity as shown in Figure 18. In drawing this figure, the constant K was adjusted so that the relative intensities marked on the diagram agree with the relative intensities, marked on Figure 16 at large distances from the center. Comparing Figures 16 and 18, it is seen that the intensity at the center is reduced 20% when the energy is considered to be released over a length of 30 miles instead of being released at a point.

To estimate the spectrum intensity from the magnitude of the earthquake the following procedure is used. Equation (12) gives for the square root of the energy released

$$\sqrt{E} = C\ 10^{0.9M}$$

Taking the spectrum intensity proportional to the square root of the energy released gives from equation (16)

$$(S.I.)_o = C_1\ 10^{0.9M}$$

In calculating Table V, C_1 was taken to be 5.1×10^{-5} for 5 = 15 miles, giving for the value of the spectrum intensity when projected to the center,

$$(S.I.)_o = (5.1)\ 10^{(0.9M-5)} \qquad (19)$$

Equation (18) gives for $l \rightarrow o$, with Kl remaining constant,

$$S.I. = \frac{K}{1+(\frac{D}{h})^2}\,\frac{\ell}{h} = \frac{K\ell}{h[1+(\frac{D}{h})^2]}$$

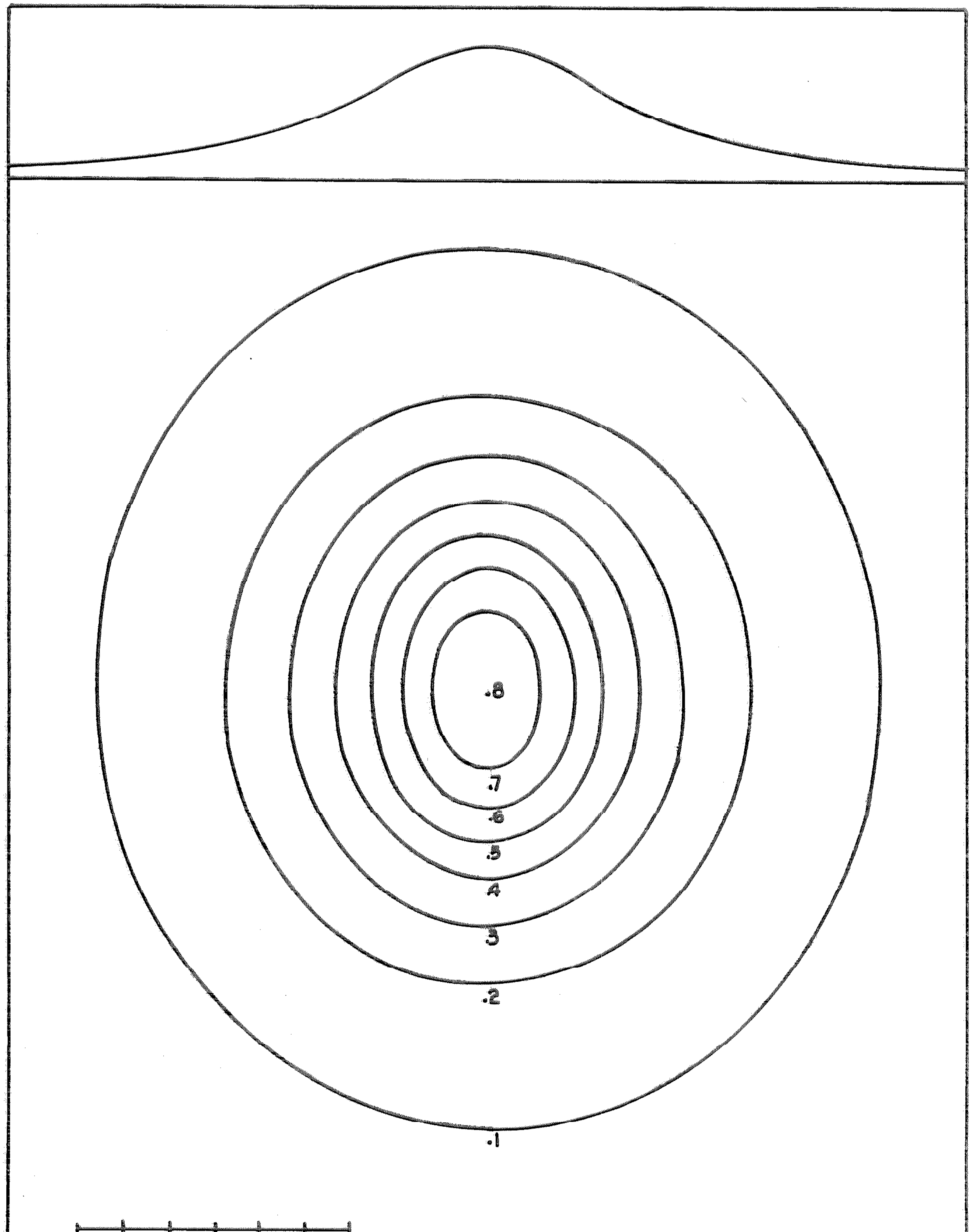
Thus

$$(S.I.)_o = \frac{K\ell}{h}$$

and

$$K = (5.1)\ 10^{(0.9M-5)}\left(\frac{h}{\ell}\right)\left(\frac{15}{h}\right)^2 \qquad (20)$$

Given the magnitude of the earthquake, equation (20) permits the constant K to be calculated and equation (18) permits the spectrum intensity to be calculated at any point from the center. It should be noted that equation (18) will not necessarily give reliable values at the center, neither



RELATIVE SURFACE INTENSITY
15 Mile Depth and 30 Mile Length of Fault

FIGURE 18

does it take into account the effect of local geological conditions. It is merely a method of calculating spectrum intensities with the reliability implied by the fit of Figure 17. The final form of the equation for calculating the spectrum intensities is

$$(\text{S.I.}) = (5.1)\, 10^{(0.9M-5)} \left(\frac{h_o}{h}\right)^2 \left(\frac{h}{\ell}\right) \frac{1}{\sqrt{1+\left(\frac{D}{h}\right)^2}} \left\{ \tan^{-1} \frac{\frac{x}{h} + \frac{\ell}{2h}}{\sqrt{1+\left(\frac{y}{h}\right)^2}} - \tan^{-1} \frac{\frac{x}{h} - \frac{\ell}{2h}}{\sqrt{1+\left(\frac{y}{h}\right)^2}} \right\} \tag{21}$$

where h_o = 15 miles. It will be noted that in the preceding section the earthquake magnitudes were computed from equation (17) which considers the fault to have zero length. Equation (21) should give better values of the magnitudes computed from the spectrum intensities if the positions and lengths of the faults were known.

In the application of equation (21) it should be noted that an error of 0.1 in magnitude corresponds approximately to an error of 25% in spectrum intensity. Also in using this equation to calculate the spectrum intensity from the magnitude there will be some indeterminacy in locating the center of the shock.

X. Variation of Surface Intensities. The preceding methods may be used to estimate the variation in the intensity of ground motion over the area in the vicinity of the origin of the earthquake. For this purpose, equation (18) is used

$$S.I. = \frac{K}{\sqrt{1+(\frac{y}{h})^2}} \left\{ \tan^{-1} \frac{\frac{x}{h} + \frac{l}{2h}}{\sqrt{1+(\frac{y}{h})^2}} - \tan^{-1} \frac{\frac{x}{h} - \frac{l}{2h}}{\sqrt{1+(\frac{y}{h})^2}} \right\} \qquad (22)$$

where x is the horizontal distance measured from the center of the fault in a direction parallel to the fault, y is the horizontal distance measured perpendicular to the fault, h is the depth and l is the length of fault. Figure (18) shows the lines of equal spectrum intensity computed from equation (22) for l = 30 miles and h = 15 miles. Remembering that equation (22) is an estimate which does not take into account the effect of local geological conditions and which may somewhat overestimate the intensity at the center, it may be said that the isoseismal lines are approximately ellipses with major axes oriented parallel to the fault. The rate of decrease of intensity is greatest at a distance h x^2+y^2 2h and beyond this distance the rate of decrease in intensity is much smaller. Figure 19 shows variation of surface intensity for a fault 120 miles long and 15 miles deep. In this case the ellipses are much more elongated as a consequence of the greater length of fault over which the energy is released.

Taking Figure 18 as the approximate variation of the surface intensity for the El Centro, 18 May 1940 shock and taking the distance of El Centro to the center of the shock to be 30 miles would place El Centro approximately on the 0.2 isoseismal line. There would thus be an approximately circular area of 30 miles radius with surface intensity equal to or greater than that experienced at El Centro. The area of higher surface intensity is thus quite localized for a distance greater than approximately 45 miles radius the intensity is less than one half what it was at 30 miles radius. Figure 18 also indicates that at the precise center of the shock the intensity was four times what it was at El Centro. However, this must be considered an upper limit for there is some evidence that indicates that the peak intensities may be lower at the center than is indicated in Figure 18 although this is not definitely established. The fact that there is a close agreement between the magnitudes of earthquakes computed by equation (22) and the G-R magnitudes, particularly in the cases where the same earthquake was recorded at two different points (March 10, 1933 and October 2, 1933), indicates that equation (22) is relatively reliable down to distances of perhaps 15 miles from the center.

It should be noted that the ellipses shown in Figure 18 are only approximations to the actual surface intensity experienced during an earthquake. Local geological conditions may well give rise to errors of 25% in the values computed from equation (22) and this much error would distort the isoseismals to quite irregular curves.

XI. Maximum Surface Intensities. In Section IV it was pointed out that the undamped spectrum intensity and the 0.2 damped spectrum intensity do not bear a fixed relation to each other. The magnitude of the undamped spectrum intensity depends upon the magnitude of the ground acceleration and the duration of the earthquake, whereas the magnitude of the 0.2 damped spectrum intensity depends upon the magnitude of the acceleration but is essentially independent of the duration of the earthquake, at least for the larger shocks. This is shown in Figure 10 where the 0.2 damped intensities are plotted against the undamped intensities and it can be seen that the points lie along a curved line. The fact that this line approaches a horizontal asymptote indicates that for such earthquakes there is an upper bound for the magnitude of the ground acceleration. That this upper bound exists is to be expected since there is an upper bound for the magnitude of stress that can be developed in the earth's crust without faulting.

The data plotted in Figure 10 are for intensities approximately 30 miles from the center of the shock with the individual data points representing a considerable variation in distance. To adjust the data to a standard distance for all the shocks, the intensities were projected back to the respective centers by means of equation (18) taking l = 30 miles. The intensities are plotted in Figure 20. The curve shown in Figure 20 has the equation

$$y = \frac{x}{2} - \left(\frac{x}{14}\right)^2 \qquad (23)$$

where y is the 0.2 damped intensity and x is the undamped intensity. The maximum value of y given by this equation is 12.25 for x = 49 which is approximately 50% larger than are the intensities of the two El Centro shocks when projected back to the center. This agrees with the extrapolation of the data of Figure 10 which was made in section IV where an estimated maximum intensity approximately 50% greater than the El Centro shock was found. The Figure 12.25 would mean 0.2 damped intensity approximately 5-6 times as great as that at the town of El Centro, 18 May 1940. This value of 12.25 is the estimated maximum at the precise center of the shock and, therefore, is not a good estimate of the maximum intensity to be expected over a city. From Figure 18, it is seen that something of the order of 60% of this is a more reasonable estimate of the maximum intensity to be expected on a city. This corresponds to a 0.2 damped intensity of 7.3 which is approximately 2 1/2 times that experienced at El Centro, 18 May 1940 or Long Beach 10 March 1933.

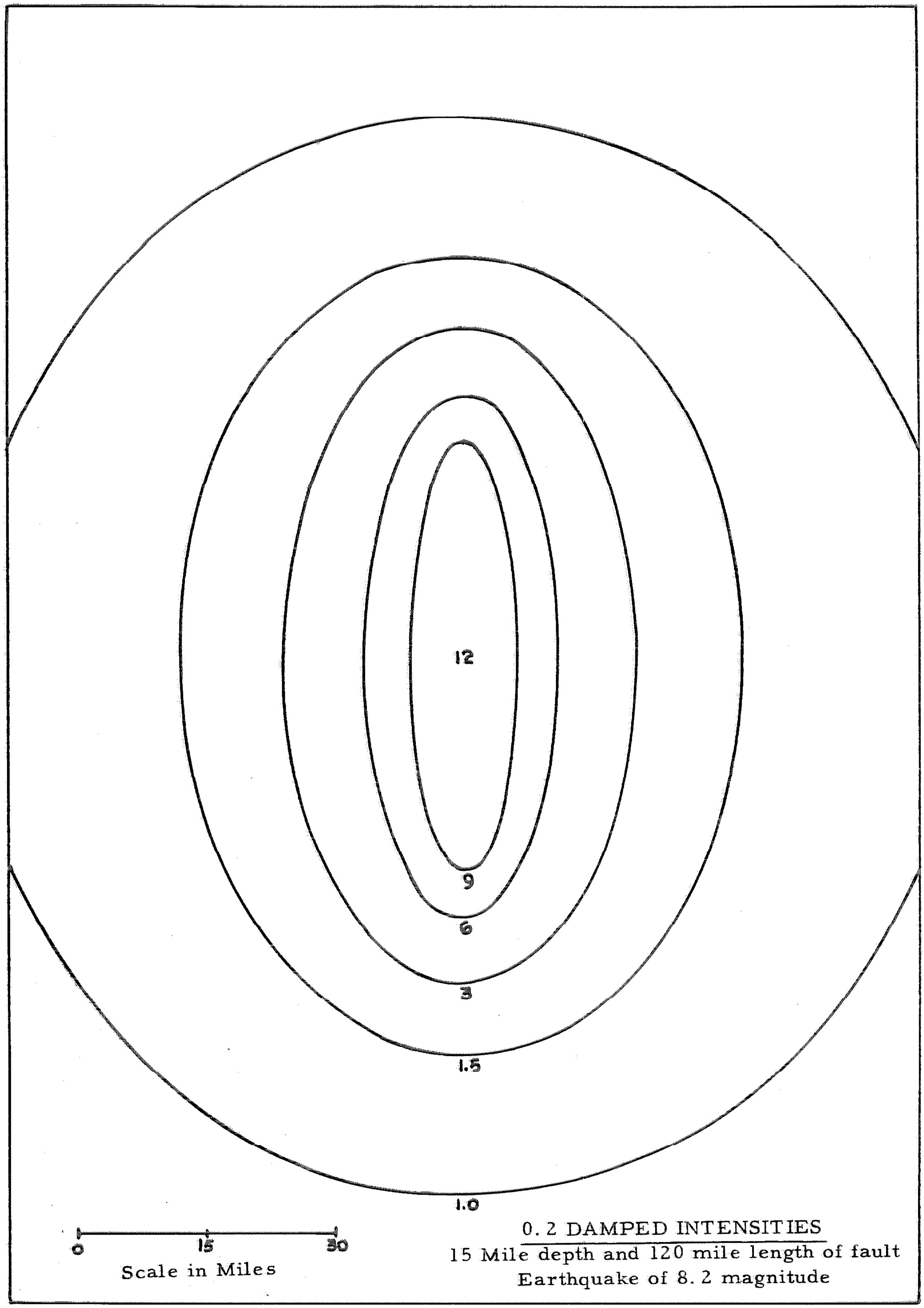

12
9
6
3
1.5
1.0
0
15
30
Scale in Miles
0.2 DAMPED INTENSITIES
15 Mile depth and 120 mile length of fault
Earthquake of 8.2 magnitude

FIGURE 19

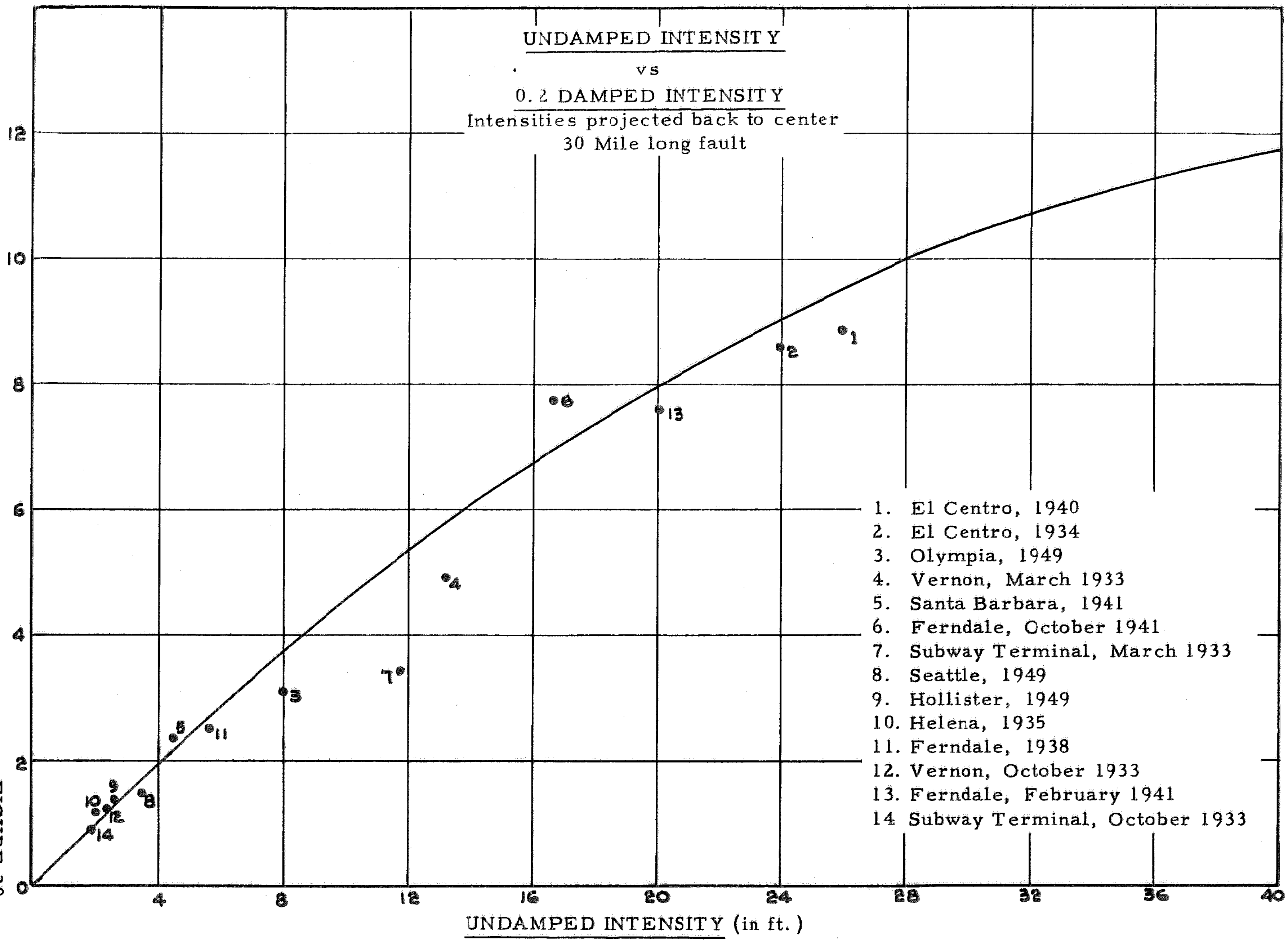

UNDAMPED INTENSITY
vs
0.2 DAMPED INTENSITY
Intensities projected back to center
30 Mile long fault
0.2 DAMPED INTENSITY (in ft.)
UNDAMPED INTENSITY (in ft.)
0
2
4
6
8
10
12
4
8
12
16
20
24
28
32
36
40
1
2
3
4
5
6
7
8
9
10
11
12
13
14
1. El Centro, 1940
2. El Centro, 1934
3. Olympia, 1949
4. Vernon, March 1933
5. Santa Barbara, 1941
6. Ferndale, October 1941
7. Subway Terminal, March 1933
8. Seattle, 1949
9. Hollister, 1949
10. Helena, 1935
11. Ferndale, 1938
12. Vernon, October 1933
13. Ferndale, February 1941
14 Subway Terminal, October 1933

FIGURE 20

If the surface intensities are computed for a large earthquake, say of magnitude 8.2 (San Francisco, 1906), for a depth of 15 miles and length of fault 120 miles, by means of equation (18) there are obtained the lines of equal surface intensities shown in Figure 19. If the maximum 0.2 damped intensity at the center is taken to be 12, the intensities at the other lines are as marked in the figure. If Figure 19 is used to estimate the surface intensity at San Francisco, taking the center of the shock at the point of maximum surface displacement of the fault (Head of Tomales Bay), there is obtained a range of values of 0.2 damped intensities of 3 to 6 which are from approximately equal to double that experienced at El Centro, 18 May 1940 or Long Beach, March 10, 1933.

XII. Maximum Ground Accelerations. As was shown in Figure 13, when the maximum recorded ground acceleration is plotted against the 0.2 damped spectrum intensity the points lie approximately along a straight line, or more nearly along the line:

$$y = \frac{1}{20}\left(1 + 1.85x - \frac{1}{1+x}\right)$$

where y is the maximum recorded acceleration and x is the 0.2 damped spectrum intensity. If this equation is used to extrapolate to the maximum acceleration corresponding to a 0.2 damped intensity of 12 (see Section XI), there is obtained an acceleration of 1.1 g. Figure 19 then shows the corresponding maximum ground accelerations if that at the center is taken to be 1.1 g. This maximum (1.1 g) acceleration is approximately 33% greater than the maximum computed for the El Centro 18 May 1940 earthquake at the Center of the shock. At a distance of 30 miles from the center of the shock the corresponding maximum acceleration would be 0.43g instead of 0.33 g actually recorded at El Centro. The Figure 1.1 g at the center of the shock is perhaps not proper for an estimate of the maximum that might be expected over a city. If for this estimate there is taken 60% of the value at the center there is obtained an acceleration of 0.66 g.

XIII. Frequency of Occurrence of Strong Motion Earthquakes. A listing of strong motion earthquakes is given by Gutenberg and Richter[6] covering the years 1904 to 1946. A frequency graph of earthquakes is shown in Figure 21, where the height of each bar corresponds to the number of earthquakes per tenth magnitude, for example, 8 world earthquakes having a magnitude $8.1 \leq M < 8.2$ occurred during the years 1904 - 1946. There is a separate graph showing the frequency of occurrence of California earthquakes during the same years. The respective frequencies are listed in Table VI[7].

The equation of the curve fitted to the world earthquakes in Figure 21 is

$$y = 16x - (3.75)^2 x^2 + (3.11)^3 x^3 \qquad (24)$$
$$x = (8.7 - M)$$

This should be adjusted so that the area under the curve between $7.0 \leq M \leq 8.7$ is equal to 629, the number of earthquakes for this period. This is achieved by multiplying by 10.

The number of earthquakes of magnitude between M_1 and M_2 is under the curve

$$f = \frac{1}{8.6}\left\{16x - (3.75)^2 x^2 + (3.11)^3 x^3\right\} \qquad (25)$$
$$x = (8.7 - M)$$

The number of California earthquake having magnitudes greater than 7.0 is insufficient to give a good estimate of frequency of occurrence, but assuming that the number having magnitudes greater than 6.0 is a fair estimate of the average number to be expected during a 43-year interval and assuming that equation (24) is valid for magnitudes equal to and greater than 6.0, there is obtained the following expression for the frequency of California earthquakes.

(26)

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6 B. Gutenberg and C. Richter, Seismicity of the Earth, Princeton University Press, 1949.

7 The world earthquakes are listed in Seismicity of the Earth only for 1918-1946 for magnitudes 7.0 to 7.7. These frequencies were multiplied by $\frac{43}{29}$ to bring the number to a 43 year interval.

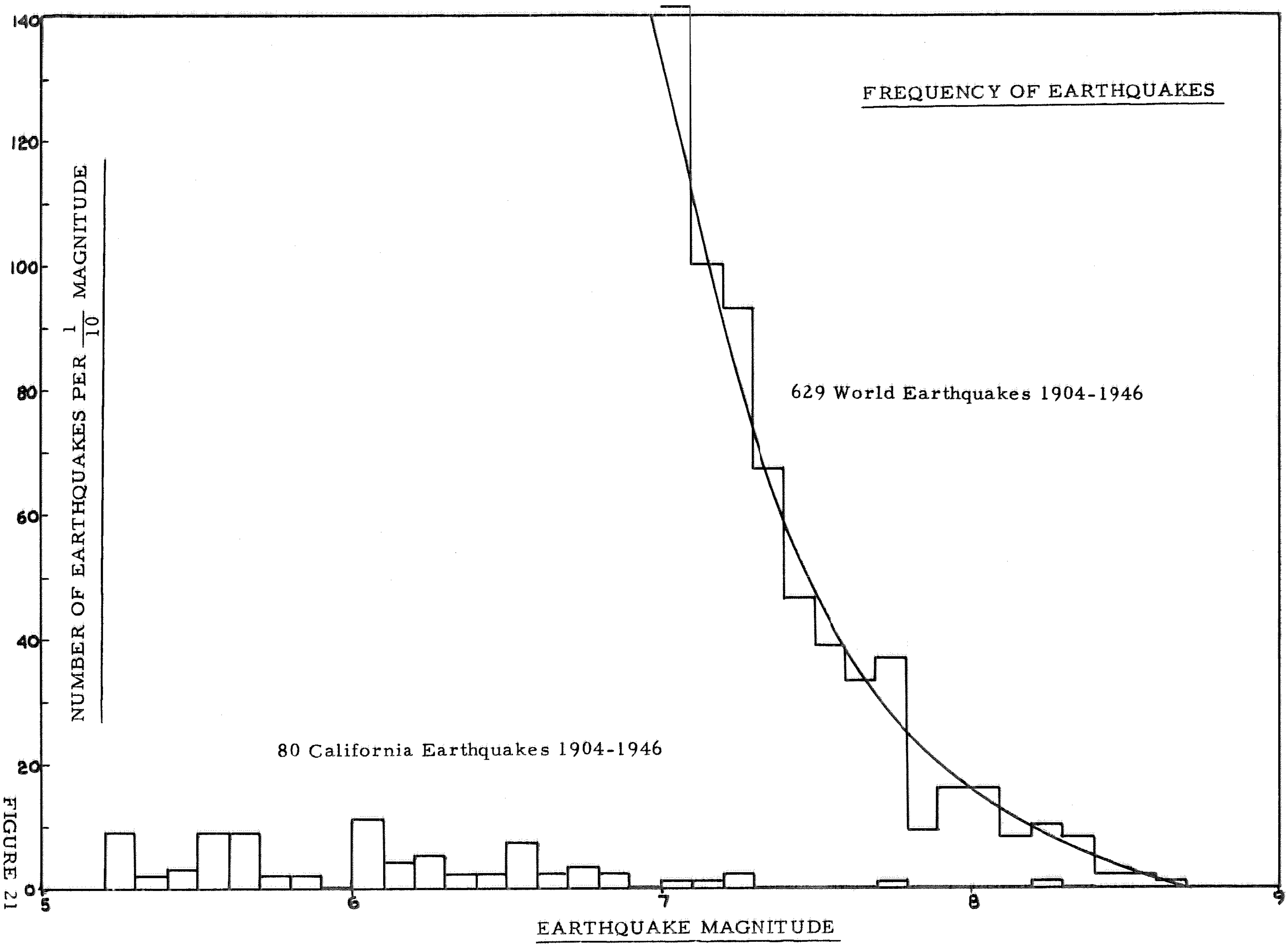
FREQUENCY OF EARTHQUAKES
629 World Earthquakes 1904-1946
80 California Earthquakes 1904-1946
NUMBER OF EARTHQUAKES PER $\frac{1}{10}$ MAGNITUDE
EARTHQUAKE MAGNITUDE
0
20
40
60
80
100
120
140
5
6
7
8
9

FIGURE 21

TABLE VI

Frequency of Occurrence of Strong Motion Earthquakes
1904 - 1946.

Earthquake Magnitude	World Earthquakes	California Earthquakes
8.8 - 8.7	1	0
8.7 - 8.6	2	0
8.6 - 8.5	2	0
8.5 - 8.4	8	0
8.3 - 8.2	10	1
8.2 - 8.1	8	0
8.1 - 8.0	16	0
8.0 - 7.9	16	0
7.9 - 7.8	9	0
7.8 - 7.7	37	1
7.7 - 7.6	33	0
7.6 - 7.5	39	0
7.5 - 7.4	46	0
7.4 - 7.3	67	0
7.3 - 7.2	93	2
7.2 - 7.1	100	1
7.1 - 7.0	142	1
7.0 - 6.9	-	0
6.9 - 6 8	-	2
6.8 - 6.7	-	3
6.7 - 6.6	-	2
6.6 - 6.5	-	7
6.5 - 6.4	-	2
6.4 - 6.3	-	2
6.3 - 6.2	-	5
6.2 - 6.1	-	4
6.1 - 6.0	-	11
6.0 - 5.9	-	0
5.9 - 5.8	-	2
5.8 - 5.7	-	2
5.7 - 5.6	-	9
5.6 - 5.5	-	9
5.5 - 5.4	-	3
5.4 - 5.3	-	2
5.3 - 5.2	-	9
	629	80

The area under this curve between the points M_1 and M_2 is then an estimate of the average number of earthquakes, having magnitudes between M_1 and M_2, to be expected in California during a 43-year interval. This assumes that the occurrence of California earthquakes follows the same pattern as world earthquakes.

It is seen in Figure 21 that there is a very rapid diminution in frequency of occurrence for increasing magnitudes and that the chart clearly pinches off at magnitude 8.7. This may be interpreted as a statement that there is an extremely small probability that an earthquake of magnitude greater than 8.7 will occur. The largest California earthquake was the 1906 San Francisco shock which had an assessed magnitude of 8.2. It is thus of magnitude very close to the maximum during the 43-year period.

XIV. Probability of Occurrence of Strong Motion Earthquakes. If it is assumed that California earthquakes have the same pattern of occurrence as world earthquakes and the future earthquakes will follow the same pattern as past earthquakes, equation (25) may be used to estimate the probability of occurrence of California earthquakes. Equation (25) describes a 43-year period, so if corrected to an arbitrary period of years it may be written

$$E.N. = \frac{Y}{(43)(8.6)} \int_0^{8.7-M_1} \left\{ 16x - (3.75)^2 x^2 + (3.11)^3 x^3 \right\} dx$$

or

$$E.N. = \frac{Y}{(43)(8.6)} \left\{ 16\frac{x^2}{2} - (3.75)^2 \frac{x^3}{3} + (3.11)^3 \frac{x^4}{4} \right\} \quad (26)$$

where E.N. is the expected number of earthquakes, during a period of Y years, which have a magnitude greater than M_1. This equation is to be interpreted as stating that in the long run there will be, on the average, E.N. earthquakes of magnitude M_1 or greater, per Y year period. The numerical values of equation (26) are listed in Table VII.

TABLE VII

Probable Number of Earthquakes in California

Of Magnitude greater than	Per 25 year Period	Per 50 year Period	Per 100 year Period	Per 200 year Period
6.0	25	50	99	198
6.2	18	36	73	146
6.4	13	26	53	106
6.6	9.3	19	37	74
6.8	6.4	13	26	51
7.0	4.3	8.6	17	34
7.2	2.6	5.2	10	21
7.4	1.7	3.4	6.7	13
7.6	.97	1.9	3.9	7.8
7.8	.51	1.0	2.0	4.1
8.0	.28	.56	1.1	2.2
8.2	.13	.26	.51	1.0
8.4	.042	.084	.17	.34

Table VII shows that an earthquake of magnitude 8.2 or greater (San Francisco, 1906) is expected to occur in California on the average of once every 200 years.

An earthquake of magnitude 6.7 (El Centro, 1940) or greater is expected to occur in California on the average of 63 times every 200 years. Shocks of magnitude 6.25 (Long Beach, March 10, 1933) or greater are expected 138 times in 200 years.

It was shown in Figure 18 and 19 that the area of high intensity is quite localized, so that although Table VII shows that on the average California earthquakes of magnitude 6.0 or greater will occur at a rate of approximately one per year, the probability that the localized region of high intensity will cover a metropolitan area is relatively small. Approximate estimate of the frequency with which a California city is expected to experience a strong earthquake can be made as follows. From Figures 18, 19, 20 and 21 it can be estimated that earthquakes of magnitude 6.0 and greater will on the average have an area of roughly 2,000 square miles subjected to a ground motion corresponding to a 0.2 damped spectrum intensity of approximately 3.0 or greater, that is, to a ground motion of severity similar to or greater than that at El Centro, 18 May 1940 or Long Beach, 10 March 1933. Taking the area of California as 150,000 square miles and assuming an equal likelihood of occurrence throughout the state, the probability that the 2000 square miles area will cover any specific point is

$$p = \frac{2000}{150{,}000} = 0.0133$$

The expected number of times that this will happen during 200 years is, using Table VII, 0.0133 x 198 = 2.6, so that on the average a metropolitan area is expected to experience ground motion of the intensity of El Centro and Long Beach or greater at a rate of approximately 2.6 times per 200 years or 3 times per 230 years. Smaller intensities can be expected to be experienced oftener and greater intensities less frequently.

XV. Conclusion. The conclusions drawn from the foregoing material are of an empirical nature and are subject to the restrictions mentioned in the body of this report. With these restrictions in mind the conclusions drawn are as follows:

1. The spectrum intensity defined in this report is a measure of the severity of the ground motion that is proportional to the maximum stresses produced in structures by the earthquake. For each earthquake the spectrum intensities corresponding to several values of damping should be computed with the intensity for 0.2 critical damping being a reasonable measure for ordinary structures.

2. The maximum 0.2 damped spectrum intensity computed from the available strong-motion accelerograms is 2.71 for the intensity at El Centro, California during the earthquake of May 18, 1940. Approximately the same intensity was estimated to have occurred in Long Beach during the March 10, 1933 earthquake. This compares with the value of 0.45 obtained for central Los Angeles during the earthquake of October 2, 1933.

3. There is a reasonably good correlation between the 0.2 damped spectrum intensity and the maximum accelerations recorded during the shock. This correlation is expressed approximately by the equation

$$y = \frac{1}{20}\left(1 + 1.85x - \frac{1}{1+x}\right)$$

where y is the maximum acceleration and x is the 0.2 damped spectrum intensity.

4. There is only an approximate agreement between the spectrum intensity of an earthquake and the Modified-Mercalli intensity. These two do not measure precisely the same thing and it is concluded that the spectrum intensity is the more meaningful for engineering design.

5. The magnitude of an earthquake, as defined by C.F.Richter, can be computed from the spectrum intensity with good accuracy and vice versa.

6. An expression is derived with which, being given the magnitude of an earthquake, the variation of spectrum intensity can be computed over the area affected by the shock.

7. It is estimated that the maximum 0.2 damped spectrum intensity to which a city may be subjected is 7.3 or approximately two and one-half times as great as the intensity at El Centro, May 18, 1940 and Long Beach, March 10, 1933.

8. It is estimated that the maximum ground acceleration to which a California city may be subjected is 0.66 g or approximately twice that experienced at El Centro, 18 May, 1940.

9. It is estimated that an earthquake of magnitude 8.2 (San Francisco, 1906) or greater is expected to occur in California on the average of once every 200 years. An earthquake of magnitude 6.7 (El Centro, 1940) or greater is expected to occur in California on the average of 63 times every 200 years. Shocks of magnitude 6.25 (Long Beach, March 10, 1933) or greater are expected at a rate of 138 times in 200 years.

10. It is estimated that an average California city can expect to experience ground motion of intensity equal to or greater than that experienced at El Centro, May 18, 1940 and Long Beach, March 10, 1933 at a rate of approximately three times per 230 year interval.

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